MEASUREMENT OF MATERIAL CONSTANTS FOR THE SURFACE IMPEDANCE BOUNDARY CONDITION

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THE SURFACE IMPEDANCE MODEL

The alternating current field measurement (acfm) technique of crack detection and measurement in metals is described elsewhere in these proceedings by Lugg [1] and by Dover [2]. In common with many eddy-current crack inspection probes, acfm probes are usually operated at a frequency f such that the electromagnetic skin-depth

$$\delta = (\mu_r \mu_0 \sigma \pi f)^{-1/2} \tag{1}$$

is much smaller than the depth of a significant crack. In this formula μ_r is the relative magnetic permeability of the test piece and σ its electric conductivity. Such thin-skin fields near a surface-breaking fatigue crack, with a uniform incident field, have been calculated by using a modified surface impedance boundary condition which includes a line source of magnetic flux to model the effect of the crack [3],[4]. With these calculations it has been possible to find the depth of fatigue cracks from a.c. field measurements. The surface tangential electric field **E** and magnetic field **H**, apart from at the crack line itself, obey the usual equations [5] of the surface impedance b.c.,

$$E_y = +Z_s H_x \tag{2}$$

$$E_x = -Z_s H_y \tag{3}$$

where x,y are the coordinates tangential to the metal surface and

$$Z_s = \frac{\mathrm{i}+1}{\sigma\delta} \tag{4}$$

is called the surface impedance. Equivalently, the boundary condition can be expressed in terms of the magnetic scalar potential ψ as

$$\frac{\partial^2\psi}{\partial z^2}-\frac{i+1}{\mu_r\delta}\frac{\partial\psi}{\partial z}=0 \qquad (5)$$

where z is the coordinate normal to the metal surface. In order to use the model it is necessary know (at least roughly) the value of $\mu_r\delta$, which depends on the material constant μ_r/σ and the operating frequency. This paper describes experimental measurements of μ_r/σ by a technique that may be used on any shape of test-piece, providing that it has a small flat area to which the necessary probes may be attached. The correct incremental permeability is measured because the field strength used is similar to that expected during the acfm test itself. The values obtained apply on the surface of the metal only and are not necessarily valid deep inside the test-piece.

EXPERIMENTAL MEASUREMENTS

A roughly uniform field was induced in the test piece with a pair of induction coils using the oscillator output of a Hewlett-Packard 4194A Gain/Phase Analyzer (fig. 1). This instrument was configured to measure the gain and phase between two input channels V_E and V_H.

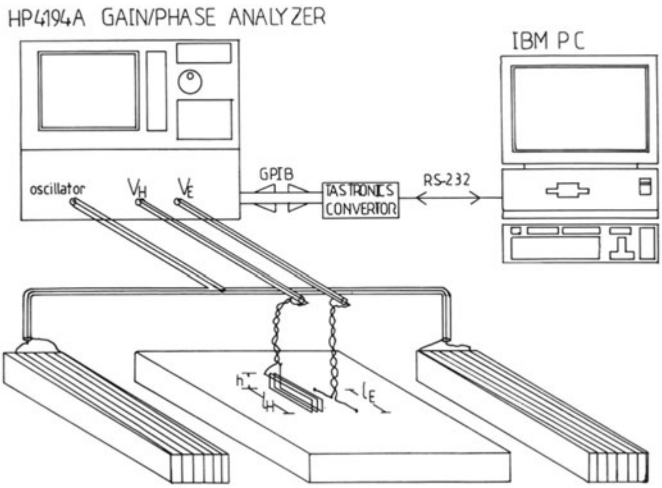


Fig. 1. Experimental apparatus

Two contacts held a distance l_E apart on an unflawed part of the surface of the test piece were connected to the first channel so that

$$V_E = l_E E_y - 2\pi i f \mu_0 H_x A \tag{6}$$

where A is the area of the loop formed by the probe leads and the test piece. The probe was designed to keep A as small as possible. A coil of width l_H, height h with N turns, placed immediately adjacent to the contacting probe was connected to the second channel so that

$$V_H = -2\pi i f \mu_0 H_x N l_H h. \tag{7}$$

The gain/phase measured between the two channels was therefore

$$V_E/V_H = \frac{A}{l_H h N} + \frac{i l_E}{l_H h N}\frac{Z_s}{2\pi f \mu_0} = \frac{A}{l_H h N} + \frac{(i-1) l_E}{2 l_H h N}\mu_r \delta. \tag{8}$$

