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CALCULATION AND USE OF THE STANDARD DEVIATION OF PARTIAL REGRESSION COEFFICIENT

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Perhaps the most fundamental concept of all statistical work is that of an infinite populaton from which more or less random samples are drawn. Any set of data or sample represents a limited experience gained from, for example, a fertility plot, a feed lot, or a breeding plot or pen. From this sample we may obtain some idea of the nature of the infinite population from which it is assumed to have been drawn and so of the probable nature of future samples. If a future sample does not agree with the expectations, it is probable that it is drawn from a different population, that is, that there is a material difference in the treatment, in the objects of the treatment, or in the methods of measuring results.

Thus we see there are two distinct problems. One of them is to determine the probability that the various statistical constants of the theoretical population will not differ from the corresponding calculated constants of the sample by more than the ratio of the calculated constant to its standard deviation, and the other is to determine the significance of differences between corresponding constants obtained from two samples. The second problem, in other words, is to determine the probability that the samples have or have not been drawn from the same population. The standard deviation (σ) lends itself to the solution of these problems.

In the illustration which follows we solve only the first problem because we have but one sample. If we assume that a given partial regression coefficient (b-coefficient) is the mean of a normally distributed population and that the distance of this mean from zero is the ratio of thd given b-coefficient to its standard deviation (b/σ_B), the probability that the corresponding b-coefficient of the population will not differ from that of the sample by more than the ratio b/σ_B may be read directly from Table II., Tables for Statisticians and Biometricians, edited by Karl Pearson. Thus, in using the table, we first calculate the ratio b/σ_B which is the x of the table; then, entering the table with this value, we read the corresponding value of $\frac{1}{2}(1 + \alpha)$ which is the probability that the corresponding b-coefficient of the population will not differ by more than b/σ_B from the one given. The assumption of normal distribution is valid only for large samples as has been demonstrated by Fisher, R. A. (1922). For small samples Student's tables should be used, Student (1917).

Our present illustration consists of 512 observations on four independent variables (A, B, C, D,) and the dependent variable X. The multiple correlation coefficient ($R_{X \cdot ABCD}$), the regression equation, and the four b-coefficients (b_{XA}, b_{XB}, b_{XC}, b_{XD}), are calculated by the tabular method presented by Wallace, H. A. and Snedecor, George W. (1925). These calculations and results are given in Tables I. and II. In order to determine the standard deviations of the four b-coefficients by this method, it is necessary also to calculate the four (n-1) order multiple correlation coefficients, that is, the multiple correlation coefficient of each independent variable with the other three after dropping the dependent variable. We represent these coefficients as follows: $R_{D\cdot ABC}$, $R_{C\cdot ABD}$, $R_{B\cdot ACD}$, $R_{A\cdot BCD}$. These calculations and results appear in Tables III., IV., V., and VI. respectively.

With the above results we are prepared to calculate the desired standard deviations. The formula we use is a modification of one presented by Kelley, Truman L. (1923) which we have adapted to the notation and machine methods of Wallace and Snedecor (1925). The formula as originally presented by Kelley (1923) is:

$$\sigma_B(01 \cdot 23 \cdots n) = \frac{\sigma(0 \cdot 123 \cdots n)}{\sigma(1 \cdot 23 \cdots n)\ \sqrt{N}}$$

which may be read,—the standard deviation of the partial regression coefficient of the dependent variable on the first independent variable is equal to the standard error of estimate of the dependent variable on all of the independent variables divided by the product of the standard error of estimate of the first independent variable on the other independent variables, and the square root of the number of observations. In accordance with a suggestion made by Dr. Kelley in a letter of November 30, 1927, the second factor in the denominator was changed to $\sqrt{N-n}$, n being the number of variables entering into the problem.

In presenting the above formula, Kelley (1923) gave the following relations:

$$\sigma(0 \cdot 123 \cdots) = \sigma(0)\sqrt{1 - R^2_{(0\,\cdot\,123\,\ldots\,n)}}$$

$$\sigma(0 \cdot 23 \cdots) = \sigma(0)\sqrt{1 - R^2_{(0\,\cdot\,23\,\ldots\,n)}}$$

Using the above relations and the suggested correction, we have:

$$\sigma_B(01 \cdot 23 \cdots n) = \frac{\sigma(0)\sqrt{1 - R^2_{(0\,\cdot\,123\,\ldots\,n)}}}{\sigma(0)\sqrt{1 - R^2_{(0\,\cdot\,23\,\ldots\,n)}}\ \sqrt{N-n}}$$

or, using the notation of Wallace and Snedecor (1925):

$$\sigma b_{XA} = \frac{\sigma_X\sqrt{1 - R^2_{X.ABCD\ldots}}}{{}_{DA}\sqrt{1 - R^2_{X.ABCD\ldots}}\ \sqrt{N-n}}$$

This form seems to be the one best fitted to our method of solution.

The next step in our illustration is to substitute the appropriate values from Tables I. to VI. inclusive in the above formula and solve for the standard deviation of each of the four b-coefficients. The arithmetic and results of these calculations are given in Table VII·

Now that we have the b-coefficients, given in the regression equation at the end of Table II, and their standard deviations, given in Table III, we are ready to obtain some idea of the nature of the theoretical infinite population in-so-far as the b-coefficients are concerned, which is the first of the two problems. This information is contained in the following table:

Coefficient	Value of coefficient	Standard deviation	$x = \frac{b}{\sigma_B}$	Probability differences will not Exceed $\frac{b}{\sigma_B}$ $\frac{1}{2}(1 + a)$
b_{XA}	.0467	.0117	3.99	.9999670
b_{XB}	.0244	.0058	4.21	·9999872
b_{XC}	—·0540	.0088	—6·14	.9999999990
b_{XD}	.9848	.0088	111.91	Certainty

The probabilities given in the last column indicate that our sample is representative of the infinite population of which it is theoretically a part. This information gives us confidence that our sample may well be used for comparison with samples similarly drawn to detect whether or not they are drawn from the same population.

By examining the formula we are able to form a basis for judgment so that we will not have to calculate the standard deviations for all samples. Since the $\sqrt{N - n}$ is a factor in the denominator of the formula, it is apparent that the size of the samples has a large influence on the standard deviation of the various b-coefficients. In a large sample in which the standard deviations of the variables are moderate in size, it is reasonable to expect that a b-cofficient of the sample will not differ significantly from that of the population. However, in small samples or in samples in which the dispersion is great, each b-coefficient should be compared with its standard deviation before attaching a great deal of significance to it.

TABLE I. CALCULATION OF CORRELATION COEFFICIENTS AND STANDARD DEVIATIONS.

		A	B	C	D	X
Sums		2560	5120	2048	2048	2048
Means		5	10	4	4	4
A	1	13824	26940	10140	10108	10196
	2	12800	25600	10240	10240	10240
	3	1024	1340	— 100	— 132	— 44
	4	32	2048	1024	1024	1024
	5		.6543	—.0977	—.1289	—.0430
B	1		55296	20470	20288	20454
	2		51200	20480	20480	20480
	3		4096	— 10	— 192	— 26
	4		64	2048	2048	2048
	5			—.0049	—.0938	—.0127
C	1			9216	8173	8113
	2			8192	8192	8192
	3			1024	— 19	— 79
	4			32	1024	1024
	5				—.0186	—.0771
D	1				9216	9190
	2				8192	8192
	3				1024	998
	4				32	1024
	5					.9746
X	1					9216
	2					8192
	3					1024
	4					32
S Dev		1.4142	2.8284	1.4142	1.4142	1.4142

TABLE II. SOLUTION OF NORMAL EQUATIONS OF (nth) ORDER ON X

Block	Line	A	B	C	D	X	S
A	1	1.0000	.6543	—.0977	—.1289	—.0430	1.3847
	2	—1.0000	—.6543	.0977	.1289	.0430	
B	3		1.0000	—.0049	—.0938	—.0127	1.5429
	4		—.4381	.0639	.0843	.0281	—.9060
	5		.5719	.0590	—.0095	.0154	.6369
	6		—1.0000	—.1032	.0166	—.0269	—1.1137
C	7			1.0000	—.0186	—.0771	.8017
	8			—.0095	—.0126	—.0042	.1353
	9			—.0061	.0010	—.0016	—.0657
	10			.9884	—.0302	—.0829	.8713
	11			—1.0000	.0307	.0842	—.8851
D	12				1.0000	.9746	1.7333
	13				—.0166	—.0055	.1785
	14				.0002	.0003	.0106
	15				—.0009	—.0025	.0267
	16				.9823	.9669	1.9491
	17				—1.0000	—.9843	—1.9842
Beta(XD) =					.9843	.9843	
Beta(XC) =				—.0540	.0302	—.0842	
Beta(XB) =			.0488	.0056	.0163	.0269	
Beta(XA) =		.0467	—.0319	—.0053	.1269	—.0430	

$$R^2 = (.0467)(-.0430) + (.0488)(-.0127) + (-.0540)(-.0771) + (.9848)(.9746)$$

$$R = \cdot 9805$$

$$\bar{X} = 4 + .0467\,\frac{1.4141}{1.4142}\,(A-5) + \cdot 0488\,\frac{1.4142}{2.8284}\,(B-10)$$

$$-.0540\,\frac{1\cdot 4142}{1\cdot 4142}\,(C-4) + .9848\,\frac{1.4142}{1\cdot 4142}\,(D-4)$$

$$\bar{X} = .0467\,A + .0244\,B - .0540\,C + .9848\,D - 2007$$

TABLE III. SOLUTION OF NORMAL EQUATIONS OF (N — 1) ORDER ON D

Block	Line	A	B	C	D
A	1	1.0000	.6543	— .0977	— .1289
	2	—1.0000	— .6543	.0977	.1289
B	3		1.0000	— .0049	— .0938
	4		— .4281	.0639	.0843
	5		.5719	.0590	— .0095
	6		—1.0000	— .1032	.0166
C	7			1.0000	— .0186
	8			— .0095	— .0126
	9			— .0061	.0010
	10			.9844	.0302
	11			—1.0000	.0307
Beta(DC) =				— .0307	— .0307
Beta(DB) =			— .0134	.0032	— .0166
Beta(DA) =		— .1231	.0088	— .0030	— .1289

$R^2 = (-.1231)(-.1289)(+ (=.0134)(-.0938) + (-.0307)(-.0186)$
$= .0177$

TABLE IV. SOLUTION OF NORMAL EQUATIONS (n — 1) ORDER ON C

Block	Line	A	B	D	C	S
A	1	1.0000	.6543	— .1289	— .0977	1.4277
	2	—1.0000	— .6543	.1289	.0977	
B	3		1.0000....	— .0938	— .0049	1.5556
	4		4281	.0843	.0639	— .9341
	5		.5719	— .0095	.0590	.6215
	6		—1.0000	.0166	— .1032	
D	7			1.0000	— .0186	.7587
	8			— .0166	— .0126	.1840
	9			— .0002	.0010	.0103
	10			—1.0000	.0307	
Beta(CD) =				— .0307	— .0307	
Beta(CB) =			.1027	— .0005	.1032	
Beta(CA) =		— .1689	— .0672	— .0040	— .0977	

$R^2 = (-\cdot1689) - (\cdot0977) + (.1027)(-.0049) + (-.0307)(-.0186)$
$= .0166$

TABLE V. SOLUTION OF NORMAL EQUATIONS OF (n — 1) ORDER ON B

Block	Line	A	D	C	B	S
A	1	1.0000	— .1289	— .0977	.6543	1.4277
	2	—1.0000	.1289	.0977	— .6543	
D	3		1.0000	— .0186	— .0938	.7587
	4		— .1066	— .0126	.0843	.1840
	5		.9834	— .0312	— .0095	.9427
	6		—1.0000	.0317	.0097	
C	7			1.0000	— .00499	.8788
	8			— .0010	— .0003	.0294
	9			— .0095	.0638	.1395
	10			.9895	.0587	1.0477
	11			—1.0000	— .0593	
Beta(BC) =				.0593	.0593	
Beta(BD) =			— .0078	.0019	— .0097	
Beta(BA) =		.6591	— .0010	.0058	.6543	

$R^2 = (.6591)(.6543) + (-.0078)(-.0938)(+ (.0593)(-.0049)$
$= .4317$

TABLE VI. SOLUTION OF NORMAL EQUATIONS OF (n — 1) ORDER ON A

Block	Line	D	C	B	A	S
D	1	1.000	— .0186	— .0938	— .1289	.7587
	2	—1.0000	.0186	.0938	.1289	
C	3		1.0000	— .0049	— .0977	.8788
	4		— .0003	— .0017	— .0024	.0141
	5		.9997	— .0066	— .1001	.8829
	6		—1.00000	.0066	.1001	
B	7			1.0000	.6543	1.5556
	8			— .0088	— .0121	.0712
	9			— .0000	— .0007	.0059
	10			.9912	.6415	1.6327
	11			—1.0000	— .6472	
Beta(AB) =				.6475	.6475	
Beta(AC) =			— .0958	.0043	— .1001	
Beta(AD) =		— .0700	— .0018	.0607	— .1289	

$R^2 = (-.0700)(-.1289) + (-.0958)(-.0977) + (.6547)(.6543)$
$= .4468$

TABLE VII. STANDARD DEVIATIONS OF b-COEFFICIENTS AND THEIR CALCULATION

$$\sigma_{b_{XA}} = \frac{\sigma_X\sqrt{1 - R^2_{X\cdot ABCD}}}{\sigma_A\sqrt{1 - R^2_{A\cdot BCD}}\ \sqrt{N - n}}$$

$$= \frac{1.4142\sqrt{1 - .9614}}{1.4142\sqrt{1 - .4468}\ \sqrt{512 - 5}} = .0117$$

$$\sigma_{b_{XB}} = \frac{\sigma_X\sqrt{1 - R^2_{X\cdot ABCD}}}{\sigma_B\sqrt{1 - R^2_{B\cdot ACD}}\ \sqrt{N - n}}$$

$$= \frac{1.4142\sqrt{1 - .9614}}{2.8284\sqrt{1 - .4317}\ \sqrt{512 - 5}} = .0058$$

$$\sigma_{b_{XC}} = \frac{\sigma_X\sqrt{1 - R^2_{X\cdot ABCD}}}{\sigma_C\sqrt{1 - R^2_{C\cdot ABD}}\ \sqrt{N - n}}$$

$$= \frac{1.4142\sqrt{1 - .9614}}{1.4142\sqrt{1 - .0166}\ \sqrt{512 - 5}} = .0088$$

$$\sigma_{b_{XD}} = \frac{\sigma_X\sqrt{1 - R^2_{X\cdot ABCD}}}{\sigma_D\sqrt{1 - R^2_{D\cdot ABC}}\ \sqrt{N - n}}$$

$$= \frac{1.4142\sqrt{1 - .9614}}{1.4142\sqrt{1 - .0177}\ \sqrt{512 - 5}} = .0088$$

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STUDIES ON THE TOXICITY OF HYDROCYANIC ACID[1]

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I.

The validity of conclusions concerning the toxicity of various substances on organisms depends upon the methods of securing and interpreting the experimental data. Experiments have been made on the toxicity of hydrogen cyanide to a species of weevil, cockroach and the rat and the collected data have been presented in a way which avoids some possible errors in interpreting toxic effects. The work which is reviewed in this paper was completed in the spring of 1927 and is in harmony, in part, with the recent published results of Brinley and Baker (1927). However, these authors report a "supertoxicity" of gas obtained from liquid hydrogen cyanide over the gas generated from calcium cyanide. The data presented here do not support his conclusion.

II.

The apparatus was designed similar to those used in war gas experiments. Fig. 1 is a diagram of the apparatus used to determine the toxicity of hydrocyanic acid gas generated from "Calcyanide." To illustrate the operation of the apparatus the course of the air and gas will be traced. As indicated by the arrow, air enters the generating train at (a). Tube (c) acts as a pressure regulator. It consists of a Carius tube with mercury in the bottom (the darkened portion) and a head of water through which excess air bubbles. Screw clamp (b) aids in regulating the flow of air. In bottles (d) and (e) the air bubbles through sulphuric acid of sufficient density to produce sixty per cent humidity. It is then forced through glass wool in bottle (f) to take up acid spray. Next it flows over sixty grams of "Calcyanide" distributed through tube (g). Here the moist air generates hydrocyanic acid from the easily hydrolyzed calcium cyanide. As the gas air mixture is forced through the flowmeter (h) the rate of flow can be determined and fixed at some mark. For the experiments recorded in this paper, a rate of forty liters per hour was generally used. The concentration of hydrocyanic acid can be determined by taking a sample through tube (i).

It is often necessary to dilute with more air, hence the y tube (k) permits part of the gas to be forced out of the laboratory into the open and part through flowmeter (l). The rate of flow can be regulated by screw clamps (m) and (n). If it is necessary to dilute the gas with more air

[1] This work was made possible through a fellowship granted by the California Cyanide Company.

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in order to obtain a desired concentration, screw clamp (o) may be opened, which will connect the dilution train with the system.

The diluted air-hydrocyanic acid-gas mixture now passes into mixing bottle (u). Excess gas mixture is permitted to escape into the open (exit). The amount which is needed for the experiment is drawn by a water suction pump into mixing bottle (w), through flowmeter (x), into exposure bottles (y) and through flowmeter (z). Tube (s) is a suction regulator. The rate of flow can be controlled by screw clamp (t) and can be determined by flowmeters (x) and (z). Any leak in the exposure train can also be detected since the two flowmeters should indicate the same rate of flow. Samples for analysis may be taken from (r).

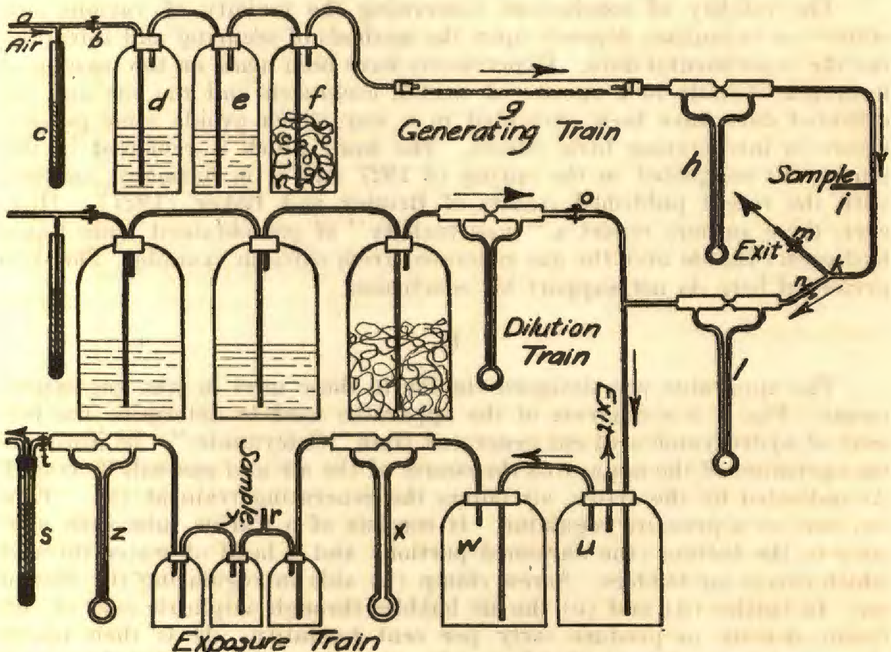

Fig. 1. Apparatus used to determine toxicity of HCN generated from calcyanide.

In all the experiments on toxicity when "Calcyanide" was used, 60 grams were distributed evenly in tube (g). This tube had a three-fourths inch bore and a length of four feet. Air (60% humidity) was passed continually through the tube, usually at the rate of forty liters per hour for three to four days. The first and second day the gas mixture obtained was generally diluted with air until the required concentration of hydrogen cyanide was reached. In most cases no dilutions were made the third and fourth day.

In order to obtain hydrocyanic acid gas from the liquid acid another generating train was substituted in the apparatus described above. Air dried by passing it through calcium chloride and concentrated sulphuric acid was bubbled through liquid hydrocyanic acid kept at a definite temperature. In most of the experiments the liquid was kept at approximately zero degrees centigrade by surrounding the container, which was inserted in a thermos bottle, by ice. The air saturated with hydrocyanic gas then passed through a flowmeter which can be considered as replacing (1) in the diagram. The rate of flow could then be determined and the gas mixture diluted to the desired strength as before. Since the vapor pressure of liquid hydrocyanic acid is known for various temperatures (Perry and Porter, 1926) it is possible to calculate the amount of gas picked up by the air and to make dilutions accordingly.

Samples of gas were analyzed by the thiocyanate method of Francis and Connel (1913) and by the standard Liebig method. The latter was found to be much simpler and more dependable than the colorimetric. It was found that the use of a solution of silver nitrate (1 cc. = 1 mg. HCN), stronger than that frequently used, and a microburette, makes it possible to obtain a good end point and at the same time to retain the accuracy sought by using a more dilute titrating solution.

The cockroach (periplaneta americana L.), the rice weevil (Sitophilus oryza L.), and the albino rat, weighing from 55 to 95 grams, were used in this work. Percentage kill was based on counts made at the time when maximum recovery was attained. Only animals which were apparently recovered were counted as living. For example, cockroaches which could move but could not walk after forty-eight hours had elapsed since they had been removed from exposure to the gas, were considered dead. Experience showed that they never fully recovered and later died.

All of the toxicity experiments recorded were carried out at room temperature varying from 23° to 28° C.

III.

There are many methods used to express results of toxicity experiments. There is the mortality rate curve so commonly used in the study of vital statistics and its integral which may be called a time course curve (see Fig. 2 and Fig. 3). Campbell (1926) has devised another method by which he plots, "speed of toxic action," against dosage, "speed of toxic action" meaning one hundred divided by the survival time. Economic entomologists often compare the effect of a toxic gas on different insects and also indicate the relative value of various toxic gases in the following way. A single time of exposure is held constant and the minimum lethal dose is determined. Tattersfield and co-workers, who worked with the effect of different sprays on Aphis rumicus L., developed a method which could be used for gaseous poisons. By holding the time of exposure constant and by varying the concentration used, the per cent kill at each concentration could be determined. The resulting curve would indicate how toxicity varies with concentration. They claim the best point for comparison between different poisons is the concentration which kills 50 per cent.

