

Using the Multivariate Spatio-Temporal Bayesian Model to Analyze Traffic Crashes by Severity

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Abstract

Unobserved heterogeneity across space, time, and crash type is often non-negligible in crash frequency modeling. When multiple crash types with spatial and temporal features are analyzed, multivariate spatio-temporal models should be considered. For this study, we analyzed the yearly county-level fatal, major injury, and minor injury crashes in Iowa from 2006 to 2015 using a multivariate spatio-temporal Bayesian model. The model adopted a multivariate spatial structure, a multivariate temporal structure, and a multivariate spatio-temporal interaction structure to account for possible correlations across injury severities over space, time, and spatio-temporal interaction, respectively. Income and weather indicators were found to have no significant effects on crash frequencies in the presence of vehicle miles traveled and unemployment rate. Both spatial and temporal effects were found to be important, and they played nearly the same roles for all three crash types in the studied dataset. Counties located in north and southwest Iowa were found to tend to have fewer crashes than the remaining counties. All three crash types generally showed descending trends from 2006 to 2015. They also had significantly positive correlations between each other in space but not in time. The crude crash rates and predicted crash rates were generally consistent for major injury and minor injury crashes but not for low-count fatal crashes. High-risk counties were identified using the posterior expected rank by the predicted crash cost rate, which was more able to truly represent the underlying traffic safety status than the rank by the crude crash cost rate.

Keywords: multivariate spatio-temporal, Bayesian, crash frequency, posterior expected rank

1 Introduction

Traffic crashes have been one of the major sources of fatalities and injuries in the United States. Crash frequency analysis is often used to identify key factors influencing the propensity of crashes, which is important for policymakers as they propose interventions to prevent road traffic crashes. However, unobserved heterogeneity is often an issue in crash frequency modeling, because many crash-related elements are often unavailable. Neglecting unobserved heterogeneity may produce biased and inefficient results (Mannering et al., 2016).

Unobserved heterogeneity may come from many sources. Crashes are usually classified into multiple types by different criteria, and their underlying correlations may produce some unobserved heterogeneity across observations when they are analyzed simultaneously (Mannering and Bhat, 2014; Mannering et al., 2016). Thus, multivariate models, such as the multivariate Poisson log-normal (MVPLN) model, are often adopted (Ma et al., 2008; El-Basyouny and Sayed, 2009; Aguero-Valverde and Jovanis, 2010; El-Basyouny et al., 2014; Zhao et al., 2017). In addition, crash frequency data are always aggregated over space and time, which may also produce unobserved heterogeneity, as crashes that occur close in space or time are very likely to share some unobserved characteristics (Lord et al., 2005; Lord and Mannering, 2010; Savolainen et al., 2011; Mannering and Bhat, 2014; Mannering et al., 2016). Previous studies have shown that spatial correlations of traffic crashes may exist across states/provinces (Erdogan, 2009; Truong et al., 2016), counties (Aguero-Valverde and Jovanis, 2006; Song et al., 2006; Eckley and Curtin, 2013), census tracts (Wang and Kockelman, 2013), traffic analysis zones (Matkan and Mohaymany, 2013), intersections (Ahmed and Abdel-Aty, 2015; Liu et al., 2015) and segments (Aguero-Valverde and Jovanis, 2008; Wang et al., 2009, 2011; Aguero-Valverde, 2011; Jiang et al., 2014; Zeng and Huang, 2014). The similarity of economy, culture, land use, weather, traffic laws, and driving behavior within a given region may explain the spatial correlations in traffic crashes. When multiple crash types with spatial correlations need to be analyzed, multivariate spatial models have been proved to be more powerful than univariate spatial models, as multivariate spatial models can account for correlations across crash types in space in addition to spatial correlations (Miaou and Song, 2005; Song et al., 2006; Aguero-Valverde, 2013; Wang and Kockelman, 2013; Aguero-Valverde et al., 2016; Barua et al., 2016). Temporal correlations of traffic crashes may exist across year (Wang and Abdel-Aty, 2006; Brijs et al., 2008; Andrey, 2010; Wang et al., 2011; Yannis et al., 2011; Matkan and Mohaymany, 2013; El-Basyouny et al., 2014), month (Quddus, 2008a; Hu et al., 2013), week (Kilamanua et al., 2011; Sukhai et al., 2011; Liu et al., 2015), and day (Brijs et al., 2008). Temporal correlations occur because many traffic-related factors, such as driver behavior, economy, weather, environment, law, and travel demand, often exhibit some temporal features. Similarly, when multiple crash types with temporal correlations need to be analyzed, multivariate temporal models should be considered, as they can account for correlations across crash types in time in addition to temporal correlations (Serhiyenko et al., 2014; Michalaki et al., 2016).

Crashes often have both spatial and temporal features. When only one crash type is analyzed, the univariate spatio-temporal modeling has been proved in some studies to be superior (Miaou et al., 2003; Aguero-Valverde and Jovanis, 2006; Truong et al., 2016; Liu and Sharma, 2017). When multiple crash types need to be analyzed, a multivariate spatio-temporal model may be needed. Ma et al. (2017) used the bivariate spatio-temporal model to analyze the daily non-injury and injury crash rates on 100 roadway segments of I70 in one year at the micro level, and Boulieri et al. (2017) used the bivariate spatio-temporal model to analyze the yearly low severity and high severity accidents of 7932 electoral wards in England from 2005-2013 considering only vehicle miles traveled (VMT). Both studies showed the superiority of the bivariate spatio-temporal model to the univariate spatio-temporal model in terms of goodness of fit.

In this study, we used the multivariate spatio-temporal Bayesian model to analyze the yearly county-level fatal, major injury, and minor injury crash frequencies in Iowa. The goal of this study was to accurately identify the long-term effects of economy and weather on crash frequency in Iowa and to explore the spatial and temporal correlations of crashes. Additionally, the counties were ranked to identify high-risk areas for safety improvement programs, as funding available for safety improvements are often limited and proper ranking can significantly influence the appropriate distribution of safety funding toward areas with more critical needs. Raw crash data-based ranking is easy to use but crude and inefficient (Miaou and Song, 2005). In Bayesian cases, one statistical ranking method is the posterior expected rank (PER), i.e. the posterior mean of the rank by ranking indicators (Miaou and Song, 2005). When rankings are the main interest, the PER method is recommended (Shen and Louis, 1998). The most common ranking indicator is crash rate, but crash rate considering crash cost by injury severity, called the “crash cost rate” in the following analysis, is strongly recommended when injury severity and associated costs are the main concerns (Miaou and Song, 2005). Thus, the PER of the crash cost rate would be used to rank the studied areas based on the predicted results of the multivariate spatio-temporal Bayesian model in this study.

2 Data Description

Traffic crash data from Iowa’s 99 counties from 2006 to 2015 were obtained from the Iowa Department of Transportation. Crashes were divided into five categories by severity: fatal, major injury, minor injury, possible injury/unknown, and property damage only. Fatal crashes, major injury crashes, and minor injury crashes were analyzed in this study, as these three types of crashes often lead to significant economic loss and casualties. VMT data for each county in each year from 2006 to 2015 were downloaded from the website of the Iowa Department of Transportation (2016). In addition, unemployment rate data were downloaded from the website of Iowa Community Indicators Program (2016), and per capita personal income data were downloaded from the website of the U.S. Bureau of Economic Analysis (2016) of the U.S. Department of Commerce. Meanwhile, weather data regarding rainfall, snowfall, and the number of days with minimum temperature exceeding 32°F (TH32) were downloaded from the website of the Iowa Environmental Mesonet (2017). These weather data are collected based on the daily climate observations from the National Weather Service’s Cooperative Observer Program. A summary of the variables is given in Table 1. All three crash types have over-dispersion, as their variances are much larger than their means. Additionally, the highest correlation among the covariates was -0.338 (between snowfall and TH32). Thus, no explanatory variables showed strong positive or negative correlations.

Table 1 Descriptive statistics of collected variables

Variables	Min.	Median	Mean	Max.	Std. Error
Fatal crash	0	2	3.383	35	3.818
Major injury crash	0	8	13.680	245	22.042
Minor injury crash	1	23	49.060	894	93.742
VMT (1,000,000 miles)	0.047	0.186	0.320	4.215	0.487

Unemployment rate (%)	2.000	4.600	4.846	10.200	1.347
Income (\$10,000)	2.247	3.877	3.877	6.464	0.666
Rainfall (inch)	17.850	38.610	38.390	64.990	8.570
Snowfall (inch)	0	35	34.560	85.100	14.377
TH32 (days)	174	221	222.600	272	15.733

Note: VMT, vehicle miles traveled; TH32, number of days with minimum temperature exceeding 32°F.

The Pearson correlation coefficients of fatal, major injury, and minor injury crashes were shown in Table 2. All three crash types were highly positively correlated. That is, locations where many fatal/major injury/minor injury crashes were observed likely also had many crashes of the other two types.

Table 2 Pearson correlation coefficients of crash counts

Pearson correlation coefficient	Fatal crash	Major injury crash
Major injury crash	0.837	
Minor injury crash	0.835	0.971

The county-level yearly average fatal, major injury, and minor injury crash counts in Iowa are shown in Figure 1. A cluster of high fatal crash frequencies can be observed in the central counties around the dark red-shaded area, where the largest city in Iowa, Des Moines, is located. A cluster of low crash frequencies can be observed in the northern and southwestern regions of Iowa (lightly shaded areas). A cluster of comparatively higher numbers of major injury crashes can also be observed in the central counties. However, no obvious clustering trends can be observed for minor injury crashes. Next, spatial correlations of crashes are examined statistically.

