A Transactive Energy Approach to Distribution System Design: Household Formulation

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***Abstract*—A household model is formulated to facilitate careful development and performance testing of bid-based transactive energy system (TES) designs with voluntary customer participation. The optimal general bid-function form for households with thermostatically controlled loads is derived from dynamic programming principles, based solely on general household thermal dynamic and welfare attributes. Quantitative forms are determined for these optimal bid functions, given quantitative forms for these attributes. These quantitative attributes are used to construct representative household types based on clusterings of correlated parameter values. Bid comparison, peak-load reduction, and load-matching test cases conducted for a 123-bus distribution system operating under a generic bid-based TES design illustrate the usefulness of our methods for ensuring TES designs are aligned with local customer goals and constraints.**

***Index Terms*—Customer-centric design, bid-based transactive energy system, optimal household bid function, household thermal dynamics, household welfare, household representative types, test-case performance evaluation**

I. Introduction

RECENT years have seen a dramatic resurgence of interest in the restructuring of electric power systems at the distribution level [1]. Researchers and practitioners have stressed the need for customer-oriented approaches encouraging households and businesses to participate more actively in distribution system operations.

This restructuring has encompassed a broad range of efforts. Technological innovations include advancements in small-scale storage and distributed generation. New operational rules include bid-based *Transactive Energy System (TES)* designs for the support of customer transactions.

A *bid-based TES design* is a hybrid collection of economic and control mechanisms that permits the reliable balancing of power demands and supplies by means of bid-based transactions [2]. Bids permit customers to express their willingness to demand power usage as a function of requested payment, and to supply generated power and/or ancillary service (e.g., demand response) as a function of offered compensation.

To date, bid-based TES design development has largely focused on the achievement of prespecified system objectives. Although system efficiency and reliability are critically important design objectives, this study stresses that the pursuit of these system objectives should be conditioned on a prior careful understanding of local customer goals and constraints in order to ensure voluntary customer participation.

This study thus considers the feasibility and desirability of undertaking bid-based TES design development from a customer-centric starting point. Proof-of-concept evidence is provided in support of the following five assertions:

- The optimal general bid-function form for customers with *thermostatically controlled loads (TCLs)* can be derived from dynamic programming principles, based solely on general customer thermal dynamic and welfare attributes.
- Quantitative forms can be determined for these optimal customer bid functions, given quantitative forms for the customers' thermal dynamic and welfare attributes.
- These quantitative customer thermal dynamic and welfare attributes can be expressed in parametric form.
- The parameters appearing in these parametric forms can be clustered into correlated subsets determining higher-level customer thermal dynamic and welfare attributes.
- This clustering can be used to define a collection of representative customer types to facilitate the development and performance testing of bid-based TES designs aligned with local customer goals and constraints.

Analytical and test-case evidence is provided in support of these five assertions for the particular case of a collection of households populating a distribution grid. Each household comprises: (i) a house with structural attributes; (ii) a set of appliances that includes an electric HVAC system with a smart price-sensitive ON/OFF controller; and (iii) a resident with comfort-cost preferences. Household thermal dynamics are expressed in terms of time-varying temperatures for inside air and inside mass. Household welfare is expressed as resident (thermal) comfort minus the net cost charged for power usage.

Section II reviews related literature. The general form of a household's optimal bid function is characterized in Section III, based on dynamic programming principles. Depending on the household's operating state, this optimal bid function expresses either power usage demand or ancillary service (power absorption) supply as a function of price.[1]

Household formulation preliminaries are given in Section IV. Continuous-time and derived discrete-time quantitative representations for the household's thermal dynamic system and welfare function are developed in Sections V and VI. Section VII derives a household's optimal bid function

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[1]Ancillary services include various forms of support for the assurance of power balance on a grid. Ancillary services in the form of dispatchable power absorption (withdrawal) are becoming increasingly important for power balance, given the increased penetration of non-dispatchable wind and solar power subject to sudden weather-induced ramping.

in quantitative form, given discretized quantitative representations for the household's thermal dynamic system and welfare function. A method is developed in Section VIII for classifying households into representative household types based on their thermal dynamic and welfare attributes.

Finally, test cases are reported in Sections IX–XII to illustrate the usefulness of these results for the development and evaluation of bid-based TES designs from a customer-centric vantage point. These test cases implement a generic bid-based TES design within a standard IEEE 123-bus distribution system populated by a mix of household types. Outcomes are reported for bid-function comparisons, peak load reduction, and load matching experiments.

Concluding remarks are given in Section XIII. Nomenclature tables for the household model are provided in Appendix A, and technical derivation and test case details are provided in Appendices B-F. Code and data for all reported test cases are provided at a website repository [3].

II. Related Literature

TES design research is rapidly expanding. However, to date, the rules governing customer participation are typically derived or directly imposed to achieve *system* efficiency and reliability objectives pre-specified by the researcher. These rules are therefore not based on a careful initial consideration of local customer goals and constraints, essential for ensuring voluntary customer participation.

TES design research is most closely associated with the Pacific Northwest National Laboratory (PNNL). As reported in [4], seminal work on transactive designs for power exchange was conducted by PNNL researchers starting as far back as 2003. More recent PNNL TES design work, including field demonstrations, is reported in [5]–[10]. TES design work by other researchers is reported in [11]–[19].

This previous work on TES design research is reviewed in [2], [7], [8], and [18]. As seen in these reviews, this previous work has largely focused on the development of simulation tools permitting the implementation and evaluation of TES designs. For example, Huang et al. [10] develop a simulation-based valuation method to compare different transactive energy schemes. They also develop an open-source simulation platform to allow agents developed on different platforms to interact with each other in a flexible manner.

However, some of this previous work focuses more specifically on the development of new transactive techniques for retail market operations. For example, Farrokh and Ipakchi [11] propose a number of ways in which transactive techniques can be extended from wholesale to retail markets, e.g., how aggregated demand-side resources can be scheduled and dispatched at wholesale in a manner similar to current wholesale resources. Chassin et al. [14] propose a transactive policy for the control of loads as demand-response resources able to provide frequency regulating services at wholesale. Mengelkamo et al. [17] propose a blockchain-based decentralized microgrid energy market facilitating peer-to-peer energy transactions between retail prosumers and consumers.

More broadly, Renani et al. [16] and Nguyen et al. [18] propose TES frameworks for end-to-end power system operations. Newly proposed forms of Distribution System Operators (DSOs) function as intermediaries between an ISO at wholesale and aggregated demand-side resources.

With specific regard to bid-based TES design, Hammerstrom et al. [5] and Fuller et al. [6] propose and implement a linear bid function for retail customers based on average retail price. Kok [13] formulates a simple bid function for retail customers with TCLs (e.g., freezers) that can easily be implemented for customers participating in his novel bid-based TES design called the PowerMatcher. Bids are demands for device power usage; ancillary service provision is not considered. The maximum price that retail customers are willing to pay for power usage is modeled as a cut-off price that increases in direct proportion to the difference between actual and desired temperature levels. This bid function form is justified on general heuristic grounds.

Nguyen et al. [18] formulate, implement, and test a version of Kok's bid-based PowerMatcher TES design for a collection of households that use a version of Kok's simple bid function as the price-sensitive controller for their electric HVAC systems. Nazir and Hiskens [19] develop a general virtual battery model for TCLs and propose a simple bid function for use as their price-sensitive controller.

The TES design work closest to the current study is by Li et al. [9]. The designer's problem is expressed conceptually as a Stackelberg game; a manager is assumed to use an electricity price signal P_c to coordinate power demands for a group of TCLs with ON/OFF controllers in order to achieve a socially efficient energy allocation subject to a peak energy constraint. Each TCL i selects a temperature setpoint to determine an energy allocation a_i that maximizes its comfort minus cost, conditional on P_c and its current state θ_i. The optimal energy allocation a_i is required to be a continuous non-increasing function of P_c, given θ_i. In contrast, the current study uses dynamic programming principles to explicitly derive the optimal bid form for an ON/OFF TCL expressing both its power usage demand and its ancillary service (power absorption) supply as state-conditioned rectilinear functions of price.

III. Household Optimal Bid: General Form

Consider a household with an electric *Heating, Ventilation, and Cooling (HVAC)* system that is controlled by a smart price-sensitive ON/OFF controller. The goal of the household is to maximize its welfare over time, measured as comfort minus cost. This section uses general dynamic programming principles to derive the optimal general bid-function form for the household's smart HVAC controller.

Let the time step during which an ON/OFF power setting is maintained for the household's HVAC system be called the *control step*. Let the time-line for the household be divided into control-steps $n = [n^{\mathsf{s}}, n^{\mathsf{e}})$. At the start-time n^{s} for each control-step n, a control signal is transmitted to the household's HVAC system to either retain or switch its current ON/OFF control setting. This control setting is then maintained for the remainder of control-step n.

The household's goal at the start-time n^{s} for each control-step n is to maximize its welfare over the next N control

steps, where N denotes the household's *look-ahead horizon*. The household at start-time n^{s} then has two possible control-relevant states. Let $\widehat{\mathrm{G}}(n{:}\mathrm{ON})$ and $\widehat{\mathrm{G}}(n{:}\mathrm{OFF})$ denote the maximum possible comfort the household forecasts it could achieve over control steps $n, n+1, \ldots, n+N-1$ if its HVAC system at time n^{s} were set to ON or OFF, respectively, and the ON/OFF HVAC controls for the remaining N-1 control steps $n+1, \ldots, n+N-1$ were then optimally set. These two control-relevant states are as follows:

X_n^{s}: *May Run as Ancillary Service Provider*
$\widehat{\mathrm{G}}(n,\mathrm{ON}) \leq \widehat{\mathrm{G}}(n,\mathrm{OFF})$

X_n^{u}: *May Run for Power Usage*
$\widehat{\mathrm{G}}(n,\mathrm{ON}) > \widehat{\mathrm{G}}(n,\mathrm{OFF})$

If the household is in state X_n^{s} at start-time n^{s}, the household will not be willing to pay a positive price for HVAC power usage during n, no matter how small. However, the household could be induced to switch (or leave) its HVAC system ON if the *price received* for this HVAC power absorption (as ancillary service supply) is *sufficiently high*. Let this sufficiently high cut-off price be denoted by $-\Pi^*(X_n^{\mathsf{s}}) \geq 0$.

Conversely, if the household is in state X_n^{u} at start-time n^{s}, the household will be willing to pay a positive price for HVAC power usage during n as long as this *price charged* is *sufficiently low*. Let this sufficiently low positive cut-off price be denoted by $\Pi^*(X_n^{\mathsf{u}}) > 0$.

Consequently, the household's optimal bid function for control-step n has the general rectilinear form depicted in Fig. 1, where $P^*(n)$ denotes the ON power consumption of the household's HVAC system during control-step n. Note the optimal bid form in the service provision state X_n^{s} constitutes a *supply* function for ancillary service (HVAC power absorption) as a function of price *received*. Conversely, the optimal bid form in the power usage state X_n^{u} constitutes a *demand* function for HVAC power usage as a function of price *paid*.

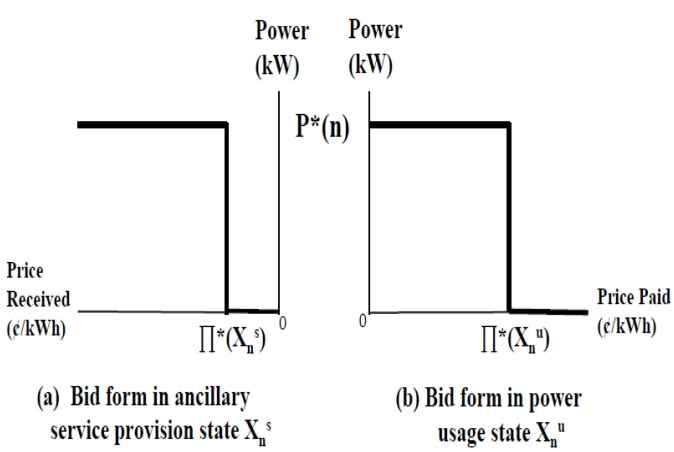

Fig. 1. A household's optimal state-dependent "May Run" bid forms for (a) ancillary service provision and (b) power usage during a control-step n. A negative price denotes a price received by the household for provision of ancillary service (HVAC power absorption). A positive price denotes a price paid by the household for HVAC power usage.

IV. Household Formulation: Preliminaries

Consider a household that consists of a resident occupying a house at a particular location subject to external weather conditions. The household has a mix of smart (price-responsive) and conventional appliances.

Specifically, the household has a smart electric HVAC system running in cooling mode with ON/OFF power settings. This HVAC system comprises a basic HVAC unit operating in parallel with a one-speed fan for air circulation. The household's conventional appliances consist of lights, clothes-washer, refrigerator, dryer, freezer, range, and microwave.[2]

The household participates in a bid-based TES design managed by an *Independent Distribution System Operator (IDSO)*. The household sends bids to the IDSO that express its demands for HVAC power usage as a function of required price payment and its supplies of ancillary service (HVAC power absorption) as a function of offered price compensation. In return, the IDSO sends price signals to the household that determine ON/OFF power control actions for the household's smart HVAC system.

