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Welfare maximization, pricing, and allocation with a product performance or environmental quality standard: Illustration for the gasoline and additives market

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Abstract

Programming models approximate market prices and quantities when regulations constrain firm choices, because market outcomes result when welfare is appropriately defined and includes performance and environmental constraints. This study discusses market operation in quality-constrained sectors, like gasoline and additives; processors expand output until marginal processing cost equals the processing margin between product revenues and raw material costs; retailers who buy gasoline and additives from processors and sell blended retail gasoline price sales at a marginal cost that includes the blended input value plus adjustments for values of constrained attributes; and market supplies and demands of measurable attributes like octane are balanced. This method can enhance predictions about the effects of new policies that regulate product quality. Analysis can now include price and output adjustment in factor and product markets, and the competitiveness of new processes and products.

Keywords: Processing; Blending; Pricing; Regulation; Gasoline additives; Ethanol

1. Introduction

Some energy market analyses focus on the effects of large price changes in international markets. Multi-sector econometric models (Broadman and Hogan, 1988) or computable general equilibrium models (Uri and Boyd, 1999; Kemfert and Welsch, 2000; Breuss and Steininger, 1998) help to evaluate the overall consequences for a country's energy sector and macro economy.

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Mathematical programming models also remain useful, especially when environmental or performance constraints limit production choices in a sector. But sector models are typically extensions of firm problems. For instance, the objective functions for energy sector analysis typically concern processor profits or costs, input prices are given, and product demands are taken as inelastic (Vlachou et al., 1996, p. 346; Manne, 1958, p. 69; Manne, 1956, p. 126). Under these assumptions, consumer price adjustments are synonymous with firm cost adjustments. For better understanding of markets and price relationships in the presence of environmental regulation, inelastic factor supplies and price responsive product demands should be taken into account.

Mathematical programming models of markets have also been a mainstay in applied economic research. These models exploit the fact that the equilibrium of a perfectly competitive market or sector is implied by maximum welfare allocations (Samuelson, 1952, p. 292). One advantage of spatial equilibrium models is estimation of market entry and exit prices (Takayama and Judge, 1971). Similarly, sector models evaluate the competitiveness of value-added enterprises such as processing sectors in a market setting (McCarl and Spreen, 1980, pp. 90–91; Takayama and Judge, 1964, p. 356).

In some cases where constraints are imposed by a government policy, market programming models suggest that markets maximize the welfare that can be obtained with the policy in place. For instance, Bawden (1966, p. 867), Takayama (1967), and Takayama and Judge (1971, p. 203) have shown that welfare maximization, constrained by trade policy, is consistent with market equilibrium; many of the situations encountered in international trade have been considered, including the fixed import duty, the variable levy, the fixed export subsidy, and the fixed import quota. Cox and Chavas (2000, p. 93) show the equivalence of welfare maximization and market outcomes, when welfare is constrained by a government sanctioned system of price wedges and the net extractions of taxes (contributions of subsidies) are subtracted from (added to) sector welfare. Research on the relevance of programming models for situations where the market must perform in harmony with government policy, however, is still incomplete.

This paper demonstrates the usefulness of programming models for markets where quality and environmental restrictions impinge on market outcomes. Quality standards are an increasingly common form of market intervention, as a means of ensuring product performance, food safety, and environmental compatibility. The method applies to any sector with factor supply, processing, and product demand. But fuels and additives illustrate some important regulation, quality, and pricing issues. First, a model of the consumer demand for gasoline, the production and blending of the intermediate products (additives and refinery gasoline), and the demand/supply for inputs to gasoline production (petroleum, natural gas and byproducts, and biomass) is discussed. Second, a welfare function and a quality restriction on the octane of gasoline is specified for the fuel sector. The first order conditions for this problem are shown consistent with a competitive market and the effects of the quality constraint on market pricing of gasoline is discussed. Third, a numerical example is presented to illustrate the tractability of the programming problem, and to indicate the effects of quality restriction on market pricing. In this fashion, we demonstrate that programming models with quality regulation, a refinery process, factor markets and product markets are tractable, prior to undertaking large simulation experiments. Also, we shed light on factor and product price relationships in the presence of quality or environmental restrictions.

