Verification of a Mathematical Model to Predict Tractor Tipping Behavior

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BECAUSE of its available axle torque and geometric configuration, the usual farm tractor is not a particularly stable vehicle. In view of the increasing emphasis being placed on safety and comfort, a knowledge of the degree of stability to be expected from a tractor under various conditions of dynamic loading might be an aid in designing safer and more functional tractors. One method of attaining this knowledge would be the use of a mathematical model. The advantages offered by a mathematical model over prototype testing include greater speed, lower costs, and, more importantly, the ability to perform preliminary evaluation of proposed designs.

Throughout the past 45 years the mechanics of the unsprung wheel tractor has received considerable study. In the 1920s the quantitative approach to the problem began with the work of McKibben (8).* Worthington (10) and Buchele (3) later analyzed the effects of pneumatic tires and soil variables, respectively, on the farm tractor. For reasons of simplicity these studies were limited to steady-state conditions. Raney and associates (9) established a mathematical model for simulating the vibratory behavior of a farm tractor, but because linear differential equations were used, the model is limited to only small amplitude vibrational studies.

The work of Goering (4) in 1965 was the first to provide a mathematical model with the capability of completely predicting two-dimensional tractor mechanics. The model, involving nonlinear differential equations, is not restricted by assumptions of zero or constant accelerations or by small angular movement. However, before such a model can be usefully applied in the design of tractors, the accuracy of its predictions must be measured and the validity of the model verified.

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* Numbers in parentheses refer to the appended references.

The objectives of the paper are to report the instrumentation and techniques that were used in verifying Goering's mathematical model and to present a comparison of actual backward tractor tipping behavior with response predicted by the model.

The Mathematical Model

The mathematical model was based on certain geometrical requirements, Newton's laws, and empirical information about several of the tractor's components. Only a list of the assumptions made in developing the model and the final set of equations as programmed into the computer are given here. Those interested in a more complete discussion of the model should consult references (4), (5), and (7).

The following simplifying assumptions were made in the development of the model:

1. All motion of the tractor occurs in a plane normal to the supporting surface.
2 The laws of Newtonian mechanics are valid for measurements made on the tractor relative to the earth.
3. Only the tractor tires are deformable; the remaining parts of the tractor are rigid bodies.
4. The rear wheels have negligible mass but appreciable mass moments of inertia.
5. The front wheels have negligible mass and negligible mass moments of inertia.
6. The front wheels are ground-driven only.
7. Damping in the pneumatic tires is viscous damping.
8. All wheel ballast is solid, not liquid.
9. The tractor has an unlocked differential; i.e., torque in each drive wheel is equal.

The set of the equations used in programming the computer follow. The terms of the equations are defined in Table 1 and principal parameters used in the model and free body diagrams of the major tractor components are shown in Figs. 1 and 2.

The moment equation for a rear wheel was as follows:

$$I_2 \ddot{\phi}_2 = T - Z_{21}(R_{23} - R_{22}) \quad . \quad [1]$$

The above equation makes the additional assumptions that $Z_{21} = Z_{22}$, and $X_{21} = 0$.

The moment equation for the chassis was expressed as:

$$\begin{aligned} I_{\Theta}\, d\dot{\Theta} &= 2(X_{12} - \mu_4 Z_{12})\, R_{41}dt + 2Tdt - (W_1X_{11}) \cos(\Theta + \delta_x)\, dt \\ &+ (W_1Z_{11}) \sin(\Theta + \delta_x) dt \\ &+ \frac{W_1X_{11}}{g} \ddot{z}_1 (\cos\Theta)\, dt \\ &- \frac{W_1Z_{11}}{g} \ddot{z}_1 (\sin\Theta) dt \\ &+ \frac{W_1Z_{11}}{g} \ddot{x}_1 (\cos\Theta) dt \\ &+ \frac{W_1X_{11}}{g} \ddot{x}_1 (\sin\Theta) dt \\ &- (H_{13})\, p \sin(\Theta + \Theta_{13}) dt \quad . \; . \quad [2] \end{aligned}$$

The summation of forces in the x and z directions reduced to the following two equations:

$$\begin{aligned} d\dot{z}_1 &= -2(g/W_1)\, R_{21}dt - 2(g/W_1)\, R_{41}dt + (g \cos\delta_x) dt \\ &+ (g/W_1)\, p\, (\sin\Theta) dt \quad . \; . \; . \; . \quad [3] \end{aligned}$$

$$\begin{aligned} d\dot{x}_1 &= 2(g/W_1)\, (R_{23} - R_{22}) dt - 2(g\mu_4/W_1) R_{41}dt - (g \sin\delta_x) dt \\ &- (g/W_1)\, p\, (\sin\Theta) dt \quad . \; . \; . \; . \quad [4] \end{aligned}$$

The kinematic relationship for front and rear wheel vertical velocities became:

$$dz_4 = \dot{z}_1 dt - (H_{12} \cos\Theta_{12})\, \dot{\Theta} dt \quad \ldots\ldots \quad [5]$$

$$dz_2 = \dot{z}_1 dt + (H_{11} \cos\Theta_{11}) (\cos\Theta)\, d\Theta - (H_{11} \sin\Theta_{11}) (\sin\Theta)\, d\Theta \quad \ldots\ldots \quad [6]$$