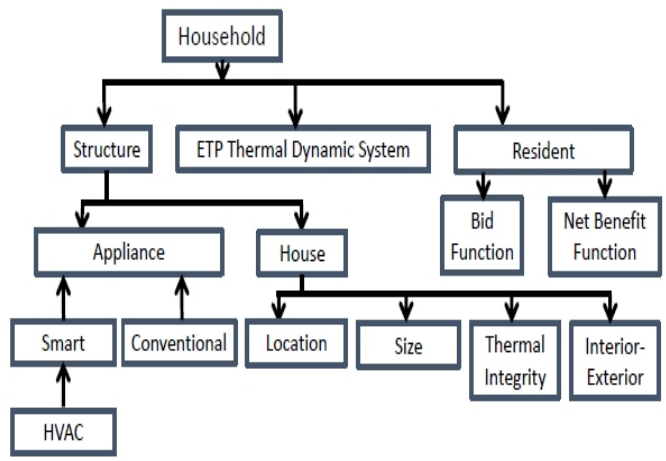

Fig. 2. Classification of household physical and behavioral attributes.

Figure 2 classifies the household's physical and behavioral attributes into conceptually distinct categories. Downward-pointing arrows denote "has a" relationships and upward-pointing arrows denote "is-a" relationships.

The 'Structure' of the household is characterized by appliance and house attributes. Appliance attributes include appliance mix and appliance features. House attributes include location, size, thermal properties, and interior-exterior features such as window framing and glazing.

The 'Resident' of the household is characterized by bid and net benefit functions. The 'Bid Function' expresses the resident's demand for HVAC power usage or supply of ancillary service (HVAC power absorption), conditional on price signals and current operating conditions. The 'Net Benefit Function' expresses resident welfare as benefit net of cost. Benefit is measured by thermal comfort. Cost is measured by charges for power usage net of payments for ancillary service.

[2]The methods developed in this study for optimal bid formulation and type classification can be applied for households with HVAC systems running in heating as well as cooling mode, and with arbitrary mixes of conventional appliances. Specific appliance assumptions are made here to enable a concrete demonstration of these methods.

The household's thermal dynamics are expressed as a dynamic system with two state variables: internal air temperature, and internal mass temperature. Starting from initial conditions, the motion over time of these two state variables is determined by external weather effects and by HVAC ON/OFF power control actions. This thermal dynamic modeling is carefully based on the household's 'Structure' and 'Resident' attributes.

V. Household Net Benefit and Thermal Dynamics in Continuous-Time Form

A. Household Net Benefit

The net benefit of the household over any designated time interval is measured by the (thermal) comfort attained by the household minus the net cost charged to the household for HVAC power consumption.

As in [12], [20], the discomfort of the household at each time s is measured by the discrepancy between inside air temperature $T_a(s)$ and the *bliss temperature* TB at which the household attains maximum comfort[3] G_{max}. Specifically, measuring time at the granularity of seconds, the comfort (utils) of the household over any time interval $[t_0, t)$ with length $\Delta t \equiv t - t_0$ (seconds) is measured as follows:

$$\mathrm{G}(t_0, t) = \int_{t_0}^{t} \Big(G^{\mathsf{max}} - h(s) \cdot [T_a(s) - \mathrm{TB}]^2 \Big) \, ds \ . \tag{1}$$

The comfort function (1) is characterized by positive parameters G_{max} and $\{h(s) \mid s \in [t_0, t)\}$. The inside air temperature $T_a(s)$ is determined by the thermal dynamics of the household at each time s. Note that $G^{\mathsf{max}}\Delta t$ is the maximum comfort the household can attain during interval $[t_0, t)$, achieved when inside air temperature is maintained at the household's bliss temperature TB during this entire interval.

The net cost (cents) charged to the household for HVAC power consumption during time interval $[t_0, t)$ is:

$$\mathrm{C}(t_0, t) = \int_{t_0}^{t} [K_h \pi(s)] P(s) \, ds \ . \tag{2}$$

In (2), $P(s)$ (kW) denotes the household's total HVAC power consumption at time s, including fan operations. The term $K_h \pi(s)$ (cents/kWs) denotes the retail power price $\pi(s)$ (cents/kWh) converted by K_h into cents/kWs. If $\pi(s) > 0$, $\pi(s)$ denotes the price *paid* by the household at time s for HVAC power usage. Conversely, if $\pi(s) < 0$, -$\pi(s)$ denotes the price *received* by the household at time s as compensation for the provision of ancillary service (HVAC power absorption).

[3] Given suitable regularity conditions, the benefit (utility) that a consumer would obtain from the consumption of a bundle of goods and services at some point in time can be derived from the preferences expressed by this consumer between variously offered pairs of lotteries that include these bundles as lottery payoffs; see Chipman et al. [21]. These expressed preferences are used to construct a *utility function* $U{:}X \to R$ that represents the consumer's preferences between bundles in the following sense: bundle x is preferred to bundle x' if and only if $U(x) \geq U(x')$. The utility function U is only unique up to a positive transformation $\phi(U)$ with $\phi' > 0$. Hence, if a most preferred bundle exists, this bundle can be assigned an arbitrary U-value without affecting the information content of U. For the case at hand, the household resident's maximum comfort (utility) is obtained when inside air temperature equals the resident's bliss temperature. This maximum comfort level, denoted by G_{max}, is assigned an arbitrary positive utility ("utils") value.

The welfare of the household over time interval $[t_0, t)$ is then measured by *net benefit* (utils), defined to be the weighted difference between comfort (1) and net cost (2):

$$\mathrm{NB}(t_0, t) \ = \ \mathrm{G}(t_0, t) - \mu \cdot \mathrm{C}(t_0, t) \ . \tag{3}$$

The positive weight factor μ (utils/cent) in (3) denotes the household's *marginal utility of money*, a standard economic welfare concept used to transform prices measured as money per quantity unit into prices measured as benefit (utility) per quantity unit.[4]

B. Household Thermal Dynamics

The thermal dynamics of the household are represented by means of the household ETP model [23] formulated in GridLAB-D (GLD) [24], [25]. This GLD Household ETP Model is a differential system that describes the motion over time of a household's inside air temperature $T_a(t)$ (^oF) and inside mass temperature $T_m(t)$ (^oF).

Assuming time is measured at the granularity of seconds, this ETP model takes the following form: For all $t \geq t_0$,

$$\dot{T}_a(t) = \frac{K_h}{C_a} \Big(U_a [T_o(t) - T_a(t)] + H_m [T_m(t) - T_a(t)] + Q_a(t) \Big) \ ; \tag{4}$$

$$\dot{T}_m(t) = \frac{K_h}{C_m} \Big(H_m [T_a(t) - T_m(t)] + Q_m(t) \Big) \ . \tag{5}$$

In this differential system, $T_o(t)$ (^oF) denotes *outside* air temperature at time t, $Q_a(t)$ (Btu/h) denotes the total heat flow rate to the household's inside *air* mass at time t, and $Q_m(t)$ (Btu/h) denotes the total heat flow rate to the household's inside *solid* mass at time t.

Equations (4)-(5) can equivalently be expressed in the following matrix form: For all $t \geq t_0$,

$$\dot{x}(t) = A K_h x(t) + B K_h v(t) \ , \tag{6}$$

$$\text{where: } A = \begin{bmatrix} -\frac{U_a + H_m}{C_a} & \frac{H_m}{C_a} \\ \frac{H_m}{C_m} & -\frac{H_m}{C_m} \end{bmatrix} \ ;$$

$$B = \begin{bmatrix} \frac{U_a}{C_a} & \frac{1}{C_a} & 0 \\ 0 & 0 & \frac{1}{C_m} \end{bmatrix} \ ;$$

$$x(t) = \begin{bmatrix} T_a(t) \\ T_m(t) \end{bmatrix} \ ;$$

$$v(t) = \begin{bmatrix} T_o(t) \\ Q_a(t) \\ Q_m(t) \end{bmatrix} \ .$$

[4] In economics, a standard budget-constrained utility maximization problem for a consumer requires, as a first-order necessary condition, that $\mu^{\mathsf{m}}\pi = \partial U(q)/\partial q$. Here $U(q)$ measures the benefit (utility) to the consumer of consuming a good Q in amount q, π denotes the unit price of Q, and the *marginal utility of money* μ^{m} is the dual variable for the consumer's budget constraint evaluated at the optimal solution point. More precisely, assuming income is measured in dollars, μ^{m} (utils/\$) is the rate of change of maximized utility with respect to a change in the consumer's income. In the first-order necessary condition, μ^{m} converts a price π measured in \$ per unit of Q into a price $\mu^{\mathsf{m}}\pi$ measured in utility ("utils") per unit of Q. For further discussion, see any standard microeconomic textbook; e.g., Varian [22, Chapter 7].

Form (6) expresses the dynamic state equations for the GLD Household ETP Model as a nonhomogenous first-order linear differential system with state vector $x(t)$, state matrix AK_h, and time-varying coefficient vector $BK_h v(t)$.

The heat flow rates $Q_a(t)$ and $Q_m(t)$ appearing in the differential system (6) are time-t endogenous variables determined by the following simultaneous relationships:

$$Q_a(t) = [1-f_i]Q_i(t) + [1-f_s]Q_s(t) + [1-f_{\mathsf{ac}}]Q_{\mathsf{hvac}}(t) \quad (7)$$

$$Q_m(t) = f_i Q_i(t) + f_s Q_s(t) + f_{\mathsf{ac}} Q_{\mathsf{hvac}}(t) \quad (8)$$

In these relationships, $Q_i(t)$ (Btu/h) denotes internal heat gain from household occupants and non-HVAC equipment at time t, $Q_s(t)$ (Btu/h) denotes internal heat gain from solar radiation at time t, and $Q_{\mathsf{hvac}}(t)$ (Btu/h) denotes internal heat gain from HVAC and fan operations during time t. The weights f_i, f_s, and f_{ac} are decimal percentages.

Recall that the household's HVAC system is assumed to run in cooling mode.[5] The internal heat gain term $Q_{\mathsf{hvac}}(t)$ in equations (7) and (8) can then be expressed explicitly in terms of ON/OFF HVAC power control actions as follows:

$$Q_{\mathsf{hvac}}(t) = \Big(-\,\text{HVACPow}(t) + \text{FanPow} \Big) \cdot u(t) \ , \quad (9)$$

where: -[HVACPow(t)] (Btu/h) denotes heat loss from the ON operation of the HVAC system running in cooling mode; FanPow (Btu/h) denotes heat gain from the ON operation of the 1-speed fan;[6] and $u(t)$ is a binary 0-1 (OFF/ON) HVAC power consumption control variable.

The expressions HVACPow(t) and FanPow in (9) take the form

$$\text{HVACPow}(t) \ = \ K(t) P_{\mathsf{hvac}}(t) \ ; \quad (10)$$

$$\text{FanPow} \ = \ K P_{\mathsf{fan}} \ , \quad (11)$$

where: $P_{\mathsf{hvac}}(t)$ (kW) and P_{fan} (kW) denote the time-t ON power consumption (kW) of the basic HVAC unit and fan, respectively; and $K(t)$ (Btu/[h-kW]) and K (Btu/[h-kW]) convert this power consumption (kW) into heat gain (Btu/h). The *total power consumption* of the HVAC system (including fan) at time t, if ON, is thus given by

$$P(t) \ = \ P_{\mathsf{hvac}}(t) + P_{\mathsf{fan}} \ . \quad (12)$$

The constant terms $\{C_a, U_a, C_m, H_m, f_i, f_s, f_{\mathsf{ac}}, P_{\mathsf{fan}}\}$ and variables $\{Q_i(t), Q_s(t), K(t), P_{\mathsf{hvac}}(t)\}$ appearing in relationships (4)–(12) are determined as functions of base (user-set) parameters and forcing terms in the GLD Household ETP Model [24]. See Tesfatsion and Battula [25] for a careful derivation and explanation of these functional dependencies.

VI. Household Net Benefit and Thermal Dynamics in Discretized Form

A. Overview

This section expresses the household's net benefit function and thermal dynamic system presented in Section V in approximate discretized form. To this end, the time-line $[t_0, +\infty)$ is partitioned into control-steps n of equal length $\Delta\tau$ (seconds), where $1/\Delta\tau$ is the rate at which the household's HVAC system receives ON/OFF power control signals.

Specifically, each control-step $n \geq 0$ takes the form $n = [n^{\mathsf{s}}, n^{\mathsf{e}})$, where the *start-time* n^{s} and *end-time* n^{e} are defined as follows:

$$n^{\mathsf{s}} \ = \ t_0 + n\Delta\tau \ ; \quad (13)$$

$$n^{\mathsf{e}} \ = \ t_0 + [n+1]\Delta\tau \ . \quad (14)$$

A function $f{:}[t_0, +\infty) \ \rightarrow \ R$ can then be expressed in a discretized form $f^*(n)$ that comports with this partitioning, as follows: For each control-step $n \geq 0$,

$$f^*(n) \ \equiv \ f(n^{\mathsf{s}}) \ . \quad (15)$$

B. Discretized Household Net Benefit Function

By the Mean Value Theorem, the household's comfort (1) measured over any control-step $n \geq 0$ can equivalently be expressed as follows: There exist a time point $\bar{t} \in [n^{\mathsf{s}}, n^{\mathsf{e}})$ such that

$$\text{G}(n^{\mathsf{s}}, n^{\mathsf{e}}) \ = \ [G_{max} - H(\bar{t})]\Delta\tau \ , \quad (16)$$

where

$$H(\bar{t}) \ = \ h(\bar{t}) \big[T_a(\bar{t}) - \text{TB}\big]^2 \ . \quad (17)$$

As in [12, Section IV], we approximate the mean value $H(\bar{t})$ by a weighted average $\widehat{H}^*(n)$ given by

$$\Big(h_1 \big[T_a^*(n) - \text{TB}\big]^2 + h_2 \big[E_n[T_a^*(n+1)] - \text{TB}\big]^2 \Big) \ , \quad (18)$$

where the weights h_1 and h_2 are positively valued. The term $E_n[T_a^*(n+1)]$ in (18) denotes the household's forecast at the start-time n^{s} of control-step n for the future inside air temperature $T_a^*(n+1)$ to be realized at the start-time of control-step n+1.[7] The household's *forecasted comfort* for control-step n, calculated at the start-time n^{s} of n, is given in approximate discretized form by

$$\widehat{\text{G}}^*(n) \ = \ [G_{max} - \widehat{H}^*(n)]\Delta\tau \ . \quad (19)$$

Also, the household's *forecasted net cost* (2) for control-step n, calculated at the start-time n^{s} of n, is given in approximate discretized form by

$$\widehat{\text{C}}^*(n) \ = \ [K_h \pi^*(n)] P^*(n) \Delta\tau \cdot u^*(n) \ . \quad (20)$$

In (20), the power control action $u^*(n)$ equals 1 (or 0) if the household's HVAC system is switched or left ON (or OFF) at the start-time n^{s} for control-step n, and $P^*(n)$ denotes the total power consumption of the household's HVAC system at the start-time n^{s} for control-step n if the HVAC system is switched (or left) ON. The term $K_h \pi^*(n)$ (cents/kWs) denotes the retail power price $\pi^*(n)$ (cents/kWh) for control-step n converted by K_h into cents/kWs.