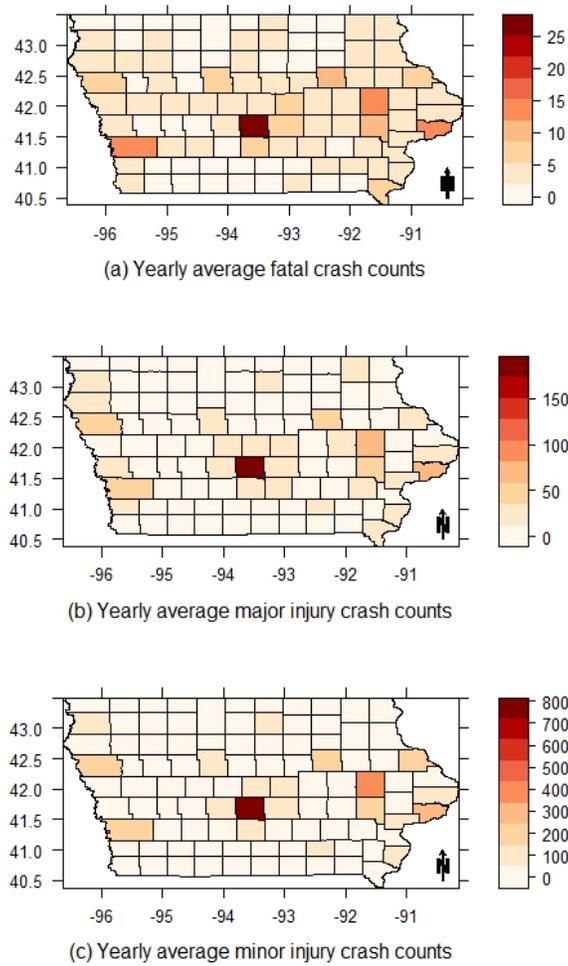


Figure 1 County-level yearly average fatal, major injury, and minor injury crash counts (2006–2015)

Moran's I statistic is commonly used to test spatial correlations in traffic crash analyses (Quddus, 2008b; Guo et al., 2010; Xie et al., 2014; Zeng and Huang, 2014). The global Moran's I is defined as (Anselin, 1988):

$$I = \frac{n \sum_i \sum_j \omega_{ij} (y_i - \bar{y})(y_j - \bar{y})}{\sum_{i \neq j} \omega_{ij} \sum_i (y_i - \bar{y})^2} \quad (1)$$

where n is the total number of observations; y_i and y_j are the values of observation i and observation j , respectively; \bar{y} is the average value of observations; and ω_{ij} is the spatial weight between observations i and j .

Negative Moran's I values indicate negative spatial autocorrelation, positive Moran's I values indicate positive spatial autocorrelation, and zero indicates no spatial autocorrelation. The z -score of Moran's I shows if the spatial autocorrelation is significant.

The global Moran's I statistics of crashes in each year from 2006 to 2015 were calculated using the "spdep" package (Bivand and Piras, 2015) in the R platform (R Core Team, 2016) with queen contiguity spatial weights, where counties with a shared border or vertex were considered neighbors. When areas were neighbors, the spatial weights were 1; otherwise, they were 0. The results are shown in Table 3.

Table 3 Global Moran's I statistics of crash counts in each year

Year	Fatal crash		Major injury crash		Minor injury crash	
	Standardized Moran's I	p -value	Standardized Moran's I	P -value	Standardized Moran's I	p -value
2006	1.986	0.024*	1.752	0.041*	0.971	0.166
2007	2.091	0.018*	1.555	0.060	1.141	0.127
2008	1.520	0.064	0.688	0.246	0.871	0.192
2009	1.661	0.048*	1.181	0.119	0.764	0.222
2010	2.486	0.006*	1.586	0.056	1.106	0.134
2011	1.919	0.027*	1.883	0.031*	1.101	0.136
2012	1.240	0.108	2.017	0.022*	1.108	0.134
2013	2.387	0.009*	2.218	0.013*	1.555	0.060
2014	1.241	0.107	1.877	0.030*	1.252	0.105
2015	2.300	0.011*	2.770	0.003*	1.468	0.071

Note: * significant at $p = 0.05$.

Fatal crash and major injury crash counts showed significant spatial autocorrelations in seven and six out of 10 years, respectively, at a 95% confidence level, but minor injury crash counts did not show any significant spatial autocorrelations at a 95% confidence level in any year. Additionally, the p -values of fatal crash and major injury crash counts were much smaller than those for minor injury crashes. Thus, fatal crashes and major injury crashes were highly likely to be spatially correlated compared to minor injury crashes. These trends may be site specific. As an example, Aguero-Valverde and Jovanis (2006) found injury crash frequencies to have a significant spatial correlation and fatal crash frequencies to not be significantly correlated in counties of Pennsylvania. Although minor injury crash frequencies did not show significant spatial autocorrelations, it does not mean the absence of spatial autocorrelation for minor injury crashes; they may still have weak spatial correlations. The different strengths of spatial autocorrelations imply that the three crash types may have different spatial model parameters.

Temporal correlation was not directly tested, as there were only 10 time points for each crash type. However, as shown in Figure 2 by the yearly state-level counts of all three crash types from 2006 to 2015, they all generally exhibited descending trends, with some dipping and heaving and different descending rates.

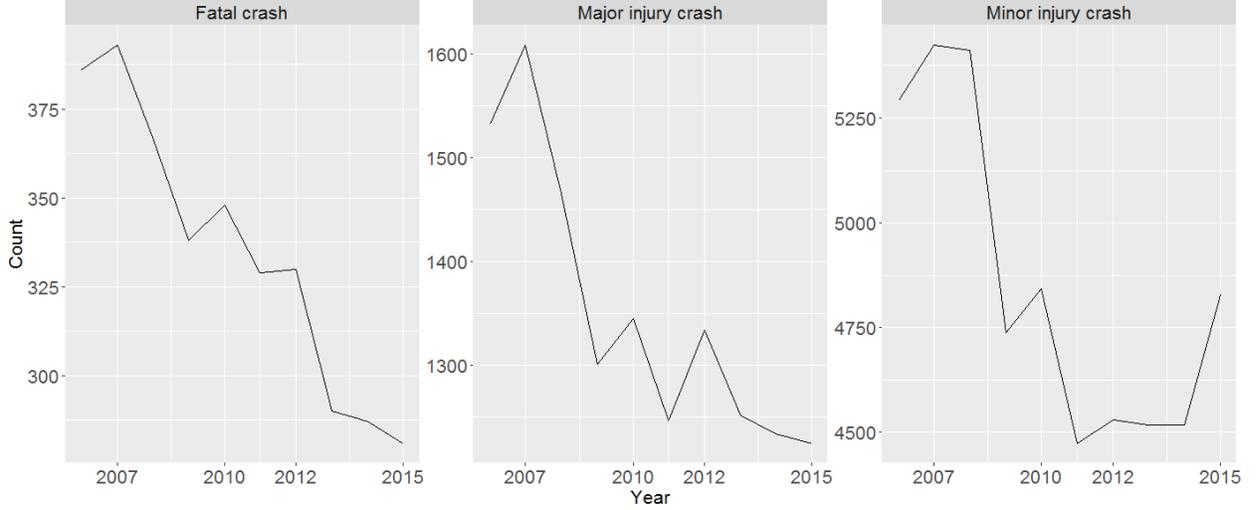


Figure 2 Iowa state-level yearly crash counts (2006-2015)

3 Methodology

3.1 Statistical Framework

The statistical framework used a Bayesian hierarchical architecture, including both spatial and temporal as well as spatio-temporal interaction components. The statistical model is presented in equations (2) and (3) (Ma et al., 2017):

$$y_{stk} \sim \text{Poisson}(\lambda_{stk}) \quad (2)$$

$$\log(\lambda_{stk}) = \alpha_k + X_{st}^T * \beta_k + u_{sk} + v_{sk} + \phi_{tk} + \theta_{tk} + \eta_{stk} \quad (3)$$

where s is the space number, i.e. county number in this case, 1, 2, ..., 99; t is the time point, i.e. year number in this case, 1 (2006), 2 (2007), ..., 10 (2015); k is the crash injury severity number, 1 (fatal crash), 2 (major injury crash), 3 (minor injury crash); y_{stk} is the crash count of injury severity k of space s in time t ; λ_{stk} is the mean crash frequency of injury severity k of space s in time t ; α_k is the intercept term of crash type k ; $\beta_k = (\beta_{k1}, \beta_{k2}, \dots, \beta_{km})$ is the m -dimensional regression coefficient vector of crash type k and m is the number of covariates; $X_{st} = (X_{st1}, X_{st2}, \dots, X_{stm})$ is the m -dimensional covariate vector of space s in time t ; u_{sk} is the structured spatial random effect of crash type k in space s ; v_{sk} is the unstructured spatial random effect of crash type k in space s ; θ_{tk} is the structured temporal random effect of crash type k in time t ; ϕ_{tk} is the unstructured temporal random effect of crash type k in time t ; and η_{stk} is the spatio-temporal interaction effect of crash type k in space s and time t .

The spatial component of each observation consisted of two parts: $u_{sk} + v_{sk}$, and the temporal component also consisted of two parts: $\phi_{tk} + \theta_{tk}$.

3.1.1 Spatial Component

3.1.1.1 Univariate Spatial Model

The spatial component of each observation, $u_{sk} + v_{sk}$, was assumed to follow the Besag-York-Mollie (BYM) model (Besag et al., 1991). The BYM model has been proved to be powerful in traffic crash analysis (Aguero-Valverde and Jovanis, 2006; Wang et al., 2013; Xie et al., 2014; Boulieri et al., 2017; Ma et al., 2017). For the BYM model, the structured spatial effect, u_{sk} , is modeled using an intrinsic conditional autoregressive (ICAR) structure, and the unstructured spatial effect, v_{sk} , follows a normal distribution.

$$u_{sk} | u_{(i \neq s)k} \sim N\left(\frac{\sum_{i \in N(s)} u_{ik}}{\#N(s)}, \frac{\sigma_v^2{}^k}{\#N(s)}\right) \quad (4)$$

$$v_{sk} \sim N(0, \sigma_v^2{}^k) \quad (5)$$

where $N(s)$ is the neighbors of space s ; $\#N(s)$ is the number of neighbors of space s , and $\sigma_v^2{}^k$ and $\sigma_v^2{}^k$ are two independent variances of crash injury severity k in space.