As previously stated, one of the purposes of this paper is to indicate a method of study of toxicity experiments. The time course curve, like the one in Fig. 2, is very common in biological literature. The sigmoid curve of the physiologist and the logarithmic curve of the bacteriologist are examples. That their shape is affected by many factors is evident. In bacteriological work a change in the reaction of the medium, and in insecticidal feeding experiments the contents of the stomach at time of poisoning, etc., would affect the course of the curve. The factor which is usually stressed is the variation and distribution of resistances. The shape of the curve is probably due to a summation of all the factors which enter into the reaction of an organism with its "poisoned" environment. Its chief value would seem to be expressed in the last statement.

Such curves would be determinants for those of the type shown in Fig. 4. In this case we have picked the one hundred "per cent kill" point for

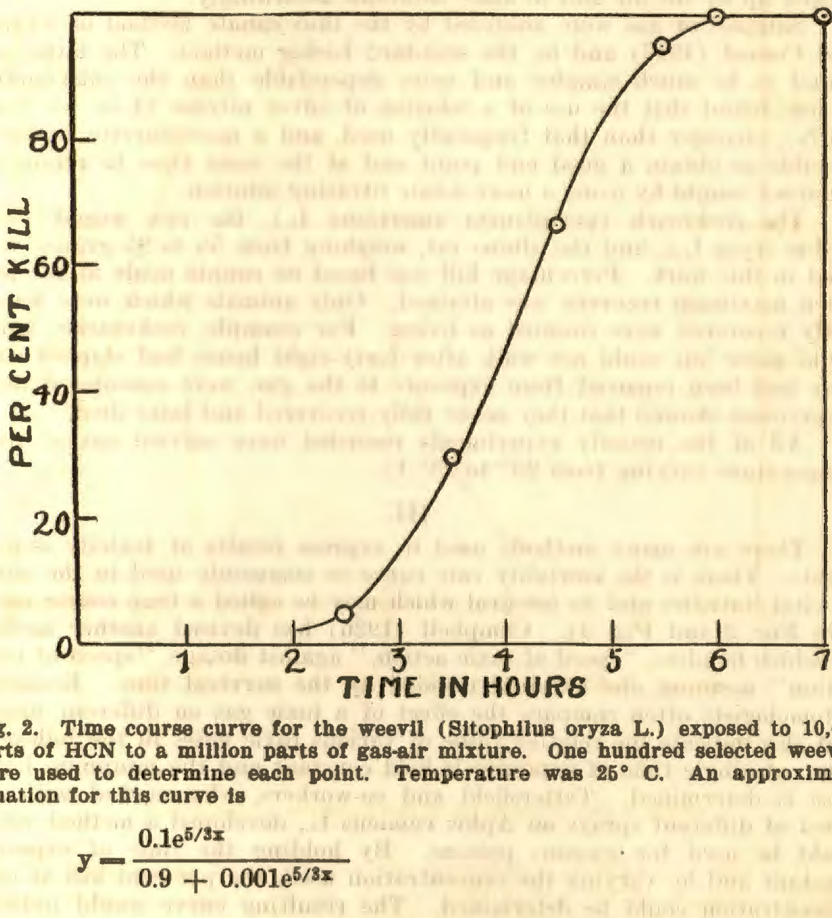

Fig. 2. Time course curve for the weevil (Sitophilus oryza L.) exposed to 10,460 parts of HCN to a million parts of gas-air mixture. One hundred selected weevils were used to determine each point. Temperature was 25° C. An approximate equation for this curve is

$$y = \frac{0.1e^{5/3x}}{0.9 + 0.001e^{5/3x}}$$

comparison. Brinley and Baker have pointed out the significance of the latter type of curve for the dosage scale, i. e., there is a limit to the concentration which would be of practical value. The equation, "toxicity = concentration $\times$ the time" will hold within certain limits which depend upon the course of the curve. The range to which it will apply will vary considerably; for instance, the variation shown by curves, a, b, and c. The method of comparing fumigants by determining the concentration necessary to produce one hundred per cent kill at one set time is open to criticism. It may only be accurate for the time of exposure used. For example, a certain concentration of carbon disulphide and the same amount of ethyl formate may give one hundred per cent kill in twenty-four hours, while at a higher concentration one may be a much more or a less efficient insecticide than the other. Finally, the large differences in resistance to the various gases shown by the various experimental animals should be noted.

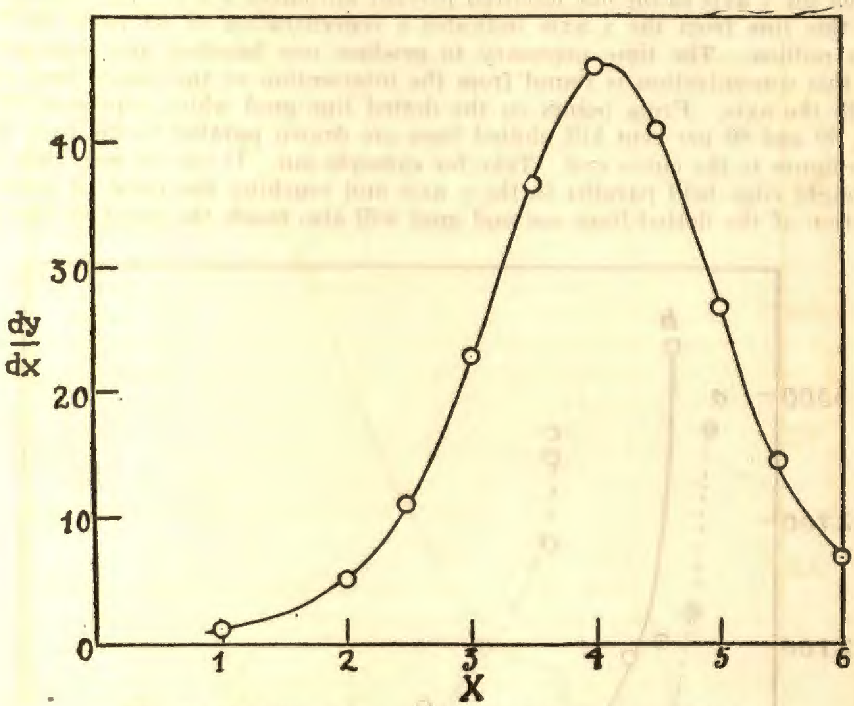

Fig. 3. Mortality rate curve obtained from the equation

$$\frac{dy}{dx} = \frac{5}{3}\,\frac{0.1e^{5/3x}}{(0.9 + 0.001e^{5/3x})^2}$$

which is the differential of the equation under Fig. 2.

It is believed that by combining the two types of curves used in the work a summary of the action of a poison on an organism can be made. This is done in Fig. 5 from data collected on the toxicity of hydrogen cyanide to the cockroach. The figure is plotted in three dimensions and includes the factors, time, concentration, and per cent kill which are commonly treated. The curve (a c b) is the same as the one shown in Fig. 4. Curve (c n d) is the same type as shown in Fig. 2. A comparison between concentrations and time at any per cent kill can be made. This is done by the use of curve (e n f) which is drawn through the points for sixty per cent kill. It should be noted that the slope of the time course curve such as (c n d) is affected by concentration and the surface which it defines is similarly affected. It is obvious that changes in the course of such curves produced by factors such as variations in resistance would become more apparent at the lower concentrations. The value of the solid figure can be summarized for the discussion of one series of toxicity tests when experimental conditions are kept constant. A study made on just a section of the figure might lead to erroneous conclusions.

To illustrate the construction of figure 5, a dotted line gc is drawn from the y axis to the one hundred percent kill curve a b c. The distance of this line from the x axis indicates a concentration of 69 parts HCN per million. The time necessary to produce one hundred per cent kill at this concentration is found from the intersection of the dotted line ch with the axis. From points on the dotted line gmd which represent 20, 40, 60 and 80 per cent kill, dotted lines are drawn parallel to the base of the figure to the curve end. Take for example mn. It can be seen that a straight edge held parallel to the y axis and touching the point of intersection of the dotted lines mn and gmd will also touch the point on the z

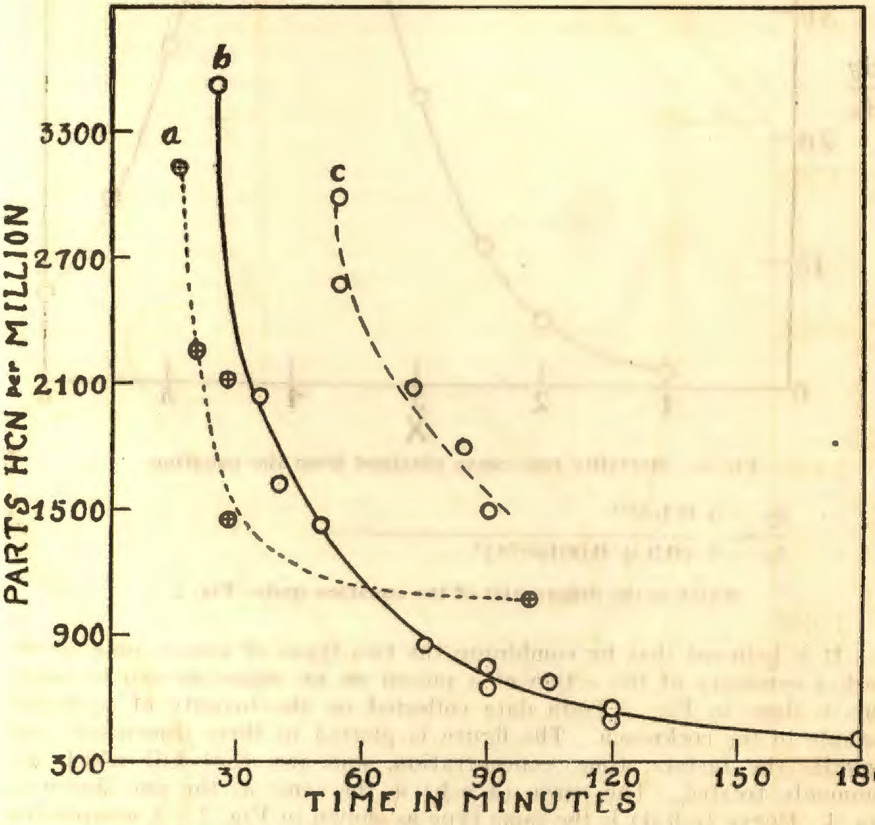

Fig. 4. Concentrations are plotted against the time interal necessary to produce 100 per cent kill. Curve a is for the albino rat, weighing from 55 to 95 grams, b for the cockroach (Periplaneta americana L.) and c for the rice weevil (Sitophilus oryza L.). The cockroach curve is plotted against the axis as indicated. In order to express the other curves on the same chart, it was necessary to multiply the values obtained for the rat by four and to divide those obtained for the weevil by five.

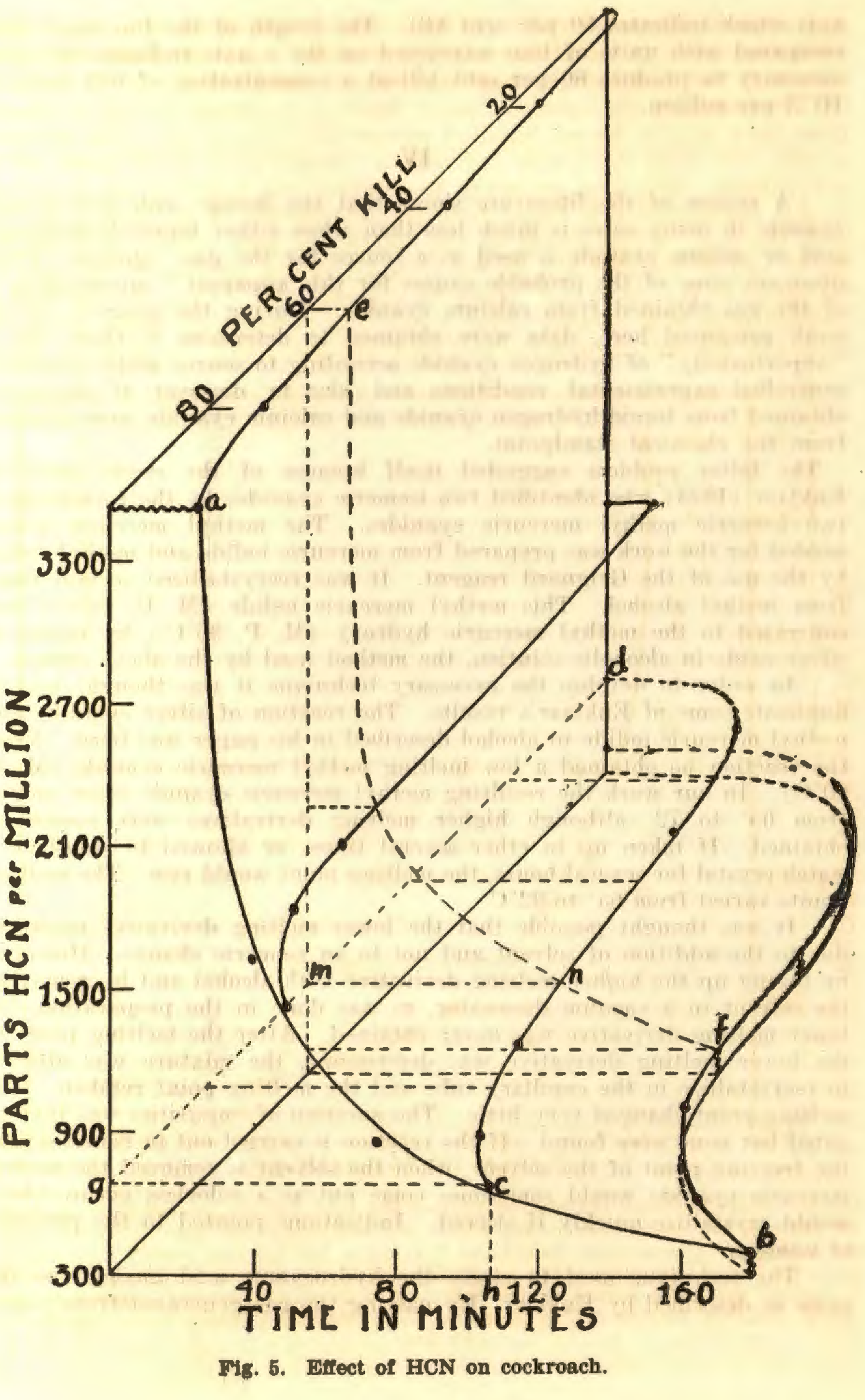

Fig. 5. Effect of HCN on cockroach.

axis which indicates 60 per cent kill. The length of the line m n when compared with units of time expressed on the x axis indicates the time necessary to produce 60 per cent kill at a concentration of 689 parts of HCN per million.

IV.

A review of the literature shows that the dosage scale for calcium cyanide in many cases is much less than when either liquid hydrocyanic acid or sodium cyanide is used as a source for the gas. Quayle (1927) discusses some of the probable causes for this apparent "supertoxicity," of the gas obtained from calcium cyanide. During the progress of the work presented here, data were obtained to determine if there was a "supertoxicity" of hydrogen cyanide according to source under carefully controlled experimental conditions and also to discover if the gases obtained from liquid hydrogen cyanide and calcium cyanide were isomeric from the chemical standpoint.

The latter problem suggested itself because of the recent work of Enklaar (1923) who identified two isomeric cyanides by the formation of two isomeric methyl mercuric cyanides. The methyl mercuric iodide needed for the work was prepared from mercuric iodide and methyl iodide by the use of the Grignard reagent. It was recrystallized several times from methyl alcohol. This methyl mercuric iodide (M. P. 45°C.) was converted to the methyl mercuric hydroxy (M. P. 96°C.) by means of silver oxide in alcoholic solution, the method used by the above author.

In order to develop the necessary technique it was thought best to duplicate some of Enklaar's results. The reaction of silver cyanide with methyl mercuric iodide in alcohol described in his paper was tried. From the reaction he obtained a low melting methyl mercuric cyanide (M. P. 60°C). In our work the resulting methyl mercuric cyanide often melted from 65° to 72° although higher melting derivatives were sometimes obtained. If taken up in ether several times, or allowed to stand on a watch crystal for several hours, the melting point would rise. The melting points varied from 65° to 92°C.

It was thought possible that the lower melting derivative might be due to the addition of solvent and not to an isomeric change. However, by taking up the higher melting derivative with alcohol and by removing the solvent in a vacuum desiccator, as was done in the preparation, the lower melting derivative was never obtained. After the melting point of the lower melting derivative was determined, the mixture was allowed to recrystallize in the capillary tube and the melting point retaken. The melting point changed very little. The question of impurities was investigated but none were found. If the reaction is carried out in benzene near the freezing point of the solvent, when the solvent is removed the methyl mercuric cyanide would sometimes come out as a colorless syrup which would crystallize quickly if stirred. Indications pointed to the presence of isomers.

The technique used to study the hydrocyanic acid gasses was the same as described by Enklaar. By passing the gas generated from liquid hydrocyanic acid through the apparatus as for a toxicity experiment, then into an alcoholic solution of methyl mercuric hydroxide, the higher melting methyl mercuric cyanide predominated, (M. P. 89°-91°C.). The gas generated from "Calcyanide" was treated in a similar manner and again the higher melting derivative was obtained (M. P. 91°-92°C.). It should be noted that the derivative obtained from "Calcyanide" generally melted slightly higher than the other but the difference was not sufficient to warrant a conclusion that any apparent "supertoxicity" of one gas over the other was due to isomeric hydrocyanic acids.

The killing properties of the two gases were next investigated. It was soon found that the points for one hundred per cent kill resulting from the use of either source of the gas would fit into a smooth curve. See the data in Table I. The cockroach (*Periplaneta americana L.*) was used to obtain this data and it is included in curve a, Fig. 4. Any irregularities which were noted were not consistent enough to point to a "supertoxicity" of either gas and were probably due to biological or other factors.

TABLE I. The Toxicity of Gas from Liquid HCN and from Calcyanide on the Cockroach (*Periplaneta americana* L.).

Conc. of hydrocyanic acid in parts per million	Time in min. necessary for 100% kill	Gas generated from
3523	25	"Calcyanide"
1435	50	"
869	75	Liquid HCN
764	90	"Calcyanide"
689	105	"
660	90	"
563	120	Liquid HCN
505	120	"Calcyanide"
407	180	Liquid HCN

SUMMARY

1. Variations in the shape of the time course curve are probably due to a summation of all the factors which enter into the reaction of living tissue with its "poisoned" environment.

2. Change in concentration of a poison is related to the time necessary to produce a constant per cent kill by curves which are determined by the time course curves. The former are of value for determining the effect of different poisons on one organism or the effect of one poison on different organisms.

3. By combining these two types of curves into a solid figure a summation can be made of numerous series of toxicity data.

4. The data presented in this paper do not indicate a "supertoxicity" or in fact any difference, either chemical or in toxic effect, between gas

generated from liquid hydrocanic acid and that generated from "Calcyanide."

I am indebted to Dr. R. M. Hixon and Dr. C. J. Drake for helpful advice and encouragement throughout the research upon the toxicity of hydrocyanic acid gas.

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ON CERTAIN POWER SERIES WITH POSITIVE COEFFICIENTS

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If we attempt to find a power series which will formally satisfy an ordinary differential equation of degree higher than the first, it usually happens that we must obtain each coefficient from a formula involving all the preceding ones. The question of the interval of convergence of power-series defined in such a way has not, I believe, been considered to any extent. The present paper will show some of the methods available for examining particular cases. Whether or not these methods should be of use in establishing a general theory, they will undoubtedly be suggestive of ways of attacking other problems of this type.

I.

If, in the equation

$$Ax^2y'^2 + 2(A+B)xyy' + (A+B+C)y^2 - y' = 0, \tag{1}$$

where A, B, C are constants, we assume that

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots, \tag{2}$$

we find that the coefficients must satisfy the equation

$$a_n = \sum_{k=0}^{n-1} b_{n,k}a_ka_{n-k-1}, \tag{3}$$

where

$$b_{n,k} = \frac{A(k+1)(n-k) + Bn + C}{n}. \tag{4}$$

It will be noticed that $nb_{n,k}$ is the most general quadratic polynomial in k, in which k and $(n - k - 1)$ are interchangeable.

We will now assume that A, B, and C are non-negative, real constants.

In certain special cases, which will be considered first, the general coefficient can be found in closed form; hence the interval of convergence, or a safe approximation of it, can be found directly.

Case 1. $A = 0$.

Without essential loss of generality, we can assume that $a_0 = 1$. This we shall do throughout this section.

In the case now under consideration

$$a_n = \frac{1}{n!}(B+C)(2nB+2C)((2n-1)B+3C) \quad . \quad . \quad .$$
$$((n+2)B+nC) \tag{5}$$

Formula (5), easily verified for $n = 1$ or 2, is proved in general by mathematical induction. If it is assumed correct for subscripts less than n, we derive from (3) and (4)

$$a_n = \frac{B+C}{n}[nB+C]\left\{\frac{1}{(n-1)!}[(2n-2)B+2C][(2n-3)B+3C]\right.$$

$$\ldots [(n+1)B+(n-1)C] + (B+C)\frac{1}{(n-2)!}[(2n-4)B$$

$$+2C][(2n-5)B+3C]\ldots[nB+(n-2)C] + \frac{1}{2!}(B+C)$$

$$(4B+2C)\frac{1}{(n-3)!}[(2n-6)B+2C]\ldots[(n-1)B+(n-3)C]$$

$$+\ldots\ldots$$

$$+\frac{1}{(k-1)!}(B+C)[(k+1)B+(k-1)C][(k+2)B$$

$$+(k-2)C]\quad\ldots\quad[(2k-2)B+2C]\;\frac{1}{(n-k)!}[(2n-2k)B+C]$$

$$\left.[(2n-2k-1)B+3C]\quad\ldots\quad[(n-k+2)B+(n-k)C]\right\}. \qquad (6)$$

The second members of (5) and (6) must now be proved identical. They are both homogeneous polynomials of degree n in B and C. That they differ, at most, by a factor independent of B and C, we prove by showing that, if any factor of the second member of (5) is 0, the same is true of the second member of (6).