Consequently, the household's *forecasted net benefit* for control-step n, calculated at the start-time n^{s} of n, is given in approximate discretized form by

$$\widehat{\text{NB}}^*(n) \ = \ \widehat{\text{G}}^*(n) - \mu \widehat{\text{C}}^*(n) \ . \quad (21)$$

[5] The case in which the HVAC system is running in heating mode can easily be handled as well.

[6] The GLD Household ETP Model [24] implements a 1-speed air-circulation HVAC fan to be ON if and only if the basic HVAC unit is ON.

[7] The precise manner in which this forecasted inside air temperature is determined is carefully explained in Appendix B.

C. Discretized Thermal Dynamics

To obtain an approximate discretized form for the household's thermal dynamics, the state-vector derivative $\dot{x}(t)$ in (6) is first approximated by a forward finite difference:[8]

$$\dot{x}(t) \;\approx\; \frac{x(t+\Delta\tau)-x(t)}{\Delta\tau} \tag{22}$$

for each $t \in [t_0, +\infty)$. The thermal dynamic system (6) is then approximated by the following system of difference equations: For each control-step $n \geq 0$,

$$x^*(n+1) = x^*(n) + A_D x^*(n) + B_D v^*(n)\ , \tag{23}$$

where

$$A_D \;=\; AK_h\Delta\tau\,; \tag{24}$$

$$B_D \;=\; BK_h\Delta\tau\,; \tag{25}$$

$$x^*(n) \;=\; \begin{bmatrix} T_a^*(n) \\ T_m^*(n) \end{bmatrix}; \tag{26}$$

$$v^*(n) \;=\; \begin{bmatrix} T_o^*(n) \\ Q_a^*(n) \\ Q_m^*(n) \end{bmatrix}. \tag{27}$$

Finally, the simultaneous equations (7) through (10) are represented as follows. For each control-step $n \geq 0$:

$$Q_a^*(n) = [1-f_i]Q_i^*(n) + [1-f_s]Q_s^*(n) + [1-f_{\mathsf{ac}}]Q_{\mathsf{hvac}}^*(n)\,; \tag{28}$$

$$Q_m^*(n) = f_i Q_i^*(n) + f_s Q_s^*(n) + f_{\mathsf{ac}} Q_{\mathsf{hvac}}^*(n)\,; \tag{29}$$

$$Q_{\mathsf{hvac}}^*(n) = \Big(-\,\text{HVACPow}^*(n) + \text{FanPow}\Big)u^*(n); \tag{30}$$

$$\text{HVACPow}^*(n) = K^*(n)P_{\mathsf{hvac}}^*(n); \tag{31}$$

$$\text{FanPow} = KP_{\mathsf{fan}}; \tag{32}$$

$$P^*(n) = P_{\mathsf{hvac}}^*(n) + P_{\mathsf{fan}}\,. \tag{33}$$

VII. Optimal Bid Function Derivation

A. Overview

The general form of the optimal bid function for our modeled household, conditional on its operating state, is depicted in Fig. 1. This section derives explicit expressions for the optimal cut-off prices $-\Pi^*(X_n^{\mathsf{s}})$ and $\Pi^*(X_n^{\mathsf{u}})$ for this bid function, assuming the household has a one-step look-ahead horizon (N=1).[9]

More precisely, the optimal cut-off prices $-\Pi^*(X_n^{\mathsf{s}})$ and $\Pi^*(X_n^{\mathsf{u}})$ are derived as functions of the "base parameters" characterizing the household's forecasted net benefit function and thermal dynamic system in the discrete-time forms presented in Section VI.[10] Consequently, as a preliminary step, we first explain more carefully the meaning of a "base parameter."

B. Household Welfare and Thermal Dynamic Attributes in Base Parameter Form

Consider a household whose forecasted net benefit function and thermal dynamic system take the discrete-time parameterized forms presented in Section VI. The *base parameter set* $\mathbb{BP}$ for this household is then defined by the following three conditions. (i) Each element of $\mathbb{BP}$ is a parameter appearing in the household's forecasted net benefit function or thermal dynamic system; (ii) Each parameter appearing in the household's forecasted net benefit function and thermal dynamic system can be expressed as a function of one or more parameters in $\mathbb{BP}$; (iii) No parameter in $\mathbb{BP}$ can be non-trivially expressed as a function of other parameters in $\mathbb{BP}$.

Thus, in standard mathematical terms, $\mathbb{BP}$ constitutes a *basis set* for the parameters appearing in the household's forecasted net benefit function and thermal dynamic system. Let β denote the household's *base parameter vector* consisting of all of the elements of $\mathbb{BP}$. A complete listing of the components of β, together with their descriptions and units of measurement, is given in Table XII in Appendix A.

C. Derivation of Optimal Cut-Off Prices

Let a control-step $n \geq 0$ be given. The power level $P^*(n,\beta)$ corresponding to $P^*(n)$ in Fig. 1 denotes the ON power consumption of the household's HVAC system running in cooling mode during n, as determined by (33).

Suppose the household's HVAC system is switched (or left) OFF at the start-time n^{s} for control-step n. Let the household's resulting forecasted net benefit (21) be denoted by:

$$\widehat{\text{NB}}^*(n,\beta,\text{OFF}) \;=\; \widehat{\text{G}}^*(n,\beta,\text{OFF}). \tag{34}$$

Conversely, suppose the household's HVAC system is switched (or left) ON at n^{s}. Let the household's resulting forecasted net benefit (21) be denoted by:

$$\begin{aligned}\widehat{\text{NB}}^*(n,\beta,\text{ON}) &= \widehat{\text{G}}^*(n,\beta,\text{ON}) - \mu\text{C}^*(n,\beta,\text{ON}) \\ &= \widehat{\text{G}}^*(n,\beta,\text{ON}) - \mu K_h \pi^*(n) P^*(n,\beta)\Delta\tau\end{aligned} \tag{35}$$

The goal of the household is to maximize its forecasted net benefit during control-step n. Consequently, the household will be willing to switch (or leave) its HVAC system ON during n if and only if

$$\widehat{\text{NB}}^*(n,\beta,\text{OFF}) \;\leq\; \widehat{\text{NB}}^*(n,\beta,\text{ON}). \tag{36}$$

Substituting (34) and (35) into (36), and rearranging terms, condition (36) is equivalent to

$$\pi^*(n) \;\leq\; \frac{\Big[\widehat{\text{G}}^*(n,\beta,\text{ON}) - \widehat{\text{G}}^*(n,\beta,\text{OFF})\Big]}{\mu K_h P^*(n,\beta)\Delta\tau} \equiv F_n(\beta). \tag{37}$$

For later purposes, using (19), note that (37) reduces to

$$\pi^*(n) \;\leq\; \frac{\Big[\widehat{H}^*(n,\beta,\text{OFF}) - \widehat{H}^*(n,\beta,\text{ON})\Big]}{\mu K_h P^*(n,\beta)}. \tag{38}$$

[8] The use of this simple Euler Method to discretize a household's thermal dynamics suffices for the purposes of this study. However, reduced approximation error can be obtained by augmenting this first-order "predictor" method with a "corrector" method. For further discussion of approximation methods for systems of differential equations, see any basic text such as Lambert [26]. For online lecture notes, see Süli [27].

[9] Extension to an N-period look-ahead horizon with N>1 is conceptually straightforward but computationally more demanding.

[10] For concreteness, the continuous-time and discrete-time thermal dynamic systems derived for a household in Sections V-B and VI-C assume the household's HVAC system runs in cooling mode. This assumption is retained in the present section as well. However, the method used here to derive the household's optimal bid cut-off prices can easily be modified to handle the case in which the household's HVAC system runs in heating mode.

It follows from (37) that the household can be in only one of two possible control-relevant states at the start-time n^{s} for control-step n:

$X_n^{\mathsf{s}}(\beta)$: *May Run as Ancillary Service Provider*
$\widehat{\mathrm{G}}^*(n,\beta,\mathrm{ON}) \leq \widehat{\mathrm{G}}(n,\beta,\mathrm{OFF})$

$X_n^{\mathsf{u}}(\beta)$: *May Run for Power Usage*
$\widehat{\mathrm{G}}^*(n,\beta,\mathrm{ON}) > \widehat{\mathrm{G}}(n,\beta,\mathrm{OFF})$

The service state $X_n^{\mathsf{s}}(\beta)$ corresponds to X_n^{s} in Fig. 1; it is the β-dependent household state in which the household *is not* willing to pay for power usage during control-step n but is willing to provide ancillary service (HVAC power absorption) during n in return for sufficiently high compensation. It follows from the derivation of relation (37) that the household is in a service state $X_n^{\mathsf{s}}(\beta)$ at the start of control-step n if and only if $F_n(\beta)$ in (37) is less than or equal to zero. In this case the household can be induced to switch (or leave) its HVAC system ON during n if and only if the *price received* for ancillary service, $-\pi^*(n)$, is at least as high as the non-negative cut-off price $-\Pi^*(X_n^{\mathsf{s}}(\beta))$ given by $-F_n(\beta)$.

Consequently, the form of the household's optimal bid function in a service state $X_n^{\mathsf{s}}(\beta)$ takes the rectilinear form depicted on the left-hand side of Fig. 1. This optimal bid function constitutes a *supply function* for ancillary service as a function of price *received*.

Conversely, the power-usage state $X_n^{\mathsf{u}}(\beta)$ corresponds to X_n^{u} in Fig. 1; it is the β-dependent household state in which the household is willing to *pay* for *power usage* during control-step n. It follows from the derivation of relation (37) that the household is in a power-usage state $X_n^{\mathsf{u}}(\beta)$ at the start of control-step n if and only if $F_n(\beta)$ in (37) is strictly greater than zero. In this case there is a range of positive prices $\pi^*(n)$ for power usage during control-step n that the household is willing to pay, bounded above by the positive cut-off price $\Pi^*(X_n^{\mathsf{u}}(\beta))$ given by $F_n(\beta)$.

Consequently, the household's optimal bid function in a power usage state $X_n^{\mathsf{u}}(\beta)$ takes the rectilinear form depicted on the right-hand side of Fig. 1. This optimal bid function constitutes a *demand function* for HVAC power usage as a function of price *paid*.

Finally, note that the IDSO does not need to pay a household for OFF ancillary service when the household is in an ancillary service state $X_n^{\mathsf{s}}(\beta)$; the IDSO simply needs to set an ancillary service compensation price that is below the household's optimal bid cut-off price $-\Pi^*(X_n^{\mathsf{s}}(\beta))$. Similarly, the IDSO does not need to pay a household for OFF ancillary service when the household is in a power usage state $X_n^{\mathsf{u}}(\beta)$; the IDSO simply needs to set a power-usage price that is above the household's optimal bid cut-off price $\Pi^*(X_n^{\mathsf{u}}(\beta))$.

Explicit expressions for the optimal bid cut-off prices $-\Pi^*(X_n^{\mathsf{s}}(\beta))$ and $\Pi^*(X_n^{\mathsf{u}}(\beta))$ as functions of the components of the base parameter vector β are provided in Appendix C.

VIII. Household Type Classification

A. Overview

As explained in Section VII-B, the physical and behavioral attributes of our modeled household are characterized by a base parameter vector β. This section develops a method for classifying our modeled households into representative household types in accordance with the values set for the components of β.

As will be demonstrated in Sections IX–XII, these household types can be used to construct physically and economically meaningful mixes of households to populate distribution systems for bid-based TES design studies.

B. Household Type Construction

As shown in Fig. 2, the 'Structure' attributes of our modeled household are divided into 'Appliance' and 'House' attributes. Let β^a denote the components of β that correspond to 'Appliance' attributes, and let β^h denote the components of β that correspond to 'House' attributes. Finally, let β^r denote the components of β that correspond to 'Resident' attributes.

A *Household Type* is then defined by three aspects: *Appliance Type* (β^a), *House Type* (β^h), and *Resident Type* (β^r). A complete description of the components of $\beta = (\beta^a, \beta^h, \beta^r)$, classified by attribute type, is given in Table XII in Appendix A.

