

EDDY-CURRENT PROBE INTERACTION WITH SUBSURFACE CRACKS

John R. Bowler

Department of Physics
University of Surrey
Guildford
Surrey
GU2 5XH
United Kingdom

INTRODUCTION

Electric current will flow around an open crack in a conductor and give rise to very abrupt variations in the field. If the crack has a negligible opening it acts as a surface barrier where the field is virtually discontinuous. Effectively the crack is then equivalent to a layer of current dipoles with the dipole orientation normal to the surface and pointing upstream. An integral equation for the dipole density has been derived for an idealised subsurface crack using the Green's function method [1]. Numerical solutions have been found by assuming a piecewise constant dipole density and satisfying boundary conditions on the crack at a finite number of points. Here we shall develop the theory further, making use of a knowledge of the dipole distribution for a given incident field, to calculate probe impedance changes ΔZ , due to subsurface cracks.

An advantage of the present approach is that the unperturbed incident field may be found quite independently of the scattering problem. Here we consider axially symmetric probes and use both analytical and finite element methods to calculate the fields in the absence of defects. There are well-known closed form expressions for the field of a cylindrical air-cored probe whose axis of symmetry is normal to the surface of a conductor [2]. However the usual analysis is incomplete. By taking the derivation a stage further the air-cored probe field is given by integral expressions containing Struve functions. We also determine ΔZ for a probe with a ferrite core and a probe with a ferrite core and ferrite shield. The unperturbed fields in these cases being calculated using a general purpose two-dimensional finite element code [3].

FIELD EQUATIONS

As is common in scattering problems, we formally write the total

electric field as the sum of an incident and a scattered field. Thus,

suppressing the time harmonic phase factor $e^{-i\omega t}$, the electric field in air ($j = 1$), and in a half space conductor ($j = 2$) is

$$\underline{E}_j(\underline{r}) = \underline{E}_j^i(\underline{r}) + \underline{E}_j^s(\underline{r}) \quad j = 1, 2 \quad (1)$$

\underline{E}_j^i being the field in the absence of the defect and \underline{E}_j^s the scattered field.

Assuming that the scatterer is a virtually closed crack at an open surface S_0 completely embedded in the conductor, the scattered field may be expressed as [1],

$$\underline{E}_j^s(\underline{r}) = i\omega\mu_0 \int_{S_0} \underline{G}_j(\underline{r}, \underline{r}') \cdot \underline{p}(\underline{r}') dS' \quad (2)$$

where $\underline{p} = \hat{n}p$ is the current dipole density on S_0 and \hat{n} is a unit vector normal to S_0 (figure 1). \underline{G}_j is a half-space dyadic Green's function for a source in the conductor [4]. (2) assumes that the discontinuity in the tangential field at the crack ΔE_t , may be written as

$$\Delta E_t = -\frac{1}{\sigma} \nabla_t p \quad (3)$$

where ∇_t is the gradient tangential to S_0 , σ being the electrical conductivity. The magnetic field is assumed to be continuous at S_0 . For a known dipole distribution the field anywhere in the conductor can be found using (2) or the jump in the field at the defect can be determined from (3). Essentially the introduction of p reduces a three-dimensional vector field problem to one of finding a surface scalar distribution with a corresponding reduction in the computation needed.

The dipole density can be found by applying the condition that the normal component of the total electric field at the crack is zero. Thus

$$\hat{n} \cdot \underline{E}^{\pm}(\underline{r}) = 0 \quad (4)$$

where the \pm sign refers to limiting values on either side of S_0 . The

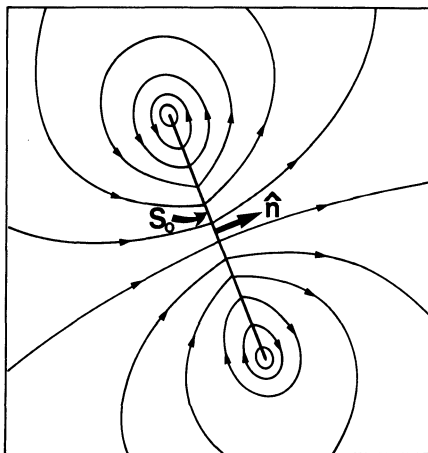


Figure 1. Scattered electric field schematic

crack faces actually acquire a small surface change in an eddy current field and hence a charging current flows normal to the crack but in the quasi-static limit this current component is negligible. Combining (1) (2) and (4) gives an integral equation for p that may be solved numerically [1] using standard boundary integral methods [5].