Equations expressing the support force on the front and rear wheels became:

$$R_{41}dt = \begin{cases} f_4\,(D_4 + z_4 + s_4)\, dt + (dz_4 + ds_4)\, C_4, & (D_4 + z_4 + s_4) \geqq 0 \\ 0 & , (D_4 + z_4 + s_4) \leqq 0 \end{cases} \quad \ldots\ldots \quad [7]$$

$$R_{21}dt = \begin{cases} f_2\,(D_2 + z_2 + s_2) \mathrm{dt} + (dz_2 + ds_2)\, C_2, & (D_2 + z_2 + s_2) \geqq 0 \\ 0 & , (D_2 + z_2 + s_2) \leqq 0 \end{cases} \quad \ldots\ldots \quad [8]$$

The equation used to obtain the differential $d\dot{x}_2$ was expressed as:

$$d\dot{x}_2 = d\dot{x}_1 + H_{11} \cos(\Theta + \Theta_{11}) \dot{\Theta}^2 \, dt + H_{11} \sin(\Theta + \Theta_{11}) \, d\dot{\Theta} \quad [9]$$

The moment equation for a rear wheel and the following relationships were required to generate values for $d\dot{\phi}_2$ and $(R_{23} - R_{22})dt$

$$(\beta_2 - \mu_2) = f\ (TR_2) \quad [10]$$

$$d(TR_2) = -\ (1/l_{20})\ d(\dot{x}_2/\dot{\phi}_2) \quad [11]$$

$$(R_{23} - R_{22})dt = (\beta_2 - \mu_2)\ R_{21}dt \quad [12]$$

These equations were programmed on a digital computer using Fordan (2). Fordan is a Fortran system that uses an analog programming method of setting up a problem, which is solved on a digital computer.

Instrumentation of the Test Vehicle

The prototype tractor used in this study was a John Deere 3010 diesel. The only difference between this prototype and tractors commonly used in farm operations was the large amount of ballast placed on the rear of the tractor, the purpose of which was to facilitate rearward tipping so as to provide an easily reproducible test. Two I-beams attached to the tractor (Fig. 3) supported the chassis ballast at the rear of the tractor and also served the very important function of limiting rearward tip of the chassis to approximately 25 deg.

The mathematical model required several input parameters, which required the taking of appropriate measurements on the prototype vehicle. The prototype vehicle also served as the source of actual field test data with which to compare the predictions made by the mathematical model. To obtain these parameters and data, the prototype tractor required considerable instrumentation.

To measure the rear axle torque, strain gages were attached to the 45-deg helices of the rear axle. A commercial brush and slip-ring assembly, recessed in a hole in the end of the axle, was used to transmit the torque signal from the rotating axle to the non-rotating chassis.

One Wheatstone bridge circuit was composed of two gages cemented to the front and two cemented to the rear of the axle housing. These gages measured the bending stresses in the housing due to horizontal forces acting on a rear wheel in the direction of travel. Bending stresses in the axle housing induced by vertical support forces acting on the rear wheel were measured by two gages mounted on the top and two mounted on the bottom of the axle housing.

It was necessary to have a signal proportional to the rotational speed of the rear wheel relative to the chassis. This signal was obtained (from the rear axle) by a chain drive to a d-c tachometer generator.

To record the time required to engage the clutch, a microswitch, which triggered an event marker on the Dynograph oscillograph, was mounted above the clutch pedal. The event marker was triggered as soon as the clutch became fully engaged. A floating bicycle-wheel device was used to obtain the true forward travel speed of the tractor. The tachometer generator, which was friction driven on the periphery of

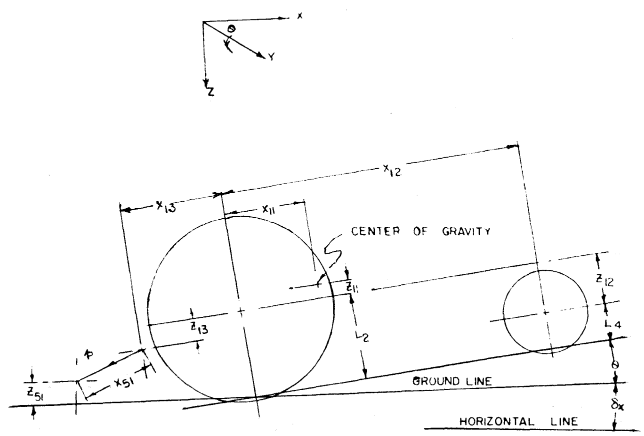

FIG. 1 The principal parameters and the coordinate system used in the model.