To be physically and economically meaningful, the base parameters comprising β for any given modeled household must be configured in a correlated manner. For example, it would be empirically problematic to assume that a household with a small-sized house, located in a temperate climate, has a large powerful HVAC system.

For the purposes of this study, a household's *Structure Quality Type* is characterized by its HVAC Type (β^{hvac}) and its House Type (β^h), where β^{hvac} consists of all base parameters in the household's Appliance Type β^a that correspond to the attributes of its HVAC system. For example, as shown in Table XIV in Appendix D, β^{hvac} includes an HVAC system's coefficient of performance (Cooling_COP) and over-sizing factor (OSF). As shown in Table XV in Appendix D, a household's House Type β^h characterizes the location, size, thermal integrity, and interior-exterior attributes of its house.

More precisely, a house's 'Location' is characterized by base parameters for position aspects, such as the voltage level of the distribution system to which the house is connected. A house's 'Size' is determined by base parameters characterizing number of stories and the width, length, and height aspects of ceilings, walls, and floors. A house's 'Thermal Integrity' is characterized by base parameters for thermal factors such as percentages of heat gain from equipment operations and solar radiation. Finally, a house's 'Interior-Exterior' attributes are characterized by base parameters for factors such as (window) glazing and number of doors.

Table I, below, illustrates how different Structure Quality Types can be constructed using different correlated settings for the base parameters in β^{hvac} and β^h, with all remaining elements of β maintained at fixed value settings. For example, a household has a *Low* Structure Quality Type if it has a 'Small' sized house, 'Poor' thermal integrity, 'Poor' interior-exterior features, and a 'Poor' quality HVAC system. It has a *Medium* Structure Quality Type if it has a 'Normal' sized house, 'Normal' thermal integrity, 'Normal' interior-exterior features, and a 'Normal' HVAC system. It has a *High* Structure

Quality Type if it has a 'Large' sized house, 'Good' thermal integrity, 'Good' interior-exterior features, and a 'Good' quality HVAC system.

TABLE I
STRUCTURE QUALITY TYPE CHARACTERIZATIONS

Structure Quality	Size	Thermal Integrity	Interior-Exterior	HVAC
Low	Small	Poor	Poor	Poor
Medium	Normal	Normal	Normal	Normal
High	Large	Good	Good	Good

Specific correlated parameter settings that could be used to characterize the Structure Quality Type classifications in Table I are given in Tables XVII through XX in Appendix F. These correlated parameter settings are used to implement Structure Quality Type classifications for the test cases reported below in Sections IX–XII.

IX. TEST CASES: PURPOSE AND COMMON FEATURES

A. Overview

As indicated in Table II, this study reports three types of test cases to demonstrate the usefulness of our household formulation, optimal bid function derivation, and household type construction method for the customer-centric development and evaluation of bid-based TES designs.

Test Case 1 focuses on the following issue: Under what conditions, and to what extent, does it matter that households participating in bid-based TES designs use bid functions optimally derived as functions of their own local welfare and thermal dynamic attributes? As will be seen, this use can significantly improve welfare outcomes for households under certain operating conditions.

Test Case 2 and Test Case 3 illustrate how simulated distribution systems, populated with systematically varied mixes of our household types, can be used to evaluate the performance capabilities of IDSO-managed bid-based TES designs. The specific focus of these test cases is the ability of the IDSO to use bid-conditioned price signals to achieve desired peak-load reductions and to shape distribution loads to match target load profiles.

TABLE II
TEST CASE DESCRIPTIONS

Test Case	Description
Test Case 1	*Bid Function Performance Comparisons:* Compares welfare outcomes under our optimal household bid function and alternative household bid functions.
Test Case 2	*Peak Load Reduction:* Illustrates use of our Household Types in a TES design to test the IDSO's ability to achieve peak load reduction.
Test Case 3	*Target Load Matching:* Illustrates use of our Household Types in a TES design to test the IDSO's ability to achieve target load matching.

For each test case, the distribution system operates over a 123-bus distribution grid during hot summer weather. The distribution grid is populated by a mix of 927 households, each modeled using our household formulation depicted in Fig. 2, and each implemented in part using the GridLAB-D (GLD) House Object [28].[11]

Each household has a smart (price-responsive) HVAC system running in cooling mode. Each household also has conventional (non-price-responsive) appliances consisting of lights, clothes-washer, refrigerator, dryer, freezer, range, and microwave. All households are participants in a IDSO-managed bid-based TES design that manages their power consumption.

The IDSO is located at a substation of the 123-bus distribution grid that connects the distribution system to a transmission system. Wholesale power is supplied to the distribution system through this T-D interface.

The next two subsections provide fuller descriptions of the 123-bus distribution grid and bid-based TES design used for all test cases. The final subsection describes base parameter values, forcing terms, and initial conditions that are maintained for all test cases.

B. Test Case Grid Populated by Households

The 123-bus distribution grid used for all of our test cases is depicted in Fig. 3. This grid modifies the standard IEEE 123-bus distribution grid [29] in three ways.

First, the load connected at each bus of the standard IEEE 123-bus distribution grid is replaced with household load. Specifically, 927 households are distributed across the 123 buses of the distribution grid in proportion to the original loads, which are then omitted. Second, the distribution grid is connected to a transmission system through a substation at bus 150. Third, the distribution system is managed by an IDSO operating at this substation.

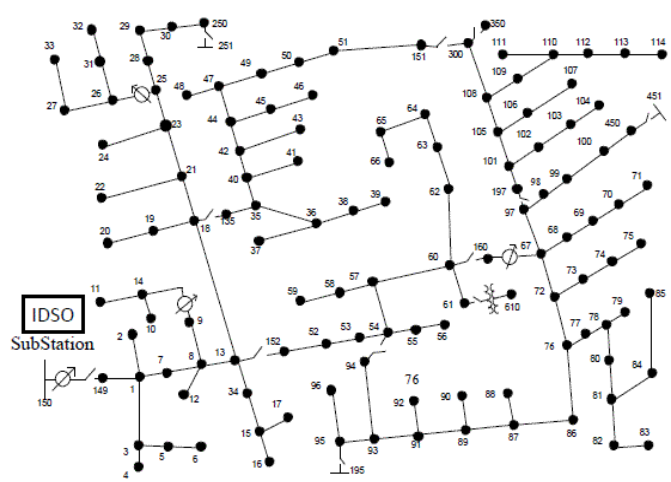

Fig. 3. The IDSO-managed 123-bus distribution grid used for all test cases.

[11] See Appendix E for GLD House Object implementation details.

C. The Five-Step TES Design

The bid-based TES design used for all test cases reported in this study is referred to below as the *Five-Step TES Design*. As discussed more fully in earlier work [18], this design is a variant of the well-known PowerMatcher TES design developed by Koen Kok [13] to balance the power transactions of numerous power consuming and producing devices operating within an electrical infrastructure.

The Five-Step TES design extends PowerMatcher in three key regards. First, each household's bid function can take the form of an ancillary service offer or a power usage demand, depending on local operating conditions. This bid function is characterized by state-conditioned cut-off prices for ancillary service provision and power usage, together with a state-conditioned forecast for the ON power consumption (kW) of the household's HVAC system. Second, the entity that manages the Five-Step TES Design is an IDSO that functions at a substation as a T-D linkage entity able to purchase and sell power within a wholesale power market. Third, the IDSO has a fiduciary responsibility for the efficiency and reliability of distribution system operations.

The Five-Step TES Design consists of five iterated steps, as follows:

- **Step 1:** The HVAC system controller for each household h collects data on the state of h at a *Data Check Rate*.
- **Step 2:** The HVAC system controller for each household h forms a state-conditioned bid function Bid(h) for power usage and/or service provision and communicates it to the IDSO at a *Bid Refresh Rate*.
- **Step 3:** The IDSO combines all received household bid functions Bid(h) into an aggregate bid function, AggBid, at an *Aggregate Bid Refresh Rate*.
- **Step 4:** The IDSO uses the aggregate bid function AggBid to determine and communicate price signals back to households at a *Price Signal Rate.*
- **Step 5:** The HVAC system controller for each household h inserts its latest received price signal into its latest refreshed state-conditioned bid function Bid(h) at a *Power Control Rate*, which triggers an ON/OFF power control action for the HVAC system.

To ensure the Five-Step TES Design permits the IDSO to fulfill its fiduciary responsibilities, careful attention must be paid to the timing of the signals propagated back and forth between the IDSO and the household HVAC system controllers. This timing depends on the five action rates for the five steps of this design. In addition, communication and action time-delays must be taken into account.

Suppose, for simplicity of exposition, that all five action rates for the Five-Step TES Design are given by $1/\Delta t$ for a common time-step Δt. Let the time-delay between Step j and Step $j+1$ in any given iteration of the five steps be denoted by ϵ_j for $j = 1, \ldots, 5$, where "Step 6" is equated with "Step 1" in the subsequent iteration. Suppose the summation of the time-delays ϵ_j for $j = 1, \ldots, 5$ does not exceed Δt. Finally, let $t_j = t_{j-1} + \epsilon_j$ for $j = 1, \ldots 5$. The iterated staggered implementation of Steps 1-5 for the Five-Step TES Design can then be depicted as in Fig. 4.[12].

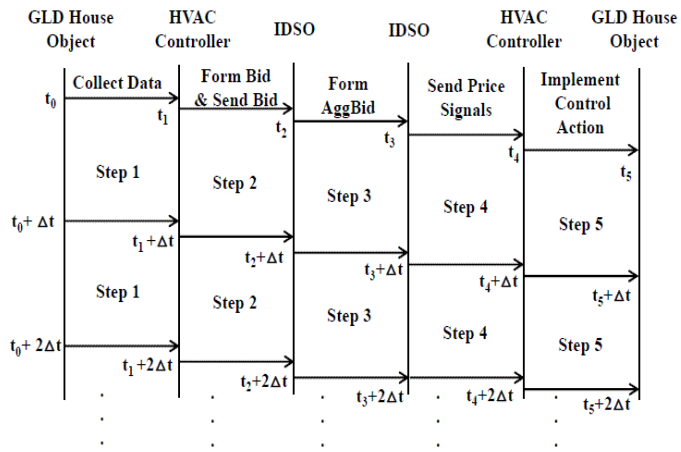

Fig. 4. Staggered implementation of the five steps comprising the Five-Step TES Design.

D. Maintained Base Parameter Values, Forcing Terms, and Initial Conditions

For all test cases, the length $\Delta\tau$ of each control-step $n \geq 0$ is set to 300s. The base parameter location attributes specified for each household are for Des Moines, Iowa, USA.

Also, for all test cases the base parameters NOC and f_{oc} appearing in each household's Resident Type β^r are set to 1 and 1.0. The setting NOC = 1 indicates the household has a single resident (occupant), and the setting $f_{\mathsf{oc}} = 1.0$ indicates this resident occupies the house 100% of the time. In addition, for each household: G_{max} = 3.3333 (utils/s); TB = 72 (^oF); and h_1 = h_2 = 0.0017 (utils/[s - $(^oF)^2$]).

The five action rates characterizing the Five-Step TES Design described in Section IX-C are set as follows for all test cases: Data Check Rate = 1/300s; Bid Refresh Rate = 1/300s; Aggregate Bid Refresh Rate = 1/300s; Price Signal Rate = 1/300s; and Power Control Rate = 1/300s.

The weather-related forcing terms used for all of our test cases consist of time series data for outside temperature, solar flux, and humidity for hot summer days in Des Moines, Iowa, USA [30]. For example, Fig. 5 depicts the specific weather-related forcing terms used for our Test Case 2 results. Outside temperature and solar flux values appear on the primary (left) vertical axis, and humidity values appear on the secondary (right) vertical axis.[13]

Finally, all of our test cases are initialized at the simulation start-time t_0 by setting the inside air and mass temperatures for each household equal to the household's bliss temperature TB. Outcomes for the first simulated day are then ignored in order to allow diversity in these temperatures to develop across the simulated households.

[12]See Appendix E for GLD implementation details

[13]See Appendix E for additional details regarding our test-case implementation of weather-related forcing terms.

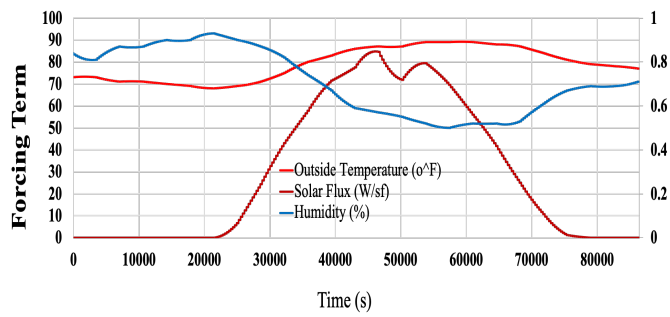

Fig. 5. Weather-related forcing terms (outside temperature, solar flux, and humidity) for a 24-hour day used for Test Case 2.

X. Test Case 1 Outcomes: Bid Function Performance Comparisons

A. Test Case 1a: Optimal vs. Non-Optimal Cut-Off Bid Prices

The optimal form of a household's bid function for any control-step $n = [n^{\mathsf{s}}, n^{\mathsf{e}})$ is depicted in Fig. 1 and carefully derived in Section VII. The optimal bid cut-off price $-\Pi^*(X_n^s) \geq 0$ is the minimum price the household is willing to *receive* for service provision (power absorption) if it is in an ancillary service state X_n^{s}. The optimal bid cut-off price $\Pi^*(X_n^u) \geq 0$ is the maximum price the household is willing to *pay* for power usage if it is in a power usage state X_n^{u}.