Two counties adjacent to each other were considered to be neighbors; otherwise, they were not neighbors. The ICAR part accounted for unobserved heterogeneity produced by possible spatial correlations between counties, and the unstructured part was responsible for county-specific heterogeneity. In the univariate BYM model, both the structured and unstructured spatial effects across crash injury severities were assumed to be independent for each observation.

3.1.1.2 Multivariate Spatial Model

The multivariate BYM (MBYM) model, shown in equations (6) and (7), is the extension of the BYM model in multivariate cases (Boulieri et al., 2017; Ma et al., 2017):

$$u_s | u_{(i \neq s)}. \sim N\left(\frac{\sum_{i \in N(s)} u_i}{\#N(s)}, \frac{\Sigma_v}{\#N(s)}\right) \quad (6)$$

$$v_s \sim N(0, \Sigma_v) \quad (7)$$

where $u_s = (u_{s1}, \dots, u_{sk})$ is the k -dimensional structured spatial random effects of space s ; $v_s = (v_{s1}, \dots, v_{sk})$ is the k -dimensional unstructured spatial random effects of space s ; $N(s)$ is the neighbors of space s ; $\#N(s)$ is the number of neighbors of space s ; and Σ_u and Σ_v are the two independent $k * k$ variance–covariance matrices in space.

The MBYM model consisted of a multivariate ICAR component and a multivariate normal (MVN) component. Different from the univariate BYM model, both the structured and unstructured spatial random effects of each observation are correlated across crash injury severities. Thus, they could account for possible unobserved heterogeneity across crash injury severities in space for each observation.

3.1.2 Temporal Component

3.1.2.1 Univariate Temporal Model

The structured temporal effect of each observation, φ_{tk} , was modeled with the 1st order random walk (RW1) structure. The unstructured temporal effect of each observation, θ_{tk} , followed a

normal distribution. This temporal component was still called the RW1 model in the following analysis, although it actually consisted of a RW1 model and an error term.

$$\varphi_{tk}|\varphi_{(-t)k} \sim \begin{cases} N\left(\varphi_{(t+1)k}, \sigma_{\varphi}^2{}^k\right) & \text{for } t = 1 \\ N\left(\frac{\varphi_{(t-1)k} + \varphi_{(t+1)k}}{2}, \frac{\sigma_{\varphi}^2{}^k}{2}\right) & \text{for } t = 2, 3, \dots, 9 \\ N\left(\varphi_{(t-1)k}, \sigma_{\varphi}^2{}^k\right) & \text{for } t = 10 \end{cases} \quad (8)$$

$$\theta_{tk} \sim N(0, \sigma_{\theta}^2{}^k), \quad (9)$$

where $\sigma_{\varphi}^2{}^k$ and $\sigma_{\theta}^2{}^k$ are two independent variances of crash injury severity k in time.

It was easy to find that the RW1 model was a special case of applying the ICAR model shown in equation (4) in time. In the univariate RW1 model, both the structured and unstructured temporal effects across crash injury severities were assumed to be independent for each observation.

3.1.2.2 Multivariate Temporal Model

The multivariate RW1 (MRW1) model is the extension of the RW1 model to multivariate cases (Boulieri et al., 2017; Ma et al., 2017) and is defined as:

$$\varphi_{t.}|\varphi_{(-t).} \sim \begin{cases} N(\varphi_{(t+1).}, \Sigma_{\varphi}) & \text{for } t = 1 \\ N\left(\frac{\varphi_{(t-1).} + \varphi_{(t+1).}}{2}, \frac{\Sigma_{\varphi}}{2}\right) & \text{for } t = 2, 3, \dots, 9 \\ N(\varphi_{(t-1).}, \Sigma_{\varphi}) & \text{for } t = 10 \end{cases} \quad (10)$$

$$\theta_{t.} \sim N(0, \Sigma_{\theta}) \quad (11)$$

where $\varphi_{t.} = (\varphi_{t1}, \dots, \varphi_{tk})$ is the k -dimensional structured temporal random effects of time t ; $\theta_{t.} = (\theta_{t1}, \dots, \theta_{tk})$ is the k -dimensional unstructured temporal random effects of time t , and Σ_{φ} and Σ_{θ} are the two independent $k * k$ variance–covariance matrices in time.

The MRW1 model consists of a multivariate RW1 component and a MVN component. Different from the univariate RW1 model, both the structured and unstructured temporal random effects of each observation were also correlated across crash injury severities. Thus, they could account for possible unobserved heterogeneity across crash injury severities in time for each observation.

3.1.3 Spatio-Temporal Component

The spatio-temporal interaction effect of each observation across crash injury severities, $\eta_{(st).}$, was used to account for unobserved heterogeneity not explained by other components. It was assumed to follow a zero-mean multivariate normal distribution.

$$\eta_{(st).} \sim N(0, \Sigma_{\eta}) \quad (12)$$

where $\eta_{(st).} = (\eta_{st1}, \dots, \eta_{stk})$ is the k -dimensional spatio-temporal interaction effect of each observation and Σ_{η} is a $k * k$ variance–covariance matrix.

The structure of Σ_η could account for the remaining possible correlations across crash injury severities for each observation.

To select an appropriate model, there were four models built in this study, as shown in Table 4.

Table 4 Summary of models developed in this study

No	Model	Spatial component	Temporal component	Spatio-temporal component
1	$S_{BYM}T_{RW1}$	BYM	RW1	MVN
2	$S_{BYM}T_{MRW1}$	BYM	MRW1	MVN
3	$S_{MBYM}T_{RW1}$	MBYM	RW1	MVN
4	$S_{MBYM}T_{MRW1}$	MBYM	MRW1	MVN

Note: BYM, Besag-York-Mollie; MBYM, multivariate BYM; RW1, 1st order random walk; MRW1, multivariate RW1; MVN, multivariate normal.

3.2 Priors Settings

All four models were built within the Bayesian hierarchical structure. The priors of parameters were set as:

$$\alpha_k \sim \text{Uniform}(-\infty, +\infty) \quad (13)$$

$$\beta_{.j} \sim N(0, \Sigma_\beta) \quad (14)$$

$$\sigma_v^2{}^k, \sigma_v^2{}^k, \sigma_\varphi^2{}^k, \sigma_\theta^2{}^k \stackrel{iid}{\sim} \text{Inverse} - \text{Gamma}(1, 0.0005) \quad (15)$$

$$\Sigma_\beta, \Sigma_v, \Sigma_v, \Sigma_\varphi, \Sigma_\theta, \Sigma_\eta \stackrel{iid}{\sim} \text{Inverse} - \text{Wishart}(I_k, k) \quad (16)$$

where $k(= 1, 2, 3)$ is the crash injury severity number, 1 for fatal, 2 for major injury, and 3 for minor injury; $j(= 1, 2, 3, 4, 5, 6)$ is the j th covariate number; $\beta_{.j} = (\beta_{1j}, \beta_{2j}, \beta_{3j})^T$ is the regression coefficient vector of the j th covariate across crash injury severities; Σ_β is the variance-covariance matrix of the regression coefficients of covariates across crash injury severities; $\sigma_v^2{}^k$, $\sigma_v^2{}^k$, $\sigma_\varphi^2{}^k$, and $\sigma_\theta^2{}^k$ are independent variances of the structured spatial effects, unstructured spatial effects, structured temporal effects, and unstructured temporal effects of crash injury severity k in univariate models, respectively; $\Sigma_v, \Sigma_v, \Sigma_\varphi$, and Σ_θ are independent variance-covariance matrices of the structured spatial effects, unstructured spatial effects, structured temporal effects, unstructured temporal effects in multivariate models, respectively; Σ_η is the spatio-temporal interaction effects; and I_k is the k -dimension identity matrix.

The regression coefficients (β) were given a multivariate normal prior with means being zero to accommodate their possible correlations across crash injury severities. A flat prior was set for intercept terms (α) to ensure identifiability of the model (MRC Biostatistics Unit, 2004). All the variances were set to have a minimally informative prior of *Inverse-Gamma*(1,0.0005) (Blangiardo et al., 2013), which also had been proved to be effective in our former study (Liu and

Sharma, 2017). All the variance–covariance matrices were assigned an inverse-Wishart prior with the scale matrix being an identity matrix and a degree of freedom of k to provide weak information.

