ELASTIC WAVE SCATTERING FROM A ROUGH STRIP-LIKE CRACK

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INTRODUCTION

Ultrasonic nondestructive testing is a widely used method for searching for defects, e.g. cracks, in the nuclear power industry. Even though the method can be considered as well-established the theoretical understanding is far from complete, especially when more complicated situations are considered. Consequently, it is advantageous to have access to a good mathematical model of the testing procedure. Such a model can, for instance, be used to perform parametric studies, to develop testing procedures, for qualification purposes, and for education. Furthermore, systematic use of a well tested and validated simulation program will most likely result in a better physical understanding of the process. It should also be emphasized that experimental work is a very expensive alternative to mathematical modeling.

Several more or less refined models have been developed during the last decades. Some models utilize high frequency approximations like GTD [1,2] or the Kirchhoff approximation [3-5]. An alternative strategy involves the solution of the elastodynamic wave equation by some kind of volume discretization technique, e.g. the finite element method [6-9] or EFIT (elastodynamic finite integration technique) [10-11]. These methods have the indisputable advantage that complicated geometries can be handled as well as inhomogeneous or anisotropic materials. On the other hand, the number of nodes easily becomes excessively large, which means that three-dimensional problems in most cases lead to unreasonable execution times and memory requirements. Another possibility is to use the boundary element method [12] which leads to a reduction in the dimensionality of the problem.

The present work is part of a larger project where a complete model of the entire ultrasonic testing procedure is developed [13-14]. The model includes a transmitter and a receiver, scattering from various types of defects, and calibration. The scattering problems are solved by surface integral methods, which means that no volume discretization is necessary. This leads to a reduction of computer execution times and memory requirements. However, the method is restricted to geometrically simple defects in homogeneous components. Recently, scattering from a smooth strip-like crack has been incorporated into the model [15]. The crack may be located close to a free surface or, in a limiting case, be surface-breaking.

Most studies of elastic wave scattering from defects have been concerned with smooth surfaces. However, all cracks are more or less rough, and it may be expected that roughness will cause the energy to be scattered in a more diffuse manner thereby decreasing the detectability of the defect. Elastic wave scattering from rough cracks has previously been treated by approximate methods like the Kirchhoff approximation [16-17].

In this work the solution for scattering from a smooth strip-like crack [15] is extended to include the effects of roughness by a perturbation approach. It is assumed that the RMS height of the irregularities is small compared to the wavelength. Furthermore, it is assumed that the small slope approximation is valid, i.e. the RMS height is small compared to the

Review of Progress in Quantitative Nondestructive Evaluation, Vol 17
Edited by D.O. Thompson and D.E. Chimenti, Plenum Press, New York, 1998

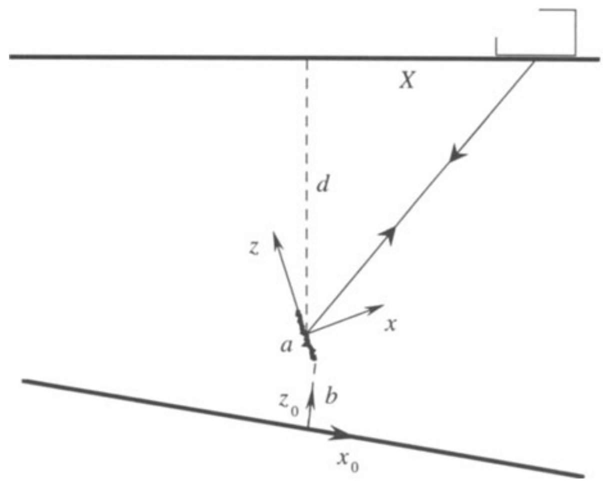

Figure 1. Geometry of the rough strip-like crack and the probe.

correlation length of the rough surface. The solution to the scattering problem is combined with a model for a conventional contact probe acting in pulse-echo mode.

STATEMENT OF THE PROBLEM

We consider a rough strip-like crack of width a and infinite length according to Fig. 1. The material is assumed to be linearly elastic, homogeneous, and isotropic with Lamé constants λ and μ, and density ρ. We assume time-harmonic conditions with an angular frequency ω, and the factor $\exp(-i\omega t)$ is suppressed throughout. The wavenumbers for longitudinal and transverse waves are denoted by k_p and k_s, respectively.