A relationship between the dipole density and the probe impedance change due to a crack can be derived using a reciprocity theorem. Thus

$$I^2 \Delta Z = \int_{S_0} \underline{E}^i(\underline{r}) \cdot \underline{p}(\underline{r}) dS \quad (5)$$

where the integration is over the surface S_0 and I is the probe current. This power balance relationship is a generalisation of a result obtained by Burrows for point scatterers [6].

Incident Field

In order to determine ΔZ , the incident field E^i must be specified for a given source. Expressions for the field due to cylindrical air-cored probes of annular cross-section are well-known from the work of Dodd and Deeds [2], but here a modified equation is used. The electric field of an annular coil may be found by a superposition of solutions for a circular current filament [7], often referred to as a "delta function coil", or one may take a more fundamental approach and integrate over the source region using the appropriate Green's function. Either procedure leads to a radial integral that has often appeared in the literature without being evaluated in terms of standard functions. To define this integral, suppose ρ is the radial source coordinate, and a_1 and a_2 the external and internal coil radii respectively, then we find [8]

$$\int_{a_2}^{a_1} \rho J_1(\kappa \rho) d\rho = a_1^2 \chi(a_1 \kappa) - a_2^2 \chi(a_2 \kappa) \quad (6)$$

where J_1 is a first order Bessel function and

$$\chi(s) = \frac{2\pi}{s} [J_1(s) H_0(s) - J_0(s) H_1(s)] \quad (7)$$

H_0 and H_1 being Struve functions. The electric field in a half-space conductor is then given by

$$\underline{E}_2^i(\rho, Z) = \frac{i\omega\mu_0 IN \hat{\phi}}{(a_1 - a_2)b} \int_0^\infty \frac{1}{\kappa + \gamma} \sinh(b\kappa) [a_1^2 \chi(a_1 \kappa) - a_2^2 \chi(a_2 \kappa)] J_1(\kappa \rho) e^{\gamma Z - \kappa h} d\kappa \quad (8)$$

with $\gamma = (\kappa^2 - i\omega\mu_0\sigma)^{1/2}$, where the root with a positive real part is used. $2b$ is the axial length of the coil, h is the height of the centre of the coil above the conductor, $\hat{\phi}$ is an azimuthal unit vector and N the number of turns.

To compare an air-cored induction coil with shielded and non-shielded ferrite cored probes, we have calculated the azimuthal field in each case using a finite element package and checked the air-cored probe results from the finite element calculations against equation (8). The accuracy is mesh-dependent but in the conductor was found to be better than 0.2%.

For each of the cases examined the coil geometry was the same (figure 2) and the same ferrite core parameters were used in modelling both shielded and non-shielded probes. The ferrite was assumed to have linear material properties, a relative permeability of 220 and a conductivity of 10^{-7} S.m^{-1} . A coil current density of 1 A mm^{-2} was assumed, at a frequency of 1 MHz giving a skin depth in the test material of 0.55mm. These values correspond roughly to test conditions suitable for detecting small surface or near surface defects in nickel alloys.

Contour diagrams of the ferrite probe of the azimuthal fields are shown in figure 3. Clearly the shield has the effect of confining the field and locally increasing its intensity, particularly near the surface of the test piece immediately below the coil (figure 4a). However, below about 0.2mm from the surface, the non-shielded probe produces a greater field intensity (figure 4b), with obvious consequences for the relative sensitivity to subsurface defects.