3.3 Initial Values Settings

Markov chain Monte Carlo (MCMC) simulation was used to get the posterior distributions of parameters and desired latent variables for this study. To start MCMC simulations, initial values have to be given for each unknown parameter and latent variable. Good initial values help MCMC simulation converge quickly to the true distributions of parameters, whereas bad initial values may make MCMC simulation converge slowly and even become stuck at some data points. OpenBUGS, a popular Bayesian analysis software using MCMC simulation, was used in this study (Lunn et al., 2009). When initial values are not given, OpenBUGS randomly generates initial values, which usually works after long MCMC iterations. However, that was not the case in this study, as some variances and variance–covariance matrices of some chains were often stuck at some points using the randomly generated initial values of OpenBUGS. Thus, the posterior distributions of parameters did not converge well. Finally, we ran the MCMC simulation twice. The results of the first MCMC simulation were used as the initial values for the second MCMC simulation, which converged very well. Based on the first MCMC simulation result, initial values of the second MCMC simulation were set as:

$$\begin{aligned} \sigma^2_v = (\sigma^2_{v^1}, \sigma^2_{v^2}, \sigma^2_{v^3}) &= \left(\frac{1}{5}, \frac{1}{1500}, \frac{1}{1500}\right), & \sigma^2_\nu = (\sigma^2_{\nu^1}, \sigma^2_{\nu^2}, \sigma^2_{\nu^3}) &= \left(\frac{1}{1800}, \frac{1}{5}, \frac{1}{600}\right), & \sigma^2_\varphi = \\ (\sigma^2_{\varphi^1}, \sigma^2_{\varphi^2}, \sigma^2_{\varphi^3}) &= \left(\frac{1}{2000}, \frac{1}{2000}, \frac{1}{2000}\right), & \sigma^2_\theta = (\sigma^2_{\theta^1}, \sigma^2_{\theta^2}, \sigma^2_{\theta^3}) &= \left(\frac{1}{2000}, \frac{1}{2000}, \frac{1}{2000}\right), & \Sigma_v^{-1} = \\ \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, & \Sigma_\nu^{-1} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, & \Sigma_\varphi^{-1} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}, & \text{and } \Sigma_\theta^{-1} = \begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}. \end{aligned}$$

3.4 Model Checking and Comparison

Deviance information criteria (DIC) is a generalized version of Akaike information criterion (AIC) for evaluating hierarchical models (Spiegelhalter et al., 2002). The deviance is defined as $D(\theta) = -2\log(p(y|\theta))$, where y is the data, θ is the unknown parameters, and $p(y|\theta)$ is the likelihood function. DIC is defined as (Spiegelhalter et al., 2002):

$$DIC = D(\bar{\theta}) + 2pD = \bar{D} + pD \quad (17)$$

where $\bar{\theta}$ is the posterior mean of the parameters; $D(\bar{\theta})$ is the deviance at the posterior mean of the parameters, a measure of data fit; pD is the effective number of the model, a measure of complexity computed as the difference between \bar{D} and $D(\bar{\theta})$; and \bar{D} is the mean of the sampled deviances from MCMC simulations.

Bayesian models with smaller DIC values are desired. Roughly, differences of more than 10 might definitely rule out the model with the higher DIC, differences between 5 and 10 are substantial, and differences less than 5 might mean that the models are not significantly different (MRC Biostatistics Unit, 2004).

Although DIC can be used for model comparison, it cannot evaluate the quality of fit of a model with observed data. Posterior predictive density is often used for checking the assumptions of a

model and its goodness of fit. Assume there is a test statistic $D(y, \theta)$, which is a summary function. If the model is correct, one can use the posterior predictive distribution to generate replicated values y^{rep} , which are expected to be close to the observed data y^{obs} . The test statistic is used to check the assumption under investigation and measure discrepancies between the data and the model (Gelman et al., 1996). Based on the posterior predictive distribution, the posterior predictive p -value is defined as (Meng, 1994)

$$\text{Posterior } p - \text{value} = P(D(y^{rep}, \theta) > D(y^{obs}, \theta) | y^{obs}) \quad (18)$$

P -values around 0.5 indicate that the distributions of the replicated and observed data are close, whereas values close to zero or one indicate differences between them (Gelman et al., 1996). In this study, the mean values of crash frequencies were used as the test statistic, as the mean is the major parameter for a Poisson model.

3.5 Random Effects Analysis

3.5.1 Spatial Fraction Analysis

For spatial analysis, one point of interest is to identify the contribution of the structured spatial effects, σ_v^2 , over the total marginal spatial variability, $\sigma_v^2 + \sigma_\nu^2$ (Boulieri et al., 2017). The spatial fraction of interest is given by

$$\text{frac}_v = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\nu^2} \quad (19)$$

When the spatial fraction is close to 1, the structured spatial effects explain most of the variability of the model in space. Otherwise, the unstructured spatial effects play the main role.

3.5.2 Temporal Fraction Analysis

Similarly, the temporal fraction is defined as the variance of structured temporal effects, σ_ϕ^2 , over the total marginal temporal variability, $\sigma_\phi^2 + \sigma_\theta^2$:

$$\text{frac}_\phi = \frac{\sigma_\phi^2}{\sigma_\phi^2 + \sigma_\theta^2} \quad (20)$$

When the temporal fraction is close to 1, the structured temporal effects explain most of the variability of the model in time. Otherwise, the unstructured temporal effects play the main role.

3.5.3 Spatial and Temporal Effects Comparison

When both spatial and temporal effects exist, it is also of interest to determine their relative importance. The relative importance of spatial effects is defined as the variance of spatial effects over the marginal variability of spatial and temporal effects:

$$\text{frac}_{\frac{s}{s+t}} = \frac{\sigma_v^2 + \sigma_\nu^2}{\sigma_v^2 + \sigma_\nu^2 + \sigma_\phi^2 + \sigma_\theta^2} \quad (21)$$

3.6 PER by Total Crash Cost Rate

In the “safety improvement candidate location” methods of Iowa (Pawlovich, 2007), the costs of fatal, major injury, and minor injury crashes were set as 200, 100, and 10 units, respectively. They were adopted to calculate the total crash cost rate as shown in equation (22), where crash rate was

the crash count per million VMT. The PER using the predicted total crash cost rate as well as the crude rank using the crude total crash cost rates would be computed and compared, respectively. The county ranked as 1st had the largest total crash cost rate.

$$\text{Total crash cost rate} = \text{Fatal crash rate} * 200 + \text{Major injury crash rate} * 100 + \text{Minor injury crash rate} * 10 \quad (22)$$

4 Results

All four models were implemented in OpenBUGS in R (R Core Team, 2016) through “R2OpenBUGS” (Sturtz et al., 2005). OpenBUGS uses the Metropolis-Hastings algorithm to sample data. Three simulation chains were run with 50,000 iterations for each chain, the first 25,000 samples were discarded as burn-in and the remaining 25,000 samples were retained to get the posterior distributions of parameters with a thinning interval of 5. Thus, 5,000 samples were recorded per chain. On an Intel(R) Xeon(R) CPU at 3.70 GHz with 16 GB random access memory, it took about 3.5 hours to run each model. The trace plots of estimated parameters showed that posterior samples converged well after the burn-in iterations. In addition, the Gelman and Rubin’s convergence diagnostic, i.e. potential scale reduction factors of variables, was also calculated to check the convergence of multiple chains (Gelman and Rubin, 1992). The DIC values for the four models listed in Table 4 are shown in Table 5.

Table 5 DIC values of four models

No	Model	DIC
1	$S_{BYM}T_{RW1}$	14,330
2	$S_{BYM}T_{MRW1}$	8,519
3	$S_{MBYM}T_{RW1}$	10,970
4	$S_{MBYM}T_{MRW1}$	8,371

Compared to the $S_{BYM}T_{RW1}$ model, the DIC values of both the $S_{MBYM}T_{RW1}$ and the $S_{BYM}T_{MRW1}$ models were much smaller. In addition, the DIC value of the $S_{MBYM}T_{MRW1}$ model was much smaller than that for the $S_{MBYM}T_{RW1}$ and the $S_{BYM}T_{MRW1}$ models. This implied that unobserved heterogeneity across crash injury severities existed in both space and time, thus the $S_{MBYM}T_{MRW1}$ model was preferred for this study. In addition, the posterior p -values of the mean values of fatal, major injury, and minor injury crashes were 0.500, 0.497, and 0.495, respectively, close to 0.5, which meant that the $S_{MBYM}T_{MRW1}$ model matched the data well. Mean and 95% credible interval (CI) values of estimated parameters of the $S_{MBYM}T_{MRW1}$ model are shown in Table 6.

Table 6 Estimated parameters of the $S_{MBYM}T_{MRW1}$ model with all covariates

Variables	Fatal crash		Major injury crash		Minor injury crash	
	Mean	95% CI	Mean	95% CI	Mean	95% CI
Intercept	0.168	(-1.591, 1.630)	2.185*	(1.212, 3.095)	3.028*	(2.505, 3.611)
Income	0.081	(-0.054, 0.228)	-0.036	(-0.119, 0.053)	0.018	(-0.052, 0.085)
Unemployment rate	0.037	(-0.029, 0.102)	-0.055*	(-0.103, -0.009)	-0.062*	(-0.092, -0.033)
Rainfall	-0.005	(-0.013, 0.003)	0.001	(-0.004, 0.006)	0.002	(-0.002, 0.005)

Snowfall	-0.002	(-0.006, 0.003)	-0.001	(-0.004, 0.002)	0.000	(-0.002, 0.001)
TH32	0.001	(-0.005, 0.008)	0.000	(-0.004, 0.004)	0.000	(-0.002, 0.002)
VMT	0.732*	(0.538, 0.903)	0.970*	(0.746, 1.160)	1.132*	(0.893, 1.301)
σ^2_v	0.305	(0.103, 0.644)	0.412	(0.116, 0.903)	0.491	(0.136, 1.088)
σ^2_v	0.124	(0.067, 0.203)	0.188	(0.100, 0.299)	0.231	(0.124, 0.362)
$frac_v$	0.681	(0.383, 0.890)	0.651	(0.314, 0.889)	0.643	(0.305, 0.887)
σ^2_ϕ	0.261	(0.081, 0.759)	0.253	(0.078, 0.708)	0.246	(0.078, 0.698)
σ^2_θ	0.229	(0.074, 0.634)	0.214	(0.071, 0.576)	0.218	(0.072, 0.589)
$frac_\phi$	0.527	(0.219, 0.816)	0.533	(0.228, 0.820)	0.525	(0.226, 0.809)
$frac \frac{s}{s+t}$	0.487	(0.235, 0.718)	0.573	(0.314, 0.792)	0.618	(0.368, 0.813)
σ^2_η	0.047	(0.031, 0.067)	0.029	(0.022, 0.037)	0.018	(0.014, 0.022)

Note: CI, credible interval; TH32, number of days with minimum temperature exceeding 32°F; VMT, vehicle miles traveled; σ^2_v , σ^2_v , σ^2_ϕ , σ^2_θ , and σ^2_η are variances; $frac_v$ is the spatial fraction; $frac_\phi$ is the temporal fraction; $frac \frac{s}{s+t}$ is the relative importance of spatial effects;

*covariates significant at the 95% credible interval.