This section addresses the following question: Suppose the household uses non-optimal bid cut-off prices $-\Pi^{\mathsf{s}} \geq 0$ and $\Pi^{\mathsf{u}} \geq 0$ for control-step n in place of the optimal bid cut-off prices $-\Pi^*(X_n^s)$ and $\Pi^*(X_n^u)$. How would this affect the household's actual net benefit (NB) for control-step n?

The household's use of non-optimal bid cut-off prices for control-step n in place of optimal bid cut-off prices will not affect the household's NB unless it changes the household's power control action $u^*(n)$. Whether this change in control action occurs will depend, in turn, on the *retail price* (RP) offered for ancillary service ($RP < 0$) or charged for power usage ($RP > 0$) during control-step n.

If $RP < 0$ and the household is in a power usage state X_n^{u}, the household will attain a higher NB if it switches (or leaves) its HVAC system ON rather than OFF, regardless of the exact magnitude of the ancillary service price compensation $-RP > 0$. The exact magnitude of the household's optimal bid cut-off price $\Pi^*(X_n^u)$ for power usage is thus irrelevant. Conversely, if $RP > 0$ and the household is in an ancillary service provision state X_n^{s}, the household will attain a higher NB if it switches (or leaves) its HVAC system OFF rather than ON, regardless of the exact magnitude of RP. The exact magnitude of the household's optimal bid cut-off price $-\Pi^*(X_n^s) \geq 0$ for ancillary service compensation is thus irrelevant.

Consequently, in either of these situations the household's NB will not be affected by the use of a non-optimal bid cut-off price in place of an optimal bid cut-off price.

Suppose $RP > 0$ and the household is in a power usage state X_n^{u}. Suppose, also, that the household is using an arbitrary bid cut-off price Π^{u} for power usage. The household will then switch (or leave) its HVAC system ON ($u^*(n)$=1) during control-step n if and only if $RP \leq \Pi^{\mathsf{u}}$.

Consequently, a change from an arbitrary bid cut-off price Π^{u} to the optimal bid cut-off price $\Pi^*(X_n^{\mathsf{u}})$ will affect the household's NB if and only if one of the following two conditions holds:

(a) Π^{u} *is too high*: $\Pi^*(X_n^{\mathsf{u}}) < RP \leq \Pi^{\mathsf{u}}$
(b) Π^{u} *is too low*: $\Pi^{\mathsf{u}} < RP \leq \Pi^*(X_n^{\mathsf{u}})$

In case (a), the optimal power control action for the household is to switch (or leave) its HVAC system OFF; however, the use of the arbitrary bid cut-off price Π^{u} results in its HVAC system being set ON. In case (b) the optimal power control action for the household is to switch (or leave) its HVAC sytem ON; however, the use of the arbitrary bid cut-off price Π^{u} results in its HVAC system being set OFF.

A similar analysis can be conducted for the case in which $RP < 0$ and the household is in an ancillary service provision state X_n^{s}. A change from an arbitrary bid cut-off price $-\Pi^{\mathsf{s}}$ to the optimal bid cut-off price $-\Pi^*(X_n^{\mathsf{s}})$ will affect the household's NB if and only if one of the following two conditions holds:

(c) $-\Pi^{\mathsf{s}}$ *is too high*: $-\Pi^*(X_n^{\mathsf{s}}) \leq -RP < -\Pi^{\mathsf{s}}$
(d) $-\Pi^{\mathsf{s}}$ *is too low*: $-\Pi^{\mathsf{s}} \leq -RP < -\Pi^*(X_n^{\mathsf{s}})$

In case (c), the offered service compensation price $-RP$ is at or above the minimum acceptable compensation price $-\Pi^*(X_n^{\mathsf{s}})$, indicating the HVAC system should be set ON for service provision. However, the arbitrary cut-off price $-\Pi^{\mathsf{s}}$ indicates the offered compensation price $-RP$ is too low and the HVAC system should be set OFF. Conversely, in case (d) the offered compensation price $-RP$ is below the minimum acceptable compensation price $-\Pi^*(X_n^{\mathsf{s}})$, indicating the HVAC system should be set OFF; yet the arbitrary cut-off price $-\Pi^{\mathsf{s}}$ indicates the offered compensation price $-RP$ is sufficiently high and the HVAC system should be set ON.

The only remaining issue, then, is the size of the gain (if any) in the household's NB for control-step n if the household switches its power control action due to a switch from a non-optimal to an optimal bid cut-off price.

Tables III–VIII report household NB effects under various test-case treatments. For each test case, NB outcomes for two non-optimal bid cut-off prices (cents/kWh) are compared with the NB outcome for the optimal bid cut-off price (cents/kWh). The two non-optimal bid cut-off prices are chosen in such a way that one results in the same power control action as the optimal bid cut-off price while the other results in a different power control action.

Net benefit results are reported in Tables III–IV for a High Structure Quality Type household. The retail price RP = 10 (cents/kWh), inside air temperature $T_a^*(n)$ = 75.12 (^oF), and outside weather $T_o^*(n)$ = 79 (^oF) are held constant; note that the inside air temperature $T_a^*(n)$ is greater than the household resident's bliss temperature TB = 72 (^oF). The treatment factor is the household's marginal utility of money μ (utils/cent) in modified form μ^{m} (utils/\$), where $\mu^{\mathsf{m}} = \mu \times$ 100cents/\$. Recall that μ is the comfort-cost tradeoff factor in the household's net benefit function (21).

As seen in Tables III–IV, when μ^{m} = 100, household NB is not very sensitive to a switch from a non-optimal to an optimal

bid cut-off price. However, when μ^{m} is instead set to 1000, the effect of such a switch on NB is potentially much greater.

TABLE III
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN POWER USAGE STATE X_n^{u} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^{m} = 100, RP = 10, $T_a^*(n) = 75.12 >$ TB, AND $T_o^*(n) = 79$.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
25.00	No	72.06	991.97
19.27	Yes	72.06	991.97
1.00	No	75.49	989.04

TABLE IV
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN POWER USAGE STATE X_n^{u} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^{m}=1000, RP = 10, $T_a^*(n) = 75.12 >$ TB, AND $T_o^*(n)$ = 79.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
25.00	No	72.06	963.56
1.93	Yes	75.49	989.04
1.00	No	75.49	989.04

In Table V, all treatment factors are the same as in Table IV with one exception: the retail price RP is increased to 15 (cents/kWh). Comparing outcomes in these two tables, it is seen that the switch from a non-optimal to an optimal bid cut-off price now has a potentially larger effect on NB.

TABLE V
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN POWER USAGE STATE X_n^{u} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^*=1000, RP =15, $T_a^*(n) = 75.12 >$ TB, AND $T_o^*(n)$ = 79.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
25.00	No	72.06	947.78
1.93	Yes	75.49	989.04
1.00	No	75.49	989.04

In Table VI, all treatment factors are the same as in Table V with one exception: the outside temperature $T_o^*(n)$ is increased from 79 (^oF) to 85 (^oF). Comparing outcomes in these two tables, this change in outside weather has a relatively small impact on the resulting NB effects.

TABLE VI
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN POWER USAGE STATE X_n^{u} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^*=1000, RP = 15, $T_a^*(n)$ > TB, AND $T_o^*(n)$ = 85.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
25.00	No	72.84	945.47
2.63	Yes	76.24	986.14
1.0	No	76.24	986.14

Finally, Tables VII–VIII report household NB effects of a switch from non-optimal to optimal bid cut-off prices for a household in a service provision state X_n^{s} when the household's marginal utility of money is increased from μ^{m}=100 (utils/\$) to μ^{m}=1000 (util/\$). To emphasis that the focus of these two tables is on ancillary service prices, not power usage prices, the retail price and ancillary service bid cut-off prices are reported in their negative forms $RP \leq 0$ and $\Pi^*(X_n^{\mathsf{s}}) \leq 0$.

The ancillary service compensation price is maintained at RP = -10 (cents/kWh), the inside air temperature is maintained at $T_a^*(n) = 70.15$ (^oF), and the outside air temperature is maintained at $T_o^*(n)$ = 79 (^oF). As previously observed for the power usage cases, the gain in NB resulting from a switch from a non-optimal to an optimal bid cut-off price is potentially more substantial for the higher μ^{m} level.

TABLE VII
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN ANCILLARY SERVICE STATE X_n^{s} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^*=100, RP = -10, $T_a^*(n)$ = 70.15 < TB, AND $T_o^*(n)$ = 79.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
-1.00	No	70.81	990.82
-31.44	Yes	70.81	997.58
-40.00	No	67.39	997.58

TABLE VIII
COMPARISON OF HOUSEHOLD NET BENEFIT (NB) IN ANCILLARY SERVICE STATE X_n^{s} FOR OPTIMAL AND NON-OPTIMAL BID CUT-OFF PRICES, GIVEN: μ^*=1000, RP = -10, $T_a^*(n)$ = 70.15 < TB, AND $T_o^*(n)$ = 79.

Cut-Off	Optimal?	$T_a^*(n+1)$	NB
-1.00	No	70.81	1019.25
-3.14	Yes	67.39	1019.25
-40.00	No	67.39	997.58

B. Test Case 1b: Comparative Bid Function Performance Tests

The optimal bid function derived in this study is compared against the following bid function proposed by Nguyen et al. [18]:

$$\pi^s(\mathrm{T}_a) = \theta^s \Big[\frac{\mathrm{TB} - \mathrm{T}_a}{\mathrm{TB} - \mathrm{TMin}}\Big] \quad \text{for} \quad \mathrm{TMin} < \mathrm{T}_a \leq \mathrm{TB}\ ; \quad (39)$$

$$\pi^u(\mathrm{T}_a) = \theta^u \Big[\frac{\mathrm{T}_a - \mathrm{TB}}{\mathrm{TMax} - \mathrm{TB}}\Big] \quad \text{for} \quad \mathrm{TB} < \mathrm{T}_a < \mathrm{TMax}\ , \quad (40)$$

where θ^s and θ^u are positively-valued scaling parameters. The test case parameter values maintained for the alternative bid function described by (39) and (40) are: TMin = 68^oF, TB = 72^oF, TMax = 76^oF, and $\theta^s = \theta^u$ = 20 (cents/kWh).

Differences in actual net benefit (NB) during a control-step n are calculated for the two bid functions under variously set values for the household's marginal utility of money μ^m (utils/\$) and the Structure Quality Types of households. Specifically, this NB difference is calculated as the NB attained using our optimal bid function minus the NB attained using the bid function described by (39) and (40). The initial inside air temperature and initial outside weather temperature for control-step n are set at $T_a^*(n)$ = 74.67 (^oF) and $T_o^*(n)$ = 79 (^oF), respectively, for both bid functions.

Figure 6 reports outcomes for this test case. It is seen that our optimal bid function results in higher NB for all tested μ^{m} values, and the improvement in NB performance is larger for larger μ^{m} values. Moreover, this same pattern holds across all three tested settings for household Structure Quality Type.

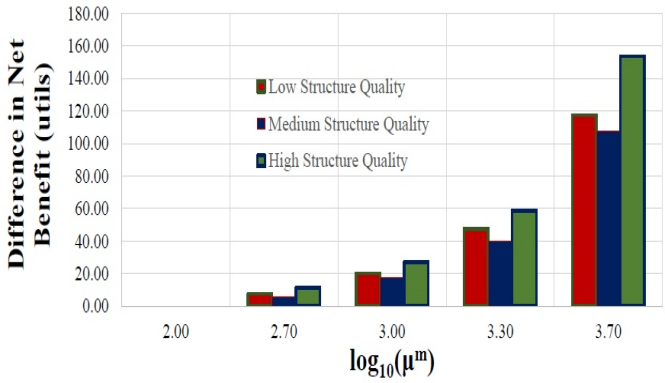

Fig. 6. Difference in net benefit between our proposed optimized bid function and a bid function developed by [18] under varied settings for household marginal utility of money μ^m (utils/$) and Structure Quality Type.

XI. Test Case 2 Outcomes: IDSO Peak Load Reduction Capabilities

This section demonstrates how the household formulation and household type construction method developed in earlier sections of this study can facilitate the customer-centric performance evaluation of IDSO-managed bid-based TES designs. The specific performance to be evaluated is as follows: Can the IDSO use retail price signals to achieve target peak-load reductions? A key finding is that the ability of the IDSO to achieve this system objective depends strongly on the mix of house quality types.

Two types of test cases are reported. The first type (Test Case 2a) investigates the IDSO's peak-load reduction capabilities when households are differentiated by their Structure Quality Type. The second type (Test Case 2b) investigates the IDSO's peak-load reduction capabilities when households are differentiated by their Resident Type.

The IDSO on day D forecasts household peak load for day D+1. The IDSO uses this forecast to determine a target peak load for day D+1, given by forecasted peak load reduced by a designated percentage. During day D+1 the IDSO then uses latest refreshed household bids to send an appropriate sequence of retail price signals to households to maintain total household load at or below this target peak-load level.

Test Case 2a: IDSO Peak Load Reduction Capabilities for Different Household Structure Quality Types

The treatment factor selected for Test Case 2a is the Structure Quality Type of households. Our general method for classifying households into Structure Quality Types is explained in Section VIII.

Specific characterizations for the Low, Medium, and High Structure Quality Types used for the test cases reported in this section can be found in Appendices D and E. The marginal utility of money μ for each household is set to 1 (utils/cent). All other household attributes are set at the maintained values listed in Section IX-D. The IDSO implements the Five-Step TES Design for the management of household power consumption.