The hypothesis will hold true if

$$(n+1+j)B + (n+1-j)C = 0 \qquad [j = 0, 1, 2, \ldots, n-1]. \quad (7)$$

Let us, then, set $C = -\dfrac{n+1+j}{n+1-j}B$ in the second member of (6). The result consists essentially of the sum

$$\sum_{k=1}^{n}\frac{1}{(k-1)!}[kj-(n+1)][kj-2(n+1)]\ldots[kj-(k-2)(n+1)]\cdot$$

$$\frac{1}{(n-k)!}[(n-k+1)j-(n-k-1)(n+1)]\;[(n-k+1)j$$
$$-(n-k-2)(n+1)]\;\ldots\;[(n-k+1)j-(n+1)]$$

$$= \sum_{k=1}^{n}\frac{(-1)^k}{(k-1)!(n-k)!}[(n-k+1)j-(n-k-1)(n+1)]$$

$$[(n-k+1)j-(n-k-2)(n+1)] \quad . \quad . \quad . \quad .$$
$$[(n-k+1)j-j(n+1)][(n-k+1)j-(j-1)(n+1)]^2 \quad . \quad . \quad .$$
$$[(n-k+1)j-(n+1)]^2[(n-k+1)j] \quad . \quad . \quad . \quad .$$
$$[(n-k+1)j+(k-j-2)(n+1)] \,. \tag{8}$$

If, now, we name

$$nj - (n-2)\ (n+1) = N_j,$$

and

$$N_j[N_j + (n+1)]\ [N_j + 2(n+1)] \ldots$$
$$\ldots [N_j - (n-j-3)\ (n+1)] = F_j\ (N_j),$$

our sum can be written

$$\frac{1}{(n-1)!}\sum_{k=1}^{n} \binom{n-1}{k-1}(-1)^k F_j(N_j + (k-1)(n-j+1)) \cdot$$
$$[-jk+(n+1)][-jk+2(n+1)] \ldots .$$
$$\ldots [-jk+(j-1)(n+1)] \,. \tag{9}$$

$\binom{r}{s}$ denotes the binomial coefficient $\dfrac{r(r-1)\cdots(r-s+1)}{s!}$.

F_j is a polynomial of degree $(n-j-2)$.

It is important to notice that the product of the brackets in (9) is a linear combination of the following $\binom{j+1}{2}$ numbers, with coefficients independent of k:

$$1,$$
$$(k-1),\qquad (k-1)(k-2),\ \ldots .\ ,(k-1)(k-2)\ \ldots\ (k-j-1),$$
$$(n-k),\qquad (n-k)(k-1),\ \ldots .\ ,(n-k)(k-1)\ \ldots\ (k-j+2),$$
$$(n-k)(n-k-1),\ (n-k)(n-k-1)(k-1),\ \ldots$$
$$(n-k)(n-k-1)\ \ldots\ (k-j+3),$$

....

$$(n-k)(n-k-1)\ \ldots\ (n-k-j+2) \tag{10}$$

The proof of this statement involves the fact that $\binom{j}{2}$ is also the number of different products of type $k^p n^q$ occurring in the product of brackets in (9). It involves, further, the fact that the determinants of several systems of linear equations, needed in finding the coefficients, are all different from 0. A typical determinant is

$$\begin{vmatrix} 1 & \sum\limits_{1}^{j-1} a & \sum\limits_{1}^{j-1} a_1a_2 & \dots & \prod\limits_{1}^{j-1} a \\ 1 & \sum\limits_{0}^{j-2} a & \sum\limits_{0}^{j-2} a_1a_2 & \dots & \prod\limits_{0}^{j-2} a \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sum\limits_{2-j}^{0} a & \sum\limits_{2-j}^{0} a_1a_2 & \dots & \prod\limits_{2-j}^{0} a \end{vmatrix},$$

which equals

$$(-1)^{\binom{j}{2}} 2^2 \cdot 3^3 \dots (j-2)^{j-2} (j-1)^{j-1}.$$

If, now, $\binom{n-1}{k-1}$ is multiplied by any of the numbers (10), the result is the product of a binomial coefficient of similar type, and of factors independent of k.

E. g., $\binom{n-1}{k-1}(n-k)(n-k-1)(k-1)$

$$= \frac{(n-1)!}{(k-1)!(n-k)!}(n-k)(n-k-1)(k-1)$$

$$= \frac{(n-1)!}{(k-2)!(n-k-2)!} = \binom{n-4}{k-2}(n-1)(n-2)(n-3).$$

The upper index of the binomial coefficient is reduced by the number of factors involving k,— at most $(j-1)$; accordingly it remains $(n-j)$ or more.

The terms of (9) being separated into $\binom{j+1}{2}$ sums, according to the linear factors involving k, it turns out that each of these sums is an $(n-j)$th, or higher, successive difference of a polynomial of degree $(n-j-2)$. The value of (9) is accordingly 0, as the proof required.

The second members of (5) and (6) differ at most, then, by a factor independent of B and C. That this factor is 1 can be seen most easily from the fact that 1 is the coefficient of C^n in both expressions.

Information on the interval of convergence of the series considered can now be obtained from the test-ratio. (5) gives us:

$$\frac{a_{n+1}}{a_n} = \left[\frac{n+3}{n+1}B + C\right]\left\{\left[1+\frac{2B}{2nB+2C}\right]\left[1+\frac{2B}{2nB+3C}\right] \dots \left[1+\frac{2B}{2nB+nC}\right]\right\} \tag{11}$$

The limit of the first factor, as n becomes infinite, is $(B+C)$.

The other factors are certainly less than $\left[1+\frac{2B}{2nB+2C}\right]^{n-1}$, of which the limit is e.

Accordingly the original series converges, at least, for

$$-e^{-1}(B+C)^{-1} < x < e^{-1}(B+C)^{-1}. \tag{12}$$

This interval of convergence can, of course, be widened by closer attention to $\frac{a_{n+1}}{a_n}$. If, for instance, we consider the effect of replacing the product of two factors equally distant from the ends of the brace in (11) by

$$\left\{1+\frac{2B}{2nB+(\frac{n}{2}+1)C}\right\}^2,$$ we find an interval of convergence extending as far as

$$(B+C)^{-1}\,e^{\frac{-B(32B^2+33C^2)}{8(4B+C)\,(B^2+C^2)}}$$

Case 2. $B=C=0$.

$$a_n=\frac{(2A)^n\,(n+1)^{n-2}}{n!}, \tag{13}$$

as can be established by mathematical induction.

$$\frac{a_{n+1}}{a_n}=\frac{2A(n+2)^{n-1}}{(n+1)^{n-1}}.$$

As $\lim\limits_{n\to\infty}\left(\frac{n+2}{n+1}\right)^{n-1}=\lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n+1}\right)^{n+1}\frac{(n+1)^2}{(n+2)^2}\right]=e,$

the interval of convergence extends to $\pm\frac{1}{2Ae}$.

The series converges at both ends of this interval, since $\left(1+\frac{1}{n+1}\right)^{n+1}<e$, and $\frac{(n-1)^2}{(n-2)^2}$ is the test-ratio of a convergent series.

We have, then, as the exact interval,

$$-\frac{1}{2Ae}\leqq x\leqq\frac{1}{2Ae}. \tag{14}$$

Case 3. $C=0$.

By methods similar to those used in Case 1, the value of the general coefficient

$$a_n=\frac{2^n(n+1)^{n-2}}{n!}(A+B)^{\frac{n+1}{2}}\left[A+B-\left(\frac{n-2}{n+1}\right)^2B\right]^{1/2}$$
$$\left[A+B-\left(\frac{n-4}{n+1}\right)^2B\right]^{1/2}\cdots \tag{15}$$
$$\left[A+B-\left(\frac{-n+2}{n+1}\right)^2B\right]^{1/2}$$

can be shown to depend on the formula

$$\sum_{k=0}^{n}\Bigg[\frac{1}{[(n-2p+1)(k+1)+(n+2)k]}\cdot\frac{1}{[(n-2p+1)(n-k+1)+(n+2)(n-k)]}\binom{n}{k}$$
$$[(n-2p+1)(k+1)+(n+2)k]\,[(n-2p+1)(k+1)+(n+2)(k-2)]\ \ldots$$
$$[(n-2p+1)(k+1)-(n+2)(k-2)]\,[(n-2p+1)(n-k+1)$$
$$+(n+2)(n-k)]\,[(n-2p+1)(n-k+1)+(n+2)(n-k-2)]\ \ldots$$
$$[(n-2p+1)(n-k+1)-(n+2)(n-k-2)]\Bigg]$$
$$=(-1)^{p+1}\frac{(n+2)^{n-1}}{2^{n-3}(n-2p+1)}\;\frac{(2n-2p)!\,(2p-3)!}{(n-p)!\,(p-2)!} \tag{16}$$

Now this formula is a special case of the much more general one, recently proved by Hasse and Bessel-Hagen :—

$$\sum_{j+k=n}\left[\frac{x}{zj+x}\binom{zj+x}{j}\frac{y}{zk+y}\binom{zk+y}{k}\right]=\frac{x+y}{zn+x+y}\binom{zn+x+y}{n}$$

Their ingenious proof of this identity, which Professor Hasse was so kind as to communicate to me, will appear in the Jahresbericht der Deutschen Mathematiker-Vereinigung.

In this formula n, j, k are non-negative integers, such that $j+k=n$, and x, y, z are variables. For our case, we let

$$x=y=\frac{n-2p+1}{2n+4},\ z=\frac{2n-2p+3}{2n+4}.$$

Using (15), let us investigate the condition for the convergence of the series. It is convenient to consider the product of two successive test-ratios

$$\frac{a_{n+2}}{a_n}=\frac{\left[A+B-\left(\frac{n}{n+3}\right)^2 B\right]\left[A+B-\left(\frac{n-2}{n+3}\right)^2 B\right]\cdots}{\left[A+B-\left(\frac{n-2}{n+1}\right) B\right]\left[A+B-\left(\frac{n-4}{n+1}\right) B\right]\cdots}\cdot$$
$$4(A+B)\frac{(n+3)^n}{(n+1)^{n-1}\,(n+2)}.$$

The second fractional factor approaches e as n becomes infinite. For an estimate of the first fraction, we consider the series, in powers of $\frac{B}{A+B}$, giving the logarithms of the factors in numerator and denominator. Addition of these series gives a result which is increased when replaced by the approximation for the first two terms,

$$\left(-\frac{1}{3}\frac{B}{A+B}-\frac{n}{10(n+1)}\left(\frac{B}{A+B}\right)^2\right).$$

Allowing n to become infinite, and using the corresponding exponential function, we find that $\frac{a_{n+2}}{a_n} < 4(A+B)e^{\frac{6A+5B}{3A+3B}}$, and hence that the power-series converges when

$$|x| < \frac{1}{2\sqrt{A+B}\, e^{1-\frac{B}{6A+6B}}}. \tag{17}$$

Case 4:

Without finding any closed form for a_n in the case where A, B and C are arbitrary positive numbers, we can readily see that the series is still convergent in the interval (17).

By the addition of C, $b_{n,k}$ is increased in the ratio

$$1+\frac{C}{A(k+1)(n-k)+nB} \leqq 1+\frac{C}{n(A+B)}.$$

Accordingly, if $\bar{a}_i$ is obtained from a_i by the substitution of 0 for C,

$$\frac{\bar{a}_1}{a_1} < 1+\frac{C}{A+B} = \lambda_1$$

$$\frac{\bar{a}_2}{a_2} < \left(1+\frac{C}{A+B}\right)\left(1+\frac{C}{2(A+B)}\right) = \lambda_2$$

....

$$\frac{\bar{a}_i}{a_i} < \left(1+\frac{C}{A+B}\right)\left(1+\frac{C}{2(A+B)}\right)\ldots\ldots\left(1+\frac{C}{i(A+B)}\right) = \lambda_i$$

....

That is, the terms of

$$a_0+a_1x+a_2x^2+\ \ldots\ldots\ +a_ix^i+\ \ldots$$

will be no greater than the corresponding ones of

$$\bar{a}_0+\bar{a}_1\lambda_1x+\bar{a}_2\lambda_2x^2+\ \ldots\ +\bar{a}_i\lambda_ix^i+\ \ldots$$

The test-ratio of the latter series is

$$\frac{\bar{a}_{n+1}}{\bar{a}_n}x\left(1+\frac{C}{(n+1)(A+B)}\right);$$

from which we can deduce its convergence for all values of x within the interval (17).

II.

Another power-series, each of whose coefficients depends on all the preceding ones, is obtained as a particular solution of the differential equation

$$2x\left(x\frac{dy}{dx}-y+1\right)^2-\frac{dy}{dx}=0 \tag{18}$$

If we choose that, in the series

$$y=a_0+a_1x+a_2x^2+\ \ldots\ +a_ix^i+\ \ldots\ ,$$

$a_0=0$, it follows that $a_i=0$ whenever i is odd. The other coefficients are determined by the equation

$$a_{2n}=\frac{1}{n}\Big[(2n-3)a_{2n-2}+(2n-5)a_2a_{2n-4}+3(2n-7)a_4a_{2n-6}+\ \ldots$$

$$+(2k-1)(2n-2k-3)a_{2k}a_{2n-2k-2}+\ \ldots\Big], \tag{19}$$

where $a_2=1$.

The interval of convergence is found by comparison with the series

$$1+\frac{2-s}{2}+\frac{2-s}{2}\cdot\frac{3-s}{3}+\frac{2-s}{2}\cdot\frac{3-s}{3}\cdot\frac{4-s}{4}+\ \ldots\ ,$$

which serves as the basis for Raabe's test, being convergent when $s>1$.

The series under discussion will converge if

$$a_{2n}x^{2n}<a_4x^4\frac{2-s}{2}\cdot\frac{3-s}{3}\ \ldots\ldots\ \frac{n-1-s}{n-1}.$$

Assuming this inequality to hold for $n<N$, and setting $\frac{1}{x^2}=\lambda$, we ask under what conditions it will be true also for $n=N$. We find

$$\frac{(N-1)(2N-3)}{(N-1-s)N}\lambda^2+\frac{(N-2)(N-1)(2N-5)}{(N-2-s)(N-1-s)N}\lambda$$

$$+\sum_{k=4}^{N-1}\frac{(N-k)(N-k+1)\ \ldots\ldots\ (N-1)}{(N-k-s)(N-k-s+1)\ \ldots\ (N-1-s)}\cdot$$

$$\frac{(2-s)(3-s)\ \ldots\ (k-2-s)}{2\cdot 3\ \ldots\ (k-2)}\cdot\frac{(2N-2k-1)(2k-3)}{N}<\lambda^3 \tag{20}$$

If, now, we allow N to become infinite, we must make sure that the terms in the summation of the first member approach those of a convergent series. Comparison with the test-ratio of $\Sigma\frac{1}{n^u}$ (where $u<1$) shows that a sufficient condition for convergence is that $s>\frac{3}{2}$.

The truth of the inequality obtained from (20) by allowing N to become infinite would not, of course, prove (20) itself. It is readily found, however, that the effect of an increase of N by a unit is, in the first place, to increase the number of terms in the first member of (20); and, in the second place, to increase the $(k-1)$th term by a positive multiple of

$$k^2(2sN-2s)+k(3sN-3N-s)+(2N^3+2N^2-sN).$$

The discriminant of this polynomial in k is negative for $N>2$; the polynomial, being positive when k is sufficiently large, is always positive. From the truth of the limiting form of (20) we can, then, infer that of (20) itself.

The infinite series in the inequality in question can again be replaced by an excessive approximation of $\Sigma\frac{1}{n^u}$ (we may choose $u=\frac{9}{8}$) obtained by evaluation of $\int_2^\infty \frac{dx}{x^u}$. The resulting inequality is, if s is taken as $\frac{7}{4}$,

$$\lambda^3-2\lambda^2-2\lambda-152>0.$$

Solution of the corresponding equation shows that this is true, and the original series accordingly convergent, if $|x|<.4$.

THE FORECASTING OF ECONOMIC PHENOMENA

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During the past ten years there has developed a widespread interest in the forecasting of prices and other economic phenomena. Many different methods, some scientific, some semi-scientific, and many entirely empirical, have been used in these efforts to anticipate the future. The results have been as diverse as the methods used. Many have proved themselves inadequately related to the basic forces. A good many have given favorable results under limited sets of conditions, but not a few seem fairly reliable under the varying conditions of the economic world.

It is not surprising that economists and others have shown an active interest in the problem of forecasting. In economic life there are relatively few things that are subject even to a moderate degree of control by the producer or the consumer in his individual capacity. Further, relatively few of the forces beyond his individual control have proven amenable to organized effort. Even here the extension of control is slow and difficult because of the tenacity with which many producers cling to the prerogatives of individual economic freedom.

To the individual, therefore, the immediately practical problem becomes largely one of adjusting himself to the conditions around him. If oats are cheap there is little that he can do toward obtaining a higher price, but he may be able to restrict his acreage of oats and to increase that of some other and more profitable crop. If the price of corn is high and hogs low he can raise fewer hogs and feed them to lighter weights. But there is no way, by positive action, in which he alone can lower the one and raise the other. In fact, while this might be in the interest of one group of producers it would be contrary to the interests of others.

The production of our common economic wares requires a considerable period of time. Hence the adjustment may well come too late and may meet with a maladjustment in the opposite direction. What the producer needs, therefore, is a cue to the reshaping of his plans in time to make them effective while the indicated conditions continue. This implies the use of foresight as to the consequences of activity still in its developmental stages. In short, it implies forecasting.

The layman may well demand that the economist give him some insight into the probable course of events. It is the economist's business to study and attempt to understand economic phenomena. And what general object has the understanding of these forces except to make use of them? If the economist really does comprehend the forces and phenomena with which he is dealing, should he not also perceive their general drift for a short time in the future? It is in the effort to answer this challenge that the methods of forecasting have been or are being developed.

Nature of the Forecasting Problem:

Economic forecasting attempts to anticipate the changes in prices or rates of activity in industry or in other series with economic signficance. If we examine some of these series, for instance, the prices of hogs, the production of pig iron, the loading of freight cars, or the volume of bank clearings, we find that each one of them shows a wide, and often abrupt, variation from month to month and from year to year. When plotted on cross section paper such a series of monthly or yearly data shows the irregular sawtooth contour which has become familiar through graphs published in papers and magazines. In these graphs the curves frequently seem to rise and fall erratically from month to month and it may seem hard to discover any really systematic variation.

In many series, however, a more careful examination will show a more or less regular oscillation from year to year, and often in addition a consistent trend upward or downward over a considerable period of time. The continued upward or downward movement is known as the secular trend, while the recurring rise and fall within years is called the seasonal variation. After these more or less regular variations are removed from the data there will be found to remain a series of irregular fluctuations which may or may not contain a rythmic or cyclical oscillation more or less uniform in periodicity. If such a periodicity is not present the residual fluctuations may appear to have no regular or systematic variations at all. These seemingly erratic variations may account for a large part of the total fluctuation of the series. The question may well be raised as to how it would be possible to forecast with any degree of nearness to the actual facts as they develop from month to month.

Generally a study of related phenomena will suggest the causes for the more outstanding fluctuations in prices or other economic series. Thus variations in crop yields may explain some of the fluctuations in livestock prices at a later time. Variations in the weather during the growing season explain variations in yields and in prices of crops at later periods. A characteristic of economic phenomena is that they are to a very large degree interrelated. Thus in explaining the price of hogs we find one influence to come from the size of the corn crop. But the size of the corn crop affects also the price of cattle, and the price of various other things into which corn enters as a raw material. It also influences the prices of those things for which corn may under some conditions be substituted. There is a close sympathy between the prices of corn and of oats and barley, a more remote one between corn and wheat, and between corn and the forage crops.

In such ways as these the lines of causation in economic activity branch out into a network which unites the entire economic system into an organic whole. Each industry must be regarded as functioning with regard to other industries. Any noteworthy event in one may be expected to have more or less definite repercussions in related fields. Thus a coal strike may be expected to tie up manufacturing and to reduce the traffic on railroads. An invention which cheapens steel may be expected to result in the use of more steel in practically all those industries where this metal serves as a raw material. A rise in the price of beef may be expected to lead to a larger demand for pork and to affect the price of that meat accordingly. The economic world is essentially a unit and this fact goes a long way toward making forecasts of economic development possible.

Economic changes, however, are not to be regarded simply as steps in the reestablishment of a static equilibrium. It is true that if conditions in each industry were to remain unchanged for a considerable period of time we might very well expect a condition in which there would be no further acceleration or retardation of activity. The difficulty with viewing the economic complex in this manner is that no condition approaching the static is ever reached or, because of the nature of the forces and influences concerned, is ever to be expected.

The concept of the supply of a commodity which is easiest to grasp is that of a certain quantity available at one price and of larger quantities available at each higher price. But a moment's reflection will show that this is only part of the truth. The supply might better be regarded as a *rate of flow* than as an amount in existence. If there develops a larger demand for radio sets it will be possible to deliver to consumers only the number already manufactured and existing in the stocks and warehouses at the present time. For a continuously larger delivery it would be necessary to go further back toward the source and increase the rate of flow of the radio sets from the factories. In a similar way if an unusually large output of pork depresses the price, no lasting relief is to be expected to the farmers unless measures proposed reduce the rate of flow of pork from the farms.

On the other side, the simple concept of demand is that of a list of different amounts which will be bought and consumed at different prices, each lower price resulting in the purchase of a larger amount. This is not so far afield as the correspondingly naive concept of supply. It is true that the rate can adjust itself more quickly than supply as far as those consumers are concerned who are already acquainted with the commodity. But the same lowering of price which increases sales to old users is likely to result in some who were previously unacquainted with it gradually adding to the list their demands as they discover the satisfactions to be obtained.

The adjustment is not simply the immediate seeking of a new but previously ascertainable equilibrium. Rather, there is a growth into a new situation in which this simpler adjustment is only one element. The same is true of reactions on the side of the supply to an even greater degree and in a more permanent sense. The increase of a price is likely to result in additions to factories, the building of new stores, the plowing up of new land, or the production of new equipment. These additions to the productive plant, once they are made, may be expected to continue to produce the commodities for which they were intended even after the price, perchance, declines again to the old level or below it. In other words, the initial change in price has more or less permanent influences which themselves compose part of the economic situations in future times.

Here there appears a deeply significant characteristic which differentiates economic phenomena from the mechanical. The economic processes are irreversible. Economic changes propagate themselves forward into new situations which contain elements produced by those changes themselves, and which therefore prevent a turning back to the former situation and prevent identical reactions recurring should the same stimulus be applied again. The element of time which serves as one dimension in economic activity is closely associated with this characteristic.

The opportunty for forecasting comes from the fact that economic reactions contain a large element of inertia and habit. Having continually reacted in an essentially similar manner to similar stimuli, persons continue from inertia to follow paths of action which vary but little in direction or rate of progress within reasonably short periods. Modifications in the economic organism such as those suggested above do not as a usual thing change the nature of the reaction to a given stimulus, but only accelerate or retard them for a limited period of time. Forecasting is permitted by the fact that the reactions may be expected to follow a fairly definite path and to continue for an appreciable period of time.