4.1 Regression Coefficients Analysis

The intercept term was insignificant for fatal crashes but was significant for major injury and minor injury crashes. As expected, VMT showed significant positive effects for all three crash types. In addition, both intercept and VMT coefficients increased as crash injury severity decreased, which was consistent with the magnitude of crash counts.

Income was statistically insignificant for all three crash types, although income had generally increased for counties in Iowa from 2006 to 2015. Unemployment rate did not have significant effects on fatal crash counts but did have significantly negative effects on major injury and minor injury crash counts; that is, the number of major and minor injury crashes decreased as the unemployment rate increased. The unemployment rate has been thought to have mixed effects on traffic crash frequencies (Wagenaar, 1983; Leigh and Waldon, 1991). On one hand, high unemployment is associated with more mental stress in the population, related to both job loss and fear of job loss, which could lead to more aggressive driving patterns and more traffic crashes (Wagenaar, 1983). On the other hand, high unemployment also brings with it less driving and thus fewer traffic crashes (Wagenaar, 1983; Leigh and Waldon, 1991). The latter seemed to predominate in Iowa, which was consistent with what was found in Michigan, where unemployment had negative effects on crash counts (Wagenaar, 1983).

Rainfall, snowfall, and TH32 did not show significant effects on any crash type. Although these weather indicators had great variability within the time span studied, they were not related to traffic safety problems in the long term. Adverse weather, such as snowstorms and flooding, may result in more crashes in the short term but may also reduce people's travel in the following time, leading to lower crash numbers. The two opposite effects seemed to offset each other.

It should be noted that all regression coefficients were assumed to be fixed for this study as shown in equation (3). That is, the effects of covariates on crash frequencies were thought to be homogeneous over space and time. However, these effects might be heterogeneous in practice in the presence of spatial instability and temporal instability, where fixed parameters models might

produce biased coefficient estimates and incorrect inferences (Mannering et al., 2016; Mannering, 2018). For example, snowfall might affect crash frequencies differently in rural areas and urban areas due to different travel demands and travel modes. Thus, spatio-temporal-varying parameter models might be considered to get more accurate results in future studies.

Because most covariates were found to be insignificant, the $S_{MBYMT_{MRW1}}$ model was re-run with significant variables, and the results were shown in Table 7. The posterior p -values of the mean values of fatal, major injury, and minor injury crash counts for the new model were 0.493, 0.495, and 0.500, respectively, which meant it fitted the data well. Mean and 95% CI values of estimated parameters were found to be generally consistent with those shown in Table 6.

Table 7 Estimated parameters of the $S_{MBYMT_{MRW1}}$ model with significant covariates

Variables	Fatal crash		Major injury crash		Minor injury crash	
	Mean	95% CI	Mean	95% CI	Mean	95% CI
Intercept	0.685*	(0.395, 0.983)	2.073*	(1.726, 2.430)	3.153*	(2.850, 3.468)
Unemployment rate	–	–	–0.051*	(–0.095, –0.006)	–0.060*	(–0.092, –0.030)
VMT	0.794*	(0.594, 0.978)	1.039*	(0.798, 1.251)	1.222*	(0.954, 1.452)
σ^2_v	0.281	(0.098, 0.591)	0.366	(0.118, 0.847)	0.431	(0.136, 1.039)
σ^2_v	0.127	(0.070, 0.203)	0.192	(0.105, 0.297)	0.234	(0.130, 0.358)
$frac_v$	0.662	(0.373, 0.878)	0.622	(0.322, 0.878)	0.614	(0.311, 0.878)
σ^2_φ	0.248	(0.079, 0.695)	0.246	(0.078, 0.682)	0.244	(0.076, 0.707)
σ^2_θ	0.213	(0.072, 0.565)	0.209	(0.071, 0.554)	0.200	(0.070, 0.519)
$frac_\varphi$	0.530	(0.225, 0.817)	0.533	(0.232, 0.818)	0.538	(0.241, 0.818)
$frac \frac{s}{s+t}$	0.487	(0.246, 0.715)	0.563	(0.311, 0.781)	0.609	(0.356, 0.809)
σ^2_η	0.047	(0.031, 0.067)	0.029	(0.022, 0.037)	0.018	(0.014, 0.022)

Note: CI, credible interval; VMT, vehicle miles traveled; σ^2_v , σ^2_v , σ^2_φ , σ^2_θ , and σ^2_η are variances; $frac_v$ is the spatial fraction; $frac_\varphi$ is the temporal fraction; $frac \frac{s}{s+t}$ is the relative importance of spatial effects; *covariates significant at the 95% credible interval.

4.2 Random Effects Analysis

4.2.1 Spatial Random Effects Analysis

For the $S_{MBYMT_{MRW1}}$ model, the spatial fraction values for fatal, major injury, and minor injury crashes were 0.662, 0.622, and 0.614, respectively. This means that, for all three crash types, unobserved heterogeneity in space existed both between counties and within individual counties, and the structured spatial effects played slightly more important roles than did the unstructured spatial effects. Shown in Figure 3 are the exponentials of the posterior means of the structured spatial effects ($\exp(u_{sk})$) for each county for all three crash types; counties with $\exp(u_{sk})$ lower than 1 tended to have fewer crashes and counties with $\exp(u_{sk})$ greater than 1 tended to have more crashes. It is found that the counties located in the north and southwest regions of Iowa tended to have fewer fatal, major injury, and minor injury crashes. For fatal crashes, this finding is generally consistent with the empirically observed fatal crash distribution shown in Figure 1(a). However, for major injury and minor injury crashes, it is not obvious to see these trends in Figure 1(a) and

Figure 1(b). This finding is a good example showing that one main benefit of spatial analysis is the identification of the underlying spatial clustering of crashes.

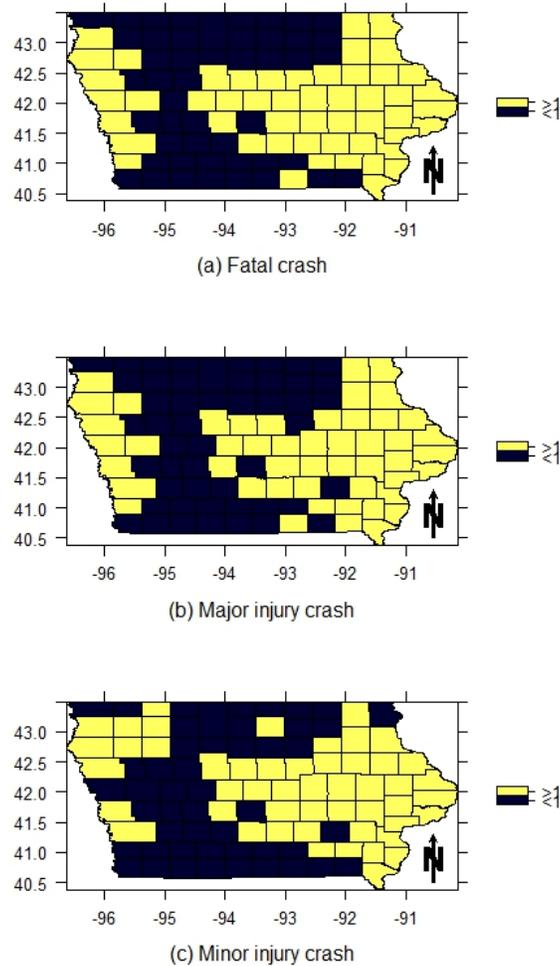


Figure 3 Exponential posterior means of the structured spatial effect ($exp(u_{sk})$) of crashes in Iowa

Moran's I statistics of residuals of the $S_{MBYM}T_{MRWI}$ model were calculated to see if they still had spatial correlations. As shown in Table 8, the residuals of fatal and major injury crashes did not show any significant spatial correlation at a 5% significance level for any year. In addition, the p -values were considerably larger than those shown in Table 3, which meant that unobserved heterogeneity in space was nearly completely covered by the spatial component. The p -values of Moran's I test for the residuals of minor injury crashes also increased considerably in most years, which meant that the weak spatial autocorrelations of minor injury crashes were also eliminated. However, there were some exceptions in 2006, 2007, and 2011 for minor injury crashes, when the raw crash data did not show significant spatial autocorrelations, whereas their residuals showed significant spatial autocorrelations. It is thought that minor injury crashes might have trivial spatial autocorrelation in these three years but did have non-trivial spatial correlations in other years.

However, because the $S_{MBYM}T_{MRW1}$ model assigned fixed spatial random effects to the data for each year, the residuals would also have spatial effects as the complement in these three years. This needs further investigation to determine the true reason. This finding implies the importance of checking the necessity of adopting spatial models in crash analysis. We suggest making spatial tests before and after spatial analysis to justify the utilization of spatial models. In general, the spatial component covered nearly all unobserved heterogeneity of crashes in space. The results also generally verified the effectiveness of the spatial model.

Table 8 Moran's I test results for the residuals of the $S_{MBYM}T_{MRW1}$ model

Year	Fatal crash		Major injury crash		Minor injury crash	
	Standardized Moran's I	P -value	Standardized Moran's I	P -value	Standardized Moran's I	P -value
2006	-1.285	0.901	0.003	0.499	1.835	0.033*
2007	-0.195	0.577	-0.944	0.828	3.434	0.000*
2008	-0.912	0.819	-0.130	0.552	1.174	0.120
2009	-0.647	0.741	0.865	0.194	0.096	0.462
2010	0.660	0.255	1.323	0.093	-0.407	0.658
2011	0.099	0.461	0.489	0.313	3.586	0.000*
2012	0.232	0.408	-0.171	0.568	-0.789	0.785
2013	0.495	0.310	-0.021	0.508	-1.019	0.846
2014	-0.430	0.666	0.355	0.361	-0.669	0.748
2015	-1.409	0.921	-0.285	0.612	-1.579	0.943

* Significant at $P = 0.05$.