Figures 7–9 report the ability of the IDSO to achieve a 0.5MW peak-load reduction by means of retail prices communicated to households with optimally formulated bid functions. These retail prices take either a flat-rate form (10 cents/kWh) or a peak-load pricing form. All households have the same Structure Quality Type, either Low, Medium, or High. The variation observed in Fig. 10 in the peak-load retail prices required to achieve a targeted 0.5MW peak-load reduction indicates that household Structure Quality Type should be given careful consideration in any peak-load reduction effort.

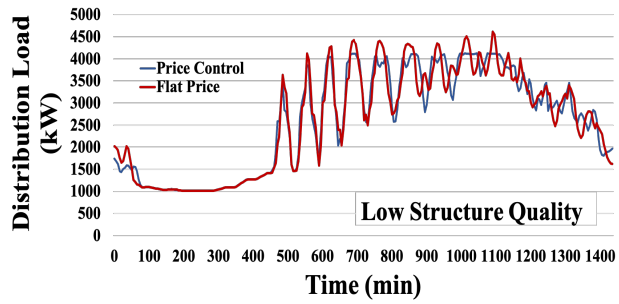

Fig. 7. IDSO's ability to achieve a 0.5MW peak-load reduction using flat-price vs. peak-load pricing when all households have Low Structure Quality.

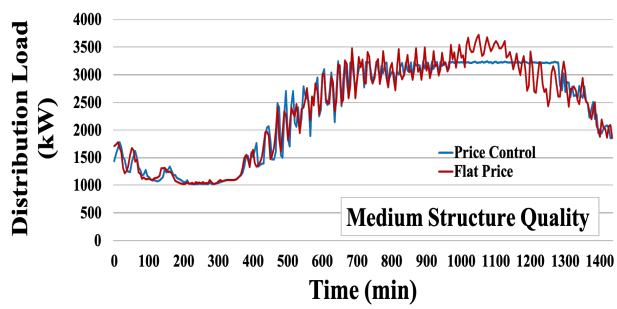

Fig. 8. IDSO's ability to achieve a 0.5MW peak-load reduction using flat-price vs. peak-load pricing when all households have Medium Structure Quality.

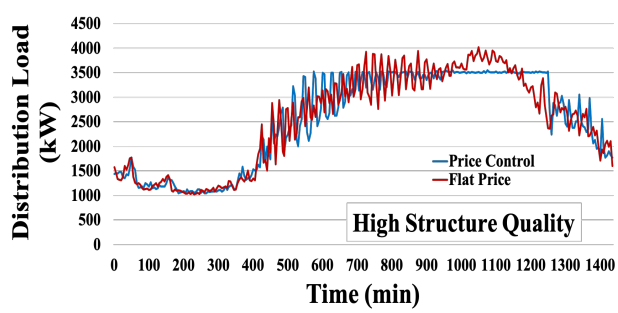

Fig. 9. IDSO's ability to achieve a 0.5MW peak-load reduction using flat-price vs. peak-load pricing when all households have High Structure Quality.

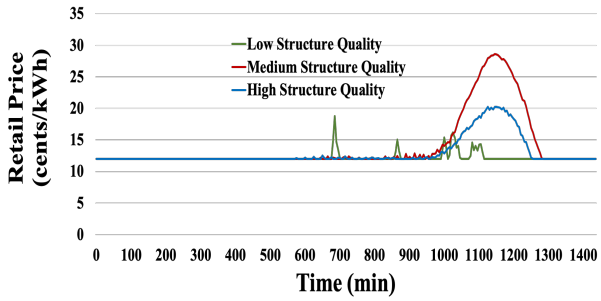

Fig. 10. Peak-load retail price signals used by the IDSO to achieve a 0.5MW peak load reduction when all households have the same Structure Quality Type (either all Low, all Medium, or all High).

Test Case 2b: IDSO Peak Load Reduction Capabilities for Different Household Resident Types

The treatment factor selected for Test Case 2b is Resident Type; specifically, a household resident's marginal utility of money μ (utils/cent) = $\mu^{\mathsf{m}} \times \$/100$cents. Three values for μ^{m} (utils/\$) are considered: 10, 100, and 1000. Recall from Section V-A that μ determines a household's comfort-cost tradeoffs for determination of its net benefit.

All households are configured to have a High Structure Quality Type. All other household attributes are set at the maintained values listed in Section IX-D. The IDSO implements the Five-Step TES Design for the management of household power consumption.

Figure 11 reports the retail prices the IDSO communicates to households to achieve a 0.5 MW peak-load reduction, given three different settings for μ^{m}. The needed retail prices for the lowest setting μ^{m}=10 are seen to be substantially higher than for the other two settings. This finding follows directly from the role μ plays in the determination of a household's willingness to sacrifice comfort for lower cost; the lower the value of μ, the higher the retail price must be to induce any given MW reduction in its power usage.

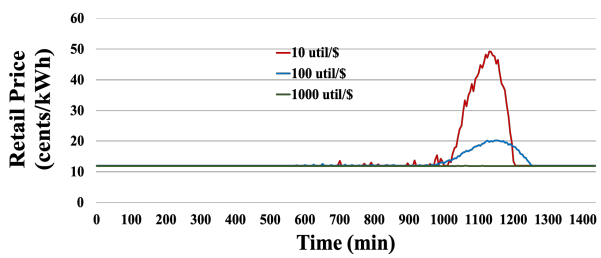

Fig. 11. Retail price signals used by the IDSO to achieve 0.5MW peak-load reduction for High Structure Quality Type households, given three different settings for household marginal utility of money μ^{m} (utils/\$).

Figure 12 reports the retail prices the IDSO communicates to households to achieve three different peak-load reductions, 0.3MW, 0.4MW, and 0.5MW, assuming μ^{m}=10 for all household residents. The retail prices needed to achieve a 0.5MW peak-load reduction are seen to be relatively high. The retail price signals needed to achieve the more moderate 0.3MW and 0.4MW peak-load reductions are substantially lower.

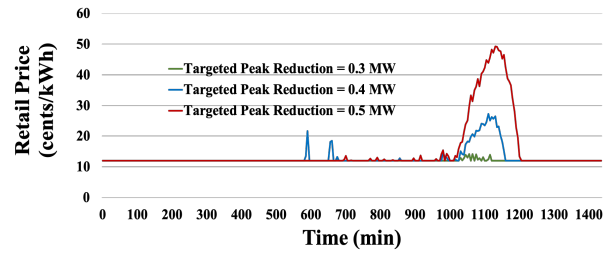

Fig. 12. Retail price signals used by the IDSO to achieve 0.3MW, 0.4MW, and 0.5MW peak load reductions for High Structure Quality Type households, each with a marginal utility of money μ^{m}=10 (utils/\$).

XII. Test Case 3 Outcomes: IDSO Load Matching

This section demonstrates how the household formulation and type construction method developed in this study can facilitate the customer-centric performance evaluation of IDSO-managed bid-based TES designs with regard to another IDSO capability: namely, the use of retail price signals to match distribution system load to a target load profile.

As in all previously reported test cases, the IDSO manages a Five-Step TES design for the 123-bus distribution grid depicted in Fig. 3. The IDSO is located at a substation (bus 150) that connects the distribution system to a transmission system. Wholesale power is supplied to the distribution system through this substation.

However, in this section the IDSO is now modeled to be an active participant in a wholesale *day-ahead market (DAM)* operating over the transmission grid. On each day D the IDSO submits a fixed demand bid into the DAM consisting of a forecasted 24-hour household load profile for day D+1. On day D+1 the IDSO attempts to ensure that actual household load does not deviate from the fixed demand bid it submitted into the DAM on day D.

Figure 13 reports load-matching outcomes for an illustrative case in which the distribution grid is populated with a mixture of households with Low, Medium, and High Structure Quality Types.[14] All households have the same maintained Resident Type with a marginal utility of money μ = 1 (utils/cent).

As seen in Fig. 13, the IDSO is successfully able to use retail price signals on day D+1 to match household load to the load profile it submitted to the day-D DAM as its fixed demand bid. The retail price signals used by the IDSO to achieve the good load matching depicted in Fig. 13 are shown in Fig. 14.

[14]The Structure Quality Type of each household connected at each grid bus is configured as Low, Medium, or High with probabilities (1/3, 1/3, 1/3).

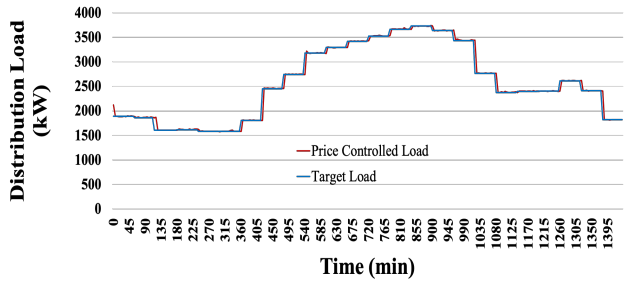

Fig. 13. Ability of the IDSO to match total household load on day D+1 to a target load profile, given by the IDSO's fixed demand bid submitted into a day-ahead market on day D.

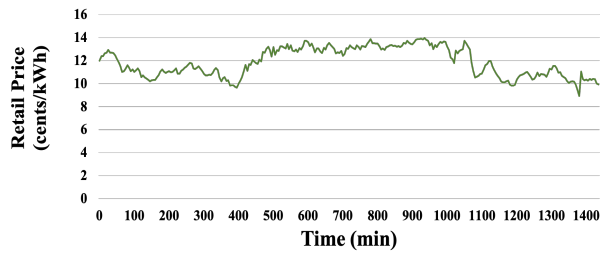

Fig. 14. The retail price signals communicated by the IDSO to households on day D+1 to match total household load to the IDSO's day-D DAM fixed demand bid, depicted as the target load profile in Fig. 13.

Suppose, instead, that the IDSO submits into the day-D DAM the fixed demand bid (load profile) depicted in Fig. 15. Once again, as seen in this figure, the IDSO is successfully able to use retail price signals on day D+1 to match total household load to this target load profile.

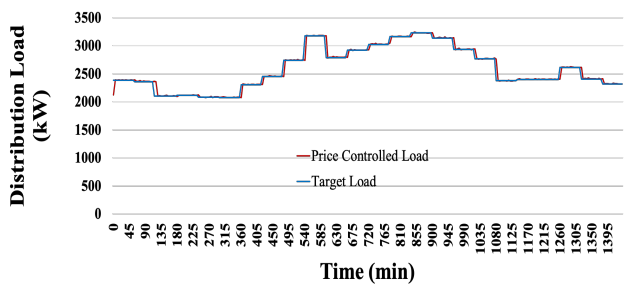

Fig. 15. Ability of the IDSO to match total household load on day D+1 to a different target load profile, i.e., a different fixed demand bid submitted into the day-D DAM.

The retail prices used by the IDSO to accomplish the good load matching depicted in Fig. 15 are shown in Fig. 16. The negative retail price signals seen in Fig. 16 indicate that the IDSO must purchase ancillary services (power absorption) from households during some control-steps in order to match its target load profile.

XIII. Conclusion

This study formulates methods to facilitate the development and evaluation of bid-based TES designs that are well aligned with local customer goals and constraints.

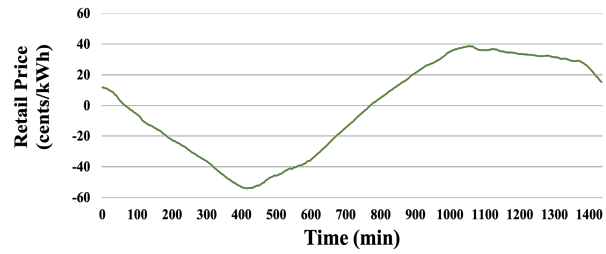

Fig. 16. The positive and negative retail price signals communicated by the IDSO to households on day D+1 to match total household load to the target load profile depicted in Fig. 15.

The optimal general form of a household's bid function is derived from dynamic programming principles, based solely on the household's general thermal dynamic and welfare attributes. This optimal bid function permits the household to demand power usage as a function of required price and to supply ancillary service (power absorption) as a function of offered price compensation.

It is then shown how this optimal bid function can be derived in explicit quantitative form, given quantitative representations for the household's thermal dynamic system and welfare function. A method is also developed for the systematic construction of representative household types based on appliance, house structure, and household resident attributes expressed in base-parameter form.

Test cases are conducted for a 123-bus distribution grid populated by a mix of household types with variously specified thermal dynamic and welfare attributes. The distribution system is managed by an IDSO using a bid-based TES design. The reported test case findings demonstrate the usefulness of our methods for the evaluation of bid-based TES designs from a customer-centric vantage point.

Future work will use the results of this study to undertake systematic evaluations of IDSO-managed bid-based TES designs that rely on voluntary customer participation. Particular attention will be focused on the ability of the IDSO to use aggregated household bids to offer flexible ancillary services into a wholesale power market.

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APPENDIX A: HOUSEHOLD NOMENCLATURE

Tables IX-XI below provide symbols and descriptions for all of the terms appearing explicitly in the representations used in this study for household thermal dynamics and household welfare.

TABLE IX
HOUSEHOLD THERMAL DYNAMICS: FACTORS AND VARIABLES.