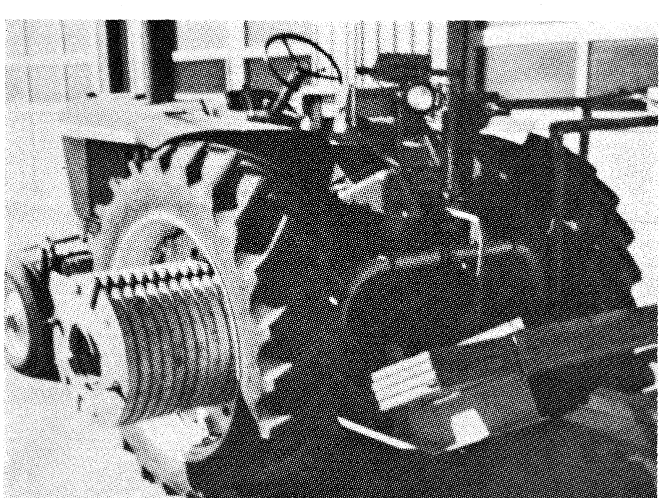

FIG. 3 Tractor heavily ballasted with eleven pair of cast iron wheel weights and steel planks across I-beams.

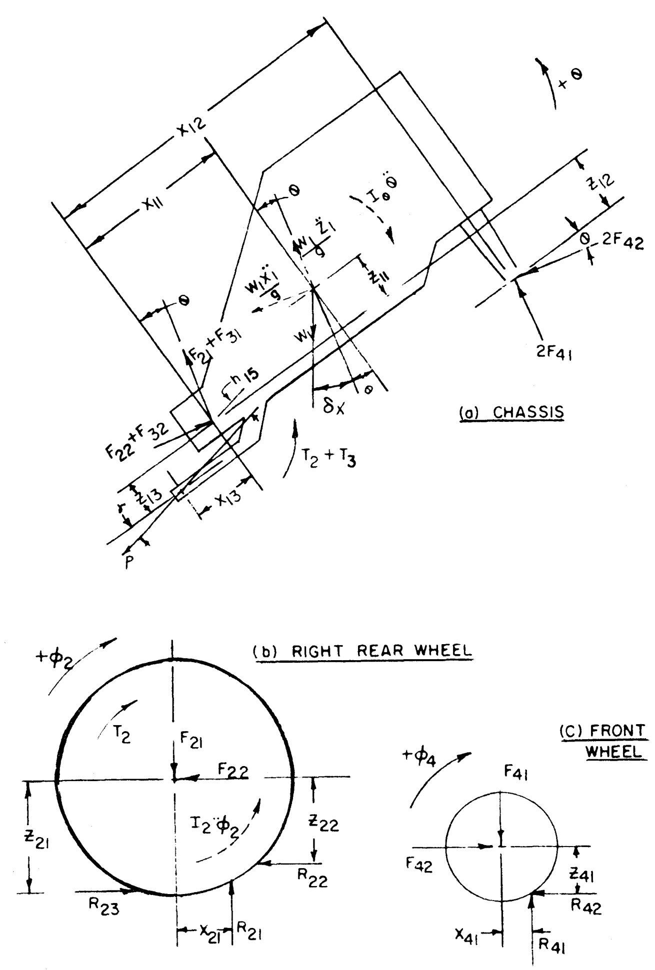

FIG. 2 Free body diagrams of the major components in the tractor.

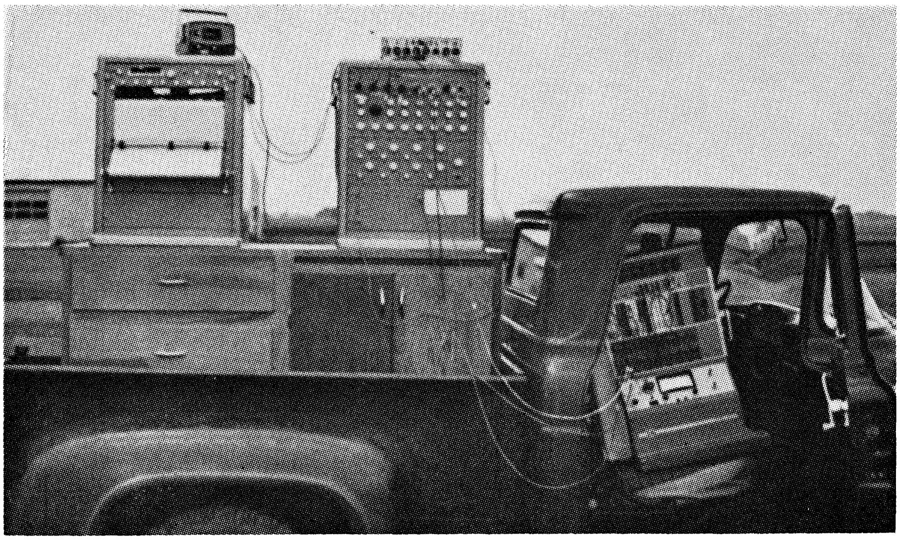

FIG. 4 The eight channel recorder and analog computer mounted on truck for field verification tests.

the bicycle tire, provided an electrical signal proportional to the rotational speed of the wheel. This device was used only during test runs in which traction-slip data was being measured.

An angular accelerometer (attached to the fender of the tractor) was used to provide a signal proportional to tipping acceleration.