PROBE RESPONSE

The dipole density was found as before [1], using the moment methods [5], with S_0 divided into n square patches and p approximated by assuming it to have a constant value p_α , over each patch ($\alpha = 1, n$). The field equations (1) and (2) are then used to get a matrix equation for p_α by demanding that (4) is satisfied at n matching points at the centre of each patch. Finally the impedance perturbation is calculated from a discretised version of (5). Thus

$$\Delta Z = \frac{A}{I^2} \sum_{\alpha=1, n} E_\alpha^i p_\alpha \quad (9)$$

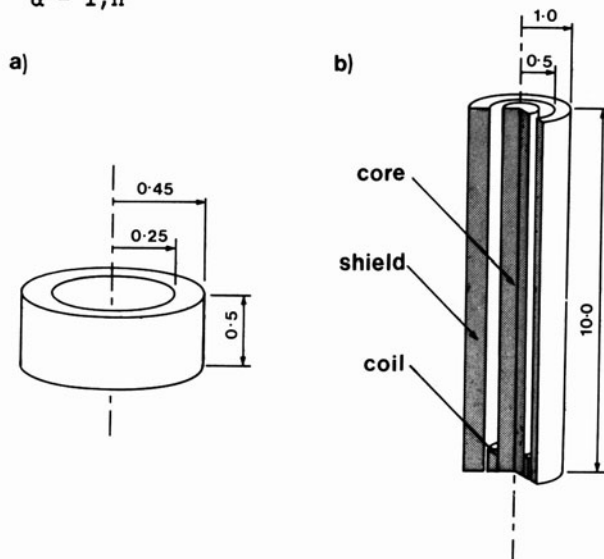


Figure 2. (a) coil, (b) shielded probe. Dimensions in mm.

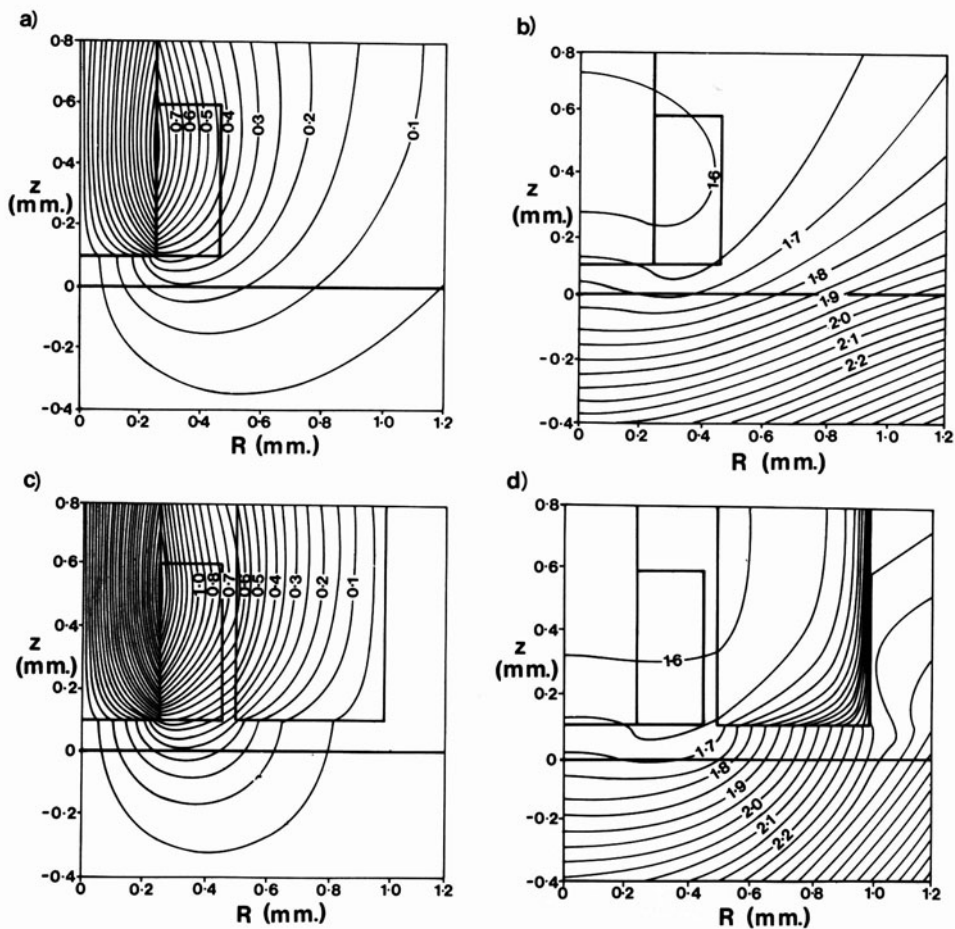


Figure 3. Electric field contours (a) modulus (Vm^{-1}) (b) phase (rads.) referred to source current - nonshielded ferrite probe. (c) modulus and (d) phase contours - shielded probe.