2. Factor-product relationships in the fuel and additives market

The main material and product flows in the gasoline complex are shown in Chart 1. Starting with the factor inputs on the LHS, crude petroleum (Q_o) is the main input into the refinery. Several types of refinery gasoline (Q_r) are produced and then blended into automobile fuel (Q_s). Each type of refinery gasoline has unique qualities; some perform well (e.g., have high octane) in a gasoline engine but burn dirty from an environmental viewpoint; some have moderate performance characteristics and burn clean; some have

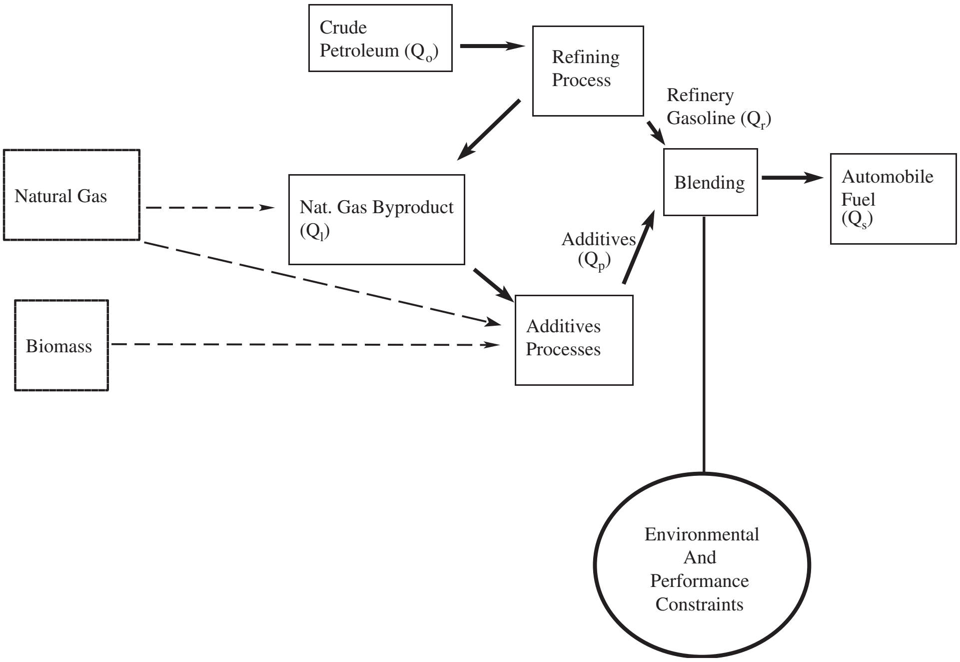

Chart 1. Factor-produced relationships in the gasoline/additives market.

marginal performance characteristics and still burn dirty. Gasoline additives ($Q_{\rm p}$) are produced because they have more desirable performance and/or environmental properties. The additives are blended into motor gasoline, sometimes at slightly higher cost than other gasoline components, to improve the characteristics of refinery gasoline. Several input chemicals (Q_ℓ) are used in the production of additives. Many of the input chemicals are byproducts of natural gas production. Others can be produced directly from natural gas. The supply of some input chemicals is also supplemented by the byproducts of petroleum refining. Biomass (including corn) is also an input for one gasoline additive. In Chart 1, the natural gas and biomass blocks are shown by dotted lines because these factors are implicit.

3. Supply and demand

Three sets of supply and demand functions are needed to specify a maximization problem and market model: consumer demand, processing supply (marginal cost), and factor supply. The notation used for market relationships, maximization constraints, and technology parameters is described in Tables 1, 2 and Appendix Table 5.

Consumers require different gasoline grades according to the performance characteristics of their automobile. The price-dependent demand function for grade i is

$$P_{si} = \alpha_{si} - \beta_{si} Q_{si}. \tag{1}$$

No substitution between grades is assumed because technology defines an appropriate quality.

Fixed proportions production processes are adequate for this problem. Most additive processes combine two or more chemicals to make a third chemical; they are 'constructive' processes. Hence the supply (marginal cost (MC)) function for additives processing is stated in terms of the production of the (one) additive output. The processing supply curve for additive *i* is

$$\mathrm{MC}_{\mathrm{p}i} = \alpha_{\mathrm{p}i} + \beta_{\mathrm{p}i} Q_{\mathrm{p}i}. \tag{2}$$

The MC function includes wages and utilities but does not include the cost of material inputs. The cost of the input chemicals, expressed on a per unit *output* basis, must be added to the processing to obtain the MC of additive production. In the case of constant MC, the slope of the above supply function is zero ($\beta_{\mathrm{p}i} = 0$).