The instruments at the data-collection center are shown in Fig. 4. The two units on the rear of the truck comprise an Offner Dynograph, type R direct writing oscillograph. One unit contains the amplifiers and power supplies. The other unit contains the paper drive and the pen motors. The unit in the cab (Fig. 4) is a TR-20 analog computer. It was used to integrate the signal from the angular accelerometer to provide angular velocity and displacement signals.

A communications system was required to transmit the various signals from the tractor to the Dynograph recorder. The cable used for this purpose contained 32 wires shielded in pairs. Junction boxes were required at the tractor and recorder for connecting the individual instrument circuits to the cable. The junction box at the recorder included a means for connecting a digital voltmeter across the power source being used to excite any of the strain gage circuits. Thus any of the excitation voltages could be monitored at any time during a test.

Parameter Determination

As previously mentioned, the model required several parameter inputs concerning the prototype tractor. A summary of the required input parameters and the quantities for which predictions were made appears in Table 2. The purely geometric relationships on the tractor were determined with the aid of a tape measure, a plumb bob, and a chalk line.

It was necessary to know the mass moment of inertia of a rear wheel and wheel weights. The method used to determine the mass moment of inertia of a rear wheel was to jack the rear wheel off the ground, bring the wheel up to speed, and then engage the brake. Axle torque and angular acceleration were monitored during the subsequent deceleration of the wheel. The inertia of the wheel was calculated as the ratio of the torque to acceleration. The mass moment of inertia of a wheel weight was found by swinging the weight as a compound pendulum. Once the period of oscillation was established, the standard physical pendulum formula with a transfer term to move the reference axis to the center of gravity was used to calculate the mass moment of inertia.

The tractor was weighed to find the chassis weight and the location of the center of gravity. The vertical location of the center of gravity was obtained by measuring the weight transferred to the rear wheels as the tractor was tilted by a lift truck. The angle of tilt and weight on the rear wheels were recorded and the necessary calculations made to establish the vertical location of the chassis center of gravity (1, pp. 314-316).

Two methods of determining the mass moment of inertia of the tractor chassis were used. A vibratory method (6), which basically entailed measuring the pitch frequency of the chassis as the tractor vibrated on linear springs, was one method used. These springs consisted of a pair of simply supported beams with provisions for concentrated loading at the center of the span. Each beam had two strain gages attached to it and these were interconnected to measure the total load at the center of the span, regardless of its distribution between the two beams. The frequency of vibration of the spring and tractor could be determined from signals provided by the gages. The spans of both springs were infinitely variable and thereby also their spring rates. This adjustment capability was required because this method of mass moment of inertia determination required that the vibrating system be uncoupled. The condition for uncoupling is satisfied when

$$K_f = K_r X_{11}/(X_{12} - X_{11}) \quad . \ . \ [13]$$

Having the spring rates adjusted properly, the tractor was placed on the springs. The tractor was excited into vibration by simply pushing downward on the front of the tractor and releasing. The signal for pitch frequency was obtained by running the signals from the two strain-gaged springs through a differential amplifier. Once the pitch frequency, $\sigma_{n\Theta}$, is known, the mass moment of inertia about the centroidal axis is given by

$$I_{\Theta t} = \frac{K_f X_{12}(X_{12} - X_{11})}{\sigma^2{}_{n\Theta}} \quad . \ . \ [14]$$

The springs used in the vibratory method were only strong enough to support the unballasted tractor and therefore many calculations were necessary to compute the effect of the added ballast on mass moment of inertia. The second method of mass moment of inertia determination had the ability to handle a fully ballasted tractor and also served as a check on the values obtained by the vibratory method.

The second method was based on the principles of the physical pendulum. To convert the entire tractor into a physical pendulum, an upright chassis support was bolted to the I-beams on both sides of the tractor at the horizontal location of center of gravity. A 4-in.-square block of steel was bolted to the upright so as to form a knife edge. The blocks could be moved up and down on the chassis supports, thereby permitting the length of the pendulum arm (distance from knife edge to tractor's center of gravity) to be varied in 3-in. increments. Ground supports on which the knife edge rested were made of 2″ x 4″ steel bar with a horizontal plate on top that could move vertically in the rectangular support tube. A five-ton hydraulic jack, located at the bottom of each steel bar, forced the bar upward, thereby lifting the tractor off the supporting surface (Fig. 5).

Once the tractor had been raised approximately 2.5 in. off the ground, it could be swung about the knife edge