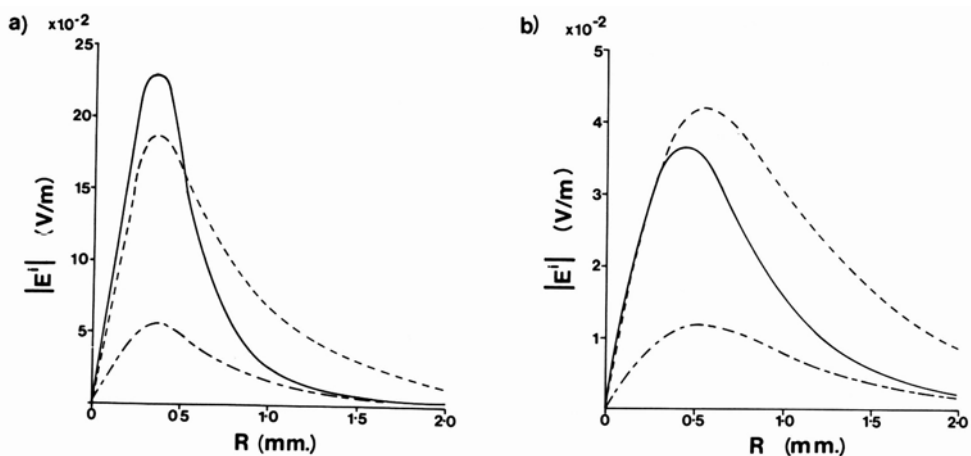


Figure 4. Modulus of electric field. Solid line - shielded probe, dashed line - nonshielded probe, long and short dashes - air-cored probe. (a) at the surface of the conductor ($z = 0$) (b) below the surface ($z = -0.4$ mm).

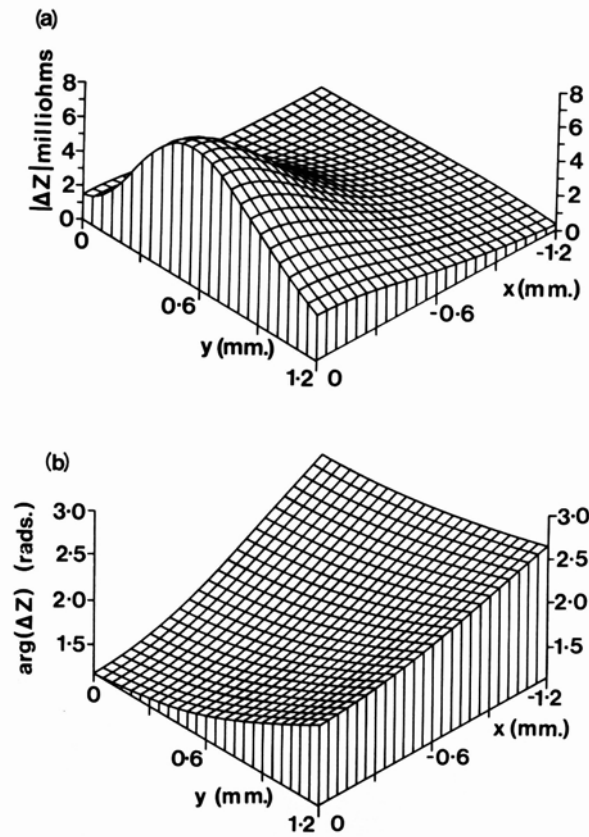


Figure 5. Impedance variations, nonshielded ferrite cored probe. Lift off 0.1mm. Crack in y-z plane.