The equation for the crack surface is

$$x = \varepsilon a \eta(y, z) \tag{1}$$

where ε is a small number and $\eta(y, z)$ is a function to be defined in the next section. The center of the crack is located at a distance d below a free surface (the scanning surface). A conventional ultrasonic contact probe acting in pulse-echo mode is assumed to scan along a line which is perpendicular to the y-axis. The crack may either be an interior crack or be located close to a free surface as shown. The distance from the lower edge of the crack to the free back surface, when it is present, is denoted by b. The back surface may be tilted with respect to the scanning surface. Later we shall assume that the distance d from the crack to the scanning surface is very large compared to the wavelength and to the width of the crack.

It should be noted that even though the crack has an infinite extension in the y-direction, the problem is not two-dimensional, since the field transmitted by the probe is fully three-dimensional. Furthermore, the properties of the rough surface are dependent on the y-coordinate as indicated by Eq. (1).

MODELING OF THE ROUGH SURFACE

Rough surfaces can in principle be divided into two categories: randomly rough surfaces and deterministic surfaces, e.g. periodic surfaces. Randomly rough surfaces, which can be modeled by numerical simulation [17-18], have the advantage of being realistic. However, the lack of an analytical expression tends to complicate matters. The computational effort may be considerable unless other simplifications like the Kirchhoff approximation are utilized. Deterministic surfaces, on the other hand, are analytically tractable but suffer from the lack of resemblance to real surfaces. In this paper we try to utilize the advantages of both approaches. The rough surface is modeled by a superposition of deterministic surfaces that

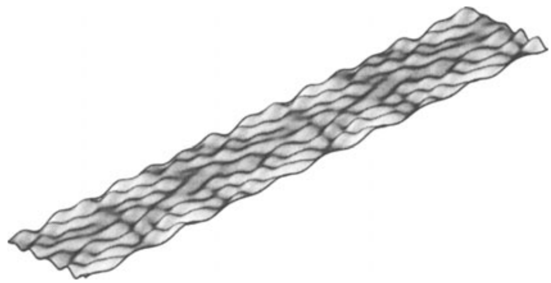

Figure 2 Example of a simulation of a rough strip-like crack according to Eq. (2).

are randomly translated with respect to each other. As a result we obtain a surface with a very simple mathematical expression. Still the surface is sufficiently irregular to serve as a model for a real rough surface.

The function $\eta(y, z)$ in Eq. (1) is chosen as

$$\eta(y, z) = \sum_{\mu,\nu} w_{\mu\nu} \sin (k_{y\mu}y + \varphi_{\mu\nu}) \sin (k_{z\nu}z + \theta_{\mu\nu}). \tag{2}$$

Here $\varphi_{\mu\nu}$ and $\theta_{\mu\nu}$ are random numbers such that $0 \leq \varphi_{mn}, \theta_{mn} \leq 2\pi$. The quantities $w_{\mu\nu}$ are weights that are normalized so that $\Sigma w_{\mu\nu}^2 = 1$. It is easy to estimate the RMS height of the irregularities, which is $\sigma = \varepsilon a/2$. In the numerical examples given below the weights have been chosen equal, but other choices are equally possible. For instance the weights can also be taken as random numbers.

The parameters $\lambda_{y\mu} = 2\pi/k_{y\mu}$ and $\lambda_{z\nu} = 2\pi/k_{z\nu}$ can be regarded as "wavelenghts" for the sinusoidal surfaces that are superposed. By choosing different ranges for $\lambda_{y\mu}$ and $\lambda_{z\nu}$ it is also possible to simulate a surface with anisotropic roughness. The correlation lengths in the y- and z-directions can be estimated by plotting the correlation functions, e.g.,

$$C(y, y') = \frac{1}{\sigma^2} \langle x(y, z)\, x(y', z) \rangle = \sum_{\mu} \cos\, ((y - y')/\lambda_{y\mu}). \tag{3}$$

The correlation length can then be estimated as the value of $|y - y'|$ where the value of the correlation function has dropped to 1/e. If the "corrugation lengths" $\lambda_{y\mu}$ and $\lambda_{z\nu}$ are chosen equally spaced it turns out that the corresponding correlation lengths are approximately equal to the arithmetic means of the two sets of corrugation lengths, which may be used as a rule of thumb. It is important that the small parameter ε is chosen so that the RMS-value σ is small compared with the correlation lengths, since the perturbation approach that is used cannot be justified otherwise.