The Time Factor in Economic Life:

The factor of time is one of the most important in economic activity, and is at the same time one of the most neglected as concerns any explicit statement of its consequences in economic literature. There has been a tendency to regard economic situations in a purely static or cross sectional manner. That is, the efforts to explain price situations have proceeded in the same manner as might be used in explaining the positions of a series of levers and weights in a mechanical contrivance in which a movement of one would be accompanied simultaneously by corresponding changes in others.

But this is far from the case in the economic world. The analogy would be more nearly correct if a time device were attached to the machine so that the moving of a lever would be followed after a time interval by the lifting of a weight, which at some still later time would result in the compression of a piston, and so on at various intervals until the original energy was dissipated.

But analogies are always more or less incomplete and unsatisfactory. It is likely that a better understanding may be gotten from some of the phenomena themselves. Each stimulus applied in economic activity may be expected to have little effect at once, but to set in motion a chain of influences which lead to subsequent events.

Thus if the price of corn is high in the winter and spring it may be expected shortly to curtail the feeding of cattle and hogs and to some extent to reduce the breeding of sows for fall pigs. It would also be expected to result in a greater amount of substitution of oats and other feeding stuffs in place of corn. Also, the high price would naturally be expected to lead to the planting of a somewhat greater acreage of corn. But this would have no direct influence on the supply or price until some six or seven months later. In the meantime, the changing appearance of the growing crop would result in the price rising or falling from week to week.

As time passes still further and the new and larger crop becomes available (assuming favorable weather conditions) the price of corn may be expected to decline as the crop comes to market or its existence becomes evident. Now with lower priced corn there will be a tendency to substitute it in place of other grains. There would be a greater number of cattle put on feed with correspondingly lower cattle prices six or seven months later. We would also expect the breeding of a greater number of sows for the following spring pig crop. This last would result in a larger amount of pork on the market after still another year, that is, eighteen months after the planting of the large corn crop. And finally if the planting of corn had been overdone the first year and price unduly depressed we might expect some readjustment and restriction in the planting in the second year.

Plainly, the corn, hog and cattle price situations just referred to cannot be considered without regard to the time factor and the length of time that has passed since the stimulus occurred and the readjustment began. In the fall, as the large corn crop begins to be harvested, the situation is not adequately described by pointing out the then existing prices, the amount of corn available for sale, and the numbers of hogs and cattle in the country.

These facts are but passing events resulting from a continuous flow of forces. In a short time each fact will be more or less changed. The corn supply will be increased as the harvest proceeds. The number of hogs will change as the earlier influences on breeding have effect, and will continue to change for some months as the number of pigs farrowed reflects the variations in the corn supply and price which occurred earlier. While the adjustment is in progress and even while its results are being felt, other stimuli, perhaps of a different nature, are pretty sure to occur and set in motion still other processes of adjustment. In this manner the economic situation remains in a continuous condition of change. Indeed, it can scarcely be said that the economic situation as separated from its developmental process has any great significance.

It may be compared to the cross section of a swiftly flowing stream, in which the water rises and falls from time to time, and from time to time overflows or recedes within its banks. If it can be imagined that in addition the current is frequently shifting and eddying from side to side of the channel, then a cross section of the stream at some particular moment may become a fairly good analogy of the economic situation of a given time. A study of unconnected cross sections of our economic world can give no more adequate conception of the forces at work and their probable outcomes than could a similar series of cross sections of the hypothetical stream taken without regard to the conformation of the channel above or below or the amount of fall per mile.

If a concept can be gained of economic situations as developmental phases in the continuous shifting of forces, we shall be much nearer a real understanding of the problem. A shifting flow of forces operating with time as one of its dimensions is a concept which we are not likely to grasp with any degree of assurance, if indeed we are really able to grasp it at all. But the effort will certainly be worthwhile, for the essence of economic activity and the source of the multitude of situations will be found to lie here and not in the mechanistically conceived and isolated cross section.

If an analogy were to be sought for economic readjustments it would more readily be found in biological forms than in mechanical ones. In fact, economics is, in a broad sense, biological. The forces which dominate buying and selling are the forces of human life. The economic organism itself displays many of the characteristics of a growing biological form. Its form is given it by continuously operating forces. Each situation is a developmental phase which will in some degree be outworn after the lapse of a little time. Further, each adjustment in response to changed conditions may be expected to have continuing effects in one direction, or in several, which propagate themselves forward into the future and which, as in biological organisms, modify the form of that generation of economic institutions as they develop.

This discussion of the nature of the economic processes may seem unduly long. But it is hardly to be expected that successful devices for the measurement or for the anticipation of price or other phenomena could be developed or understood without understanding the general nature of the flow of economic forces and also the environment and the institutions in which they work. With such a survey we are much better able to evaluate both the devices used and the results obtained.

The Development of Forecasting Methods:

The methods that have been used in attempts at forecasting have been many and diverse. But for the most part they can be grouped into a small number of general types. Of these perhaps six may be mentioned as including the greater part of the field. These are, first, forecasting by analogy with earlier periods; second, forecasting by the projection of an existing trend; third, forecasting from the appearance of events or of combinations of events which were observed to stand in earlier periods in certain empirical relationships to the price or other phenomena in question. A fourth method of forecasting has been by the use of mathematical formulae based on quantitative but empirical relationships with related factors. A fifth method is the analysis of the economic situation generally by unquantitative methods and the deduction of the probable outcome in more or less general terms as a result of this study. What approaches another method is the use of a mathematical formula along with an analytical study of the whole situation and the qualification of the results by pointing out the probable effects of influences which it was not possible to express mathematically.

Of the general methods mentioned, the process of forecasting by analogy stands on the weakest ground. This usually consists in comparing the period under consideration with some period in the past in which the prices or other phenomena behaved in an apparently similar manner, and then assuming that the outcome of the present situation will be parallel or very similar to the earlier one. Generally little or no causal relationship is discovered to offer any reason why the outcome in question should be as predicted. The superficial similarity of movement of series is generally the only basis for the forecast. Yet in spite of the lack of demonstrable causal relationship, this method has been carried to the extent of superimposing graphically a series referring to one period on top of the graph of the same series for an earlier period, and then attempting by analogy to forecast the movement of the present series. This was done numerous times after the European war, when price series of the period were plotted along with similar data for the period after the Civil War and forecasts attempted from the resulting graph.

This type of forecast is based on no particular reasoning except the assumption that there have been similar appearing movements of prices in various epochs, and on the unsupported assumption that "history repeats itself." In every attempt at interpreting events or in projecting their consequences into the future there are implied assumptions as to the identity of the influences at work and as to the manner in which these operate in accomplishing their results. The examination of the data at hand involves inferences as to causation and as to the time periods concerned. The dependability of the results rests largely on the choice of alternative inferences. Usually only a few of these are capable of being tested or demonstrated by statistical means. Others must be chosen on the basis of unquantitative observation. Here the need for sound judgment as to practical affairs, and at the same time an ability to penetrate through external appearances to the underlying flow of forces is essential to success. The simple inference that a price will behave as it did in the past certainly cannot serve as the basis for a forecast in an economic world in which no two situations result from exactly similar chains of causation.

A method quite similar to the forecasting by analogy is in the projection of existing trends into the future. This really rests on no more solid basis than the assumption that the price or other series in question will continue to change in the same direction and at the same rate as in the past. Generally, however, a more discreet use of the method has been made. It has seldom been used in unmodified form except in the forecasting of the growth of population or some other similar mass.

There is likelihood of relatively small error if the projection of trend is carried into the future for only a short period. The mass of population already existing will compose the larger part of the predicted number and its inertia lends considerable stability to its growth. Such extrapolations as to the growth of population are usually done by a "curve of growth" formula. But there is no good reason for expecting that a particular type of population will necessarily follow some particular growth curve. Also new cycles of growth may at any time be initiated by some new discovery in industry or science. The application is therefore safe only for a very limited future period. When an attempt is made to project a logistic curve for some decades or a century into the future the proceeding ceases to have any real or even probable significance.

The third method of forecasting mentioned was from observation of empirical relationships between events. Many of these have been observed and used in an effort to forecast without resorting to mathematical or analytical methods. Thus it has been observed that a rapid rise in interest rates precedes a decline in security prices and a decline in business activity, and that stock prices have generally risen during periods when rates on ninety day loans were lower than on long time high grade bonds.

It has been observed that the rates of activity in certain sensitive industries have forecasting value. The fluctuation in the production of pig iron has been used in several statistical services and in several forecasting attempts. Efforts have been made to demonstrate that stocks and bond prices would reach a high point fourteen months after the preceding low point in the production of pig iron and would reach a low point eighteen months after the high point in a period of increasing pig iron production.

Most of the statistical services make use of what are called business barometers. These are constructed by combining various series which seem particularly sensitive to business changes into a composite which indicates the probable combined effect of the group of influences. The Brookmire Economic Service uses a barometer composed of data on the physical volume of production of basic industries, the ratio of imports to exports, the turnover of bank deposits, and commercial paper rates. The direction of movement of the composite of these factors and the location of the curve with regard to a normal line is interpreted as forecasting the coming changes in commodity prices. "When the composite of these factors crosses the neutral zone from a favorable to an unfavorable position the upward movement of commodity prices as shown by Bradstreet's Index will, on the average, reach its highest point five months later. The same lag prevails between a favorable crossing of the neutral zone and the bottom of a downward price movement[1]."

In the Harvard Economic Service three curves are used. One of these represents factors related to the amount of speculation, a second indicates the activity of business, and the third represents the rates of interest. The forecasting in this case is partly by the direction of movement of each of these three curves with regard to the other two, partly by the amount of movement, and partly by their preceding movements. Thus a fall of the curve representing speculation indicates a coming decline of the curve representing business, and this is particularly true if the curve representing money rates rises at the same time that the speculation curve declines. If the money rate curve falls while the speculation curve rises active business in the near future is indicated. Various other combinations of movements of the curves indicate revival or recession of business after longer periods of time[2].

It will be seen that the Brookmire and Harvard forecasting devices both assume a sequence of certain types of events in a recurrent business cycle. It is assumed that variations in the supply of credit occur before variations in speculative activities and these before corresponding changes in the rate of business. Both of these services make other analyses to support or verify the indications of the barometric series used. A study of influences not included in the barometers is made and is used to modify the forecasts suggested by the empirical methods. In between the critical junctures indicated by these methods, it is the analytical study of business conditions that affords the means of interpreting current events.

Many other devices are also used. They are founded on bases of various types and with varying degrees of validity, and range all the way from comprehensive measurements of the fluctuations in many lines of business activity down to the simple assumption that a depression must be followed by prosperity, and that prosperity must be followed by a depression. However, it is clear that such means as the two described above, while empirical in form, have a decidedly useful place to fill and that they may rest on a sound analysis of causal relationships.

The Use of Correlation Methods:

The empirical methods just described are adapted to the detection of turning points in business activity and the indication of direction of trends. But they give only indefinite indications of the amount of change to be expected. Naturally the question of how much the rise or fall will be presents itself shortly after the question of direction. For the handling of this problem it is necessary to discover continuously operating series which stand in a more or less definite relationship to the series to be forecast, and whose relationships to that series are susceptible to definite quantitative measurement.

For this purpose economists have borrowed from biologists the methods of correlation[3]. In the case of biological data the methods of correlation are used to measure the degree of correspondence between variations of bio-~~logical forms from~~ their norms. The observations to be correlated here are often pairs of measurements of the same individual, such as length and thickness of stem of the same plant. In economic data, however, no such logical homogeneity of data is possible. The paired measurements to be correlated do not represent different aspects or measurements of the same thing, but at best measurements of different elements in the same situation, as the price of wheat with the price of corn at the same time. Frequently the two factors to be correlated may not pertain to the same time. A time lag may be introduced and a series of prices correlated with some other series of preceding or following months.

The method of correlation is, therefore, not quite at home in economic data. That is, it was not designed for application to data of this type. But the mechanics of the correlation process can be applied to any body of paired observations. The question is not one of feasibility of application, but of logical fitness of the method and of the meaning of the correlation coefficients after they are derived. If sound judgment is used in the selection of the data in the first place and if the results are interpreted carfully afterward, there is no necessary reason why the method should not be made use of here as well as in other fields.

One of the first economists in the United States to use the methods of correlation on economic data was Professor H. L. Moore of Columbia University, who had borrowed it from English economists and from biologists. Professor Moore applied this method in his book, "Economical Cycles: Their Law and Cause," in 1914, and later in "Forecasting the Yield and Price of Cotton" in 1917. The earlier attempts at forecasting were made with a great deal of enthusiasm. Professor Moore's methods and the lessons derived from them are interesting as illustrations of the earlier forecasting by correlation, and as showing how the original assumptions had to be modified before any great progress could be made.

In the book on "Forecasting the Price of Cotton," Moore makes the statement: "By means of the methods presently to be described, it is possible for any person (1) from the current reports of the Weather Bureau as to rainfall and temperature in the states of the Cotton Belt, to forecast the yield of cotton with a greater degree of accuracy than the forecasts of the Department of Agriculture, and (2) from the prospective magnitude of the crop, to forecast the probable price per pound of cotton with a greater precision than the Department of Agriculture forecasts the yield of the crop."

Moore then proceeded by multiple correlation methods to discover the relationships between accumulated rainfall surplus or deficiency and accumulated variations in temperatures and the yield of cotton. A formula was derived whereby yields might be forecast as early as May, but with increasing probability as the season advanced. The method necessarily assumed that the relationships were linear. That is, it was assumed that a deficiency of two inches of rainfall would have just half the influence on the cotton crop of a deficiency of four inches, or a third as much as a deficiency of six inches.

The next step was to discover in the same manner the relationship between the yield of cotton and the general price level on the one hand and the cotton price on the other. From this problem a formula similar to that used in forecasting the yield was derived for the forecasting of the price.

The optimism with which Moore stated his first conclusions was reduced later when it came to be realized that not "any person", but only experienced persons with good judgment could be expected to exercise the necessary discrimination in deciding which series to correlate; and that both judgment and experience were much needed afterward in the interpretation of the results of the correlation problem. Also, it soon became clear that the correlation coefficients measured only the degree of association between factors in the past. Thus the obtaining of high coefficients and of a close adherence between the estimates and the actual prices in the past period on which the study was based, were poor criteria of the degree to which the same formula would prove satisfactory in future periods. It was over optimistic to state that "both the yield and the price of cotton, therefore, are so much a matter of routine that they admit of prediction with a high degree of precision."

Yet the method used seemed the only feasible one for the problem at hand. The only way in which future events could reasonably be predicted was by discovering relationships which obtained in the past and which it could reasonably be assumed would continue to obtain in the future. If the forces which made prices or yields could not be determined from past data, or if their influences changed whimsically from year to year the problem was hopeless. A rational analysis of the problem indicated that these basic assumptions were well founded. This being the case, the problem had two aspects. One was the selection of the most significant from among the many series of data concurrent with the prices or yields in question. This was a problem which called for a high degree of sound judgment and a wide acquaintance with the field under investigation. Moore, who himself possessed this qualification amply, overlooked this problem and appeared to assume that it was largely a routine matter requiring no great ability.

The other problem was the development of a method capable of discovering truly or with a close degree of approximation, the relationships which really did exist between the independent series and the series to be forecast. For measuring average relationships between associated factors the method of correlation was the only one at hand and although it was originally devised for a different sort of use, still with reasonable care and particularly with intelligent interpretation of the results there was no reason why it should not do satisfactory service here. Moore deserves considerable credit for adapting the method to economic uses and making known its possibilities.

Shortly after the work done by Moore on cotton yields and prices, the Harvard Committee on Economic Research began an elaborate study of relationships between pairs of economic series, using both the simpler methods of observation and also the correlation method. However, the Harvard Committee proceeded more cautiously than Moore in the discovery of relationships and in their interpretation. Some of the implications of the correlation method as applied to economic data were called in question. A little later it was realized and pointed out that the time influence in such series prevents the consideration of economic variations of this type as random fluctuations which may be expected to assume a normal distribution of each series about its norm. This being the case the standard deviation and the probable error, which in the study of biological data are used to test the dependability of the conclusions, here cease to have any definite meaning. The only way of testing the results then is by observation and comparison with the actual phenomena as these materialize. Clearly significant mathematical tests are not yet developed.

The Harvard Committee added no new methods in the derivation of forecasting formulae. But it did perform a valuable service in the development of methods usable in the earlier stages of analysis. Improved methods were developed for the discovery of secular and seasonal trends, and in the derivation of simple correlation coefficients and the discovery of lag between time series. The work of Professor Warren M. Persons of the Harvard Committee was highly valuable in the development of these methods and also in amassing a large volume of information regarding characteristics of and relations existing among series related to a large number of our most important industries. The first volume of the Harvard Committee's publication, the Review of Economic Statistics, appeared in 1919 with a description of the methods used and an analysis of 24 important series. Others were taken up as rapidly as time permitted. With this information as a background, and by a thoroughgoing technique of analysis, the Harvard Committee has been able to render a valuable and highly dependable service in the interpretation of current economic events.

If the Harvard Committee was chary of the use of multiple correlation methods, others were not so backward. Shortly after 1920 these methods began to be used by various economists in efforts to forecast prices. In 1922 there appeared a study by Dr. Holbrook Working of the University of Minnesota, on "Factors Determining the Price of Potatoes in St. Paul and Minneapolis[4]." In this study a forecasting formula was obtained by using the regression formula from a multiple correlation problem much as was done by Moore in his study of cotton yields and prices. The factors used by Working included the yield per acre, changes in the general price level, and the long time or secular rate of change in the prices resulting from the growing demand for potatoes and the fact that the production increases less rapidly than the demand.

By means of the formula mentioned, estimates of the prices of potatoes were made annually for the crops from 1902 to 1920. In order to obtain estimates of the monthly prices the yearly estimates were multiplied by indexes of seasonal price variation. The average error in the estimates of yearly prices was 9.5 per cent of the estimates. The estimates made in the following years, however, showed wider discrepancies.

In 1923 and 1924 circulars were issued by the New Jersey Agricultural Experiment Station adapting the method and formula of Working to the estimation of the price of New Jersey potatoes and sweet potatoes[5].

In 1925 there appeared four studies of hog prices by the multiple correlation method. The first one, by G. C. Haas and M. J. B. Ezekiel, developed a formula for forecasting prices of hogs six months in advance by the use of six factors[6]. The second, by Charles F. Sarle, permitted forecasts at three months from four factors[7]. In August of the same year a forecast of hog prices for the coming eleven months was published by Ezekiel in The Agricultural Situation, a monthly publication of the Bureau of Agricultural Economics. As the actual prices materialized later they conformed to the general directions of movement forecast, but during five of the months in question deviated from the values of the forecasts by about $2.00 per hundred pounds, apparently from an inaccurate forecast in one of the factors used. The fourth hog price study by Sewall Wright involved influences on the corn crop and its prices and on the supply and price of hogs only[8].

In 1926 the present writer published a study of the prices of beef cattle in which monthly forecasts made for the period from 1922 to 1926, from data available six months in advance, differed from the actual prices by an average of 3.8 per cent of the value of the estimates[9]. In this case, as in most of the others described above, the methods of linear multiple correlation were used. The factors correlated represented related series expressed as percentage deviations from their respective secular trends. The forecast as obtained from the regression formula is therefore expressed as a percentage of the ordinates of seasonal and secular trend projected forward from the period on which the formula was based. This series of forecasts will be discussed more at length as an illustration of a method a few pages farther on.

Curvilinear Correlation Methods:

A shortcoming of the method of multiple linear correlation was soon discovered in the deviations of the forecast from the actual values, which in a good many series showed a tendency to arrange themselves in a more or less definite curve about the straight line of regression. Common sense suggested that many of the relationships between series were of a curvilinear rather than a rectilinear nature. Thus it might be expected that the increases in yields of crops from successive applications of fertilizer would not be at a uniform but at a decreasing rate. A twenty per cent shortage of the corn crop might be expected to have more than twice as much influence on the price as a ten per cent shortage. Similar curvilinear relationships might be expected to appear in nearly any correlation problem. If the correlation method were capable of expressing relationships only in terms of straight lines it might often be expected to yield forecasts wide of the mark or completely misleading.

A method of curvilinear correlation was developed by Mordecai Ezekiel and published in 1924[10]. This method involves the computation of the linear multiple correlation regression equation as a first step. From this point on the mathematical devices are largely dispensed with. The deviations of the actual prices from those estimated by the regression equation are then arranged with regard to the magnitude of each independent variable in turn, and are plotted graphically as deviations from the linear regression line. If a curvilinear relationship is present it may be expected to make itself evident by the plotted points assuming a curvilinear arrangement. An approximate curvilinear regression line may be drawn freehand through these points. New values of the estimated price may then be read from this curve and closer approximations to the true curve of regression made from the differences between these second estimates and the true prices.

By this method Ezekiel and others have been able to obtain much higher correlation between the forecasts and the actual prices. This, of course, applied particularly to the period on which the problem was based but the results obtained in forecasting future events were also improved. This method has been used in various correlation problems of the ordinary frequency distribution type. In the estimating or forecasting of prices the study by Haas and Ezekield on "Factors Affecting the Price of Hogs," a similar one by F. F. Elliott on "Adjusting Hog Production to Market Demand," and the study on "Factors Related to Lamb Prices," by Ezekiel, are the better known at the present[11].

There is little doubt that a method of curvilinear correlation will be the necessary basis of forecasting for most series. There are, of course, some cautions to be observed in its use. In the first place, care is needed in the discovery of the relationships to be sure that the body of data at hand is sufficient to show the true conformation of the curves. This is especially true of the extreme sections of the curves of regression, since there is a strong tendency for the values to be bunched toward the middle of the array with only a few cases at the extremes. In the case of linear correlation this is not so serious because the middle portion of the data is more likely to define the direction of the straight line.

A second caution which applies here somewhat more than in linear correlation is that care should be taken to see that the inflections of the curves of regression are not determined solely by data relating to brief periods of time in which abnormal economic conditions obtain and cause extreme variations in the factors concerned.

What correlation really shows and the ways in which it falls short should be kept in mind both in making the study of relationships between factors and in interpreting the results. The method is adapted simply to measuring the degree to which attributes are associated in a given mass of data. These relationships are discovered by concomitant variations of the values in question as they relate to identical items. That is, if the prices of hogs and of corn are both low or high in the same months and if a rise in one is accompanied by a rise in the other in the same month, some sort of a causal connection is thereby suggested and the degree of relationship is measured by the coefficient of correlation.