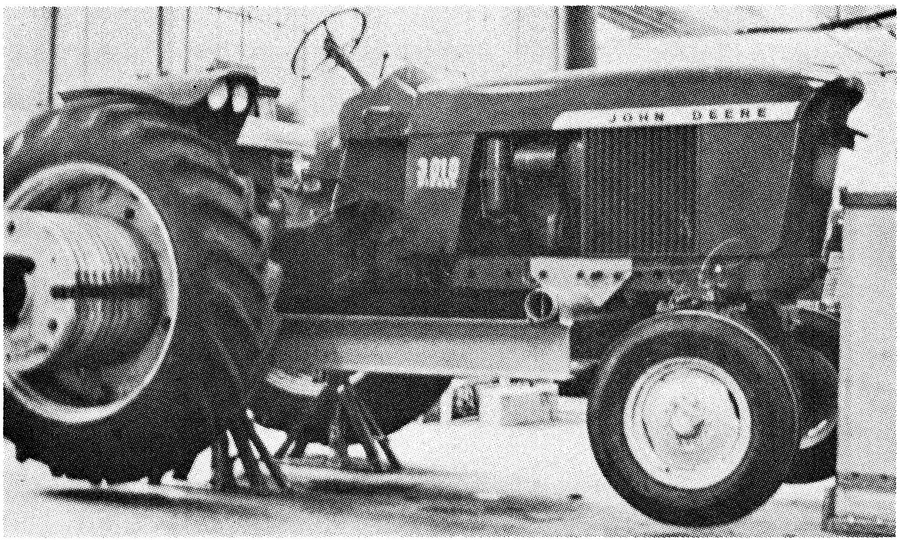

FIG. 5 Fully ballasted tractor during test to determine chassis mass moment of inertia by pendulum method.

and the period for an oscillation measured. From basic physics equations, the mass moment of inertia of a physical pendulum about a centroidal axis is given by:

$$I_{\Theta} = Wl(T^2/4\pi^2 - l/g) \quad . \ . \ [15]$$

where T is the time for a single oscillation of the tractor.

This equation involves a term l defined as the distance from the pivot point to the mass center of gravity. Since the knife edge could be moved in the vertical direction, the value of l could be varied, which varied the period of oscillation. By taking the data from two positions of the knife edge, substituting into equation 2, and setting them equal, it is possible to calculate the vertical location of the tractor's center of gravity. This served as a check on the weighing method of locating the tractor's center of gravity.

Several wheel reaction parameters were required by the model. Force-deflection curves for the front and rear tires were obtained from the tire manufacturers. Work done by Raney on finding viscous damping constants of tires similar to those used in this study was used in estimating viscous damping constants for the rear and front tires.

Traction-slip data for the various surfaces operated upon were obtained with the aid of the instrumented prototype tractor. Traction-slip data were recorded immediately after a verification test had been completed. This was done so that the traction-slip data would truly represent the surface conditions that existed at the time of the verification test. Three test runs were made on each surface to check the repeatability of the data.

The model required traction-slip data in the form of a traction coefficient versus travel reduction curve. To obtain continuous data for the complete range of the curve, a track-type tractor was used to provide a variable drawbar load for the tractor. The rear wheel rotational speed was obtained with the aid of the rear wheel tach generator sensor. The tach generator driven by the bicycle wheel was used to determine the true forward speed. Strain gages mounted on the rear axle housing and rear axle measured dynamic weight upon the rear axle and torque delivered by the rear wheel respectively. All four signals were recorded on the Dynograph.

By taking simultaneous readings of torque, rear wheel loading, actual forward velocity, and rotational velocity of the rear wheel from the chart records, the travel reduction and traction coefficient of the rear wheel could be calculated at any time. With the aid of a digital computer, these calculations were made and plotted as shown in Fig. 6.

Procedure for Verification Tests

To check the accuracy of the model, a clutch engagement test was used. No drawbar load was attached to the tractor during the clutch tests. The tractor was parked with the engine running at 1900 rpm, the transmission in fourth gear, and the clutch disengaged. The tractor operator then engaged the clutch as rapidly as possible an instant after the instrument operator started the recorder and analog computer. The rapid clutch engagement resulted in rearward tipping of the tractor (Fig. 7).

The time lag between starting the instrumentation equipment and releasing the clutch was kept as small as possible to minimize drift of the integrators. Considerable practice was required in getting the timing exact.

The clutch engagement tests were run on three different surface conditions — concrete, sod, and tilled ground. The concrete surface had been constructed specifically for these tests and had extreme surface roughness giving it a high traction coefficient. The sod surface was an area of brome grass that had been mown to a height of ap-

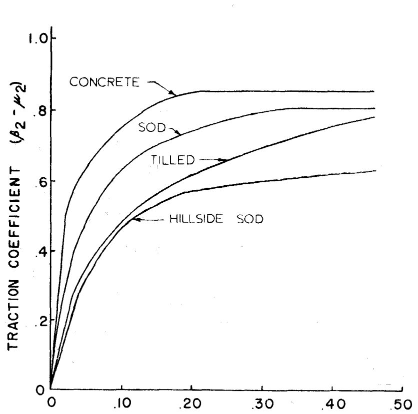

FIG. 6 Traction-slip curves for the various surfaces upon which verification tests were conducted.

FIG. 7 A typical clutch engagement test on concrete.