A portion of a rough strip-like surface as defined by Eq. (2) is shown in Fig. 2. The surface is made up by superposition of 9 sinusoidal, doubly corrugated surfaces. The width of the crack is $a = 5$ mm, and the RMS-height is 0.1 mm. The correlation lengths are approximately 3 mm along the crack and 1 mm across the crack.

THE SCATTERING PROBLEM

To solve the scattering problem we use the same approach as Bövik and Boström [15] who solved the corresponding problem for a smooth crack. The first step is to solve the scattering problem for a strip-like crack in a halfspace for an arbitrary incident field. In order to do this we derive an integral equation for the crack opening displacement (COD). In the next section we specialize to the field transmitted by an ultrasonic probe. It should be noted

that multiple scattering between the crack and the scanning surface is neglected in this approach.
The starting point is an integral representation for the total displacement field $u_n(r)$:

$$u_n^{\mathrm{i}}(r) + \frac{k_{\mathrm{s}}}{\mu} \int_{-\infty}^{\infty} \mathrm{d}y' \int_{-a/2}^{a/2} \mathrm{d}z' \, \Delta u_{n'}(r') \, (t'(\mathrm{G}(r',r)))_{n'n} = u_n(r), \qquad z_0 > 0 \tag{4}$$

where u_n^{i} is the incident field, Δu_n is the COD, $\mathrm{G}(r', r)$ is the halfspace Green tensor, and t' is the traction operator with respect to the position vector r'. The coordinates (x, y, z) and (x_0, y_0, z_0) are defined in Fig. 1. The traction $t'(\mathrm{G}(r',r))$ is evaluated for $x' = \varepsilon a\eta(y', z')$.

Operating with t and taking the limit $x \to \varepsilon a\eta(y, z)$ we obtain

$$\lim_{x\to\varepsilon a\eta} \frac{k_{\mathrm{s}}}{\mu} \int_{-\infty}^{\infty} \mathrm{d}y' \int_{-a/2}^{a/2} \mathrm{d}z' \, \Delta u_{n'}(r') \, (\Sigma(r',r))_{n'n} = -t_n(u^{\mathrm{i}})|_{x=\varepsilon a\eta} \tag{5}$$

which is the integral equation for the COD. Here we have assumed that the traction vanishes at the surface of the crack, but the solution can easily be extended to include the more general spring boundary conditions.

To solve the integral equation we first expand the Green tensor in plane waves:

$$\mathrm{G}_{n'n}(r, r') = 2\mathrm{i} \sum_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p\mathrm{d}q}{k_j h_j} f_{jn'} f_{jn}^{*} \exp\,[\mathrm{i}(h_j\,|x' - x| + p(y' - y) + q(z' - z)]$$

$$+ 2\mathrm{i} \sum_{jj'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p\mathrm{d}q}{k_j h_j} g_{jn'} R_{jj'} g_{j'n}^{\dagger} \exp\,[\mathrm{i}(q(x_0' - x_0) + p(y_0' - y_0) + q(z_0' - z_0)]. \tag{6}$$

The first term is simply the free space Green tensor, while the second term represents the influence of the back surface. The wavenumbers are numbered so that $k_1 = k_2 = k_{\mathrm{s}}$, and $k_3 = k_{\mathrm{p}}$. Furthermore, $h_j = (k_j^2 - p^2 - q^2)^{1/2}$ with the branch chosen so that $\mathrm{Im}\, h_j \geq 0$. The subscripts n and n' run through 1, 2, 3 and j and j' are summed over 1, 2, 3. The components of the reflection matrix $R_{jj'}$ as well as f_{jn} and g_{jn} are all elementary functions of p and q with the explicit expressions given in Ref. [15]. It is now straightforward to determine the components of the traction Green tensor $\Sigma_{n'n}$.