Although the presence of correlation (positive or negative) suggests causation it does not by any means establish its presence nor indicate its direction. Thus a perfect correlation would be found between the occurrence of day and of night. As far as the correlation coefficient is concerned it might either be assumed that day causes night or that night causes day, but there is nothing to indicate from the mathematical solution that both are caused by the revolution of the earth. Inference must be used from this point on in reckoning the probable direction and nature of the basic processes of causation.

In the study of economic data we find that we have a network of interrelated forces. In measuring the influences of these we have to use as measurements statistical series showing volumes of production, rates of activity, prices, etc. But each series, such as a price, which is capable of expressing the influences of one of the forces at work is pretty sure to reflect to some degree also the influence of various other forces. Thus the price of corn, which is largely a reflection of the supply of corn available, is also influenced by the number of hogs in the country which constitute to a large degree the demand for corn, as well as various other influences. In many cases, therefore, there arises a perplexing problem in the selection of the series which will best represent the influences it is desired to study, and in the avoidance of series which are too much under influences not perhaps pertinent to the question at hand. If care is not exercised here it is quite possible that the investigator may obtain spurious results which reflect influences it may be desired to omit, or give a false impression concerning those in question. Here again the satisfactory solution of the problem is seen to depend to a large degree on the amount of judgment possessed by the investigator.

Forecasting by Use of a Demand Curve:

It has been mentioned that there are many influences on prices that either cannot be expressed in quantitative terms or else occur at such rare and irregular intervals that they cannot be used as factors in a correlation problem. Thus some unusual occurrence, such as the outbreak of a war, a flood in a producing section or the outbreak of an epidemic among a class of farm animals, may occasion a change in the supply conditions which could not be handled by the usual methods of correlation, but which would have to be taken into consideration in any worthwhile forecasts of prices.

In most periods of unsettlement it seems likely that the demand condition as expressed in terms of the prevailing price level may be more likely to remain unchanged. This is particularly true if the article in question is some necessary article of food or basic raw material such as the typical farm product. It may be possible to forecast the production or supply of the crop in question by the method of correlation with arbitrary adjustments for the influences not included in the formula, or by various methods of crop forecasting. Then in order to forecast the price it would be necessary to know the relationship between supplies of different sizes and the prices at which these quantities could be sold.

In order to make use of whatever estimate might be made of the coming supply, a few efforts at forecasting have started out with a study of the basic demand conditions. These have attempted to construct a demand curve or schedule of the prices at which each possible quantity of the commodity in question could be sold, after correction is made for variations in the level of prices. An example of this method in forecasting hog prices is explained by Mordecai Ezekiel in the Journal of the American Statistical Association for March, 1927[12]. Here it was found in comparing forecasts obtained by means of the demand curve with others made by means of a definite formula derived from a correlation study, that the formula gave better results in the test period, but the shortcomings of the demand curve forecasts seemed due to errors in a government forecast of market supplies, which more careful methods in crop forecasting might be expected to correct. The possibilities of the demand curve forecasts thus seem not to be fully exploited as yet. It is likly to prove a valuable means of obtaining comparable figures for checking up on the forecasts made by formula.

The demand curve method has also been used in connection with other commodities. An interesting explanation of its possibilities and a report on the earlier stages of a study involving its used by Holbrook Working in the study of wheat prices is to be found in the July, 1927, issue of the Journal of Farm Economics[13]. Here it is pointed out that even for wheat the information as to the probable yields in most countries is often quite inaccurate and that the data on the carryover is subject to considerable error. Until these inaccuracies are largely overcome no method of forecasting could be expected to yield highly accurate forecasts.

There is a further difficulty in the demand curve pointed out by Working in that the exact form of the demand schedule is not known and that up to the present it is only possible to say that such a schedule lies within certain reasonably narrow limits. Further, it cannot be assumed that the schedule would remain fixed in its shape or location even if it could be discovered exactly for a given period of time. Changes in the per capita consumption of wheat, as well as of other products, are clearly occurring constantly. In such staple food products these changes may be assumed to be at a relatively slow rate. But this is not necessarily true of all products, and constant watchfulness would be necessary in the use of this method as well as of others to be sure that the basic relationships have not shifted between the time they were discovered and the time the application is made by their use in a forecast.

Forecasting by Analysis of the Economic Situation:

The fifth method of anticipating economic changes that was mentioned at the beginning of this section was by a careful rationalistic study of the factors of the economic situation together with its background. Again it must be remembered that an economic situation is simply a cross section in a continuous flow of forces. Each element of the situation may be regarded as actually in motion, but in our cross sectional view is caught or photographed in its position at that particular time.

The analysis which is to serve as the basis for some prediction as to future events must uncover each important line of influence at the time as indicated by prices, quantities of production, etc. But this alone will indicate little as to the future. It will be necessary also to have some means of judging from observation of past events what are the normal relationships between the elements of the situation toward which they will eventually gravitate. Even more important for the forecasts of the immediate future and more in conformity with the dynamic or biological concept of economic progress, it will be necessary to discover the direction of change through which each factor has been passing and the rate at which it has been going.

A comprehensive acquaintance with the organization of an industry and the rate at which it is capable of adjusting itself to a given condition will of course be an essential in making any predictions. Next it will be necessary to obtain the fullest possible description of the situation and of the lines of trend which the factors concerned have followed in bringing it about. Statistical information will furnish the basic material with which to work, and will yield measurements of seasonal or longer time trends, the usual relationships between factors, approximate amounts of time which usually elapse between initial stimuli and the various phases of reaction of different factors to them. Inference must be resorted to in respect to the causal relationships between factors in many cases. Unusual elements existing in the present situation must be given consideration and some judgment arrived at as to their probable influences on the course of development in question.

All of this process of analysis will be seen to proceed along no rigidly fixed path. There are no formulae which may be applied, at least in arriving at the final conclusions. The only methods prescribed are those flexible and often indefinite methods of critical scientific procedure in accordance with which any economic study needs to be made. The conclusions will necessarily be stated in rather general terms as to amounts of change. The same body of information might well serve as a basis for widely varying conclusions by different workers, depending on the importance attached to various factors. The conclusions here even more than with other methods must be taken with some regard to the investigator who has formulated them.

On the other hand, the method of analysis has a great advantage in its unlimited flexibility. Being tied down to no limited mechanical method, it is capable, in the hands of a highly competent investigator, of being adapted to any combination of circumstances that may arise. It is not necessary by this method to ignore any line of influence.

Composite Methods:

It has already been implied that each of the methods of forecasting described has its limitations and that some of them are rather serious. If correlation it used it may be found that some important lines of influence are likely to prove unmanageable and must be left out of consideration in this rather mechanical method. If the demand schedule method is used there may be serious uncertainty as to some element of the supply on which the forecast rests, or the location of the demand schedule may be misjudged to some degree. In the use of the method of analysis there is always uncertainty as to the weight that should be given to each factor concerned. Further, unless the situation seems almost identical with various others in the past, the conclusions must be phrased in such indefinite terms that it may often seem to have but little value.

A little reflection, however, will show that a thorough process of analysis is implied in the use of the correlation or in the demand curve method. These are not purely mechanical, but involve a careful study to discover which are the significant factors of causation and how much dependence can be placed in each of them in the explanation and consequently in the forecasting of changes in prices or other phenomena. Before a satisfactory formula can be put into use, it is necessary to examine a large mass of related data to discover the significant series, the degrees of time lag or lead between them, and the closeness of their relationships to the price in question. These series must then be placed in the most significant relationship to the price series and forecasts tried and tested out by comparisons with the actual prices. The differences between the forecasts and the actual prices must be compared to the amount of variation which existed between the original series and its norm, and the reduction in the standard deviation taken as one of the more significant criteria of the success of the effort.

As a result of this process of analysis it may be said that the assumptions and implications on which the forecasting formula rests are given a test which is generally more rigorous than any which could be applied by the unquantitative and less definite methods of a purely rational and unmechanical analysis. Without much doubt the weaknesses of the formula may be expected to come to light. There is therefore an advantage in combining into such a form any factors which are amenable to such methods.

The factors which cannot be expressed quantitatively in a regular and continuous statistical series must be dealt with by the less exact method of rational analysis. There will nearly always be found influences which cannot be handled by either of the two methods just discussed. But these cannot be ignored because at various times they may grow into forces which dominate the situation. The predictions of the mathematical methods must constantly be qualified in view of this unmathematical process of observation and analysis.

At the same time that forecasts are made by the method of correlation it would be desirable to make parallel predictions by the demand curve method. These may be used as a check on the other series of forecasts. It is likely also that some factors which could not be handled by the correlation method may be taken into account by estimating their probable effects on the size or yield of the crop or other supply and then interpreted into price by means of the demand schedule. We would naturally be prepared to place much more dependence on forecasts arrived at independently by two more or less independent methods than on the forecast by one alone. And where the two disagree we would be prepard to regard each with skepticism and to fall back on rational analysis.

The ideal method of forecasting will thus be a composite of all the methods available, so used as to furnish checks on each other, and to take each factor into consideration by the method most appropriate to it. Further, the results must constantly be compared to the actual prices as they materialize and the methods used revised to eliminate defects, or include new data.

An Example of Price Forecasting:

In Fig. 1 is shown a comparison of two series of forecasts of cattle prices made six months in advance. One is based on a formula obtained by linear correlation, and the other by curvilinear methods. The dotted line, based on linear correlation, will be observed to follow the actual prices closely only for part of the period concerned, and then to indicate the direction of the price movement generally, but not the amount.

Both of these forecasts were based on the same series of factors related to cattle prices. One of these is the price of corn taken seven months before the cattle price in question, since corn is the most important single material used in the fattening of cattle. The typical feeding period is about six months, but the highest correlation is obtained with a seven months lag. This is probably because the farmers who feed cattle are influenced in their decisions by the outlook for corn at a time slightly before they buy the feeder cattle.

A second factor was the margin or difference in price between feeders and fat cattle six months in advance of the cattle price. The amount of premium obtained on the fat cattle over the feeders is second only to the price of corn as a determinant of the profit in cattle feeding. Its highest correlation with cattle prices was obtained at six months.

A third factor was an index of the condition of the forage and pasture crops in the corn belt seven months before the cattle price. The seven months lag seems explainable on the same grounds as in the case of the corn price.

A fourth factor is the condition of the ranges in the western states in which the feeders are raised. This factor showed the closest relationship —an inverse one—ten months in advance of the cattle price. It should be remembered that a change in the condition of the ranges, as from a rain

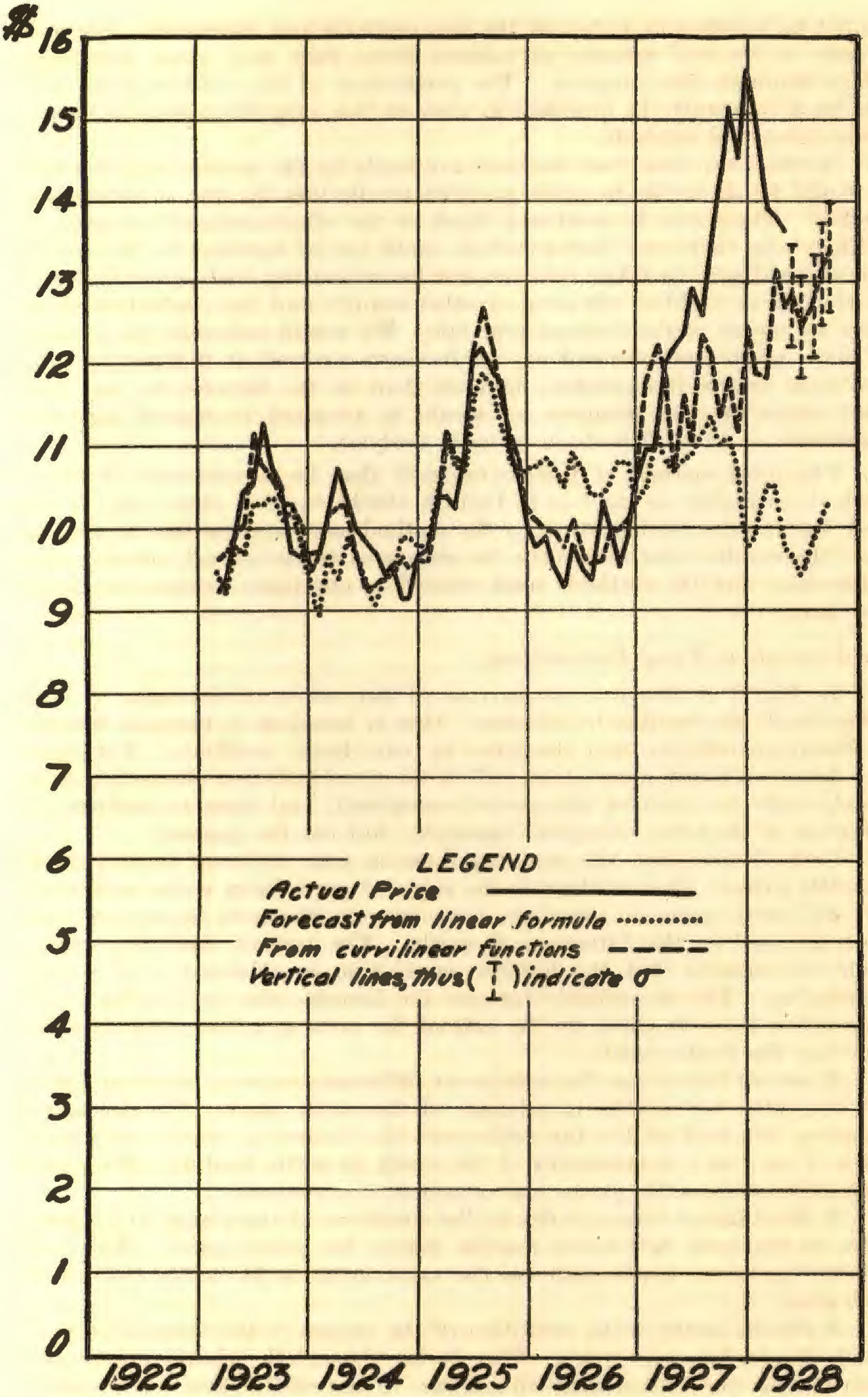


Fig. 1. Cattle prices forecast from data available six months in advance.

or a drought, continued to influence the condition of the western feeders and other cattle for some time and does not reach its maximum effect until a somewhat later date.

The fifth and sixth factors have to do with the shipment of feeder cattle from the stockyards into the feeding sections. One of these consists in the number of feeder cattle shipped per million of population, the figures being taken six months before the cattle price in question. The other is intended to represent the changing number of cattle in the country and consists of the number of feeders shipped during the twelve months ending six months before the price to be forecast.

In discovering the relationships on which forecasts could be based percentage deviations of the factors from their respective averages were put in a multiple correlation equation with the prices of cattle at the periods in the future named. From this equation, or rather from the series of simultaneous equations involved, the following prediction formula was derived:

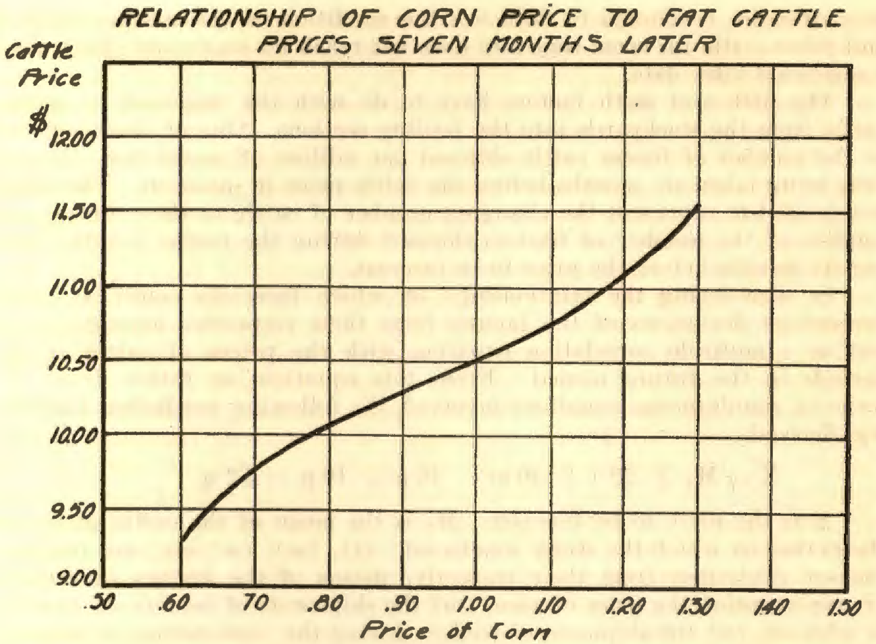
$$\overline{X} = M_x + .22\ i + .40\ m - .16\ n - .10\ p - .52\ q.$$

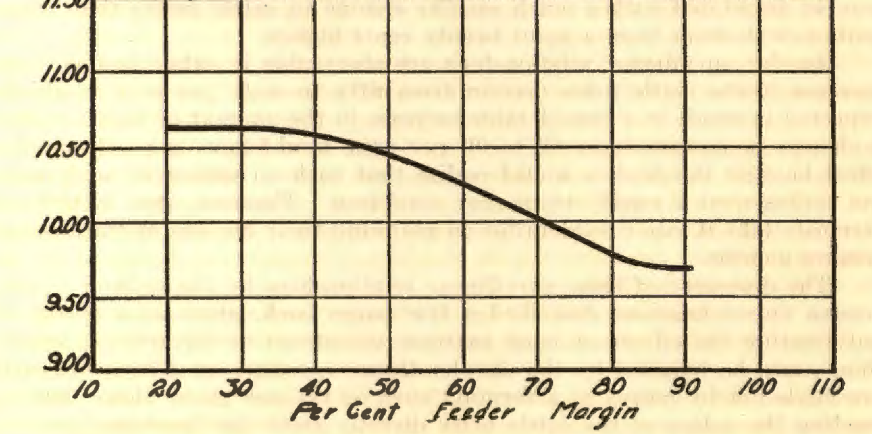
$\overline{X}$ is the price to be forecast. M_x is the mean of the cattle prices in the period on which the study was based. (i), (m), (n), etc., are the percentage deviations from their respective means of the factors concerned, (i) representing the price of corn, (m) the shipments of feeders six months in advance, (n) the shipment of feeders during the year ending six months before, (p) the feeder margin, (q) the condition of the western ranges, and (r) the condition of the forage and pasture crops in the corn belt.

It will be observed that while the forecasts obtained from this formula conform fairly well to the actual prices from 1923 to 1925, they depart from them more and more widely in 1926 and 1927. This seems to be partly because of wide deviations of some of the factors from their means during the later years. The relationships found by the linear correlation method simply represent the average change in the cattle price associated with a unit change in the independent factor. Actually, as is shown in the curve in Fig. 2, a rise of ten cents in the corn price from its average may be associated with a much smaller change in cattle prices than a ten cents rise starting from a point twenty cents higher.

Similar curvilinear relationships are observable in other factors. An increase in the cattle price margin from fifty to sixty per cent would be expected to result in a considerable increase in the amount of feeding. But a change in margin from 80 to 90 per cent would have a much smaller effect because the feeders would realize that such an unusually wide margin represented a purely temporary condition. Therfore, they would not seriously take it into consideration in planning their feeding operations for coming months.

The discovery of these curvilinear relationships by the method of successive approximations described a few pages back, gives us a means of anticipating the effects of more extreme variations in the related factors than could be handled by the simpler linear correlation. These forecasts are made not by means of a formula such as the one given above, but by reading the values of the cattle price directly from the functional curves, such as are shown in Figs. 2 and 3. Thus if the price of corn is 90 cents, we find by interpolation on the functional curve in Fig. 2 that cattle may


RELATIONSHIP OF CORN PRICE TO FAT CATTLE PRICES SEVEN MONTHS LATER
Cattle Price
$
12.00
11.50
11.00
10.50
10.00
9.50
9.00
.50
.60
.70
.80
.90
1.00
1.10
1.20
1.30
1.40
1.50
Price of Corn
Fig. 2.


RELATIONSHIP OF FEEDER MARGIN TO FAT CATTLE PRICES SIX MONTHS LATER
Cattle Prices
$
12.00
11.50
11.00
10.50
10.00
9.50
9.00
10
20
30
40
50
60
70
80
90
100
110
Per Cent Feeder Margin
Fig. 3.

be expected to be about $10.35. If the feeder margin is 50 per cent, we read from the curve shown in Fig. 3 that the price of cattle most often associated with this margin is $10.45. The influences of the various factors are discovered in this same way and are combined into their composite price influence which is the forecast in question.

The forecasts from the curvilinear relationships will be observed from Fig. 1 to conform much more closely to the actual prices than did those from the linear relationships in 1926 and 1927. A noteworthy discrepancy still exists, however, from the middle of 1927 to the spring of 1928. This evidently reflects the dominance during this period of forces which were not present during the base period, and which might, not improbably, disappear after a short time. Incidentally, the divergences under these conditions may be expected to help in locating the deficiencies of the group of influences used and in this way prove highly valuable.

In this case the cause of the discrepancy seems to be in the fact that in the late spring of 1927 a large number of ranchers and farmers decided, because of the government reports of smaller numbers of cattle in the country, and continuously rising prices, that it was a good time to stock up more heavily on cattle. Consequently a large number of heifers and cows that would otherwise have been sold were kept for breeding purposes. This further shortened the receipts of cattle at the markets and resulted in the prices rising even further until the end of 1927.

A similar influence was observed in the early 1880's, particularly in 1882, when the rapid development of a demand for cattle to stock new range territory in the west, as well as a similar demand for more stock in older sections of the country, caused cattle prices to climb rapidly for about two years. For a few months in 1882 a level of prices was reached much above the years preceding or following. Much the same thing occurred during the late 1890's, particularly in 1899. But clearly these influences operate only at widely separated and irregular periods. They may be explained more by misinformation, or by over response of the farmers to correct information than to any element inherent in the situation itself.

It might be possible to discover factors of a psychological nature associated with these particular reactions on the part of the farmers. But this might be expected to prove very difficult even if data were available covering the periods in which such incidents occurred. In the present case there was no such reaction in the base period of 1923 to the middle of 1927, and no comprehensive data on several of the most important other factors in the earlier periods when these events did occur. Influences not present in the base period could not, of course, be used in forecasting in later years. This would therefore seem to be a case calling for the use of the rational analysis already discussed which should accompany the forecasts by the more mechanical methods.