4.2.2 Temporal Random Effects Analysis

For the $S_{MBYM}T_{MRW1}$ model, the temporal fraction values for fatal, major injury, and minor injury crashes were 0.530, 0.533, and 0.538, respectively. The structured temporal effects and the unstructured temporal effects played nearly the same roles for all three crash types. Thus, unobserved heterogeneity in time existed both between years and in individual years. Shown in Figure 4 are the exponentials of the posterior means of the structured temporal effects ($\exp(\varphi_{tk})$) in each year for all three crash types. All three crash types generally showed descending trends from 2006 to 2015, whereas major injury and minor injury crashes had some fluctuations.

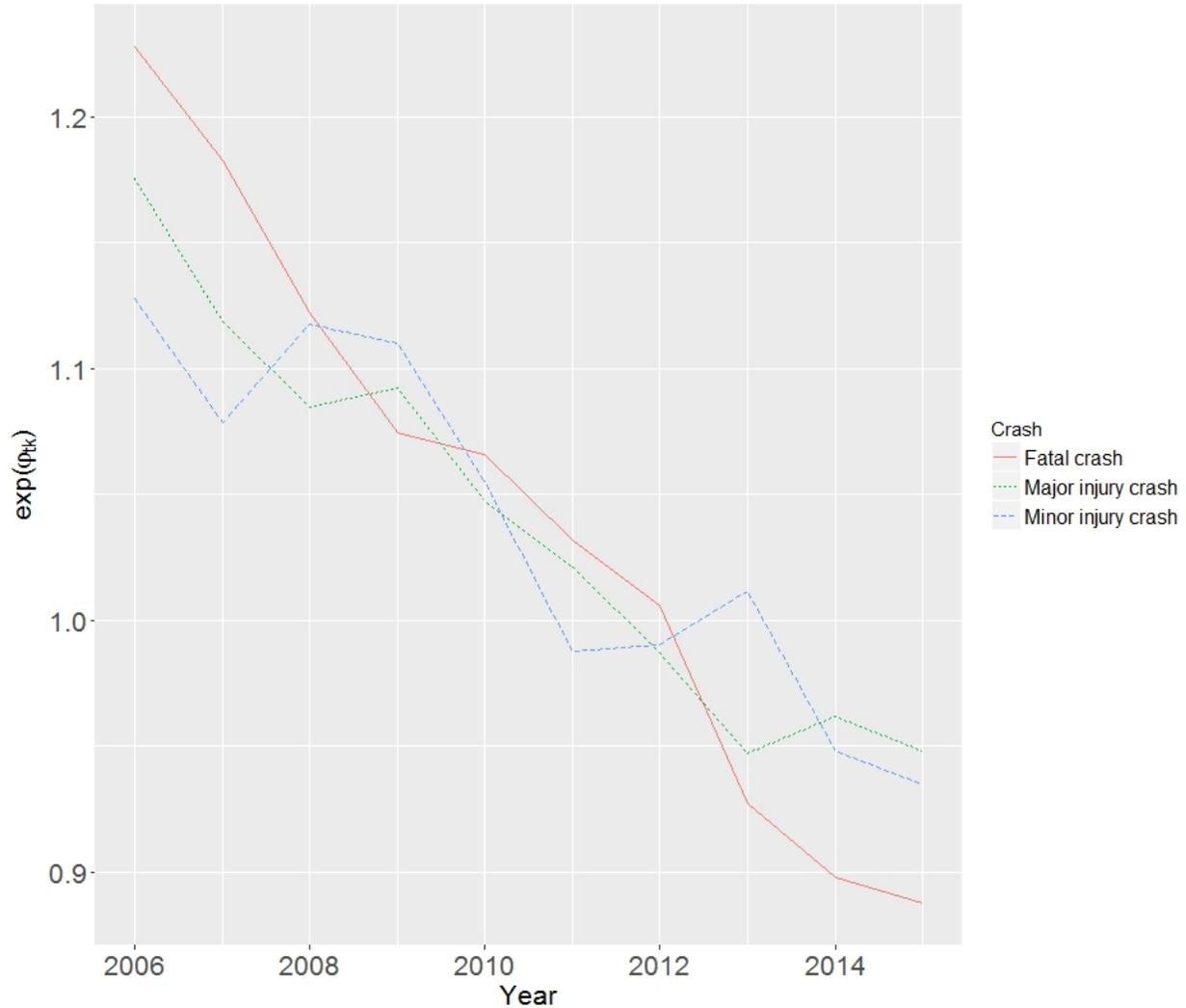


Figure 4 Exponential posterior means of the structured temporal effects ($exp(\varphi_{tk})$) of the $S_{MBYM}T_{MRW1}$ model

4.2.3 Spatial and Temporal Random Effects Comparison

The $frac_s$ values for fatal, major injury, and minor injury crashes were 0.487, 0.563, and 0.609, respectively. This means that temporal effects played slightly more important roles for fatal crashes, whereas spatial effects played slightly more important roles for major injury and minor injury crashes. That is, the relative importance of spatial effects and temporal effects varied slightly by crash injury severity.

4.2.4 Unobserved Heterogeneity across Crash Injury Severities

The estimated variance–covariance matrices for all the random effects and the corresponding 95% credible intervals of the $S_{MBYM}T_{MRW1}$ model are shown in Table 9.

Table 9 Estimated variance-covariance matrices of the $S_{MBYM}T_{MRW1}$ model

Structured spatial effects (v_s)			
Σ_v	Fatal crash	Major injury crash	Minor injury crash
Fatal crash	0.281 (0.098, 0.591)		
Major injury crash	0.235 (0.035, 0.594)	0.366 (0.118, 0.847)	
Minor injury crash	0.253 (0.035, 0.661)	0.322 (0.063, 0.850)	0.431 (0.136, 1.039)
Unstructured spatial effects (v_s)			
Σ_v	Fatal crash	Major injury crash	Minor injury crash
Fatal crash	0.127 (0.070, 0.203)		
Major injury crash	0.110 (0.046, 0.190)	0.192 (0.105, 0.297)	
Minor injury crash	0.121 (0.051, 0.207)	0.173 (0.082, 0.279)	0.234 (0.130, 0.358)
Structured temporal effects (φ_t)			
Σ_φ	Fatal crash	Major injury crash	Minor injury crash
Fatal crash	0.248 (0.079, 0.695)		
Major injury crash	0.001 (-0.223, 0.231)	0.246 (0.078, 0.682)	
Minor injury crash	-0.003 (-0.235, 0.227)	-0.009 (-0.257, 0.208)	0.244 (0.076, 0.707)
Unstructured temporal effects (θ_t)			
Σ_θ	Fatal crash	Major injury crash	Minor injury crash
Fatal crash	0.213 (0.072, 0.565)		
Major injury crash	-0.002 (-0.193, 0.191)	0.209 (0.071, 0.554)	
Minor injury crash	-0.003 (0.182, 0.173)	-0.007 (-0.186, 0.159)	0.200 (0.070, 0.519)
Spatio-temporal interaction effects ($\eta_{(st)}$)			
Σ_η	Fatal crash	Major injury crash	Minor injury crash
Fatal crash	0.047 (0.031, 0.067)		
Major injury crash	0.002 (-0.007, 0.010)	0.029 (0.022, 0.037)	
Minor injury crash	0.000 (-0.005, 0.006)	0.006 (0.002, 0.010)	0.018 (0.014, 0.022)

Note: Values shown are the posterior mean with the 95% credible interval in parentheses; Σ_v , Σ_v , Σ_φ , Σ_θ , Σ_η are variance-covariance matrices of structured spatial effects, unstructured spatial effects, structured temporal effects, unstructured temporal effects, and spatio-temporal interaction effects, respectively.

For the $S_{MBYM}T_{MRW1}$ model, unobserved heterogeneity across crash injury severities had three sources: space, time, and spatio-temporal interaction. All the off-diagonal elements of Σ_v and Σ_v were significantly positive, which meant there were strong positive correlations across crash injury severities for both structured and unstructured spatial effects. That is, with the increase of fatal crash counts in one county, the major injury and minor injury crash counts of this county, and the fatal, major injury, and minor injury crash counts of its neighboring counties were also expected to increase. This proves the necessity of using multivariate spatial models from another viewpoint. However, none of the off-diagonal elements of Σ_φ and Σ_θ were significantly different from zero, which implied that there were no strong correlations across crash injury severities for either structured or unstructured temporal effects. However, the DIC value of the $S_{MBYM}T_{MRW1}$ model

was still much smaller than that for the $S_{MBYM}T_{RW1}$ model (shown in Table 5). This implies that, although crashes may not show strong correlations in time, their correlations may still not be ignored, as weak correlations may still explain some variability in the data. For the spatio-temporal interaction effects, major injury crashes showed significantly positive correlations with minor injury crashes, but fatal crashes did not show significant correlations with the other two crash types.