The next step is to expand the COD in a power series in the small parameter ε:

$$\Delta u_n = \Delta u_n^0 + \varepsilon\, \Delta u_n^1 + \mathrm{O}(\varepsilon^2). \tag{7}$$

Expanding $\Sigma_{n'n}$ in Taylor series around $x = x' = 0$ and $t_n(u^{\mathrm{i}})$ around $x = 0$, and substituting into Eq. (5) we obtain a sequence of equations, where the lowest order corresponds to the smooth crack. To solve the equations we use an expansion of the COD in Chebyshev functions in the z-coordinate and a Fourier transform in the y-coordinate:

$$\Delta u_n^i = \sum_m \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{k_{\mathrm{s}}} \beta_{n\,m}^{\,i}(p)\, \phi_m(z + a/2)\, \mathrm{e}^{-\mathrm{i}py} \tag{8}$$

where

$$\phi_m(z) = \begin{cases} -\dfrac{1}{m\pi} \cos\,(m \arcsin\,(2z/a - 1)), & m = 1, 3 \ldots \\ \\ \dfrac{i}{m\pi} \sin\,(m \arcsin\,(2z/a - 1)), & m = 2, 4 \ldots \end{cases} \tag{9}$$

It should be noted that the Chebyshev functions have the correct square-root behavior at the crack tips. Inserting the expansion for the COD into the integral equations, projecting on the Chebyshev functions and taking a Fourier transform with respect to y we arrive at a set of simultaneous equations with the coefficients $\beta^i_{nm}(p)$ as unknowns:

$$\sum_{nm} Q_{n'm'nm}(p)\, \beta^i_{nm}(p) = T^i_{nm}(p), \qquad i = 0, 1\ldots \tag{10}$$

Here $Q_{n'm'nm}$ is obtained as an integral which can be evaluated numerically for any given value of p. The explicit expression is given in Ref. [15]. The matrices T^i_{nm} depend on the incident field. This means that in principle the coefficients $\beta^i_{nm}(p)$ can be determined from Eq. (10) for any given incoming field. In other words, the COD is known, and the scattered field can be calculated from the integral representation. However, the simultaneous equations (10) have to be solved for every value of p, which makes this approach less useful. We shall avoid this difficulty later by assuming that the distance between the crack and the scanning surface is very large. Using a stationary phase approximation we shall see that only a limited number of p-values will contribute.

THE INCOMING FIELD

We assume that the incoming field is generated by a contact probe acting in pulse-echo mode. The probe model that is used was developed by Boström and Wirdelius [14]. The probe is modeled by the traction vector on the contact surface beneath the probe. The probe can be of any type (P, SV, SH) and of any angle. Both rectangular and elliptic contact areas are included in the model.

The direct radiated field from the probe is expressed in terms of plane waves. Since the direct field does not satisfy the boundary condition on the back surface the reflected field from the back surface has to be added. The details are given in Ref. [15], and will not be repeated here. The only additional complication is that the traction $t_n(u^i)$ on the crack surface has to be expanded in a power series in ε as was mentioned previously.

THE SIGNAL RESPONSE FROM THE RECEIVING PROBE

The electrical signal response can be modeled using an electromechanical reciprocity argument [19]. For a fixed frequency ω the change in the electrical reflection coefficient can be obtained as an integral over the crack:

$$\delta\Gamma = -\frac{i\omega}{4P} \int_S \Delta u^{(2)}_n\, t^{(1)}_n \,\mathrm{d}S \tag{11}$$

where P is the power. The superscript (1) refers to the field from the receiver without a crack, but with the back surface, while the superscript (2) corresponds to the field from the transmitter with a crack. In this case we need not distinguish between the transmitter and the receiver, of course.

The crack opening displacement is in principle determined from Eqs. (8-10) above and the result can be substituted into Eq. (11). However, it turns out that the result involves multiple integrals which are more or less awkward from a numerical point of view. To avoid this difficulty we assume that the distance d in Fig. 1 is very large compared to other distances of interest, i.e. the crack width and the wavelength. It follows that several integrals can be calculated asymptotically using the stationary phase approximation. In fact, no multiple integrals have to be calculated numerically. Furthermore, we only need to solve Eq. (10) for a discrete number of p-values corresponding to the stationary points. For the smooth crack only $p = 0$ contributes, while the stationary points for the rough crack are given by

$$p_s = \pm \frac{k_{y\mu} k_j}{k_j + k_{j'}}. \tag{12}$$

The reader is referred to Ref. [15] for a detailed discussion of the accuracy of the stationary phase approximation.

NUMERICAL RESULTS

Finally, a few numerical results will be presented. The results given here are for a 45° SV probe with a frequency of 2 MHz. The probe is rectangular with sides 5 × 5 mm. The crack has a width of 5 mm, and the plane of the crack is normal to the scanning surface. The RMS height is 0.5 mm. All results are uncalibrated.