A refinery breaks a petroleum molecule into many smaller molecules, making several types of refinery gasoline. A refinery has a 'destructive' process. Here, the supply (MC) for petroleum processing is stated in terms of the (one) crude petroleum *input*:

$$\mathrm{MC}_{\mathrm{op}} = \alpha_{\mathrm{o}} + \beta_{\mathrm{o}} Q_{\mathrm{o}}. \tag{3}$$

Again, this MC function does includes wages and utilities but excludes costs for material inputs.

A horizontal processing supply curve can occur in the short-run when all firms have the same technology (e.g., gasoline yield per barrel of petroleum) and factor prices, wages and utilities prices, are fixed because the processing sector is small relative to the labor and utilities markets. Otherwise, the short-run processing supply curve is upward sloping if processing technology is heterogenous or if the processing sector is large relative to the markets for labor, chemicals, or utilities. The short-run processing supply curve becomes perfectly inelastic (vertical) at the point where a capacity constraint is reached, regardless, of whether technology is heterogenous, or if the labor, chemicals, or utilities markets have inelastic supply curves.

The refinery is actually a collection of fixed proportion production process (Gary and Handwerk, 1994). Most of the choices concerning the product mix coming from a barrel of oil are set in the long-run period when the configuration of fixed proportion production processes is chosen. The remaining choices in short-run allocation decisions, the allocation of intermediate gas–oils for gasoline or diesel production, the sale or internal use of residual fuel oil, and the proportion of kerosine in the gasoline mix do not vary widely from year to year. Hence, the fixed proportions assumption is a good first approximation.

Most of the factor supply curves for additive inputs are likely upward sloping because they are the byproducts of natural gas production. Further, domestic production of byproducts is supplemented by imports. The price-dependent supply for additive input *i* is

$$P_{\ell i} = \alpha_{\ell i} + \beta_{\ell i} Q_{\ell i}. \tag{4}$$

For the moment, assume that the factor supply curves include one biomass based input for the production of one additive. A processing complex for ethanol and a supply curve for crude petroleum will be important for large simulation models of this sector. For now, they just add complexity without changing the basic relationships developed here. Also, the price of crude petroleum (P_{o}) is taken as exogenous.

4. Maximizing welfare

Sector welfare is consumer surplus less the operating and material costs associated with processing. The objective function states consumer welfare as the area under the product demand curves. Processing costs are given by the area under the appropriate processing supply function—processing costs for several additives processes and the refinery sector are given below. Factor costs for additive inputs are also given by the area under the appropriate supply function. Finally, expenditures defined by price times quantity are used for crude petroleum, since the petroleum price is exogenous to the refinery sector in this illustration.

The Lagrangian for the maximization problem also includes several constraints on sector welfare. Most of these constraints are supply-utilization identities for the markets in the gasoline and additives sector. To illustrate performance and environmental constraints on the market, an octane constraint on blended consumer gasoline is also included. The Lagrangian for an example is given in Eq. (5). There are three gasoline products (say, regular, midgrade, and premium); three additive inputs (say, isobutane, propylene, and corn); three additive processes (say, MTBE, alkylates, and ethanol); and three types of refinery gasoline (say, catalytic cracker, reformer, and coker). Further generalization is possible, but it becomes difficult to visualize properties for the maximization problem.