For each observation, because Σ_v , Σ_ν , Σ_φ , Σ_θ , and Σ_η are independent, the Pearson's correlation coefficients of random effects across crash injury severities can be calculated as follows:

$$\rho_{12} = \frac{\Sigma_{v_{12}} + \Sigma_{\nu_{12}} + \Sigma_{\varphi_{12}} + \Sigma_{\theta_{12}} + \Sigma_{\eta_{12}}}{\sqrt{\sigma^2_{v^1} + \sigma^2_{\nu^1} + \sigma^2_{\varphi^1} + \sigma^2_{\theta^1} + \sigma^2_{\eta^1}} \sqrt{\sigma^2_{v^2} + \sigma^2_{\nu^2} + \sigma^2_{\varphi^2} + \sigma^2_{\theta^2} + \sigma^2_{\eta^2}}} \quad (23)$$

$$\rho_{13} = \frac{\Sigma_{v_{13}} + \Sigma_{\nu_{13}} + \Sigma_{\varphi_{13}} + \Sigma_{\theta_{13}} + \Sigma_{\eta_{13}}}{\sqrt{\sigma^2_{v^1} + \sigma^2_{\nu^1} + \sigma^2_{\varphi^1} + \sigma^2_{\theta^1} + \sigma^2_{\eta^1}} \sqrt{\sigma^2_{v^3} + \sigma^2_{\nu^3} + \sigma^2_{\varphi^3} + \sigma^2_{\theta^3} + \sigma^2_{\eta^3}}} \quad (24)$$

$$\rho_{23} = \frac{\Sigma_{v_{23}} + \Sigma_{\nu_{23}} + \Sigma_{\varphi_{23}} + \Sigma_{\theta_{23}} + \Sigma_{\eta_{23}}}{\sqrt{\sigma^2_{v^2} + \sigma^2_{\nu^2} + \sigma^2_{\varphi^2} + \sigma^2_{\theta^2} + \sigma^2_{\eta^2}} \sqrt{\sigma^2_{v^3} + \sigma^2_{\nu^3} + \sigma^2_{\varphi^3} + \sigma^2_{\theta^3} + \sigma^2_{\eta^3}}} \quad (25)$$

where ρ_{12} is the Pearson correlation coefficient of random effects between fatal and major injury crashes, ρ_{13} is the Pearson correlation coefficient of random effects between fatal and minor injury crashes, and ρ_{23} is the Pearson correlation coefficient of random effects between major injury and minor injury crashes.

The posterior means and 90% credible intervals of Pearson correlation coefficients of random effects are shown in Table 10. The Pearson correlation coefficient between any two crash types was significantly positive at a 90% credible interval, but the Pearson correlation coefficient between major injury and minor injury crashes was generally larger than the other two values. That is, major injury and minor injury crashes had a stronger correlation compared to fatal crashes, which was consistent with the Pearson correlation coefficients of crash counts shown in Table 2.

Table 10 Pearson correlation coefficients of random effects across crash injury severities

Pearson correlation coefficient	Mean	90% CI
ρ_{12}	0.357	(0.047, 0.605)
ρ_{13}	0.366	(0.066, 0.605)
ρ_{23}	0.453	(0.145, 0.689)

Note: CI, credible interval; ρ_{12} , ρ_{13} , ρ_{23} , Pearson correlation coefficients between fatal and major injury crashes, between fatal and minor injury crashes, and between major injury and minor injury crashes, respectively.

4.3 Site Ranking Results Analysis

The crude crash rates and the predicted crash rates for all three crash types, which were calculated by dividing the crash counts by VMT, are shown in Figure 5. A linear regression model was built to check their correlation.

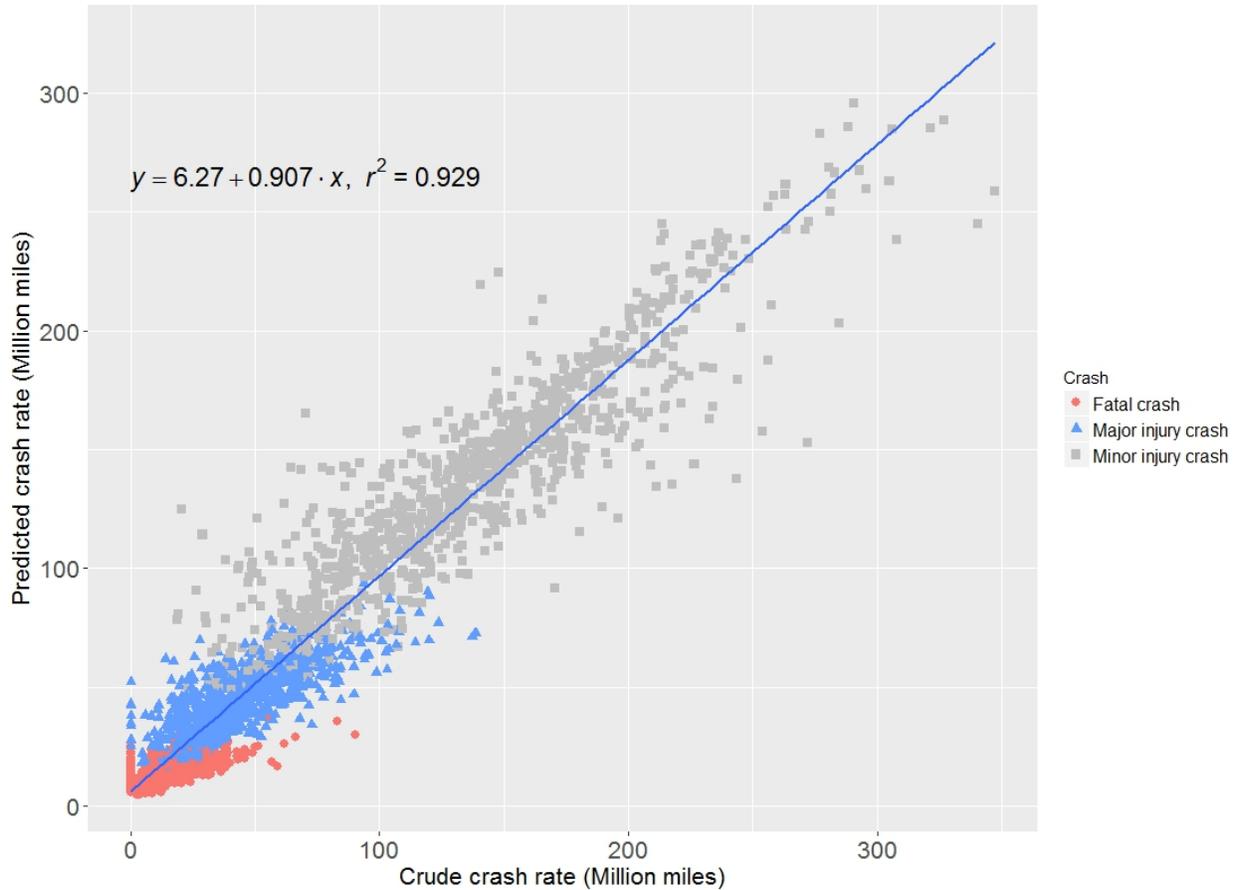


Figure 5 Crude crash rate versus predicted crash rate of fatal, major injury, and minor injury crashes

The R^2 value was 0.929, which means that the crude crash rates were generally consistent with the predicted crash rates. Specifically, for major injury and minor injury crashes, these two rates were very consistent, but for fatal crashes, they were inconsistent. Major injury and minor injury crash counts were very large, but fatal crash counts were very small, as shown in Table 1. Thus, occurrences of fatal crashes were more stochastic than major injury and minor injury crashes. It is thought that the multivariate structure could borrow information from major injury and minor injury crashes to estimate fatal crashes stably (Boulieri et al., 2017). Thus, the predicted data from the $S_{MBYMT_{MRW1}}$ model are expected to be smoother for unstable low-frequency fatal crashes, and could represent the underlying true distribution of fatal crashes better than the crude data could.

The crash cost rates directly influenced the ranking results shown in Figure 6, where x-axis showed the crude rank by the crude crash cost rate and y-axis showed the PER by the predicted crash cost rate. The two ranking methods produced consistent results for major injury and minor injury crashes but had large differences for fatal crashes, which led to different ranking results for total crashes.