A person familiar with the cattle industry and with the history of the behavior of cattle prices in past years, might have been expected to modify his forecasts in the summer of 1927. After the reports of a cattle shortage and after the farmers appeared to be holding back more than usual of certain types of breeding animals, it became possible to perceive the drift of the time, and while it would have been difficult to foretell the duration of the abnormally high prices it should have been clear that these were a matter of only a few months. At the same time, knowledge of the length of time required to increase the supply of cattle at the markets, and reports on the numbers of cattle in the country indicated that levels of prices definitely above those of the preceding years were to be expected for two or three years more.

Application and Interpretation of Forecasts:

It will be noted that the forecasts discussed up to this point have practically all referred to prices. As a matter of fact, it is in the form of probable future levels of price that we may expect forecasts to be most effective in the control of production. Each person operating a business may be expected to modify the plans of that business primarily with the aim of maximizing its profits. Thus a farmer does not reduce his acreage of corn or wheat because he believes that a smaller total crop will be wanted in the country during the coming year, but because he believes that the price and cost situation will make this an unprofitable crop for him. Further, he does not reduce an acreage merely because he fears that it, considered alone, may be unprofitable, but because he thinks that some other crop would prove more profitable or would result in a smaller loss than the one in question.

In any event, the stimulus to increased or decreased activity comes to the individual in the form of a change in prices, either realized or anticipated. With the information that the price of hogs will probably be higher than in the preceding year, the farmer decides to breed a somewhat larger number of sows and to sell less corn in the form of grain. After the pigs are farrowed there are still opportunities for adjustment between the feeding of hogs or the sale or other use of the corn. That is, the pigs may be sold in heavy weights if the price gives promise of keeping up or may be sold in lighter weights if the price declines or threatens to do so. Thus with hogs the opportunity for the use of forecasts is almost continuous throughout the production process.

But with crops the alternative must generally be decided on once and for all before planting. After that step has been taken there is little that can be done short of abandoning part of the acreage. It is true that a promise of high prices is likely to result in giving the crop slightly better care at some stages in its production, but the changes in amount of production from this cause seem decidedly small.

The significance of a forecast should not be forgotten in any effort to make use of such data. The fact that a certain price is predicted for some month does not mean that the forecaster expects the price obtained to be exactly that amount. Rather, it means that out of all the possible figures which might turn out to be the average or typical price in the month in quesion, this seems to be the most probable one. It must be remembered that as a matter of fact the forecast does not pertain so much to the future as it does to the past. The forecast which is intended to indicate the course of the price or other phenomenon for the future rests on relationships observed between it and related factors in past periods. It is made by applying these relationships to data of the recent past and assuming that they will apply to the near future in the same way as in the past period for which they were discovered.

Permanence of Price Relationships:

Of course it is not necessarily true that the relationships between the factors studied and the price in question will continue to obtain in the future to the same degree as in the past. It may be that the study did not cover all essential influences, or that new ones came into prominence which did not assert themselves in the base period. It might be expected that after the lapse of a considerable period of time, or after some economic cataclysm such as a great war, the price determining factors might be decidedly changed in their relative influences.

With these questions in mind, a price determining formula for fat cattle derived from data for the period 1903 to 1913 was used to estimate prices currently and a similar formula was used to forecast them at two months in the period from 1921 to July, 1925. The results of this experiment, reported in the Journal of Farm Economics for October, 1927, shed some interesting light on the permanence of price relationships and on the influence on them of an economic disturbance such as the European War.

"The forecasts made from data available currently were found to differ from the actual prices by an average of 6.7 per cent of the ordinates of secular trend from January, 1921, to July, 1925, the latest data available at that time. . . . . But if we average the differences by years we find them to be much larger in the two earlier and less normal years than in the later ones. The average differences are 9.8 per cent for 1921, 9.9 for 1922, 4.8 for 1923, 3.3 for 1924, and 3.5 for the first half of 1925.

"The same method was used in making forecasts at two months. The average difference in this case was 8.8 per cent. But by years the averages were 15.7 per cent for 1921, 11.3 for 1922, 7.2 for 1923, 3.0 for 1924, and 3.3 per cent for the first half of 1925[14]."

The changes in these variations seem to give some indication of the degree to which new or unusual combinations of forces dominated the cattle prices in the period just after the war. They also indicate that these unusual influences gradually disappeared during the period named, and that the old forces reestablished themselves and governed prices in pretty much the same manner as in the pre-war years. This was true in spite of some changes in the technique of cattle production and the maturing of cattle at somewhat younger ages than in the pre-war period. An appearance of considerable stability is thus given to price relationships in times reasonably free from powerful abnormal influences. But it is also indicated that in years when such unusual influences are present, resort must be had to other than mechanistic forecasting methods.

Forecasts as Centers of a Belt of Probability:

The forecast does not indicate an exact price for the month named, but rather the center of a range of possible prices within which it appears likely that the average for that month will fall. It is desirable to have some way of designating this range or belt of probability. For this purpose the standard deviation has been used in a few studies[16].

Now the standard deviation, like the correlation coefficient, is not wholly at home in series of time variables. The standard deviation measures a belt on either side of the arithmetic average which includes about sixty-eight per cent of the total number of items. For instance, if one is shooting at a mark and the gun is aimed correctly, but because of variations in the powder charge, projectile, air currents, etc., the shots are falling scattered in a normal fashion, the standard deviation as measured to either side from the mark will bound a belt within which approximately sixty-eight per cent of the shots fall. Approximately 95 per cent of the cases may be expected to fall within a range of three times the standard deviation on each side of the mark.

The standard deviation, however, rests on the assumption that the items fall in normal manner about the mean value. It cannot be assumed, however, that the items of a time series will fall about an average in the manner of a normal frequency distribution. A time series is not composed of chance variations from a norm, but of a series of unique items each of which depends for its magnitude on the place which it occupies in the sequence.

In periods of time in which no outstandingly abnormal influences are at work, it may be observed that the deviations of a time series from its norm do assume a form essentially that of the normal frequency distribution. In the usual period, therefore, the standard deviation may be used empirically with fairly good results. It is in this empirical manner that it is used in Fig. 1 to indicate the range within which we would normally expect about 68 per cent of the prices to fall during the months from July to October, 1928, for which forecasts are made.

An illustration may make the use of this device more clear. For the month of June, 1928, a forecast of $12.61 is made as the most probable price for 1200 to 1500 pound steers at Chicago. The standard deviation of the actual prices from the forecasts in the base period was 4.7 per cent of the value of the forecasts. Therefore, if the same influences determine the prices with the same proportionate weights in June, 1928, we would expect that there would be 68 out of 100 chances that the actual price would be within 4.7 per cent of $12.62, that is, between $12.03 and $13.21. In the same manner we would expect that there would be 95 chances out of 100 that the price would fall within a range of three times 4.7 per cent or within $1.79 of $12.62. Thus if the same conditions apply as in the period for which the forecasting relationships were discovered, there are 95 chances out of 100 that the price will lie between $10.83 and $14.41, and 68 chances that it will be between $12.03 and $13.21. When the forecasts are thus viewed in the light of the deviation which experience shows between them and actual prices, they lose much of the rigid and positive appearance with which the popular mind is apt to invest them.

The amount of range permitted by the standard deviation depends largely on the variation in conditions in the base period. This degree of variability determines whether the formula is sufficiently comprehensive and true to life. The width of the standard deviation will also depend on whether it is computed on the deviations between the actual and forecast prices in the base period only, or whether it includes the forecasts made by extrapolation in later months as well. Influences not found in the base period are likely to make themselves felt as time goes on, thus increasing the amount of deviations.

For the illustration given above, a standard deviation was computed from the differences between the actual and forecast prices to include the months from November, 1927, to May, 1928, as well as the base period. This standard deviation is 8.7 per cent of the values of the forecasts as compared to the 4.7 per cent for the differences in the base period alone. This may be considered as an extreme change in the range and was caused by the pronouncedly unusual influences which dominated the cattle market from the spring of 1927 to the corresponding months in 1928.

As pointed out on an earlier page, such changes in the conformity of the forecast to the actual prices may be expected at irregular but widely separated intervals. When they occur there are two possible courses of action for the research worker. He may either explain them as unusual aberrations which it is not worthwhile to attempt to include in the formula used. Or he may give such explanations as are possible at the time on rational grounds, but rework the correlation problems as soon as feasible and include factors which will reflect the new forces. This latter is the course of action which recommends itself most strongly.

The development of a method of forecasting a price or some other economic phenomenon is not a task that can be carried through to completion in a brief period of time and then regarded as finished beyond the need of further revision. The discovery of the normal relationships not only requires careful research, but, as a practical observation, leaves considerable doubt as to whether the influences found are really comprehensive. Therefore, verification from future developments is necessary. It will usually be found that some adjustment in the method is needed. An old factor may prove unreliable under changed conditions, and the need for new ones may be strongly felt. Constant watchfulness must be exercised to avoid misinterpretation. Frequent revision of the formulae to bring in the most recent data will be needed to make the forecasts most valuable.

It is clear that forecasts of the probable direction of movement of prices may be expected to prove highly valuable to the producers in their efforts to make the most effective and profitable use of their resources. The value to consumers by maintaining a more uniform flow of supply at more nearly constant prices would scarcely be less.

A great amount of research work remains to be done in the discovery of the normal relationships on which the forecasts in each different industry must rest. The degree of success obtained so far gives good promise both of the extension of forecasting methods to other fields and of an increasing accuracy in the industries in which it is already being applied. But there has been a decided change in the outlook of economists interested in forecasting since Moore expressed the opinion that it would be largely a matter of routine which might be done by anyone, once the formula was worked out. This is certainly not the case. A highly developed judgment and a deeply comprehensive acquaintance with an industry is needed both in the development of methods and in interpreting the results from their application. It is safe to say that the development and the interpretation of the forecasts promises to be for a long time a task for the skilled research worker and not for the individual farmer or business man.

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STUDIES IN HOME CANNING*

II. *Indices of Spoilage in Home Canned Foods***

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INTRODUCTION

In connection with an investigation of the keeping qualities of vegetables and meats canned by the hot water bath method (Sunderlin, Nelson and Levine, 1928), a routine examination was made of each of 2990 jars. These examinations served as bases for detection of spoilage in the canned products. Obviously there must be some criterion or criteria for separating those jars considered spoiled from the others. In this study all jars of canned foods which showed evidence of bacterial growth or changes due to bacterial growth by organoleptic, bacterial or chemical tests were pronounced spoiled. This is a more severe criterion than that used in most of the other reported investigations of spoilage in home canned foods.

HISTORICAL

In studies in home canning of foods, various criteria have been used to detect spoilage.

Biester, Weigley and Knapp (1921), using taste as the criterion, regarded those jars which were considered inedible by the majority of the judges as spoiled. They stated that in several cases the degree of spoilage was so slight that the judges disagreed.

Skinner and Glasgow (1919) made bacteriological examinations of jars of canned asparagus after nine months storage. They made aerobic and anaerobic cultures from both the unspoiled and the spoiled food. They found that jars that were apparently keeping might contain living organisms, and that spore forming aerobes such as those of the *B. subtilis* group, *B. vulgatus* and *B. megatherium* were the types most often found associated with the spoilage of asparagus. They also noted that those jars treated with vinegar (when canned) showed the presence of organisms less frequently than did those not so treated.

Normington (1919) in the examination of home canned peas noted physical appearance, titrated for acidity, made aerobic and anaerobic plate counts as well as direct microscopic counts, and made two sets of gelatin agar shakes, one of which was heated to 80° C., while still liquid, to determine the presence of spores. She later made a chemical analysis of two cans of spoiled canned peas which had been inoculated with "Bacillus A",

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an organism previously isolated from spoiled canned peas. The appearance and odor of the inoculated peas were almost normal, but the peas were found to be softer and more watery on mashing. The juice was cloudy. The taste of the peas was somewhat flat. Gas analyses showed a high percentage of carbon dioxide and hydrogen and a small percentage of oxygen and nitrogen. An increase in acidity was shown by both titration and pH. Creatinin and ammonia were increased in the spoiled peas. She suggested that the determination of creatinin and ammonia, especially the former, may serve to detect bacterial decomposition in canned peas.

Edmondson, Thom and Giltner (1922) noted the condition of the jar and its contents, the hydrogen ion concentration, the types of bacteria present in the juice as determined by Gram stained smears, and types of bacteria found in aerobic and anaerobic cultures. Only part of the jars canned were opened and subjected to these tests. The remaining jars were examined for physical evidence of spoilage such as gas production, foul odor, and disintegration of the material. They stated that "the correlation between these findings and the bacteriological results was so close that for practical purposes the material could be judged as spoiled or not by a physical examination of the jar." They applied the word spoiled to those jars in which there was an active growth of anaerobic bacteria.

The examination of commercially canned foods and the detection of spoilage in foods in general are so closely related to the subject in hand that they are briefly reviewed in the appendix of this paper.

EXPERIMENTAL

METHODS

One jar from each boiler of canned foods was opened as soon as possible after canning and examined bacteriologically and chemically to serve as a basis for comparison later. The jars stored were observed at regular intervals and any showing evidence of spoilage were immediately opened and examined. The remaining jars were opened at the termination of the experiment, after a storage period of five to nine months.

The spoiled jars included 63 of asparagus, 166 of string beans, 49 of Swiss chard, 190 of sweet corn, 26 of tomatoes, 75 of beef and 46 of pork, making a total of 615 jars. These were generally from underprocessed materials, although in many cases other factors such as the use of vegetables which had been held for several days and storage of the processed materials at high temperatures exerted a determining effect.

The observations and tests made on each jar in the routine examination were as follows: appearance, suction, odor, pH, titrable acidity, amino N and ammonia as shown by Sörenson formol titration, microscopic examination of the sediment, bacterial counts on plates at 37° and 20° C., and in dextrose broth tubes. Under appearance were included gas production as evidenced by bubble formation, bulged caps or broken seals; cloudiness of the liquor; sediment; color of the product; consistency and disintegration; and formation of patches of growth.

Suction or vacuum was determined by quickly pulling the rubber from beneath the Mason cap with a pair of pliers. A sharp sound due to the inrush of air was taken to indicate a satisfactory vacuum. A spurting of the liquid from the jar indicated pressure, while an absence of sound was recorded as "no suction". When a perfect seal was secured during the canning process the new Mason caps were drawn in by suction (due to partial vacuum) as the products cooled. Any raising or bulging of the caps was indicative of pressure.

As quickly as possible after the cap was removed, the odor of the product was noted. Any apparent abnormality was recorded.

The pH of the liquid in the jars was determined colorimetrically. Phenol red, bromthymol blue, chlorphenol red, bromcresol green, methyl red and bromphenol blue, were the indicators found most suitable.

Total acidity was determined by titrating 10 c.c. of the liquid, which had been diluted with 10 c.c. of boiled neutralized distilled water containing 1% phenolphthalein, against N/20 NaOH. The figure obtained divided by two gave the percentage of normal acid.

The liquid which had been neutralized by the titration for total acidity was used for the amino nitrogen and ammonia determinations. (Sörenson formol titration.) To this liquid was added 10 c.c. of 50% neutral formaldehyde which contained 1% phenolphthalein. It was then titrated with N/20 NaOH, and multiplied by the factor 70 to obtain the number of milligrams ammonia and amino nitrogen per liter.

Gram stains were made of the sediment in the jars, obtained by means of a sterile pipette. In the microscopic examination of the stained mounts, the relative number and morphology of the organisms and the presence of spores were notd.

Bacterial counts were determined at 20° and 37° C. on Bacto nutrient agar. In the case of tomatoes, tomato agar was used for plating. Special media made from beans and corn were used at first for the growth of organisms from the spoiled beans and corn, but no advantage was observed.

Dextrose broth with Andrade's indicator in Durham fermentation tubes was employed for evidence of acid and gas production and to allow for growth of some organisms which would not grow on aerobic plates. The tubes were heated for 10 minutes in boiling water and quickly cooled before transfers of 1 c.c. of the liquid from the jars were made into them. The presence and character of growth and acid and gas production were noted after two days and again after one week at 37° C.

In determining which jars of vegetables and meats were spoiled the following criteria were taken as positive evidence of spoilage:

(1) any marked change in appearance.
(2) any change in odor.
(3) a broken seal, bulged cap or pressure on opening.
(4) a difference in pH of 0.2 or more as compared with the normal jars of the same series.
(5) a difference of 1 c.c. or more in titrable acidity or formol titration as compared with normal jars of the same series.
(6) numerous organisms showing in the stain of the sediment.
(7) excessive counts as indicated by the plate or dextrose broth count.

In assembling and studying the data the indices of spoilage were grouped under three general heads:

(1) physical evidence of spoilage (organoleptic tests), including appearance, odor and suction (vacuum) changes.
(2) chemical evidence of spoilage, including change in pH, titrable acidity, and formol titration.

(3) bacteriological evidence of spoilage, including increase in the numbers of organisms in the stain of the sediment or the living organisms in plate counts or dextrose broth.

RESULTS

Relation between physical, chemical and bacteriological indices of spoilage. The interrelationships between the physical, chemical, and bacteriological tests used to detect spoilage in 615 jars of vegetables and meats classed as spoiled are summarized in Table I. In 200 (32.5%) of the 615 jars of spoiled food examined, there was an absolute correlation between the physical, chemical and bacteriological indices of spoilage. In the different products examined, the proportion showing perfect correlations varied from less than 20% in the case of Swiss chard to more than 50% in the case of asparagus.

In 106 (17.2%) of the 615 spoiled jars there was evidence of spoilage by physical tests, but by neither the chemical nor bacteriological tests used. Of the spoiled chard 55% fell into this group, whereas, in only one jar (3.8%) of the tomatoes was the evidence of spoilage restricted to the physical tests.

TABLE I. THE NUMBER AND PERCENTAGE OF JARS SHOWING EVIDENCE OF SPOILAGE BY PHYSICAL, CHEMICAL, AND BACTERIOLOGICAL TESTS.

Indices of spoilage			Asp'agus	Beans	Chard	Corn	Tomat's	Beef	Pork	All prods.
†Py.	Ch.	Ba.	63 jars	166 jars	49 jars	190 jars	26 jars	75 jars	46 jars	615 jars
+	+	+	33(52.4)*	36(21.7)	9(18.4)	68(35.8)	7(26.9)	28(37.3)	19(41.3)	200(32.5)
+	—	—	7(11.1)	41(24.7)	27(55.1)	14(7.4)	1(3.8)	15(20.0)	1(2.2)	106(17.2)
+	+	—	4(6.3)	32(19.3)	5(10.2)	19(10.0)	0(0)	1(1.3)	4(8.7)	65(10.6)
+	—	+	11(17.5)	46(27.7)	8(16.3)	16(8.4)	15(57.7)	29(38.7)	3(6.5)	128(20.8)
—	+	+	6(9.5)	1(0.6)	0(0)	13(6.8)	0(0)	1(1.3)	8(17.4)	29(4.7)
—	—	+	2(3.2)	6(3.6)	0(0)	57(30.0)	3(11.5)	0(0)	4(8.7)	72(11.8)
—	+	—	0(0)	4(2.4)	0(0)	3(1.6)	0(0)	1(1.3)	7(15.2)	15(2.4)
+			55(87.3)	155(93.4)	49(100)	117(61.6)	23(88.5)	73(97.3)	27(58.7)	499(81.1)
	+		43(68.3)	73(44.0)	14(28.6)	103(54.2)	7(26.9)	31(41.3)	38(82.6)	309(50.2)
		+	52(82.5)	89(59.6)	17(34.7)	154(81.1)	25(96.2)	58(77.3)	34(73.9)	429(69.7)

* Numbers in parentheses indicate %.
Other numbers indicate number of jars showing positive evidence of spoilage.
†Py.—Physical Ch.—Chemical Ba.—Bacteriological.

In 65 (10.6%) of the 615 jars spoilage was evidenced by both the physical and chemical, but not by the bacteriological tests employed. Spoiled beans and corn were particularly likely to be found in this group. Thus 32 of the 65 jars in this group were beans and 19 corn.

In 128 (20.8%) jars of spoiled products the physical and bacteriological tests only were positive. There was considerable variation in the proportion of the different products falling in this group, ranging from 6.5% in the case of pork to 57.7% for tomatoes.

Twenty-nine (4.7%) of the 615 jars showed no spoilage by physical means, but did give positive evidence by both the chemical and bacteriological tests used. In general the proportion of jars falling in this group was small, ranging from practically none in chard, beans, tomatoes, and beef to 17.4% for pork.

Seventy-two of the 615 jars gave evidence of spoilage by bacteriological tests only. Fifty-seven of these were of corn; these constituted 30% of all of the spoiled corn. The physical tests, if employed alone, are apparently more likely to be misleading in the case of corn than in any of the other products studied. Very few jars of other products, aside from corn, were judged spoiled on the bacteriological basis only.

Only 15 (2.4%) of the 615 jars of spoiled food showed chemical evidence of spoilage without showing either physical or bacteriological evidence. Seven of these jars were underprocessed sausage where the only apparent abnormality was a decrease in pH. The other products showed none or very low percentages of spoilage evidenced by chemical tests alone.

In this series of observations, if we had depended on the physical tests alone as the criterion of spoilage, 116 of the 615 jars called spoiled would have been missed. The physical tests used did not include taste, and it is probable that an "off" flavor would have been apparent in many of these 116 jars, especially in those where there was a change in pH or acidity, as in the corn and the pork. (Taste was purposely omitted from the criteria, because it is felt that it is undesirable and unwise to test canned foods for spoilage by actually bringing them in contact with the mucous membranes of the mouth.) Spoilage was evidenced by physical tests in 499 jars (81.1%), by chemical tests in 309 jars (50.2%), and by bacteriological tests in 429 jars (69.7%). In general, physical evidence was a good criterion of spoilage in asparagus, beans, chard, tomato and beef, but not as good in corn and pork. Chemical evidence of spoilage showed up in 82.6% of the cans of spoiled pork, but in less than 50% of most of the products. Bacteriological evidence of spoilage was detected in only 34.7% of the spoiled chard, but in over 59% of each of the other products.

Of all the jars showing spoilage by the physical tests used (499 jars), only 66% showed evidence by the bacteriological, and 53% by the chemical tests. Of those showing spoilage by the chemical tests (309 jars), 86% showed physical signs of spoilage and 74% bacteriological. Of those showing bacteriological evidence of spoilage (429 jars), 76% showed physical evidence and 53% chemical. These relations are brought out in Table II.