The Lagrangian for this problem is

$$
\begin{aligned}
\mathscr{L} = {} & \sum_{i=1}^{3}\left(\alpha_{si}Q_{si} - \frac{\beta_{si}}{2}Q_{si}^2\right) - \sum_{i=1}^{3}\left(\alpha_{pi}Q_{pi} + \frac{\beta_{pi}}{2}Q_{pi}^2\right) - \sum_{i=1}^{3}\left(\alpha_{\ell i}Q_{\ell i} + \frac{\beta_{\ell i}}{2}Q_{\ell i}^2\right) \\
& - \left(\alpha_{o}Q_{o} + \frac{\beta_{o}}{2}Q_{o}^2\right) - P_{o}Q_{o} \\
& + \lambda_1[Q_{\ell 1} + Q_0x_1 - (r_{11}Q_{p1} + r_{12}Q_{p2} + r_{13}Q_{p3})] + \lambda_2[Q_{\ell 2} + Q_0x_2 - (r_{21}Q_{p1} + r_{22}Q_{p2} + r_{23}Q_{p3})] \\
& + \lambda_3[Q_{\ell 3} + Q_0x_3 - (r_{31}Q_{p1} + r_{32}Q_{p2} + r_{33}Q_{p3})] \\
& + \mu_1[Q_{p1} - (Z_{11} + Z_{12} + Z_{13})] + \mu_2[Q_{p2} - (Z_{21} + Z_{22} + Z_{23})] + \mu_3[Q_{p3} - (Z_{31} + Z_{32} + Z_{33})] \\
& + \theta_4[Q_oy_1 - (Z_{41} + Z_{42} + Z_{43})] + \theta_5[Q_oy_2 - (Z_{51} + Z_{52} + Z_{53})] + \theta_6[Q_oy_3 - (Z_{61} + Z_{62} + Z_{63})] \\
& + \phi_1[(Z_{11} + Z_{21} + Z_{31} + Z_{41} + Z_{51} + Z_{61}) - Q_{s1}] \\
& + \phi_2[(Z_{12} + Z_{22} + Z_{32} + Z_{42} + Z_{52} + Z_{62}) - Q_{s2}] \\
& + \phi_3[(Z_{13} + Z_{23} + Z_{33} + Z_{43} + Z_{53} + Z_{63}) - Q_{s3}] \\
& + \psi_1[(O_1Z_{11} + O_2Z_{21} + O_3Z_{31} + O_4Z_{41} + O_5Z_{51} + O_6Z_{61}) - K_1Q_{s1}] \\
& + \psi_2[(O_1Z_{12} + O_2Z_{22} + O_3Z_{32} + O_4Z_{42} + O_5Z_{52} + O_6Z_{62}) - K_2Q_{s2}] \\
& + \psi_3[(O_1Z_{13} + O_2Z_{23} + O_3Z_{33} + O_4Z_{43} + O_5Z_{53} + O_6Z_{63}) - K_3Q_{s3}]. \qquad (5)
\end{aligned}
$$

A group of supply-utilization identities is defined in Eq. (5) by Lagrange multipliers that have one Greek letter and alternative subscripts. All constraint sets follow the inequality convention of programming models for markets, namely, that supply equals or exceeds demand.

The constraints with multipliers λ_i, $i = 1,2,3$, state that the supply of a factor used in additive production, Q_{li}, is augmented by factor supply derived as a byproduct of petroleum production, Q_ox_i, where x_i is the byproduct yield from a unit of petroleum. Also, the production of additive i (Q_{pi}) times the input requirement of factor j in the production of additive i, r_{ij}, defines the derived demand for a particular factor arising from a given additive. Hence, the total derived demand for an additive is the sum of input requirements times production for each additive. Finally, supply from both sources equals or exceeds the total derived demand for an input. The multipliers λ_i represent the marginal contribution to net surplus from another unit of additive-input supply.

The other supply constraints concern blending. The variable Z_{ji} indicates the amount of gasoline component j used in product i. There are six gasoline components in the example. Components 1, 2, and 3 are gasoline additives. Components 4, 5, and 6 are types of refinery gasoline. Blending is specified as a costless activity here, but as the reader can verify, the main results are unchanged if a constant blending cost term is added for each Z_{ji}.

The constraints with multipliers ϕ_i, $i = 1,2,3$, state that the sum of the supply of gasoline components in a particular grade of fuel, Z_{ji}, equals or exceeds the demand for each grade of fuel, Q_{si}, for $i = 1,2,3$. The multipliers ϕ_i represent the marginal contribution to net surplus from another unit of blended fuel supply.

The constraints with multipliers μ_i, $i = 1,2,3$, state that the supply of additive i, Q_{pi}, equals or exceeds the sum of the demand for the additive across every grade of fuel, Z_{ji}, for $j = 1,2,3$. The multipliers μ_i represent the marginal contribution to net surplus of another unit of additive i supply.

The equations with θ_i state that the supply of refinery gasoline type i, $Q_o y_i$, equals or exceeds the sum of the demand for the refinery gasoline types across fuel grades, Z_{ji}, for $j = 4, 5, 6$. The multipliers θ_i represent the marginal contribution to net surplus of another unit of refinery gasoline i.