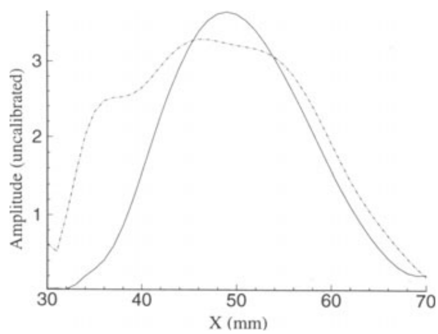

Figure 3. The uncalibrated response from a smooth crack (solid line) and from the crack depicted in Fig. 2 (dashdotted line) as functions of the position of the probe in the absence of a back surface.

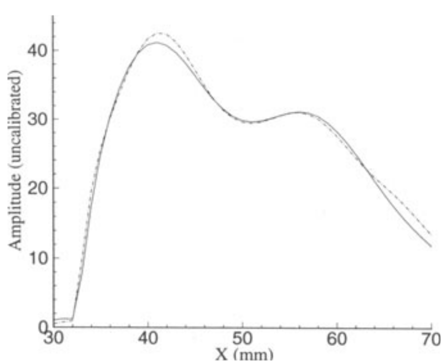

Figure 4. The uncalibrated response from a smooth crack (solid line) and from the crack depicted in Fig. 2 (dashdotted line) in the vicinity of a free back surface as functions of the position of the probe.

In Fig. 3 the response from the crack in Fig. 2 is shown as a function of the position of the probe (*X* in Fig. 1). The effect of a back surface is not taken into account. In Fig. 4 the response from the same crack in the vicinity of a back surface is shown. The distance *b* as defined in Fig. 1 is 2 mm, and the back surface is taken to be parallel to the scanning surface. Not surprisingly the effects of roughness are much smaller in this case, since the back surface has a dominant influence on the scattering process.

ACKNOWLEDGEMENTS

This work is supported by the Swedish Nuclear Power Inspectorate (SKI), which is gratefully acknowledged.

REFERENCES

1. J. M. Coffey and R. K. Chapman, Nucl. Energy 22, 319 (1983).
2. R. K. Chapman, J. Nondestr. Eval. 9, 197 (1990).
3. P. Calmon, O. Roy, and F. Champigny, in *Proc. 13th International Conference on NDE in the Nuclear and Pressure Vessel Industries*, eds.K. Iida, G. M. Light, and M. J. Whittle (ASM International, Materials Park, 1995).
4. A. Minachi, F. J. Margetan, R. B. Thompson, and M. S. Greenwood, in *Review of Progress in QNDE*, Vol. 16, eds.D. O. Thompson and D. E. Chimenti (Plenum, New York, 1997).
5. J. A. Ogilvy, Ultrasonics 27, 69 (1989).
6. W. Lord, R. Ludwig, and Z. You, J. Nondestr. Eval. 9, 129 (1990).
7. K. Harumi and M. Uchida, J. Nondestr. Eval. 9, 81 (1990).
8. A. Minachi, J. Mould, and R. B. Thompson, J. Nondestr. Eval. 12, 151 (1993).
9. M. Castaings and P. Cawley, J. Acoust. Soc. Am. 100, 3070 (1996).
10. V. Schmitz, K. J. Langenberg, and M. Krönig, in *Non-destructive examination – practice and results*, eds. E. Borloo and P. Lemaitre (EC, Luxembourg, 1994), p. 361.
11. P. Fellinger, R. Marklein, K. J. Langenberg, and S. Klaholz, Wave Motion 21, 47 (1995).
12. P. J. Schafbuch, R. B. Thompson, and F. J. Rizzo, J. Nondestr. Eval. 9, 113 (1990).
13. A. Boström, SKI Report 95:53 (Stockholm, 1995).
14. A. Boström and H. Wirdelius, J. Acoust. Soc. Am. 97, 2836 (1995).
15. P. Bövik and A. Boström, J. Acoust. Soc. Am. (in press).
16. R. A. Roberts , R. J. Brind, and J. D. Achenbach, J. Nondestr. Eval. 2, 153 (1981).
17. J. A. Ogilvy and I. D. Culverwell, Ultrasonics 29, 490 (1991).
18. J. A. Ogilvy, J. Phys. D: Appl. Phys. 21, 260 (1988).
19. B. A. Auld, Wave Motion 1, 3 (1979).