TABLE II. THE RELATIONS OF PHYSICAL, CHEMICAL AND BACTERIOLOGICAL INDICES OF SPOILAGE.

Series	Positive evidence of spoilage	Number of jars	Percentage of each series showing spoilage by tests employed		
			Physical	Chemical	Bacter'logical
I	Physical	499	100%	53%	66%
II	Chemical	309	86%	100%	74%
III	Bacteriological	429	76%	53%	100%

It appears from a consideration of the data in Tables I and II, that while physical evidence is the best single index of spoilage, it would be necessary to employ both chemical and bacteriological tests as well, if it is desired to detect all cases of spoilage.

Relative value of various determinations. A summary of the incidence of spoilage observed by the various tests employed is given in Table III. Of the different observations classed under physical evidence of spoilage (odor, appearance and suction), the odor of the product was better as an index of spoilage than appearance or suction change, except in the case of spoiled beef, where appearance was altered in more jars than was the odor. Appearance was a good index of spoilage in beans, chard and beef, but not so dependable in the other products. A change in suction was evident in 57.7% of the jars of tomatoes, but in less than 38% of the jars of any of the other materials.

The chemical tests used for the detection of spoilage (total acidity, pH and formol titration) varied greatly in their value with the different food products. The formol titration, as would be expected, was of no value as an index of spoilage in the vegetable products. There was a decided change in formol titration in beef and pork when decomposition was advanced, but this test was of no value in detecting incipient decomposition. A significant difference in formol titration was observed in only 26-29% of the samples of beef and pork which had spoiled.

The reaction as indicated by pH was found to be more suitable than the titrable acidity as an index of change in reaction in the case of asparagus, beans, chard and pork, but the reverse seemed to be true in the case of corn, beef and tomatoes.

The presence of organisms capable of growing in dextrose broth tubes was a more frequent index of spoilage than the other bacteriological tests used in the case of chard, corn, beef and pork; but in asparagus, tomatoes and beans the appearance of numerous organisms in the microscopic examination of the sediment was the most frequent bacteriological index of spoilage. In spoilage caused by bacteria, one might expect to invariably be able either to see or isolate the organisms responsible. The experience encountered in this study was that in 30% of the jars judged spoiled, organisms were not observed either by staining methods or by attempts at cultivation. This is in line with the experience of Bitting and Bitting (1917), who found that the length of time the organisms live in cans may vary from a few days to seven years, and that the bacteria may disintegrate so that they do not even show in the stain. Weinzirl (1919) also found that 6 of 20 "hard swells" failed to show organisms. The difficulty of differentiating bacteria from amorphous matter in the stains of some products like corn, the possibility of the organisms developing and dying within a few days and the likelihood of autolytic enzymes disintegrating the bacteria, as well as the chance of having organisms present which will not grow under the conditions used for cultivating them, all combine to make negative bacteriological evidence of spoilage unreliable. On the other hand, the fact that there may be living organisms (spores) in canned foods which are not considered spoiled complicates the interpretation of results. The necessity for drawing a line somewhere, in the interpretation of the significance of the bacterial counts obtained from the products tested, led us arbitrarily to consider a count of more than 25 living organisms per c.c. as evidence of probable spoilage.

In general, of the different indices of spoilage used in this series of observations, odor ranked highest. In all the products but corn and pork, the odor was altered in over 70% of those jars classed as spoiled. Considering all of the spoiled samples, change in odor was evident in 68.3% of the jars. This was more than 20% above what was obtained with any other test employed. The growth of organisms in dextrose broth tubes, presence of organisms in the stain, change in appearance, titrable acidity and pH,

TABLE III. INCIDENCE OF SPOILAGE OBSERVED BY VARIOUS PHYSICAL, CHEMICAL AND BACTERIOLOGICAL TESTS.

Index of Spoilage	Asparagus 63 jars	Beans 166 jars	Chard 49 jars	Corn 190 jars	Tomatoes 26 jars	Beef 75 jars	Pork 46 jars	Total 615 jars
Change in appearance	27(42.9)*	89(59.6)	34(69.4)	32(16.8)	10(38.5)	64(85.3)	14(30.4)	270(43.9)
Change in suction	17(27.0)	42(25.3)	15(30.6)	51(57.8)	15(57.7)	28(37.3)	15(32.6)	183(29.8)
Change in odor	49(77.8)	134(80.7)	36(73.5)	99(52.1)	23(88.5)	55(73.3)	24(52.2)	420(68.3)
Change in pH	39(61.9)	56(33.7)	13(26.5)	91(47.9)	4(15.4)	7(9.3)	33(71.7)	243(39.5)
Change in titrable acidity	28(44.4)	47(28.3)	8(16.3)	122(64.2)	6(23.1)	26(34.7)	17(37.0)	254(41.3)
Change in formol titration	0(0)	0(0)	0(0)	0(0)	0(0)	20(26.7)	13(28.3)	33(5.4)
More organisms in stain	49(77.8)	65(39.2)	8(16.3)	88(46.3)	25(96.2)	33(43.4)	17(37.0)	285(46.3)
More organisms in aerobic count	9(14.3)	29(17.5)	10(20.4)	95(50.0)	12(46.2)	49(65.4)	29(63.0)	233(37.9)
Growth in dextrose broth tubes	17(27.0)	42(25.3)	12(24.5)	123(64.7)	16(61.5)	54(72.0)	32(69.0)	296(48.1)

* Numbers in parentheses indicate percentages.

Other figures indicate number of jars showing positive evidence of spoilage.

and organisms in aerobic plate counts ranked in the given order, ranging from 48.1% to 37.9% of the jars spoiled; while the suction change and formol titration variation ranked lowest as indices of spoilage.

Relation between change in titrable acidity and pH. In many instances, there was no apparent correlation between change in titrable acidity and pH. In most products the pH of different jars from the same batch of material varied within a range of less than 0.3, whereas the titrable acidity varied through a wider range (1 to 4% normal acid). In view of the fact that the actual reaction of any given jar could not be ascertained, it was not possible to detect a change in reaction unless the change was considerable and sufficient to bring the reaction well beyond the observed range of the unspoiled samples. Thus if the unspoiled samples showed a reaction range of 1 to 4% normal acid it would be impossible to say that a spoiled jar showing 3.8% normal acid had not increased in acidity, for it might have contained but 1% initially. This was a difficulty encountered in interpreting titrable acidities.

The titrable acidity was significantly increased without a corresponding change in pH in eleven jars of spoiled corn, twenty jars of spoiled beef and five jars of spoiled pork. The pH decreased while the titrable acidity was not significantly beyond the range of reaction for unspoiled jars in ten jars of spoiled corn, seventeen jars of spoiled pork and one jar of spoiled beef. In four jars of spoiled beef there was a decrease in acidity as determined by hydrogen ion concentration accompanied by an increase in titrable acidity.

This lack of correlation between titrable acidity and hydrogen ion concentration may be explained on the basis of various types of organisms growing in buffered mediums and producing end products which have different reactions, dissociation constants, and buffer values.

In the spoiled beans and asparagus it was observed that the change in hydrogen ion concentration was well correlated with that of titrable acidity.

TABLE IV. RELATION OF FORMOL TITRATION TO SPOILAGE IN BEEF AND PORK.

Formol titration range cc N/20 NaOH for 10 cc. liquor.	Beef		Pork	
	Unspoiled	Spoiled	Unspoiled	Spoiled
	Number of jars		Number of jars	
0 - 4				1
4.1 - 8		3	2	0
8.1 - 12	156	47	12	3
12.1 - 16	53	8	160	31
16.1 - 20	14	4	8	5
20.1 - 24				0
24.1 - 28				2
28.1 - 32		1		2
32.1 - 36				1
36.1 - 40		3		1
40.1 - 44		2		
44.1 - 48		1		
48.1 - 52		2		
60.1 - 64		1		
64.1 - 68		1		
68.1 - 72		1		
96.1 - 100		1		
Total	223	75	182	46

The formol titration in beef and pork as an index of spoilage. It is evident from a comparative study of the figures for the formol titration of the spoiled and the unspoiled beef and pork that while a decided increase in formol titration indicates spoilage, a titration within the normal range does not necessarily point to the absence of spoilage. Table IV gives the distribution of formol titration results from both the spoiled and the unspoiled products. In many of the spoiled jars there was a marked increase in formol titration; on the other hand, in many jars of meat showing spoilage or incipient decomposition the formol titration did not vary from that of the unspoiled meat.

That a decided increase in amino nitrogen and NH_3 (formol titration) was closely related to advanced decomposition and that formol titration was not an aid in detecting incipient decomposition were evident from a study of the data. The relation between increase in formol titration and odor as observed with 121 jars of spoiled beef and pork is shown in Table V.

TABLE V. THE RELATION OF FORMOL TITRATION AND ODOR IN SPOILED MEAT.

Odor	No. of jars	Formol titration: Number showing distinct deviation from average of unspoiled material	Formol titration: Average increase over unspoiled jars (Mg.N per liter).
No change	42	4	7
Slightly off	36	2	0
Distinctly off or putrid	43	27	980

It was found that the jars of beef and pork which showed decided physical evidence of spoilage, particularly disintegration, and were consequently opened before the end of the experimnt (19 days to 5 months) showed, as a rule, a much greater increase in formol titration than those jars in which physical evidence of spoilage was not marked, and which, in consequence, were stored for the entire duration of the experiment (8-9 months). This relationship is brought out by the figures given in Table VI.

TABLE VI. THE RELATION OF FORMOL TITRATION AND PHYSICAL EVIDENCE OF SPOILAGE.

	No. of jars	Formol titration: Number showing distinct deviation from average of unspoiled material.	Formol titration: Average increase over unspoiled jars (Mg.N per liter).
Jars opened because of distinct evidence of spoilage	45	27	860.0
Jars opened at close of experiment	76	6	4.9

The relation between increase in acidity and informol titration in spoiled meat. There was a correlation between the change in total acidity and the change in formol titration in the jars of spoiled beef and pork. This is shown in Table VII.

TABLE VII. RELATION BETWEEN INCREASE IN ACIDITY AND IN FORMOL TITRATION IN 121 JARS OF SPOILED MEAT

Number of jars	Average deviation from unspoiled meat	
	Titrable acidity*	Formol titration†
3	—80	7.7
78	0	5.6
16	52	154.0
12	138	700.0
8	258	2282.0
4	482	1890.0

* In terms of cc. normal NaOH to neutralize 1 liter of sample.

† Expressed in mg.N per liter.

In the 78 jars of spoiled meat where the total acidity was similar to the unspoiled meat, there was almost no change in formol titration. In the 16 jars that showed an increase in acidity of 52 c.c. N acid per liter there was a corresponding increase of 154 mg. per liter in amino nitrogen and ammonia (formol titration). In the 12 jars that showed an increased acidity of 138 c.c. there was an increase of 700 mg. N in the formol titration. In the 8 jars that showed an increase of 258 c.c. n acidity the formol titration increased to the extent of 222 mg. amino nitrogen and ammonia per liter. These figures show a positive correlation between increase in titrable acidity and in the development of amino nitrogen and ammonia as determined by the formol titration.

A heat resistant anaerobic Actinomyces found in spoiled beans. Actinomycete-like forms were observed in the stain of the sediment of 30 jars of the spoiled string beans, which had been processed from one to four hours in the boiling water bath. Spoilage of canned products due to *Actinomyces* was not anticipated, as the thermal death time of the *Actinomyces* spores is usually given as not longer than one hour at 80° or 90° C. Bergey (1919), however, noted two thread-like thermophilic organisms which required two hours at 100° C. for killing. It is also true that most species of the genus *Actinomyces* are aerobic forms and thus would not grow in the sealed jars. Of the 64 species of *Actinomyces* listed in Bergey's Manual of Determinative Bacteriology (1923), only one is described as anaerobic and three as microaerophilic.

The beans in which the actinomycete-like forms were observed had been either underprocessed, kept one or more days after gathering before canning, or incubated at 37° C.

In two of the jars incubated at 37° C., a clump of *Actinomyces* growth as large as a pea was visible on a bean submerged in the liquid. This growth was cream, pink and white in color and consisted of smooth, compact nodules forming a mass with an irregular surface.

That the organisms grew anaerobically or at least under partial anaerobic conditions in the jars was shown by the evidence of suction as the jars were opened.

In view of the fact that the center of the jar of beans processed in a boiling water bath reaches the boiling temperature in less than one hour, it seems that in the two jars processed for four hours the organisms resisted the boiling temperature for over three hours. Of the remaining jars showing these organisms, seven had been boiled for three hours, seven for two and one-half hours, eight for two hours, and six for one hour.

Transfers were made from the two jars showing nodular growths into thirteen different kinds of media, including the standard culture media as well as bean agar, beans in large tubes, carrot plugs, glycerin agar, potato agar, casein digest agar, and 2% milk agar. The media were incubated at 37° C. under aerobic and anaerobic conditions.

The only growth which resembled the original clump was secured after an incubation period of five weeks in a Novy jar in which the air had been displaced with hydrogen, at 37° C. One colony was observed on a veal infusion agar plate. A stain of this colony showed the actinomycete-like organisms, and transfers were made into veal infusion agar and bean agar on plates and tubes and into sterilized string beans in large tubes. After one month in a Novy jar at 37° C., a small clump developed in one tube of beans. A stain of this showed the characteristic structures of the *Actinomyces.*

Due to the length of time it takes for visible growth to occur and the irregularity of occurrence of growth in the tubes inoculated, a detailed study of the cultural characteristics has not been possible.

SUMMARY AND CONCLUSIONS

1. Data regarding the indices of spoilage in 615 jars of spoiled canned foods are presented.

2. The criteria used for judging spoilage are described. These included physical tests such as appearance, odor and suction; chemical tests for total acidity, pH and the formol titration; and bacteriological tests such as microscopic examination of the sediment, plate count at 37 and 20° C., and growth in dextrose fermentation tubes.

3. Physical evidences were more frequent indices of spoilage than either the chemical or bacteriological tests used. No one group of tests was sufficient to detect all cases of spoilage. It was necessary to employ physical, chemical and bacteriological tests to detect all spoilage.

4. Of the different tests used, change in odor ranked highest with respect to the detection of spoilage. Change in suction and in formol titration ranked lowest.

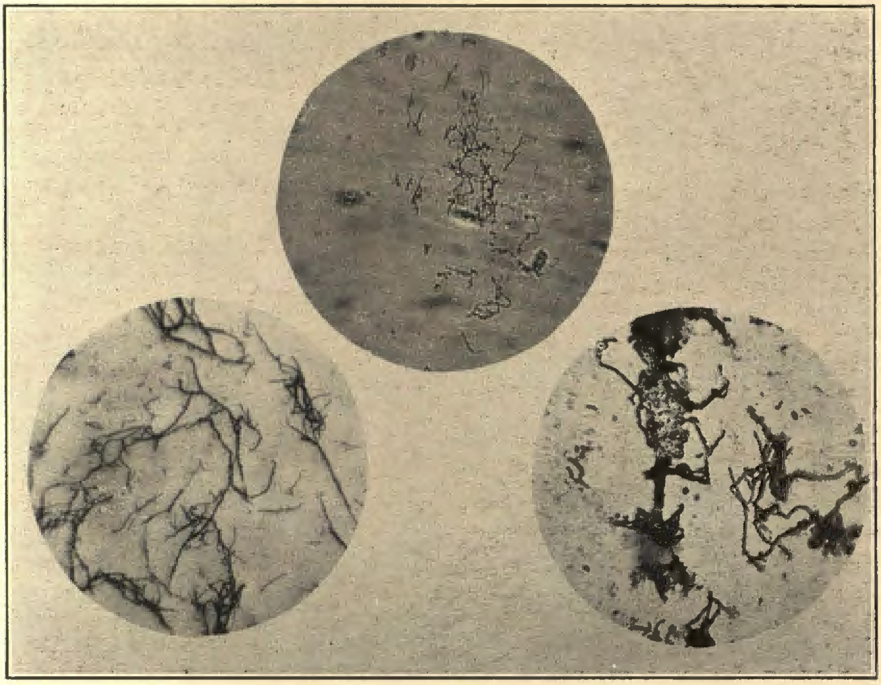
5. In spoiled beans and asparagus, change in hydrogen ion concentration was well correlated with change in titrable acidity. In spoiled beef, pork and corn, there was no apparent correlation between change in titrable acidity and in hydrogen ion concentration.

6. A decided increase in formol titration indicated spoilage in beef and pork, but a titration within the normal range did not necessarily indicate absence of spoilage. That the increase in formol titration was an index of advanced decomposition of meat was shown by the correlation of increase in this determination with pronounced odor and with distinct physical evidence of spoilage. Formol titration, however, was not suitable as an index of slight or incipient spoilage.

7. There was a correlation between increase in titrable acidity and increase in formol titration in the spoiled meat.

Stained Mounts of *Actinomyces* from Jars of Spoiled Beans.

FIGURE I



8. A heat resistant anaerobic *Actinomyces* was found in jars of spoiled beans.

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APPENDIX

The following is a condensed review of part of the literature upon the examination of spoiled canned foods and the detection of spoilage in foods:

1911 Bacon and Dunbar (1911) reported that canned tomatoes, when sound, contain no volatile acids, but a considerable amount of citric acid and invert sugar, while when spoiled they contain quite large amounts of volatile acids and little or no invert sugar or citric acid.

Howard (1911) described a method for microscopic examination of fruits and tomatoes for detection of molds, yeast and bacteria, and disintegration of the plant cells.

1915 Schneider (1915) outlined the micro-analytical and bacteriological methods in the food and drug laboratories and suggested using Eber's test to detect putrefactive change in meat. The test as described appears to be a qualitative test for NH_3.

1916 Bigelow (1916) outlined the methods employed by technical laboratories in the inspection of canned foods. These methods included external appearance of container; odor, flavor and appearance of contents; microscopical and bacteriological examinations; determination of acidity; and the composition of the gas in cans with bulged ends.

Billings (1916) stated that canned goods may manifest their unsanitary condition by becoming swelled due to the development of gas within the can, or by sourness or putrefaction, or the presence of living or dead micro-organisms in excessive numbers.

Tillmans and Mildner (1916) investigated chemical means of detection of incipient decomposition in meat. The determination of ammonia and of amino nitrogen gave positive results only when the meat had reached an advanced stage of putrefaction. They devised a method for judging the condition of the meat by the amount of dissolved oxygen remaining in an aqueous extract after various periods of incubation. If the oxygen had disappeared within four hours after incubation the food was judged unfit for use.

1917 Bitting and Bitting (1917) described a method for determining spoilage and potential spoilage in packs of canned goods. They recommended the study of the history of the foods in question; selection of representative cans; observation of gross appearance; incubation of cans at 37 and 55° C. for one to fifteen days; use of a vacuum gauge to determine whether a vacuum is or is not present; testing of the gas in swelled cans for hydrogen; inspection of the contents as to color, texture, consistency, odor and flavor and for small infected spots; inspection of the can for rust spots, black patches and erosion; examination of the unstained liquid microscopically for actively motile organisms; and the inoculation of culture media under aerobic and anaerobic conditions in all cases of doubt.

Bushnell and Utt (1917) examined 52 samples of various brands of canned salmon for bacteria. They used dextrose fermentation tubes, agar plates, agar shakes, milk, and Endo agar with negative results.

Howard, Burton and Stevenson (1917) reported the results of a five-year investigation to establish a basis for judging tomato products. In their experience, tomato products from stock judged acceptable by visual inspection never showed high counts of micro-organisms, while products from stock which was not good or was improperly handled showed high counts.

1919 Brauer (1919) stated that incipient decomposition in sausage and canned foods can be detected by inoculating dextrose bouillon with a small portion of the sample, incubating at 38° C. for 24-48 hours, and noting whether or not gas is produced during this period. This method was based on the opinion that all bacteria which are harmful to health, when they grow in food, ferment dextrose broth with gas production.

Falk, Baumann and McGuire (1919) tested decomposing meat for total nitrogen, ammonia, creatine and creatinine and purine nitrogen. Different strains of organisms showed marked differences as well as similarities in changes. The purine values decreased rapidly with some organisms, and distinctly increased with others. Ammonia increased in every case and they suggested that this might be of value in determining when the meat becomes unsuitable for use.

Falk and McGuire (1919) determined the ammonia content of meat undergoing decomposition. Meat that decomposed at room temperature containing 0.3 to 0.4 mg. ammonia nitrogen per gram of meat was unfit for food, while that decomposed at 0 to 5° C. might contain as high as 3.0 mg. ammonia nitrogen per gram before the meat was unfit to eat.

Hunter and Thom (1919) emphasized the fact that "sterility is not to be confused with fitness for food." Cultures from 350 cans of salmon showed 237 unsterile cans, but only 13 with active spoilage. They stated that the presence of living bacteria has little significance as to the quality of the product when the can is opened, since some of the sterile cans were found on chemical examination to contain putrid and decomposed fish, while many of the cans from which bacteria were grown contained apparently sound fish.

Weinzirl (1919) examined bacteriologically 1018 samples of canned goods. He noted the condition of the container, appearance, consistency, odor and taste of the contents and the microscopic findings, as well as making gelatin and agar plate cultures and enrichment cultures in broth at 37 to 55° C. He listed the spoiled and suspected canned goods separately from those called commercial canned foods (evidently not spoiled), but stated that among the suspected cans were included some samples the contents of which were entirely sound. He found, as have other investigators, that sound canned foods are not always sterile, but may contain viable spores of bacteria.

1921 Bigelow (1921) discussed methods for the detection of spoilage in canned foods. He divided them into organoleptic (appearance, color, odor, taste) bacteriological, and chemical, and the examination of the can. He stated that when organoleptic examination shows abnormal products, bacteriological examination will often disclose the reasons. The hanging drop sometimes gives evidence of spoilage before sterilization. Certain products of bacterial spoilage such as high acidity may be detected by chemical methods. He stated that a qualified analyst is better able to judge of the soundness of a sample by organoleptic examination than by examination by means of bacteriological and chemical methods.

Tillmans, Strohecker and Schütze (1921) modified the Tillmans and Mildner's oxygen consumption method for detection of incipient decomposition in meat. They devised two other tests. By their nitrate reduction test, meat in incipient decay gave no nitrate reaction after four hours' incubation. The decolorization of methylene blue in less than one hour was taken to indicate incipient decomposition.