	Description
HVACPow(t)	Heat gain (Btu/h) from the ON operation of the basic HVAC unit at time t
K	Conversion factor (3412Btu/[h-kW]) that converts kW to Btu/h
K_h	Conversion factor (1h/3600s) that converts seconds s to hours h (hence 1/h to 1/s)
$K(t)$	Coefficient of performance factor (Btu/[h-kW]) for the basic HVAC unit at time t
n	Control-step $n = [n^{\mathsf{s}}, n^{\mathsf{e}})$ with length $\Delta\tau$, where $n^{\mathsf{s}} = t_0 + n\Delta\tau$ and $n^{\mathsf{e}} = t_0 + [n+1]\Delta\tau$
$P(t)$	Total power consumption (kW) of the HVAC system (including fan) if ON at time t
$P_{\mathsf{hvac}}(t)$	Power consumption (kW) of the basic HVAC unit if ON at time t
$Q_a(t)$	Total heat flow rate (Btu/h) to inside air mass at time t
$Q_{\mathsf{hvac}}(t)$	Heat flow rate (Btu/h) from the HVAC system (including fan) if ON at time t
$Q_i(t)$	Heat flow rate (Btu/h) from internal non-HVAC equipment and occupants at time t
$Q_m(t)$	Total heat flow rate (Btu/h) to inside solid mass at time t
$Q_s(t)$	Heat flow rate (Btu/h) from solar radiation at time t
t	Time point indicator (granularity of seconds)
t_0	Simulation start-time (granularity of seconds)
$T_a(t)$	Inside air temperature (^{o}F) at time t
$T_m(t)$	Inside mass temperature (^{o}F) at time t
$T_o(t)$	Outside air temperature (^{o}F) at time t
$u^*(n)$	Binary 0-1 variable denoting OFF/ON HVAC power control action for control-step n
$\Delta\tau$	Length (seconds) of each control-step $n = [n^{\mathsf{s}}, n^{\mathsf{e}})$
$\pi(t)$	Retail power price (cents/kWh) at time t

TABLE X
HOUSEHOLD THERMAL DYNAMICS: DERIVED (NON-BASIC) PARAMETERS

	Description
C_a	Heat capacity (Btu/^{o}F) of the inside air mass
C_m	Heat capacity (Btu/^{o}F) of the inside solid mass
U_a	Thermal conductance (Btu/[h-^{o}F]) between internal and external air masses
H_m	Thermal conductance (Btu/[h-^{o}F]) between inside air and solid masses
FanPow	Heat gain (Btu/h) from the ON operation of the 1-speed fan
P_{fan}	Power consumption (kW) of the 1-speed fan when ON

TABLE XI
HOUSEHOLD WELFARE: VARIABLES, FUNCTIONS, AND PARAMETERS

	Description
$\widehat{\mathrm{C}}^*(n)$	Forecasted net cost (cents) for control-step n
$\widehat{\mathrm{G}}^*(n)$	Forecasted comfort (utils) for control-step n
$\widehat{\mathrm{NB}}^*(n)$	Forecasted net benefit (utils) for control-step n
μ	Marginal utility of money (utils/cent)
$-\Pi^*(X_n^s)$	Min acceptable payment (cents/kWh) for HVAC ancillary service provision in state X_n^{s}
$\Pi^*(X_n^u)$	Max willingness to pay (cents/kWh) for HVAC power usage in state X_n^{u}.

TABLE XII
APPLIANCE, HOUSE, AND RESIDENT COMPONENTS OF A HOUSEHOLD'S BASE PARAMETER VECTOR β

β	**Parameter Name**	**Parameter Description**
β^a	AuxHeatCap	Auxiliary heating capacity (Btu/h)
	base_power	Base real power (kW) of the total load at nominal voltage (kW)
	Cooling_COP	Coefficient of performance (unit free) for HVAC system in cooling mode
	cooling_system_type	Determines type of HVAC system running in cooling mode (electric, gas)
	CoolSupplyAirTemp	Cooling supply air temperature (^{o}F)
	current_fraction	Fraction (decimal %) of the load that is constant current (p.u.)
	current_pf	Power factor (unit free) for constant current portion of load (p.u.)
	DCT	System design cooling set-point (^{o}F)
	DesignCoolCap	Design cooling capacity (Btu/h)
	DesignHeatCap	Design heating capacity (Btu/h)
	DesignHeatSetpoint	Design heating setpoint (^{o}F)
	DIG	System design internal gain (Btu/h)
	DuctPressureDrop	Duct pressure drop (inches of water)
	FanDesignPower	Designed maximum power draw (W) of the ventilation fan
	$\mathrm{fI}_{\mathsf{eu}}$	Fraction (decimal %) of non-HVAC end-use load eu internal to house
	HeatSupplyAirTemp	Heating supply air temperature (^{o}F)
	impedance_fraction	Fraction (decimal %) of load that is constant impedance (p.u.)
	impedance_pf	Power factor (unit free) for constant impedance portion of load (p.u.)
	LatCoolFrac	Fractional cooling-load increase (unit free) due to latent heat
	NEU	Number (integer) of household non-HVAC end-use loads
	OSF	Over-sizing factor (unit free)
	power_fraction	Fraction (decimal %) of the load that is constant power (p.u.)
	power_pf	Power factor (unit free) for constant power portion of load (p.u.)
β^h	CDT	System cooling design temperature (^{o}F)
	DPS	System design solar load (Btu/[h-ft^2])
	ECR	Exterior ceiling, fraction of total (decimal %)
	EFR	Exterior floor, fraction of total (decimal %)
	EWR	Exterior wall, fraction of total (decimal %)
	$f_{\mathsf{ac}}, f_s, f_i,$	Heat gain (decimal %) from ($Q_{\mathsf{hvac}}(t)$, $Q_s(t)$, $Q_i(t)$) to $Q_m(t)$
	glass_layer	String-coded window glass-layer type (ONE, TWO, ...)
	glass_type	String-coded glass type (GLASS, LOW_E,...)
	glazing_treatment	String- coded exterior window reflectivity type
	HDT	Heating design temperature (^{o}F)
	h_s	Interior surface heat transfer coefficient (Btu/[h-^{o}F-ft^2])
	I	Infiltration volumetric air exchange rate (#times per hour)
	IWR	Interior/exterior wall surface ratio (unit free)
	m_f	Total thermal mass per unit floor area (Btu/[^{o}F-ft^2])
	n_s	Number (integer) of stories
	n_d	Number (integer) of doors
	R_c	Thermal resistance ([h-^{o}F-ft^2]/Btu) of house ceilings
	R_d	Thermal resistance ([h-^{o}F-ft^2]/Btu) of house doors
	R_f	Thermal resistance ([h-^{o}F-ft^2]/Btu) of house floors
	R_w	Thermal resistance ([h-^{o}F-ft^2]/Btu) of house walls
	V_nominal	Nominal rating voltage (volts)
	WET	Window exterior transmission coefficient (decimal %)
	WF	String-coded window-frame type (INSULATED, WOOD, ...)
	WWR	Window-to-exterior-wall ratio (decimal %)
	x, y, h	Width, length, and height (ft)
	$\Delta\tau$	Length (seconds) of each control-step n
β^r	f_{oc}	Household occupancy fraction (decimal %)
	G_{max}	Max possible comfort (utils/s) of household resident during control-step n
	$h1, h2$	Weight factors (utils/[s-(^{o}F)^2]) in household's comfort function
	NOC	Number (integer) of household occupants
	SHOC	Sensible heat (Btu/h-occupant) from each occupant
	TB	Household Resident's "Bliss" internal air temperature (^{o}F)
	μ	Household's marginal utility of money (utils/cent)

Appendix B: Inside Air Temperature Forecast

The discretized thermal dynamic system for our household model, presented in Section VI-C, is a first-order difference equation system in two state variables: namely, inside air temperature $T_a^*(n)$ and inside mass temperature $T_m^*(n)$, for each control step $n \geq 0$. All forcing terms, including the heat flow rates $Q_i(t)$ and $Q_s(t)$ and the outside temperature $T_o(t)$, are held constant during each control-step n at their realized values at the start-time for n.

The 'Appliance' component (β^a), 'House' component (β^h), and 'Resident' component (β^r) of the base parameter vector β = $(\beta^a, \beta^h, \beta^r)$ for this household model are listed in Table XII. Let the first two structure components for β be abbreviated as $\beta^s = (\beta^a, \beta^h)$.

Consider the household at the start-time n^{s} of any control-step $n \geq 0$. For later purposes, this appendix section uses the household's discretized thermal dynamic system to derive the household's forecast for inside air temperature $T_a^*(n+1)$ at the start-time $(n+1)^{\mathsf{s}}$ for the future control-step n+1. This forecast is expressed as a function of: (i) the HVAC ON/OFF power control action $u^*(n)$ implemented at time n^{s}; (ii) endogenous variables determined at time n^{s}; (iii) forcing terms realized at time n^{s}; (iv) initial state conditions at time n^{s}; and (v) the base parameter structure component β^s. This forecast is as follows:

$$\begin{aligned}
&E_n[T_a^*(n+1,\beta^s)] = \\
&\left(1 - \frac{K_h \Delta\tau}{C_a(\beta^h)}\left[U_a(\beta^h) + H_m(\beta^h)\right]\right) \cdot T_a^*(n) \\
&+ \left(\frac{K_h \Delta\tau \cdot H_m(\beta^h)}{C_a(\beta^h)}\right) \cdot T_m^*(n) \\
&- \frac{K_h \Delta\tau \cdot [1 - f_{ac}]}{C_a(\beta^h)}(\text{HVACPow}^*(n,\beta^s) - \text{FanPow}(\beta^a))u^*(n) \\
&+ \frac{K_h \Delta\tau}{C_a(\beta^h)} \cdot \Big([1 - f_s]Q_s^*(n,\beta^h) + [1 - f_i]Q_i^*(n,\beta^a,\beta^r)\Big) \\
&+ \frac{K_h \Delta\tau}{C_a(\beta^h)} \cdot U_a(\beta^h) \cdot T_o^*(n)
\end{aligned} \tag{41}$$

The specific expressions for $E_n[T_a^*(n+1),\beta^s]$ for the cases when the household's HVAC is OFF ($u^*(n) = 0$) or ON ($u^*(n) = 1$) are thus given by:

$$\begin{aligned}
&E_n[T_a^*(n+1,\beta^s,\text{OFF})] = \\
&\left(1 - \frac{K_h \Delta\tau}{C_a(\beta^h)}\left[U_a(\beta^h) + H_m(\beta^h)\right]\right) \cdot T_a^*(n) \\
&+ \left(\frac{K_h \Delta\tau \cdot H_m(\beta^h)}{C_a(\beta^h)}\right) \cdot T_m^*(n) \\
&+ \frac{K_h \Delta\tau}{C_a(\beta^h)} \cdot \Big([1 - f_s]Q_s^*(n,\beta^h) + [1 - f_i]Q_i^*(n,\beta^a,\beta^r)\Big) \\
&+ \frac{K_h \Delta\tau}{C_a(\beta^h)} \cdot U_a(\beta^h) \cdot T_o^*(n)\ ;
\end{aligned} \tag{42}$$

$$\begin{aligned}
&E_n[T_a^*(n+1,\beta^s,\text{ON})] \\
&= E_n[T_a^*(n+1,\beta^s,\text{OFF})] \\
&- \frac{K_h \Delta\tau}{C_a(\beta^h)}[1 - f_{ac}](\text{HVACPow}^*(n,\beta^s) - \text{FanPow}(\beta^a))\ .
\end{aligned} \tag{43}$$

Appendix C: Explicit Derivations for the Optimal Bid Cut-Off Prices

The three components of the base parameter vector β = $(\beta^a, \beta^h, \beta^r)$ for our household model are explained in Section VIII and Table XII. As in Appendix B, let $\beta^s = (\beta^a, \beta^h)$ denote the first two structure components of β.

From (38), if the modeled household is in a "may run for service provision" state $X_n^{\mathsf{s}}(\beta)$ at the beginning of any control-step n, the optimal cut-off $\Pi^*(X_n^{\mathsf{s}}(\beta))$ is given by

$$\Pi^*(X_n^{\mathsf{s}}(\beta)) = \frac{\left[\widehat{H}^*(n,\beta,\text{OFF}) - \widehat{H}^*(n,\beta,\text{ON})\right]}{\mu K_h P^*(n,\beta^s)} \leq 0 \tag{44}$$

Conversely, if the modeled household is in a "may run for power usage" state $X_n^{\mathsf{u}}(\beta)$ at the beginning of any control-step n, the optimal cut-off $\Pi^*(X_n^{\mathsf{u}},\beta)$ is given by

$$\Pi^*(X_n^{\mathsf{u}}(\beta)) = \frac{\left[\widehat{H}^*(n,\beta,\text{OFF}) - \widehat{H}^*(n,\beta,\text{ON})\right]}{\mu K_h P^*(n,\beta^s)} > 0 \tag{45}$$

In either case, using (18),

$$\begin{aligned}
&\left[\widehat{H}^*(n,\beta,\text{OFF}) - \widehat{H}^*(n,\beta,\text{ON})\right] \\
&= h_2 \cdot \left[E_n[T_a^*(n+1,\beta^s,\text{OFF})] - \text{TB}\right]^2 \\
&\quad - h_2 \cdot \left[E_n[T_a^*(n+1,\beta^s,\text{ON})] - \text{TB}\right]^2
\end{aligned} \tag{46}$$

Substituting (42) and (43) into (46), and simplifying and rearranging the resulting terms, the right-hand sides in (45) and (44) can each be expressed as

$$\begin{aligned}
&K_{11}(\beta) \cdot K_{12}(n,\beta^s) \cdot K_h \Delta\tau \cdot \Big([T_a^*(n) - \text{TB}] \\
&+ K_{21}(\beta^h) \cdot [T_m^*(n) - T_a^*(n)] \cdot K_h \Delta\tau \\
&+ K_{22}(\beta^h) \cdot [T_o^*(n) - T_a^*(n)] \cdot K_h \Delta\tau \\
&+ K_3(n,\beta) \cdot K_h \Delta\tau \\
&- K_{12}(n,\beta^s) \cdot K_4(\beta^h) \cdot P^*(n,\beta^s) \cdot K_h \Delta\tau\Big)
\end{aligned} \tag{47}$$

where $K_{11}(\beta)$, $K_{12}(n,\beta^s)$, $K_{21}(\beta^h)$, $K_{22}(\beta^h)$, $K_3(n,\beta)$ and $K_4(\beta^h)$ are as defined in Table XIII.