Octane constraints illustrate the effects of performance or environmental restrictions. Each of these equations, indicated by the Lagrange multipliers ψ_i, for $i = 1, 2, 3$, is also a supply-demand identity. Each identity states that sum of the supply of octane (measured in, say, octane-gallons) provided to a given gasoline grade by the sum of each blending component, $O_j Z_{ji}$, equals or exceeds the demand for octane in the given gasoline grade, $K_i Q_{si}$. O_j indicates the octane content of a particular gasoline component. K_j indicates the octane performance standard for a particular grade of gasoline. The multiplier λ_i represents the marginal contribution of another unit (octane-gallon) of octane supply to net sector welfare.[1] In this fashion, a quality regulation creates a demand for a scarce resource, creating additional value for the gasoline components or additives with desired attributes, and perhaps destroying value of components without much of the desired attribute.

5. First order conditions

The first order conditions for highest net sector welfare are derivatives with respect to quantity variables and Lagrange multipliers that are determined in the market and processing system. The first order conditions for the example are given below. Eqs. (6)–(10) are obtained by differentiating quantity variables. Eqs. (11)–(15) are supply-demand identities or constraints obtained by differentiating with respect to Lagrange multipliers. The identities are stated as inequalities that allow supply to be greater than or equal to demand.

$$\frac{\partial \mathscr{L}}{\partial Q_{si}} = (\alpha_{si} - \beta_{si} Q_{si}) - \phi_i - \psi_i K_i \geqslant 0; \quad i = 1, 2, 3, \tag{6}$$

$$\frac{\partial \mathscr{L}}{\partial Q_{pi}} = -(\alpha_{pi} + \beta_{pi} Q_{pi}) - \mu_i - (\lambda_1 r_{1i} + \lambda_2 r_{2i} + \lambda_3 r_{3i}) \geqslant 0; \quad i = 1, 2, 3, \tag{7}$$

$$\frac{\partial \mathscr{L}}{\partial Q_{\ell i}} = -(\alpha_{\ell i} + \beta_{\ell i} Q_{\ell i}) + \lambda_i \geqslant 0; \quad i = 1, 2, 3, \tag{8}$$

$$\frac{\partial \mathscr{L}}{\partial Q_o} = -(\alpha_o + \beta_o Q_o) + (\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3) + (\theta_4 y_1 + \theta_5 y_2 + \theta_5 y_3) - P_o \geqslant 0, \tag{9}$$

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial Z_{ij}} &= -\mu_i + \phi_j + \psi_j O_i \geqslant 0; \quad i = 1, 2, 3, \\ &= -\theta_i + \phi_j + \psi_j O_i \geqslant 0; \quad i = 4, 5, 6, \end{aligned} \tag{10}$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_i} = Q_{\ell i} + Q_o x_i - (r_{i1} Q_{p1} + r_{i2} Q_{p2} + r_{i3} Q_{p3}) \geqslant 0; \quad i = 1, 2, 3, \tag{11}$$

$$\frac{\partial \mathscr{L}}{\partial \mu_i} = Q_{pi} - (Z_{i1} + Z_{i2} + Z_{i3}) \geqslant 0; \quad i = 1, 2, 3, \tag{12}$$

[1] A conventional form for this quality constraint in firm linear programming models, which states that recipe shares of gasoline components equals or exceeds the quality standard, can be obtained by dividing both sides of an octane constraint by Q_{sj}.

$$\frac{\partial \mathscr{L}}{\partial \theta_i} = Q_o y_o - (Z_{i1} + Z_{i2} + Z_{i3}) \geqslant 0; \quad i = 4, 5, 6, \tag{13}$$

$$\frac{\partial \mathscr{L}}{\partial \phi_i} = (Z_{1i} + Z_{2i} + Z_{3i} + Z_{4i} + Z_{5i} + Z_{6i}) - Q_{si} \geqslant 0; \quad i = 1, 2, 3, \tag{14}$$

$$\frac{\partial \mathscr{L}}{\partial \psi_i} = O_1 Z_{1i} + O_2 Z_{2i} + O_3 Z_{3i} + O_4 Z_{4i} + O_5 Z_{5i} + O_6 Z_{6i} - K_i Q_{si} \geqslant 0; \quad i = 1, 2, 3. \tag{15}$$

6. Market equilibrium

The maximum net welfare solution is the same as the market equilibrium for the sector, if the first order conditions for the maximum problem are also the equilibrium conditions for a market. This equivalence is straightforward for input suppliers and processors. But a formal profit-maximizing problem for the blender must be presented to demonstrate equivalence at the retail level.