TABLE 1. NOTATION

Body 1, the tractor chassis
Body 2, the right rear wheel
Body 3, the left rear wheel
Body 4, a front wheel
Body 5, the supporting surface
Θ, angle of rotation of the chassis about an axis parallel to the rear axle centerline
ϕ_i, angle of rotation of wheel i
x_i, displacement of the center of gravity of body i in the x-direction
z_i, displacement of the center of gravity of body i in the z-direction
s_i, the effective surface roughness "seen" by wheel i
l_{iO}, effective rolling radius of wheel i when drawbar pull is zero
Θ_{ij}, the j^{th} fixed angle on body i
X_{ij}, the j^{th} horizontal dimension on body i
Z_{ij}, the j^{th} vertical dimension on body i
H_{ij}, the j^{th} fixed hypotenuse on body i
δ_x, slope of the datum in the x-direction
μ_i, coefficient of rolling resistance of wheel i
β_i, coefficient of traction of wheel i
D_i, static tire deflection of the i^{th} wheel
C_i, viscous damping constant for wheel i
$f_i(\ )$, functional load-deflection relationship for wheel i
g, acceleration of gravity
TR_i, travel reduction of the i^{th} wheel
R_{ij}, the j^{th} supporting surface reaction acting on wheel i
p, the drawbar pull acting on the tractor
W_1, the weight of the chassis, which was assumed equal to the weight of the tractor
I_i, the centroidal mass moment of inertia of wheel i
$I_{\Theta t}$, the centroidal mass moment of inertia of the entire tractor about the pitch axis (axis parallel to the rear axle centerline)
$\sigma_{n\Theta}$, the pitch natural frequency of the tractor about the centroidal axis
I_{Θ}, the centroidal mass moment of inertia of the chassis about the pitch axis
T_i, the torque in the rear axle of wheel i
K_r, K_f, the respective spring rates of the rear and front springs used in determining the mass moment of inertia of the tractor

TABLE 2. REQUIRED INPUT PARAMETERS AND OUTPUT DATA

Required inputs	Available predicted data*
1. Vehicle wheelbase	1. Rear wheel rotational velocity
2. Location of center of gravity	2. Rear wheel thrust force
3. Vehicle weight and distribution	3. Vertical displacement of rear axle
4. Mass moment of inertia of chassis	4. Chassis tipping acceleration
5. Mass moment of inertia of rear wheels	5. Chassis tipping velocity
6. Load deflection information for tires	6. Angle of chassis tip
7. Viscous damping constant for tires	7. Forward velocity of vehicle
8. Effective rolling radius of driving tires	8. Travel reduction
9. Coefficient of rolling resistance of wheel on surface being operated upon	
10. Traction-slip curves for surface being operated upon	
11. Empirical torque data	

* All above data can be predicted at any instant in time.

proximately 4 in. The tilled area had been plowed to a depth of 7 in. and disked twice with a tandem disk. All of these test areas had zero slope. A fourth clutch-induced tipping test was made on another area of sod, which had an upward slope of 4.4 percent in the direction of travel.

The signals taken from the various sensors on the prototype tractor and recorded on the Dynograph were the following: rear axle torque, rear wheel rotational velocity, vertical load on rear axle, rotational acceleration of chassis, and time of clutch engagement. The chassis rotational velocity was obtained by integrating the electrical signal from the angular accelerometer. The angular velocity signal was then integrated once again on the analog computer to obtain angular displacement of the tractor chassis.

During the tests on concrete a mechanical tip indicator was used to check the accuracy of the accelerometer and the integrating circuits. The tip indicator was no more than a friction lever that was pushed upward as the tractor tipped and remained at its maximum height when the tractor returned to a level position. After a test had been run, the maximum angle of tractor tip could then be measured by simply checking the position of the mechanical tip indicator.

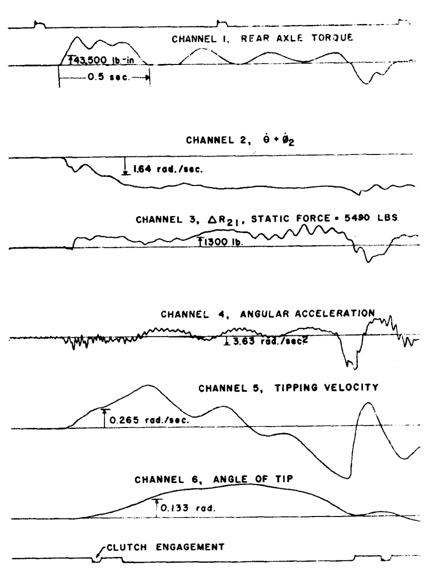

FIG. 8 Results obtained from a typical fourth gear clutch engagement test on concrete.

Comparison of Field Tipping Tests with Model Predictions

Preliminary clutch engagement tests indicated the greatest angle of tip occurred with the tractor operating in fourth gear. Consequently, fourth gear was used for all of the verification tests. Several clutch engagement tests were run on each type of surface and repeatability of the data obtained was found to be excellent.