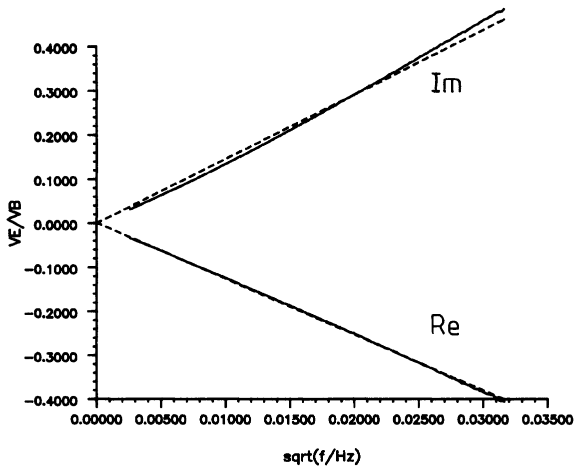
The real and imaginary parts of V_E/V_H were plotted against $f^{-1/2}$ over a range of frequencies in a variety of materials. If the permeability and conductivity are simple constants, independent of field strength, direction and frequency, the gradients should be equal and opposite and of magnitude

$$\frac{l_E}{2 l_H N h}\mu_r \delta \sqrt{f} = \frac{l_E}{2 l_H N h \sqrt{\pi \mu_0}}\sqrt{\frac{\mu_r}{\sigma}}. \tag{9}$$

In all the samples tested, the plots were close to straight lines as expected (fig. 2). Values for the constant μ_r/σ were found from the data by least squares fitting. The line for the imaginary part was constrained to pass through the origin but the line for the real part was offset, in order to allow for the term in A. Probe dimensions are given in table 1.

Table 1. Dimensions of probes

l_H	19.15 ± 0.02mm
l_E	20.0 ± 0.5mm
N	28
h	2.89 ± 0.01mm



Structural steel 50D

Fig. 2. Typical variation of gain V_E/V_H with frequency, solid lines:measured, broken:least-squares fit

PRECAUTIONS AND ACCURACY

Various forms of electric and magnetic interference are to be expected in such very low voltage a.c. measurements [6] manifesting themselves as common mode errors i.e. the signal does not exactly reverse sign when the exciting coils or measurement probes are reversed. At very high or very low frequencies, interference dominated the signal and therefore the band over which measurements were possible was restricted to 1kHz-1.5MHz at best. Electrostatic coupling occurred mainly on the contacting probe and was reduced by keeping the coaxial lead to the analyzer as short as possible (300mm) and by winding the induction coils in opposite senses and connecting them in parallel with their grounded sides next to the test-piece. Induction coils with 400 turns were used below 150kHz to give a strong signal for a small instrument current and coils with 20 turns were used at higher frequencies to avoid resonance problems. The probe lengths l_E and l_H were made nearly equal so that the two fields were being measured over the same region. Random noise, as indicated by the standard deviation associated with fitting the straight lines, was negligible. The gain/phase was measured with the exciting coils in both orientations and the mean values of the two senses, for each of the real and imaginary parts, were used for line fitting. It is hoped that these precautions led to an accuracy substantially better than the 50% figure guaranteed by the manufacturer at the signal strengths measured, which were about 10μV. A lower bound on the experimental accuracy is 2.5%, the error in the measurement of the probe length l_E.

DISCUSSION OF RESULTS

The values of $\mu_r\delta$ for the ferromagnetic metals 50D and AISI-4145 are much greater than for the other two materials, as one would expect (table 2). All the phase values are close to 135^0 which is the theoretical phase for isotropic, homogeneous and non-hysteretic materials. The value of μ_r/σ for the Dural is effectively just the resistivity $1/\sigma$ because the material is non-magnetic. For comparison, the resisivity of pure Al is $2.55\times10^{-8}\Omega m$ [7]. The bandwidth over which measurements in Dural were practicable was low because the voltages on the contacting probe were very small. In general, the technique works better on ferromagnetic metals in which it is easier to induce a field. The low frequency μ_r/σ of 50D is reduced in the heat affected zone, probably because the permeability is reduced, as found by Thompson, Allen and Turner [8]. The high frequency μ_r/σ is enhanced in the HAZ however, which is surprising.

In ferromagnetic materials it is possible for the quantity $\mu_r\delta$ to be much larger than the size of the crack, even though the skin-depth δ itself is much smaller. In this regime a special case of the surface impedance model, the "unfolding" model, may be used [4] and it is not necessary to know $\mu_r\delta$ accurately to calculate the fields. A useful conclusion from these experiments is the approximate rule that $\mu_r\delta f^{1/2}$ in ferromagnetic steels is typically about $2000\mathrm{mm}\sqrt{\mathrm{Hz}}$. For a given expected size of fatigue crack, it is therefore possible to pick an operating frequency such that the unfolding model applies. In other circumstances, the technique described here may be used to obtain a value of $\mu_r\delta$ to use in evaluating the more general surface impedance model.

Table 2. Results

Material	f/kHz	$\frac{\mu_r\delta\sqrt{f}}{\mathrm{mm}\sqrt{\mathrm{Hz}}}$	phase	$\frac{\mu_r/\sigma}{10^{-6}\Omega m}$
Structural steel 50D	1-150	2181	131.1^0	18.78
	100-1500	1859	134.2^0	13.64
High strength steel AISI-4145	1-150	2078	137.9^0	17.05
Heat affected zone of weld in 50D	1-150	1785	136.7^0	12.59
	100-1500	2155	137.6^0	18.34
Dural NS8	1-70	97.39	131.8^0	0.03738
Austenitic stainless steel	1-150	378.5	135.8^0	0.5656

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