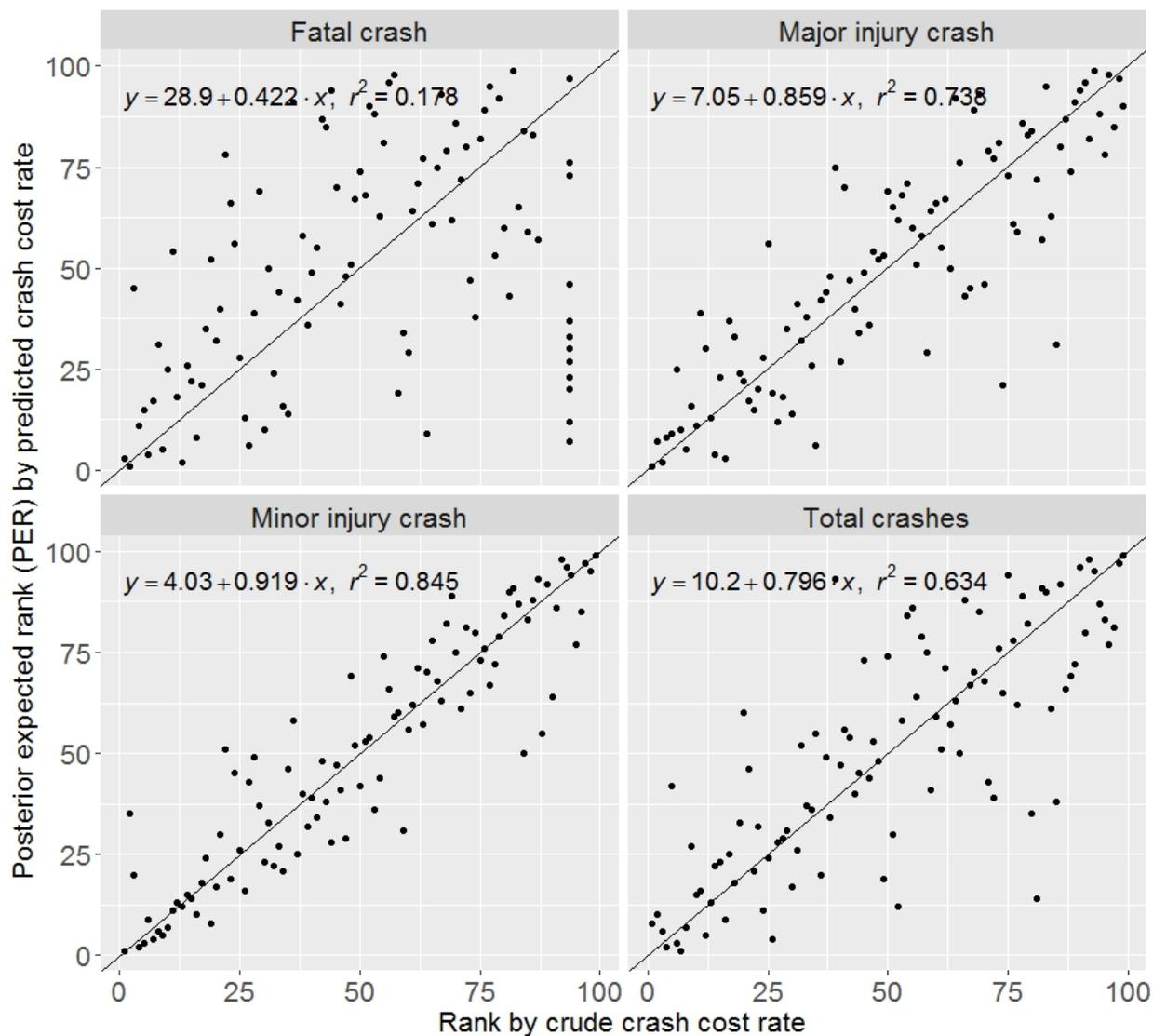
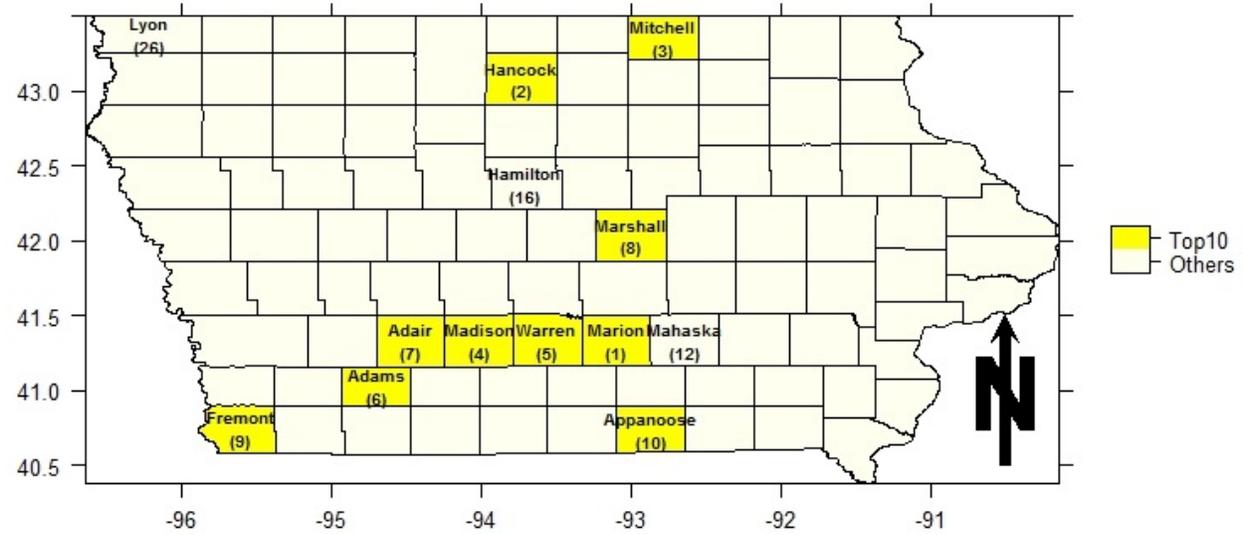
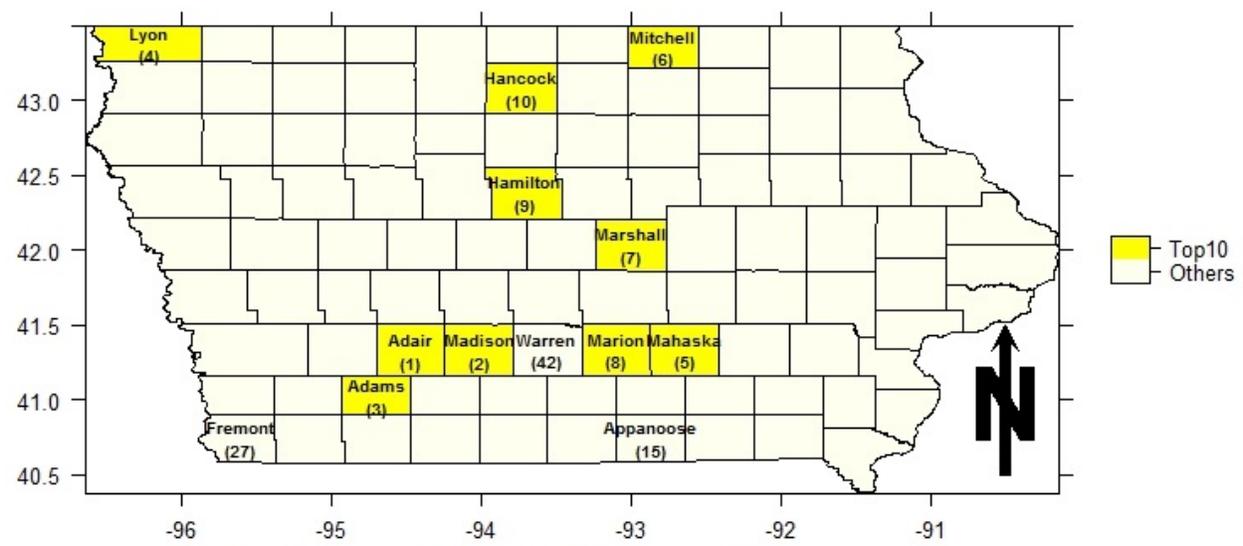


Figure 6 County rank by crude crash cost rate versus county posterior expected rank by predicted crash cost rate in 2015

The top 10 risky counties using the two ranking methods are shown in Figure 7. Of the counties ranked by these two methods, seven appeared in the top 10 for both methods, whereas three counties appeared only in the top 10 of one or the other method; Lyon, Hamilton, and Mahaska Counties were in the top 10 list using the predicted crash cost rate PER but not in the crude crash cost rate ranking. Moreover, the rank orders of the seven counties appearing in both top 10 lists were also very different. For example, the highest ranked county by the crude crash cost rate, Marion County, was ranked only eighth by the PER of the predicted crash cost rate. The big differences between the two ranking methods show the importance of the multivariate spatio-temporal Bayesian model, which is expected to better identify the underlying true status quo of traffic safety. The top 10 risky counties shown in Figure 7(b) should be the focus of future safety improvement programs.



(a) Rank by the crude crash cost rate



(b) Posterior expected rank (PER) by the predicted crash cost rate

Figure 7 Counties with the 10 highest crash cost rates using the two ranking methods

5 Discussion and Conclusions

Unobserved heterogeneity of crashes over space and time is often a big issue in crash frequency analysis. When multiple crashes are analyzed, correlations across crash types may also produce unobserved heterogeneity, which may exist in space, time, and space–time interactions. In this study, we used the multivariate spatio-temporal Bayesian model to analyze the yearly county-level fatal, major injury, and minor injury crash counts in Iowa from 2006 to 2015. Income, rainfall, snowfall, and temperature did not have significant influences on the frequencies of any of the three crash types, whereas unemployment rate showed significantly negative influences on major injury and minor injury crash counts, and VMT showed significantly positive influences on all three crash types.

All three crash types showed very strong spatial correlations. The counties located in northern and southwestern Iowa tended to have fewer crashes, whereas the remaining counties tended to have more crashes. All three crash types generally showed descending trends from 2006 to 2015. Both spatial and temporal effects were non-negligible, and they played nearly the same roles for all three crash types with slight differences. In addition, all three crash types showed significantly positive correlations between each other across space but not across time. The crude data and the predicted data were generally consistent for major injury and minor injury crashes but were very different for fatal crashes, the crude data of which were more stochastic due to the low counts. The predicted data from the multivariate spatio-temporal model were smoother than were the crude data. The crash cost rates were calculated based on crash rates and crash costs by injury severity and were used as ranking indicators. Two ranking methods, crude rank by the crude crash cost rate and PER by the predicted crash cost rate, were presented to identify the counties with higher risks for traffic safety. The two methods produced very different ranking results, and the latter method was thought to be able to better represent the true status quo of traffic safety. The ranking results would be helpful for transportation agencies drawing up traffic safety improvement programs in the future.

In future research, the data may be analyzed using smaller space and time scales, which would produce more targeted and practical findings. In addition, as shown in Table 3, the spatial correlations of all three crashes were different in different years. That is, the spatial correlations may evolve dynamically over time. Similar situations may also appear in temporal correlations, whereby the descending rates of crashes in different counties may be different. Thus, dynamic spatio-temporal models should be considered in future studies. Meanwhile, in this study, only random effects were thought to be correlated in space and time, but regression coefficients might also be correlated in space and time. Thus, future researchers may want to consider spatio-temporal-varying coefficient models. It is suggested that the review by Mannering (2018) about temporal instability in accident analysis be consulted for more ideas. All the above-mentioned directions would need more data or more complex statistical models, so computation may be a big concern, especially when using MCMC simulation to estimate Bayesian models. Some emerging fast Bayesian estimation tools, such as integrated nested Laplace approximation (Rue et al., 2009), should be considered. As was shown in this study, care should also be taken in the selection of appropriate priors and initial values for MCMC simulations. Finally, for this study we adopted two common spatial and temporal models; however, there are many other spatial and temporal models

available. Future researchers may also explore the effectiveness of other models in crash frequency analysis.

6 References

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