A typical chart from a clutch engagement test is shown in Fig. 8. This type of chart served as the source of data for the experimental curves that are compared with the predicted curves in the following graphs. Channel 2 on Fig. 8 is a trace of rear wheel speed relative to the chassis. From this trace, it was necessary to subtract the experimentally measured chassis tipping velocity (Channel 5 of Fig. 8) to obtain the absolute wheel velocity plotted in Fig. 9. Channel 3 of Fig. 8 is a record of the rear wheel support force. By using ordinates from this trace with the force-deflection curve of the rear tire, the experimental z_2 versus time trace of Fig. 10 was obtained. Only the curve for concrete is shown since all were similar. The experimental angular displacement curves in

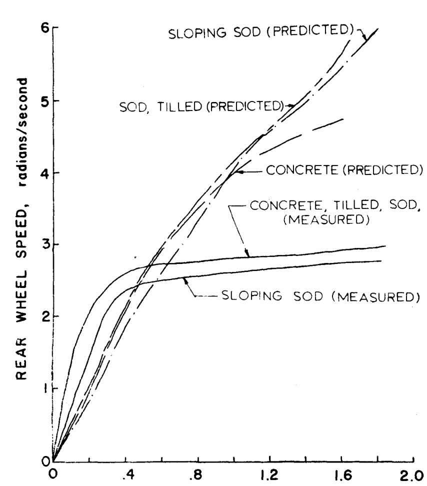

FIG. 9 Measured and predicted rear wheel rotational speed versus time for tipping tests on the various surfaces.

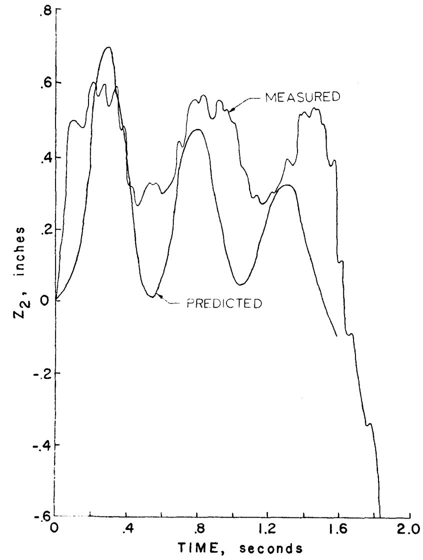

FIG. 10 Measured and predicted rear wheel vertical displacement versus time for a tipping test on level concrete.

Fig. 11 through 14 were obtained from Channel. 6.

As previously mentioned, only basic parameter information was required as input data for the model with one exception: that empirical torque versus time data had to be introduced into the system. This torque information was obtained during the verification tests and a sample trace is shown in Channel 1 of Fig. 8. Because of computer input limitations, however, the data used in the model was a smoothed version of the torque trace. An attempt was made to keep the area under the actual and smoothed torque traces equal.

The mathematical model predictions shown in Fig. 11 through 14 indicate the excellent ability of the model to predict angle of tip. The maximum angle of tip was at least within 0.015 radian (1 deg) of the experimental values measured. The time of front wheel touchdown was consistently predicted early, but within 0.1 sec of actual.

The model did not predict the oscillatory behavior of the chassis angular velocity; however, the general trend of the experimental velocity curve was duplicated quite well. The probable reason for the model's inability to predict this oscillatory motion was that a smoothed torque curve was used rather than the actual torque curve, which contained several oscillations.

It was hypothesized that the oscillations in rear axle torque were due to the response of the engine-governor system. This seemed to be indicated by the fact that torque was immediately lowered once the rear wheels attained speed and from that point on the torque seemed to fluctuate to maintain the governed rear wheel speed. To be able to predict the oscillatory behavior of the torque trace and the angular velocity curve, it will be necessary to develop an operable engine simulator.

Fig. 9 shows the predicted rear wheel speed for all surface conditions to be greater than experimental data indicate. This can also be attributed to the smoothed torque curve. During the actual verification tests, the fluctuations in rear axle torque apparently maintained the rear wheel velocity near its governed level. With the smoothed curve, there was a continuing gradually reducing torque as time elapsed. Therefore, there was always torque available throughout the entire prediction run to cause the rear wheel to accelerate.

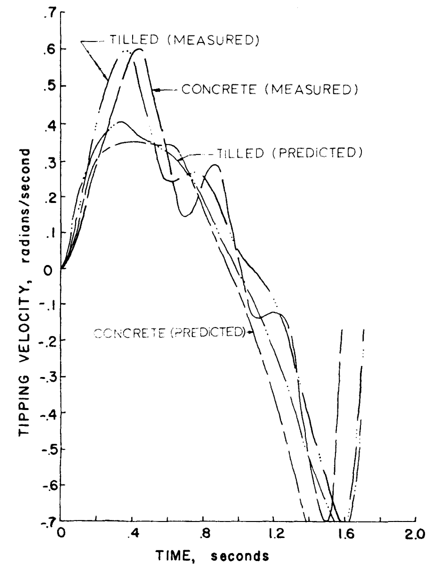

FIG. 15 Measured and predicted tipping velocities versus time for tipping tests on level concrete and tilled ground.

Conclusions and Scope of this Study

As a result of this study it was concluded that the mathematical model established for predicting the mechanics of an unsprung wheel tractor is valid and is capable of producing highly accurate predictions of vehicle behavior. The Fordan computation system provided a suitable but relatively slow method of solving the equations of the mathematical model.

The following secondary conclusions were reached:

1. The method used in determining traction-slip characteristics worked satisfactorily and gave valid, repeatable results.

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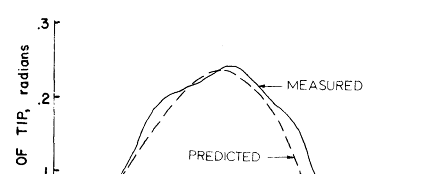

FIG. 11 Measured and predicted tip angle versus time for a tipping test on level concrete.