1922 Clough (1922) found that, in general, when more than 1.5 mg. of indol per 100 gm. is found in canned salmon, a considerable degree of decomposition has taken place. He made quantitative determinations of indol in 544 cans of salmon. As some of these had a strong tainted odor and yet contained very little indol, the absence of indol cannot be taken as evidence that decomposition has not taken place. Rather close correlation was found between the number of bacteria present from day to day during spoilage and the indol content. The indol producing power of 299 different bacterial cultures from salmon was tested and gave only 31% positive tests. The volatile nitrogen increased from day to day during salmon decomposition, but owing to its probable formation during the cooking process it was not a suitable measure of decomposition in canned salmon. It was of value in raw salmon. The free fatty acids of salmon oils gave very unsatisfactory results as a measure of decomposition.

Peterson (1922) made bacteriological and chemical analyses of normal and discolored kraut and concluded that discolored kraut was characterized by high alcohol content, low lactic acid, low sugar, and the presence of large numbers of wild yeasts.

Savage, Hunwiche and Calder (1922), in a study of the bacteriology of canned meat and fish, concluded that sterility is not in itself a reliable test of soundness and that samples cannot be justifiably condemned merely because they are not sterile.

1923 Clark, Clough, Fellers and Shostrom (1923) described their systematic method of examining canned salmon. The work included physical, bacteriological and chemical examinations.

Hunter and Linden (1923), in an investigation of oyster spoilage, found that the spoilage of shucked oysters is more or less definitely correlated with the hydrogen ion concentration of the oyster liquor. Oysters passing from good to stale changed from pH 6.1 to 5.6 while those with a pH of less than 5.0 were usually in an advanced stage of decomposition. Yeast counts, the number of lactic fermenting organisms and bacterial counts from the oyster meats were found too variable to be of significance in determining spoilage of oysters.

Schoenholz, Esty and Meyer (1923), in studying correlation of toxin production and signs of spoilage in commercially canned vegetables and fruits inoculated with detoxified spores of *Bacillus botulinus*, felt that the associtaion of physical signs of spoilage with toxicity had been over-emphasized. Asparagus, beets, spinach and string beans were sometimes toxic without showing physical evidence of spoilage.

Thom (1923) stated that "a food product should be considered spoiled when a discriminating consumer, knowing its history or handling the 'raw' product, would refuse to eat it.

Wyant and Tweed (1923), in a description of 3 cans of so-called "flat sour" canned peas and inoculations into sterile cans of vegetables of four organisms isolated, used the following tests in examination for spoilage: gas production, color, appearance, odor, consistency, pH, and growth in shake cultures in dextrose agar at 37° C. and 65° C. They considered "flat sours" to be apparently normal cans whose contents had a pH below 5.8 without the production of gas or of only a very small quantity of gas.

1924 Bidault (1924) made twenty-four different chemical tests on canned meats which had stood from one to thirty-two years. He found a slight increase with age in the amount of ammoniacal nitrogen, amino nitrogen, especially that from tryosin, and a large increase in volatile acids. The composition of the gases in the cans had changed, most of those packed ten years or more showing considerable hydrogen.

Broadhurst and Van Arsdale (1924) found that there was no close correlation between the rate of bacterial multiplication and hydrogen ion or acidity readings in food spoilage in the ice box. This was explained on the basis of mixed organisms growing in the food. They stated that "measurable chemical changes and by-products apparently lag far behind bacterial multiplication ratings or else mask each other." Unless organisms produce some marked objectionable change (musty odor, sourness), the bacteria may become innumerable before a given foodstuff will be rejected as food.

Fellers, Shostrom and Clark (1924) determined hydrogen sulfide production in bacterial cultures and canned gooseberries, salmon and shrimp. Twelve out of fifty-three organisms tested gave positive results. Canned gooseberries which had been sprayed while immature with lime-sulfur contained large quantities of H_2S when spoiled, but when normal, none. Under the conditions of the experiment no H_2S was liberated from decomposed salmon or shrimp.

Murray (1924) tested methods of detecting incipient decomposition in foods. She found the reduction of methylene blue of greater value as a standard to indicate the time of initial decomposition than other methods tried. A positive nitrate reduction was obtained earlier than a great increase in bacterial count, but the test was somewhat too sensitive to indicate incipient food spoilage. No good results were obtained from change in titrable acidity or hydrogen ion concentration.

Thom and Hunter (1924) stated that chemical methods in detecting decomposition are occasionally effective but often exceedingly difficult to interpret. "The end products of decomposition resulting from many rotting processes occur in such small amounts and are so difficult to identify that decomposition must be excessive before it can be detected readily."

Tillmans and Otto (1924) found that the flesh of fish decomposes differently from that of mammals. Incipient decomposition in fish could be detected by the determination of NH_3 and amino N, though this method fails to indicate incipient decomposition of meat. Fish with more than .03% NH_3 or more than .1% amino N was starting to spoil. Oxygen absorption could be used to detect spoilage in either meat or fish. They found the reduction of potassium nitrate gave varying values, though reduction in less than four hours might always be considered a sign of incipient decomposition. The reduction of methylene blue was considered an indication of somewhat more advanced decomposition. If the methylene blue was reduced within an hour, decomposition was definitely indicated. The determination of peptone, carbonic acid, indol, soluble nitrogen, and the ability to combine with iodine furnished varying or negative results.

1925 Almy (1925) described a method for estimation of hydrogen sulfide in proteinaceous food products and found that H_2S was formed progressively during putrefaction of beef, pork and fish.

Arbenz (1925) used the consumption of oxygen and the reduction of methylene blue for detecting the degree of putrefaction of meat. Satisfactory results were obtained in testing beef, pork, veal, mutton, horse, mince meat, sausage, pigeon and several kinds of fish.

Association of Official Agricultural Chemists (1925) described official and tentative methods for the analysis of canned vegetables and for tomato products.

Clough, Shostrom and Clark (1925) tested thirty-five food products from sixty-six sources for indol and skatol. They found no skatol, but concluded that the presence of indol may be safely taken as an evidence of some degree of decomposition.

Clough, Shostrom and Clark (1925) gave the results of an experimental pack of salmon showing the correlation between the gases present and the condition of the fish. They found that hydrogen, when present, is a positive indication of decomposition, but its absence does not signify that no decomposition has taken place.

Dill (1925) investigated the post-mortem disappearace of glycogen as a possible index to spoilage in clams. Owing to the seasonable variation of glycogen content of the fresh clams the amount present could not be used as a criterion of the freshness of the clams when canned.

Esty and Stevenson (1925) published a detailed paper on the method and diagnosis in the examination of spoiled canned foods. They developed a routine method for bacterial examination of canned foods based on years of field and laboratory experience. They suggested correlations involving historical, bacteriologic and physical data. Their routine examination includes incubation of the samples to be tested, noting the condition of the container, cleaning and opening the container, inoculating into standard plain and dextrose broths pH 7 with brom cresol purple indicator and dextrose peptic digest beef heart medium covered with petrolatum and incubating the tubes for at least one week at 37 to 55° C., noting changes in hydrogen ion concentration of the canned product, microscopic examination of stained smears from the canned product, physical examination of the contents and examination of the cultures secured. With acid foods, as tomatoes and fruits, special media having tomato juice as a base and the incubation temperature of 35 to 37° C. are employed. Their paper discusses the significance of various findings in their relation to the cause of spoilage.

Sullivan (1925) reported canned beans from a lot containing flat sours being sent to four collaborators for examination. They were judged by vacuum, odor, taste, pH, cloudiness of liquid, counts of living bacteria and microscopic stains. One collaborator found that the acidity increased, the pH decreased, and the number of living organisms increased in the spoiled cans.

1926 Cameron and Esty (1926) applied the term "flat sour" only to those spoiled products which have a distinctly sour taste (pH value not markedly above 5.0) without the evidence of gas production. They found that when canned foods spoil as a result of understerilization, "swells" result from anaerobic activity and "flat sours" denote the presence of facultative anaerobic types. The 214 cultures of "flat sour" organisms studied belonged to the facultative thermophilic or the obligative thermophilic groups.

1927 Savage (1927) emphasized the importance of carrying out an examination in a systematic manner as a regular routine for the detection of spoilage in canned foods. He included in his routine analysis the examination of the unopened tin; sterilization of the tin before opening; culturing; noting the condition of the contents, including gas escape, appearace and odor; direct microscopic examination; and chemical tests as titrable acidity and examination for tin if desirable. In rare cases chemical tests for evidence of decomposition are indicated.

Wadsworth (1927) described the method used by the N. Y. State Department of Health for examination of food in cases suspected of poisoning with *Cl. botulinum* and *Bact. enteritidis.*

1928 Thompson (1928) described a quick method for detecting spoilage in packs of canned corn. The time of sampling, number of samples, and incubation temperatures and periods were specified. Flat sours were detected by adding a few drops of Brom cresol purple indicator to the corn, when a yellow color indicated sourness. The thermophilic anaerobes were indicated by swelling of the cans on incubation at 130° F. Sulphide spoilage was shown by darkly discolored kernels and the odor of H_2S.

CHEMICAL TESTING OF NICOTINE DUSTS*

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In a preceding publication (1927) an "unaccountable loss of nicotine" was reported for a series of dusts. A number of similar observations have been reported by other workers. McDonnell and Young (1925) have pointed out that there is considerable loss of nicotine from various dusts even when stored in sealed containers for various periods of time. Thatcher and Streeter (1923) have also reported significant losses in sealed containers. In view of these conclusions in the literature, decomposition was presumed to be the explanation of the previous results.

The following paper presents evidence to show that bentonite adsorbs nicotine so firmly that it cannot be quantitatively extracted from the dust with ether or water. It can, however, be quantitatively recovered by direct steam distillation of the dust. It is apparent that when any dust shows this abnormally strong adsorption that the interpretation of the "unaccountable loss of nicotine" in other dusts as due to decomposition becomes doubtful without further evidence of the precision of the patricular analytical method used.

Decomposition of nicotine in the vapor phase has also been reported by de Ong (1923). In the previous publication it was found impossible to establish equilibrium conditions in the vapor over the dust. In the following data this is shown to be due to adsorption of nicotine vapor by rubber. An apparatus constructed entirely of glass using concentrations of nicotine vapor in equilibrium with liquid nicotine and nicotine dusts showed no decrease in concentration of the nicotine in the air mixture that could be detected over periods of forty minutes exposure. The inability to maintain such vapor concentrations has been the basis of evidence for decomposition of nicotine in the vapor phase.

It is concluded that any experimental method which permits the contact of nicotine vapor with rubber (i. e. storage of nicotine dusts in rubber stoppered bottles) or which reports the analysis of a dust by extraction with either ether or water without quantitative recovery cannot be interpreted as due to decomposition of nicotine wthout further evidence.

ADSORPTION OF NICOTINE BY BENTONITE

Previous to preparing a dust, the bentonite was extracted with ether to remove any ether-soluble products. The nicotine used was a colorless oil having a boiling point of 247° C. at 730 mm. and a density of 1.010 at 20° C. A dust containing 3% nicotine was prepared and samples allowed to remain exposed to the air for several days (nicotine is comparatively

*These studies were made possible through a Fellowship maintained by the Tobacco By-Products and Chemical Corporation.

non-volatile from bentonite dusts, the odor of nicotine completely disappearing in three days in closed containers) while other samples were kept in tightly stoppered bottles.

The samples of nicotine-bentonite dust were then extracted with ether for twenty-four hours in a Soxhlet extractor. In both the dusts exposed to the air and those in stoppered containers only 10% to 20% of the nicotine originally present was obtained by ether extraction. An examination of this ether extract failed to reveal any decomposition products of nicotine. The ether-soluble oil previously reported was found to have come from outside contamination during the extraction.

The ether-extracted dust was steam distilled and in all cases the remainder of the nicotine was quantitatively recovered. An examination of the steam distillate showed no decomposition products.

The amount of nicotine absorbed or held by bentonite is approximately a constant amount regardless of the concentration of nicotine in the dust. In a 3% dust 12% of the nicotine is extracted by ether, in a 6% dust 54% is extracted and in a 14% dust 80% is extracted; that is, in a 3% dust 0.0264 gm. of nicotine is retained per gram of bentonite, in a 6% dust 0.0276 gm. is retained and in a 14% dust 0.0280 gm. is retained. These data were obtained by extracting the nicotine-bentonite dusts for twelve hours in a Soxhlet extractor. The addition of hydrated lime (10% to 20%) increases the amount of nicotine extracted by ether.

Very little nicotine is obtained in the filtrate when a 3% nicotine-bentonite dust is washed with water. It is also noteworthy that considerable heat is evolved when pure nicotine is added to bentonite, which is indicative of an adsorption or union between the bentonite and nicotine.

The fact that there is no perceptible odor of nicotine in a 3% nicotine-bentonite dust two or three days after preparation when kept in tightly stoppered bottles, the fact of the insolubility of nicotine in ether and water, and the fact of the relatively large heat of adsorption when pure nicotine is added to bentonite indicate that the degree of adsorption is so great that it approaches a loose chemical union.

Bentonite and hydrated lime dusts have shown no significant loss of nicotine (less than 0.1%) when kept in tightly stoppered bottles for the past six months. It should be pointed out, however, that the silicotungstic acid method of analysis is a poor criterion for decomposition of nicotine. Any compound which is volatile with steam and similar in structure to nicotine will be precipitated along with nicotine by silicotungstic acid. Thus only radical changes in the nicotine molecule (which are improbable in dusts) or the formation of compounds non-volatile with steam will be noted by the present analytical methods.

ADSORPTION OF NICOTINE IN VAPOR STATE

It is stated by de Ong (1923) that if nicotine vapor diffuses in the air it is soon oxidized. As evidence of this he cites an experiment in which air was aspirated through a nicotine solution and the nicotine vapor passed through two or three feet of rubber tubing. Only a trace of nicotine vapor was recovered in the washing solution at the end of the tube, while a close correlation was found between the amount of nicotine recovered at the top of the tube and the concentration of the original solution.

Chemically, nicotine is comparatively stable toward weak oxidizing agents. It is a common characteristic of many nitrogenous bases with a pyridine nucleus to darken when exposed to light or air without any appreciable decomposition taking place. Nicotine is oxidized to oxynicotine by a 2½% solution of H_2O_2. This is the most logical oxidation product of nicotine with air acting as the oxidizing medium. No trace of this compound has been found in dusts exposed to the air or in nicotine vapor-air mixtures.

Nicotine vapor from a nicotine-hydrated lime dust (using apparatus described by Hixon and Drake) was passed through a series of 3-2 liter glass bottles. These bottles were closed with rubber stoppers with an inlet and outlet glass tube, these tubes being connected together with rubber tubing. Air was passed over the dusts at six liters per hour. One hour would then elapse from the time the nicotine entered the series of bottles until it was absorbed for analysis by washing the gases in 2N H_2SO_4 with a series of bubblers. Time was allowed in all cases for any ordinary absorption on the walls of the bottles to take place before any analyses were made. Analyses of the nicotine concentration were made before and after the vapor had passed through this series of bottles. A large decrease in concentration was always noted. This difference in nicotine concentration was also noted biologically. Rice weevil (*Calendra oryzae* L.) were placed in each of the bottles in the series. Much larger kills were always obtained in the bottles nearest the source of nicotine vapor.

These observations are in harmony with de Ong's conclusions mentioned above, but when an entire glass train with no rubber stoppers or connections was inserted no such discrepancies were found. In table I data are given upon which these statements are based. In numbers 1 and 2 the nicotine vapor was passed through a train containing rubber connections, while in numbers 3, 4 and 5 the vapor was passed through an entire glass train. One hour was required for the nicotine vapor to pass through the train in experiments number 1 and 2, while only forty minutes was required in numbers 3, 4 and 5.

TABLE I.

Experiment Number	Conc. mgs. nicotine per 10 liters air at beginnnig of train.	Conc. mgs. nicotine per 10 liters air at end of train.	% nicotine not accounted for
1	2.56	1.52	—40.6%
2	1.56	0.90	—42.3%
3	2.67	2.66	— 0.4%
4	2.99	2.97	— 0.6%
5	2.76	2.78	+ 0.7%

These experiments were duplicated using pure nicotine as the source of nicotine vapor in place of the dusts, with similar results. These data will be reported in a subsequent publication on the volatilty of nicotine. These data show that there is no rapid oxidation of nicotine vapor as suggested by de Ong, his conclusions being due to the adsorption of nicotine by rubber.

SUMMARY

1. A careful chemical study has failed to reveal any decomposition of nicotine either in dusts or in the vapor phase.

2. Dusts such as bentonite adsorb nicotine so strongly that it cannot be extracted by either water or ether.

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THE PREPARATION OF ETHYL β-FURYLACRYLATE FROM FURFURAL[1]

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INTRODUCTION

There is a large commercial demand for β-furylacrylic acid or its sodium salt. The satisfaction of this demand is contingent on the preparation of the acid or its salt at a so-called reasonable price. All commercially feasible methods for the preparation of this compound must, of necessity, start with furfural. Accordingly, a study has been made of the preparation of the acid from furfural. The stipulated prerequisites have been met as a result of the investigation reported here of the optimal conditions for the preparation of ethyl β-furylacrylate. The ester is hydrolyzed with great smoothness to give the desired acid or its salt.

The ester was first prepared by the esterification of furylacrylic acid by means of ethyl alcohol and concentrated sulfuric acid[2]. Then Claisen[3] prepared it in a 36.1% yield, by the use of the general reaction which now bears his name, from furfural, ethyl acetate and sodium. As a result of a series of experiments it is now possible to prepare the ester by the Claisen condensation in yields as high as 63.3%.

EXPERIMENTAL

The following details of a typical run incorporate the more practical conditions for the preparation of the ester. The results obtained in some other runs under varying conditions are incorporated in Table I.

In a two-liter, three-neck, round-bottom flask fitted with an efficient stirrer and a thermometer are placed 29 grams (1.25 atoms) of well-powdered sodium[4]. The flask is then cooled in a freezing mixture, and 455 c.c. or 410 g. (4.66 moles) of cold, absolute ethyl acetate is added[5]. As soon as the ester has been added, stirring is started and when the mixture reaches —10° then 96 g. (1 mole) of redistilled furfural is added drop by drop from a separatory funnel. The appearance of a reddish-brown color on the particles of sodium and a rise in temperature indicate that the reaction has started[6]. The temperature of the mixture should not be allowed

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[1] This is one of a series of studies in organic chemistry concerned with the utilization of agricultural wastes. The authors gratefully acknowledge help from the Industrial Science Research Fund for the defrayal of expenses incurred in this investigation.

[2] Marckwald, Ber., 21, 1398 (1888).

[3] Claisen, Ber., 24, 143 (1891). It has also been used in other studies by Asahina and Shibata, J. Pharm. Soc. Japan, No. 423, 391-9 (C. A. 11, 2457, 1917). The abstract of this article does not mention the method of preparation,

to go above $-5°$. After all the furfural has been added (about two hours) the stirring is continued for half an hour with the flask still surrounded with the freezing mixture.

The freezing mixture is removed and the stirring is continued for another hour at room temperature. Then, through the separatory funnel, there is added 120 c.c. or 125 g. (2.1 moles) of glacial acetic acid[7], followed by about 500 c.c. of water.

The ethyl acetate layer is separated, and the aqueous solution is extracted with 50 c.c. of ethyl acetate. After washing the combined ester portions with 100 c.c. of water, it is dried with about 75 g. of anhydrous sodium sulfate. The ethyl acetate is then distilled from a water bath, and the ethyl furylacrylate at reduced pressure from an oil bath. The yield of ethyl furylacrylate distilling as a light yellow oil at 115-119°/14 mm. is 100 g. or 60% of the theoretical amount.

General Comments: The use of much lower temperatures than those just specified did not significantly improve the yield of ester, but added considerably to the difficulties of the preparation by greatly extending the time of addition of furfural and by making the process of cooling very laborious. A run between $-14°$ and $-15°$ gave a 63.3% yield of ester. (See Table I.) If the temperature is held between 0° and 5° the yields are consistently above 50%. Substitution of part of the ethyl acetate with invert solvents like petroleum ether gave very poor yields, and the yield was also decreased by the use of smaller quantities of ethyl acetate than those specified above.

In Claisen's experiments[3] where he obtained a 36.1% yield the quantities of materials used were as follows: one mole of furfural, one atom of sodium and six moles of ethyl acetate.

The authors wish to thank George Wright for help in some of the experiments. They are also grateful to the Miner Laboratories of Chicago for liberal supplies of furfural.

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[4] The powdered sodium can be conveniently prepared by shaking molten sodium under hot xylene in a tightly stoppered 500 cc. round-bottom Pyrex flask. (See "Laboratory Manual of Organic Chemistry" by Fischer, p. 139, 2nd edition, published by John Wiley and Sons). The sodium can also be powdered by melting it under xylene in the flask in which the run is carried out. As soon as the sodium has melted, the stirrer is started and kept running until the xylene has cooled. An efficient stirrer and rapid stirring are necessary to produce finely divided sodium. The xylene may be carefully decanted after it is cold.

[5] Absolute ethyl acetate is necessary for this preparation. The grade supplied by the U. S. Industrial Alcohol Company is quite satisfactory. The purification of ethyl acetate as described by Gattermann ("Practical Methods of Organic Chemistry" p. 179, third edition, published by MacMillan and Company) and others does not seem to give as good yields as can be obtained by the use of the above mentioned grade of ethyl acetate.

[6] If a rise in temperature is not noted after about one cc. of furfural has been added, the stirring should be stopped until bubbles rise from the sodium. Stirring is then resumed. The addition of furfural should be slow.

[7] When the glacial acetic acid is added, the mixture in the flask becomes almost solid, and care must be taken to stir it well so that all of the excess sodium is destroyed; otherwise a fire may result when water is added.

TABLE I.

Furfural		Sodium		Ethyl Acetate		Temp.	% Yield	Hours stirring at room temp.
g.	moles	g	atoms	cc.	moles			
96	1	29	1.25	455	4.66	0°- 5°	48.2	0
96	1	29	1.25	455	4.66	0°- 5°	48.2	0
96	1	29	1.25	455	4.66	0°- 5°	52.1	0
96	1	29	1.25	455	4.66	0°- 5°	56.4	2
96	1	29	1.25	455	4.66	—10°- 4°	60.3	1
96	1	29	1.25	455	4.66	—15°-14°	60.3	2
96	1	29	1.25	455	4.66	—15°-14°	63.3	2.5
192	2	58	2.5	455	4.66	—10°- 5°	51	2
96	1	29	1.25	147	1.5	—15°-10°	25	2
96	1	29	1.25	195	2	—10°- 5°	45	2

SUMMARY

Some optimal conditions have been described for the preparation of ethyl β-furylacrylate from furfural, sodium and ethyl acetate.

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