TABLE XIII
Symbols and Corresponding Expressions

Symbol	Expression	Units
$K_{11}(\beta)$	$\frac{2 \cdot h_2 \cdot (1-f_{ac})}{\mu \cdot C_a(\beta^h)}$	$\frac{\text{cents}}{(\text{s-}^o\text{F-Btu})}$
$K_{12}(n,\beta^s)$	$\frac{(\text{HVACPow}^*(n,\beta^s) - \text{FanPow}(\beta^a))}{P^*(n,\beta^s)}$	Btu/kWh
$K_{21}(\beta^h)$	$\frac{H_m(\beta^h)}{C_a(\beta^h)}$	^{o}F/h
$K_{22}(\beta^h)$	$\frac{U_a(\beta^h)}{C_a(\beta^h)}$	1/h
$K_3(n,\beta)$	$\frac{[1-f_s]Q_s^*(n,\beta^h) + [1-f_i]Q_i^*(n,\beta^a,\beta^r)}{C_a(\beta^h)}$	1/h
$K_4(\beta^h)$	$\frac{(1-f_{ac})}{2 \cdot C_a(\beta^h)}$	^{o}F/Btu

Note that expression (47) has a different sign in each of the two possible states $X_n^{\mathsf{s}}(\beta)$ and $X_n^{\mathsf{u}}(\beta)$. This difference in sign arises from the terms involving differences at time-n^{s} between inside air temperature $T_a^*(n)$ and TB, inside mass temperature $T_m^*(n)$ and inside air temperature $T_a^*(n)$, and/or outside air temperature $T_o^*(n)$ and inside air temperature $T_a^*(n)$.

Finally, the following functional forms appear in the expressions given in Table XIII: $C_a(\beta^h)$; HVACPow(n, β^s); FanPow(β^a); $P^*(n, \beta^s)$; $H_m(\beta^h)$; $U_a(\beta^h)$; $Q_s^*(n, \beta^h)$; and $Q_i^*(n, \beta^a, \beta^r)$. The straightforward but lengthy calculations needed to derive these functional forms as explicit functions of the indicated components of the base parameter vector β are provided in Tesfatsion and Battula [25, Sec. 4].

Appendix D. Household Type Details

As explained in Section VII-B, our household model is characterized by a base parameter vector $\beta = (\beta^a, \beta^h, \beta^r)$. The component vectors β^a, β^h and β^r characterize the household's 'Appliance', 'House', and 'Resident' attributes, depicted in Fig. 2. A complete listing of the base parameters included in each of these three component vectors is given in Table XII.

This appendix section provides a further breakdown of the component vectors β^a and β^h, important for the construction of Household Types. Specifically, in accordance with Fig. 2, the 'Appliance' component β^a is partitioned in Table XIV into base parameters characterizing the household's smart HVAC system and conventional appliances. The 'House' component β^h is partitioned in Table XV into base parameters characterizing the household's 'Location', 'Size', 'Thermal Integrity', and 'Interior-Exterior'.

TABLE XIV
Appliance Attributes: Parameter Vector β^a

HVAC	**Conventional Appliances**
AuxHeatCapacity	base_power
Cooling_COP	current_fraction
cooling_system_type	current_pf
CoolSupplyAirTemp	$\mathrm{fI}_{\mathrm{eu}}$
DCT	impedance_fraction
DesignHeatCapacity	impedance_pf
DesignHeatSetPoint	power_fraction
DIG	power_pf
DuctPressureDrop	
FanDesignPower	
HeatSupplyAirTemp	
LatCoolFrac	
OSF	

TABLE XV
House Attributes: Parameter Vector β^h

Location	**Size**	**Thermal Integrity**	**Interior-Exterior**
CDT	h	f_{ac}	ECR
DPS	n_s	f_i	EFR
HDT	x	f_s	EWR
V_{nominal}	y	glass_layer	glazing_treatment
		glass_type	h_s
		I	IWR
		R_c	m_f
		R_d	n_d
		R_f	WET
		R_w	WWR
		WF	

A few explanatory remarks regarding the 'Location' base parameters (CDT, DPS, HDT) in Table XV might be helpful. The GridLAB-D (GLD) default values set in [24] for a household's Cooling Design Temperature (CDT) and Heating Design Temperature (HDT) denote highest and lowest temperatures recorded at a particular geographical location. The GLD default value set in [24] for a household's Design Peak Solar Radiation (DPS) is calculated as incident solar radiation on a typical clear day assuming equal window areas in each of eight cardinal directions. Thus, each of these 'Location' base parameters depends on location and is specific to a location.

Appendix E. GridLAB-D Test Case Implementation

Each household populating the 123-bus distribution system for our test cases is formulated using our household model and implemented in part by means of a GridLAB-D (GLD) House Object [28]. Key implementation details are highlighted below.

HVAC Settings:

The GLD House Object parameter *thermostat_control* is set to NONE to enable external control of the household's HVAC system. The GLD House Object parameter *cooling_system_type* is set to ELECTRIC to model the household's HVAC system as an electric system running in cooling mode. The GLD House Object parameter *fan_type* is set to ONE_SPEED to ensure that the household's ON HVAC system includes the operation of a one-speed fan for maintaining air circulation.

At the start-time n^s for each control-step $n \geq 0$, the GLD House Object communicates the current value for system_mode to the household's HVAC controller. If $n > 0$, this value gives the ON/OFF status of the household's HVAC system during the previous control-step n-1. If n = 0, this value gives the user-set ON/OFF status of the household's HVAC system at the simulation start-time t_0.

In addition, at the start-time n^s for each control-step $n \geq 0$, the GLD house object communicates current values for $T_o^*(n)$, $Q_s^*(n)$, $Q_i^*(n)$, RH$^*(n)$, and $\mathrm{V}^*_{\mathsf{actual}}(n)$ to the household's HVAC controller. See [25] for further discussion of the relative humidity forcing term RH$^*(n)$ (decimal %) and the simulated-actual voltage forcing term $\mathrm{V}^*_{\mathsf{actual}}(n)$ (volts) that enter into the determination of HVACPow$^*(n)$ in (31).

Given these operating conditions at the start-time n^s for any control-step $n \geq 0$, the household's HVAC controller (operating within a Five-Step TES Design) determines a power control action $u^*(n)$ that leaves or switches the HVAC system ON (or OFF) during control-step n if $u^*(n)$ = 1 (or $u^*(n)$ = 0). The HVAC controller then resets the GLD House Object parameter system_mode as follows to ensure the household's HVAC system runs during n in accordance with this power control action: system_mode = COOL (if $u^*(n)$ = 1) or OFF (if $u^*(n)$ = 0).

Floor and Aspect Ratio Settings:

Settings for the width, length, and story-number base parameters x, y, and n_s appearing in the 'House Type' base parameter component β^h in Table XII determine derived settings for two parameters A and R for the GLD House

Object, as follows: The house floor area A is set to *floor_area* = $x \cdot y \cdot n_s$, and the house aspect rate R is set to *aspect_ratio* = y/x.

External Forcing Terms

The data used in our test cases to calculate the discretized heat flow rate $Q_s^*(n)$ from solar radiation during each control-step n are as follows: direct normal irradiance; diffuse horizontal irradiance; day of the year; time of the day; latitude; and longitude. The data used in our test cases to calculate the discretized outside air temperature $T_o^*(n)$ during each control-step n are dry bulb temperature data.

These weather-related data are read from GLD, which in turn obtains these data from '.tmy3' files available at the NREL weather data site [30]. All weather-related data at this NREL site are hourly data. For our test case purposes we used quadratic interpolation to obtain sub-hourly weather data values.

GLD uses a Climate Object to pass climate related data to objects such as the GLD House Object. We used the GLD Climate Object to pass NREL weather data (in '.tmy3' file form) to the GLD House Object. The base parameters CDT and HDT in the 'House Type' base parameter component β^h for each household were then assigned values from these weather data.

In GLD, a house (object) is connected to a distribution system using a Meter Object. The value set by the user for the parameter *nominal_voltage* in a house's Meter Object determines the value assigned to the base parameter V_{nominal} appearing in the 'House Type' base parameter component β^h for this house.

The hourly load profiles used in our test cases to construct the base power for each household's conventional (non-price-responsive) appliances can be accessed at [31]. We "skewed" these conventional load profiles to ensure their diversity across different households.[15]

Modeling of Conventional Loads

We configured the base parameter settings for each Structural Quality Type of household to ensure that the ratio of their average HVAC load to their average conventional load was empirically reasonable for hot summer weather.

For the calibration of these ratios, we assumed that each household's Resident Type was identical, with a marginal utility of money given by μ = 1 utils/cent. We also assumed a flat-rate retail price equal to 10 cents/kWh. Finally, the location of each household was assumed to be Des Moines, Iowa, and Des Moines weather data for 02 July 2003 was used for all households.

Given these specifications, we adjusted the Structural Quality Type base parameter settings to achieve the following ratios. For a household having a Low Structure Quality Type, the ratio was calibrated to 0.42. For a household having a Medium Structure Quality Type, the ratio was calibrated to 0.37. For a household having a High Structure Quality Type, the ratio was calibrated to 0.41.

[15]Specifically, we randomly assigned an integer value between -1000 and +1000 to the GLD House Object parameter *schedule_skew* for each household. The input '.glm' files giving these assigned integer values are available at our test case code/data repository site [3].

Additional GLD House Object Details

The parameter f_{ac} appearing in the 'House Type' base parameter component β^h in Table XII is in fact hard-wired to 0 in the GLD House Object code. A user interested in setting a different value for this parameter has to modify the GLD House Object code. For all of our test cases, we used the GLD hard-wired setting $f_{\mathsf{ac}} = 0$.

GLD Implementation of the Five-Step TES design

The staggered implementation of the five steps constituting the Five-Step TES Design is illustrated in Fig. 4 for the special case in which the time-step for each action rate in each step is commonly set to Δt. The assumption of a common time-step Δt = 300s is maintained for all of our test cases. To implement this common time-step, the GLD time-step is set to Δt = 300s.

The action for Step $j+1$ is triggered by data received from the previous Step j, for $j = 1, \ldots 5$, where "Step 6" is identified with Step 1 in the subsequent five-step iteration. Step 1 repeats at each time t_0, $t_0+\Delta t$, $t_0+2\Delta t$, ..., regardless of incoming data, where t_0 is the start-time for the simulation.

At times the GLD House Object sends data to a household's HVAC controller more frequently than $1/\Delta t$, where Δt = 300s is the GLD time-step. We have therefore coded each household's HVAC controller so that it triggers a bid-refreshing action at most once every GLD time-step.

Finally, the time-delay ϵ_j between Step j and Step $j+1$ is commonly set to 1s for $j = 1, 2, 4, 5$. The time-delay ϵ_3 between Step 3 and Step 4 is set to 0.

Appendix F. Test Case Settings for Maintained Base Parameters and Treatment Factors

Tables XVI through XX show the correlated groupings of base parameter values (e.g., 'Poor,' 'Normal,' 'Good') that we used as treatment factors for our test cases. Unless otherwise indicated in Appendix E, all remaining elements of the base parameter vector β for each of our test case households are set and maintained at their GLD House Object default values.

TABLE XVI
Maintained Base Parameter Settings for Household Conventional Appliances

Parameter	Lights	Clotheswasher Refigerator Freezer Microwave	Dryer	Range
current_fraction	0.00	0.00	0.10	0.00
current_pf	0.00	0.97	0.90	0.00
$\mathrm{fI}_{\mathsf{eu}}$	0.80	0.80	0.80	0.80
impedance_fraction	1.00	0.00	0.80	1.00
impedance_pf	1.00	0.97	1.00	1.00
power_fraction	0.00	1.00	0.10	0.00
power_pf	0.00	0.97	0.90	0.00

TABLE XVII
Base Parameter Treatments for HVAC

Parameter	Poor	Normal	Good
Cooling_COP	3.5	3.9	4.1
OSF	0.0	0.1	0.2

TABLE XVIII
BASE PARAMETER TREATMENTS FOR HOUSE SIZE

Parameter	Small	Normal	Large
n_s	1	1	2
x	24	30	28
y	36	45	42

TABLE XIX
BASE PARAMETER TREATMENTS FOR HOUSE THERMAL INTEGRITY

Parameter	Poor	Normal	Good
glass_layer	TWO	TWO	THREE
glass_type	GLASS	GLASS	LOW_E_GLASS
I	1.5	1.0	0.5
R_c	19	30	48
R_d	3	3	11
R_f	4	19	30
R_w	11	11	22
WF	ALUMINUM	THERMAL_BREAK	INSULATED

TABLE XX
BASE PARAMETER TREATMENTS FOR HOUSE INTERIOR-EXTERIOR

Parameter	Poor	Normal	Good
glazing_treatment	REFL	REFL	HIGH_S
m_f	4.5	4	3.5
n_d	1	2	4
WET	1.0	0.6	0.6