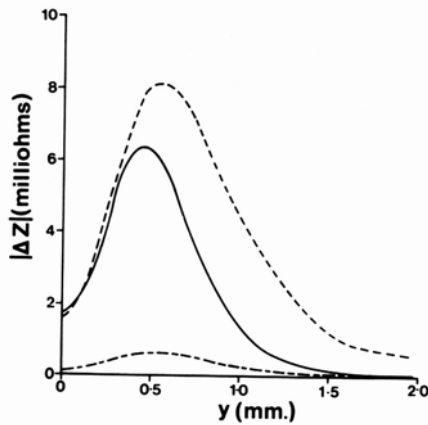


Figure 6. Impedance variations for transverse scans ($x = 0$). Solid line - shielded probe, dashed line - nonshielded ferrite probe, long and short dashes - air-cored probe.

where A is the patch area and $E_{\alpha}^i = \hat{n} \cdot E^i(\underline{r}_{\alpha})$, is the normal component of the electric field at the matching point \underline{r}_{α} .

In the present formulation discretisation of the governing equations leads to an overestimate of the dipole density and hence an overestimate of $|\Delta Z|$. These errors arise, at least partly because p is not accurately represented by a piecewise constant distribution, especially at the edge of the crack. One can always increase the number of patches to reduce these errors but only at the cost of greater computer time. As a compromise a 20×10 patch array was chosen. Based on sample calculations using a larger array, the error in $|\Delta Z|$ for the standard patch format was estimated to be 5%.

A crack was modelled as a rectangular subsurface scattering object in a plane normal to the surface of the test material. Its dimensions are $6 \times 6/2$, the longer side being parallel to the surface, at a depth of 0.46 ($6 = \sqrt{2/\omega\mu_0\sigma}$). Figure 5 shows impedance variations with displacement for the unshielded ferrite-cored probe assuming a 40 turn coil winding. Where the probe is located directly above the centre of the crack, at $(x,y) = (0,0)$, $|\Delta Z|$ has a saddle point and the overall variation exhibits the familiar double-hump pattern. Also at the saddle point we have a phase minimum related to the defect depth and skin depth 6 (figure 5b).

Figure 6 shows variations in ΔZ for transverse scans, with the axis of the probes in the plane of the defect. Changes in the air-cored probe impedance are smallest and the nonshielded ferrite-cored probe gives the largest variation. This is consistent with the comparative incident field intensities in the defect region (figure 4). Although the shielded probe response is smaller than that of the nonshielded ferrite probe, the results indicate that it has a better resolution even for subsurface defects.

CONCLUSION

The usual approach in modelling eddy current-defect interactions is to calculate the electromagnetic field and then determine the probe response from the results. It is then possible to see in detail what the scattered field is like. However, one is often only interested in finding the probe response to a narrow crack, in which case it is simpler just to calculate the jump in the field of the crack or the corresponding dipole density before evaluating ΔZ . By using a combination of boundary integral and finite element methods we have shown that this can be accomplished for realistic probe structures.

ACKNOWLEDGEMENT

The author would like to thank Dr D J Harrison (R.A.E. Farnborough, U.K.) for helpful comment.

The work was carried out with the support of the Procurement Executive, Ministry of Defence, U.K.

REFERENCES

1. J. R. Bowler, Review of Progress in Quantitative Nondestructive Evaluation Vol. 5A, D. O. Thompson and D. E. Chimenti, Eds. Plenum, New York (1986).
2. C. V. Dodd and W. E. Deeds, J. Appl. Phys. 39, 2829 (1968).
3. A. G. A. M. Armstrong and C. S. Biddlecombe, I.E.E.E. Trans. Mag. Vol. Mag-18, No.2, (1982).
4. P. Weidelt, J. Geophys. 41, 85 (1975).
5. R. F. Harrington, "Field Computation by Moment Methods", Macmillan, New York (1968).
6. M. A. Burrows, "A Theory of Eddy Current Flaw Detection". Ph.D Thesis, University of Michigan (1964).
7. P. Hammond, I.E.E. 514S, 508, (1962).
8. I. S. Gradshteyn and I. M. Ryzhik, "Tables of Integrals Series and Products", 4th Edition, A.P. (1965) p.683.