First consider input suppliers and processors. For the additive-input producers, direct inspection of Eq. (8) suggests that the MC of inputs to additive production (the inverse supply equation) equals the marginal benefit (λ_i) from using another unit of the additive-input. It is well known that MC also equals price in a competitive input market, so Eq. (8) is also the market equilibrium condition. In other words, $\lambda_i = P_{\ell i}$.

Hence, in Eq. (7), the marginal value of additive production (μ_i) equals the sum of marginal processing costs from the processing supply function plus factor costs (sum of input requirements in additive process i, r_{ij}, times input prices, $P_{\ell j}$). Further the additive price less factor costs (processing margin) equals marginal processing costs in competitive equilibrium.[2] It follows that the additive price equals the marginal value of a unit of additive supply, $P_{pi} = \mu_i$, in competitive equilibrium.

Similarly, Eq. (9) states that the marginal cost of petroleum processing equals the net marginal benefit. The MC comes directly from the petroleum processing equation. The benefits from byproduct processing, the sum of byproduct yields (x_i) times shadow byproduct values (λ_i), are actually revenues since $\lambda_i = P_{\ell i}$. Further, revenues per unit of petroleum processed less the petroleum price (processing margin) equals marginal processing cost in competitive equilibrium.[3] Hence, $\theta_i = P_{ri}$.

[2]The profit function for an additive processor includes revenues less commodity inputs less variable and fixed costs (FC): $\pi_{pi} = P_{pi}Q_{pi} - (P_{l1}Q_{l1} + P_{l2}Q_{l2} + P_{l3}Q_{l3}) - \mathrm{VC}(Q_{pi}) - \mathrm{FC}$.

A quadratic variable cost (VC) function is consistent with the linear MC function: $\mathrm{VC}(Q_{pi}) = \alpha_{pi}Q_{pi} + (\beta_{pi}/2)Q_{pi}^2$.

Also, $Q_{lj} = r_{ji}Q_{pi}$ because of the fixed proportions assumption about additive processors. So the profit function becomes $\pi_{pi} = M_{pi}Q_{pi} - (\alpha_{pi}Q_{pi} + (\beta_{pi}/2)Q_{pi}^2) - \mathrm{FC}$, the margin is $M_{pi} = P_{pi} - (P_{l1}r_{12} + P_{l2}r_{22} + P_{l3}r_{33})$. Under competition, processors maximize profits. So

$$\frac{\partial \pi_{pi}}{\partial Q_{pi}} = M_{pi} - (\alpha_{pi} + \beta_{pi}Q_{pi}) = 0 \quad \text{or} \quad M_{pi} = \alpha_{pi} + \beta_{pi}Q_{pi}.$$

This equilibrium condition says that additive processors in a competitive market expand output until the processing margin equals marginal cost.

[3]The profit function for a petroleum processor includes byproduct and gasoline revenues less petroleum expenditures, operating expenses and fixed costs:

$$\pi_o = (P_{r1}Q_{r1} + P_{r2}Q_{r2} + P_{r3}Q_{r3}) + (P_{\ell 1}Q_{\ell 1} + P_{\ell 2}Q_{\ell 2} + P_{\ell 3}Q_{\ell 3}) - P_o Q_o - \mathrm{VC}(Q_o) - \mathrm{FC}.$$

A quadratic VC function is consistent with the linear MC function:

$$\mathrm{VC}(Q_o) = \alpha_o Q_o + (\beta_o/2)Q_o^2.$$

For product pricing, consider a profit maximizing firm that buys refinery gas and additives as inputs, and then blends them. This marketing/blending firm maximizes profits subject to several of the same constraints present in the maximum welfare Eq. (5). Specifically, supply constraints for additives (μ_i), refinery gas (θ_i), and consumer gasoline blending (ϕ_i), and the quality constraint (ψ_i) must all be satisfied. The constrained profit function is the revenues from sale of consumer grade gasoline less input expenditures for additives and refinery gasoline:

$$\begin{aligned}\pi = {} & (P_{s1}Q_{s1} + P_{s2}Q_{s2} + P_{s3}Q_{s3}) - (P_{p1}Q_{p1} + P_{p2}Q_{p2} + P_{p3}Q_{p3}) - (P_{r1}Q_{r1} + P_{r2}Q_{r2} + P_{r3}Q_{r3}) \\ & + \mu_1[Q_{p1} - (Z_{11} + Z_{12} + Z_{13})] + \mu_2[Q_{p2} - (Z_{21} + Z_{22} + Z_{23})] + \mu_3[Q_{p3} - (Z_{31} + Z_{32} + Z_{33})] \\ & + \theta_4[Q_{r1} - (Z_{41} + Z_{42} + Z_{43})] + \theta_5[Q_{r2} - (Z_{51} + Z_{52} + Z_{53})] + \theta_6[Q_{r3} - (Z_{61} + Z_{62} + Z_{63})] \\ & + \phi_1[(Z_{11} + Z_{21} + Z_{31} + Z_{41} + Z_{51} + Z_{61}) - Q_{s1}] \\ & + \phi_2[(Z_{12} + Z_{22} + Z_{32} + Z_{42} + Z_{52} + Z_{62}) - Q_{s2}] \\ & + \phi_3[(Z_{13} + Z_{23} + Z_{33} + Z_{43} + Z_{53} + Z_{63}) - Q_{s3}] \\ & + \psi_1[(O_1Z_{11} + O_2Z_{21} + O_3Z_{31} + O_4Z_{41} + O_5Z_{51} + O_6Z_{61}) - K_1Q_{s1}] \\ & + \psi_2[(O_1Z_{12} + O_2Z_{22} + O_3Z_{32} + O_4Z_{42} + O_5Z_{52} + O_6Z_{62}) - K_2Q_{s2}] \\ & + \psi_3[(O_1Z_{13} + O_2Z_{23} + O_3Z_{33} + O_4Z_{43} + O_5Z_{53} + O_6Z_{63}) - K_3Q_{s3}]. \end{aligned} \tag{16}$$

The marketer's choice variables are Q_{s1}, Q_{s2}, Q_{s3}, Q_{p1}, Q_{p2}, Q_{p3}, Q_{r1}, Q_{r2}, Q_{r3}, and Z_{ij}.

The first order conditions for the profit maximizing marketer/blender are

$$\frac{\partial\pi}{\partial Q_{si}} = P_{si} - \phi_i - \psi_i K_i \geqslant 0; \quad i = 1, 2, 3, \tag{17}$$

$$\frac{\partial\pi}{\partial Q_{pi}} = P_{pi} + \mu_i \geqslant 0; \quad i = 1, 2, 3, \tag{18}$$

$$\frac{\partial\pi}{\partial Q_{ri}} = P_{ri} + \theta_{i+3} \geqslant 0; \quad i = 1, 2, 3, \tag{19}$$

$$\begin{aligned}\frac{\partial\pi}{\partial Z_{ij}} = {} & -\mu_i + \phi_j + \psi_j O_i \geqslant 0; \quad i = 1, 2, 3, \\ & -\theta_i + \phi_j + \psi_j O_i \geqslant 0; \quad i = 4, 5, 6. \end{aligned} \tag{20}$$

These conditions are the same as the welfare maximizing conditions. In fact, Eq. (17) is identical to Eq. (6), except that (17) contains one explicit price variable, while (6) contains the quantity dependent price

(*footnote continued*)

Also, $Q_o y_i = Q_{ri}$ and $Q_o x_i = Q_{li}$ because of the fixed proportions assumption about petroleum processors. So the profit function becomes $\pi_o = M_o Q_o - (\alpha_o Q_o + (\beta_o/2)Q_o^2) - \text{FC}$, where the processing margin is

$$M_o = (P_{\ell 1}x_1 + P_{\ell 2}x_2 + P_{\ell 3}x_3) + (P_{r1}y_1 + P_{r2}y_2 + P_{r3}y_3) - P_o.$$

Under competition, processors maximize profits.

So $\dfrac{\partial\pi_o}{\partial Q_o} = M_o - (\alpha_o + \beta_o Q_o) = 0$ or $M_o = \alpha_o + \beta_o Q_o$.

This equilibrium condition says that additive processors in a competitive market expand output until the processing margin equals marginal cost. In the event that a long-run analysis is appropriate, the fixed cost term in the profit function is replaced with a term that indicates the annual capital cost associated with an incremental unit of processing capacity. Then the MC term in the equilibrium condition includes operating and capital costs.

function. Both versions state that the marginal benefit (price) of consumer gasoline equals or exceeds the MC of producing another unit of consumer's gasoline. In turn, the MC from (17) consists of two components: the marginal value (ϕ_i) of another unit of blended gasoline supply and a quality adjustment that reflects the value of another unit of octane supplied ($\psi_i K_i$).