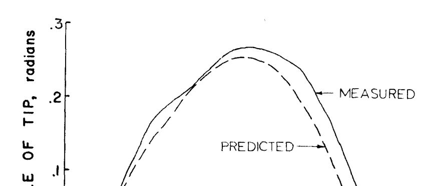

FIG. 13 Measured and predicted tip angle versus time for a tipping test on level sod.

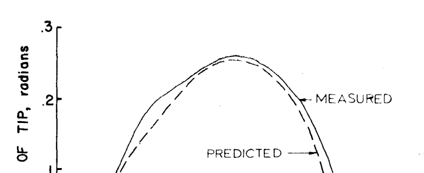

FIG. 12 Measured and predicted tip angle versus time for a tipping test on level tilled ground.

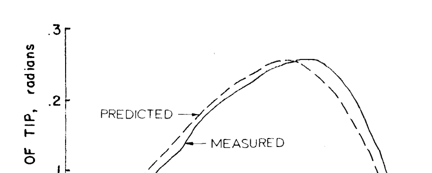

FIG. 14 Measured and predicted tip angle versus time for a tipping test on a sod slope.

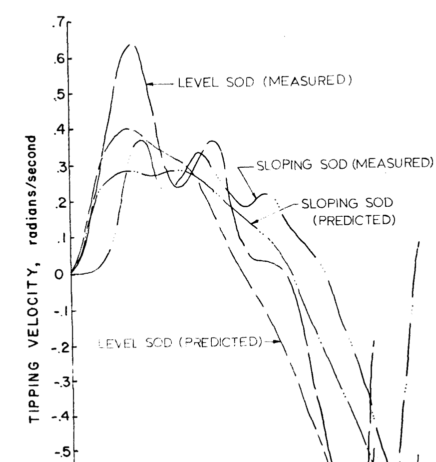

FIG. 16 Measured and predicted tipping velocities versus time for tipping tests on level and sloping sod.

MODEL TO PREDICT TRACTOR TIPPING

(Continued from page 72)

2. The physical pendulum was a simple and accurate means of determining the chassis mass moment of inertia.

3. The vibratory system of determining the chassis mass moment of inertia was found to give accurate results only if the spring rates were actually measured rather than calculated from theoretical equations.

This research work should be considered as only a single basic step in the study of the mechanics of the unsprung wheel tractor. It had considerable limitation in that it could only handle two dimensional behavior and empirical torque data was required by the model. However, these limitations can be overcome by expansion and sophistication of the model. Proper expansion of the mathematical model could allow it to predict, for example, lateral tipping, the effect of mounted implements on the tractor, or behavior of a four-wheel drive vehicle.

The computer has eliminated the necessity of making simplifying assumptions, such as small oscillations and zero or constant acceleration, formerly made so the equations could be easily solved by long hand methods. Equations that exactly express the interaction of the governing parameter can now be used to analyze the mechanical behavior of the vehicle, even if the equations are non-linear differentials.

References

1 Barger, E. C., Liljedahl, J. B., Carleton, W. M. and McKibben, E. G. Tractors and their power units, 2nd ed. John Wiley & Sons, Inc., New York, 1963.

2 Blumenberg, Wayne E. Fortran simulation of a digital differential analyzer. Unpublished M.S. thesis. Library, Iowa State University of Science and Technology, Ames, Iowa, 1965.

3 Buchele, W. F. The mechanics of a vehicle operating on a yielding soil. Unpublished mimeographed paper presented at the Society of Automotive Engineers, National Farm, Construction and Industrial Machinery Meeting, Milwaukee, Wisconsin, September 10-13, 1962. Society of Automotive Engineers, Inc., 1962.

4 Goering, C. E. The mechanics of unsprung wheel tractors. Unpublished Ph.D. thesis. Library, Iowa State University of Science and Technology, Ames, Iowa, 1965.

5 Goering, C. E. and Buchele, W. F. Computer simulation of an unsprung vehicle. **Transactions of the ASAE** 10:(2)272-280, 1967.

6 Goering, C. E., Marley, S. J., Koch, J. A. and Parish, R. L. Determining the mass moment of inertia of a tractor using floor suspension. **Transactions of the ASAE** 11:(3)416-418, 1968.

7 Koch, James A. Comparison of tractor tipping behavior with response predicted by a mathematical model. Unpublished M.S. thesis. Library, Iowa State University of Science and Technology, Ames, Iowa, 1966.

8 McKibben, E. G. The kinematics and dynamics of the wheel type farm tractor. Agricultural Engineering 8:15-16, 39-40, 58-60, 90-93, 119-122, 155-160, 187-189, 1927.

9 Raney, J. P., Liljedahl, J. B. and Cohen, R. The dynamic behavior of farm tractors. **Transactions of the ASAE** 4:(2)215-218, 221, 1961.

10 Worthington, W. H. Evaluation of factors affecting the operating stability of wheel tractors. Agricultural Engineering 30:119-123, 179-183, 1949.