Next, Eqs. (10) and (20) state that the marginal value of another unit of consumer gasoline (ϕ_j) equals the marginal benefit of the corresponding additive (μ_i) or refinery gasoline (θ_i) less a correction for the value of the octane provided by additive or refinery gas i ($\psi_i O_i$).

Eqs. (18) and (19) merely assign a market price variable to a Lagrange multiplier. Eq. (18) states that the additive price equals the marginal surplus contribution of an incremental additive unit. Eq. (19) states that the refinery gas price equals the marginal surplus contribution of an incremental unit of refinery gas.[2,3]

In the case where the quality constraint is not binding ($\psi_i = 0$), the producer and consumer price revert to a more familiar form. Specifically, Eqs. (6) and (17) state that the marginal benefit or price of another unit of consumer gasoline equals the MC (ϕ_i). Meanwhile, Eqs. (10) and (20) state that the MC of additive or refinery gasoline i equals the MC of blended consumer gasoline of grade i. That is, the consumer price of all gasoline grades are equal, and all additives and all types of refinery gasoline have the same price.

The welfare maximization problem identifies a market equilibrium, then, regardless of whether a quality restriction is present.

7. Product price and quality relationships

To investigate the price and quality relationships implied by market equilibrium, look at the marketer/blender margin between the sales price of blended gasoline and the purchase price for additives or refinery gasoline. Begin with the Eq. (6) for grade j of blended consumer gasoline when the octane constraint is binding:

$$P_{sj} = \phi_j + \psi_j K_j. \tag{6}$$

Eq. (10) for additive type i is

$$\phi_j = P_{pi} - \psi_j O_i. \tag{10}$$

Substituting yields and rearranging yields the marketing price relationship between consumer prices for gasoline and the prices paid for additives:

$$P_{sj} + \psi_j (O_i - K_j) = P_{pi}. \tag{21}$$

The second term on the left-hand side of the above equation defines a price premium (discount) that is paid for an additive i that has an octane above (below) the standard for grade j (K_j). For instance, $O_i = 113$ for ethanol and $K_j = 87$ for regular gasoline. If $\psi_j = \$.01/\text{octane gallon}$, then the ethanol price (P_{pj}) will exceed the price of regular gasoline (P_{sj}) by \$.52/gallon. A similar price relationship for the types of refinery gasoline can also be developed. The price for a low octane refinery gasoline, like coker gas, would have a price below blended consumer gasoline.

Usually, grade standards are associated with product performance or environmental attributes. Then the multiplier ψ_j represents a cost associated with producing quality. But standards are sometimes arbitrary. In this context the shadow value times octane differential represents the subsidy equivalent of the quality restriction for inputs having above average amounts of the restricted attribute. Similarly, the discount for below average inputs represents a tax associated with the quality restriction.

8. An example

An example helps to demonstrate tractability of the programming problem and to illustrate operation of markets with a constraint. We now modify a trade example that includes factors and products from p. 143 of Takayama and Judge (1971). Three importing country demand equations from their example are used for consumer gasoline demand in the new example. Similarly, three exporting country supply equations from their example are used as equations of factor supply for the additives industry. Also, processing supply functions for additives were added. The reference point values of the processing functions corresponds to the magnitude of transport costs in the trade example. Then, a petroleum price and processing function that were lower than the other factor supplies was included, and the intercepts of factor supply equations were adjusted so that all factors and processes were used in the baseline solution. The assumed parameter values for supply and demand functions are given in appendix Table 4.

The processing technology and quality assumptions correspond roughly to the actual refinery and additives processes. The yields of refinery gasoline from petroleum roughly correspond to relative yields of coker gasoline, catalytic cracker gasoline, and reformer gasoline. The octane assumptions, low, medium, and high correspond to the same processes. The refinery yields of inputs for additives processing roughly correspond to the yields of iso-butane, butylene, and propylene from oil. There are three additives processes. The first process uses equal amounts of factors 1 and 2. The second additive process uses equal amounts of all three factors. The third additive process uses only input three. Roughly, the first two processes correspond to the production of alkylates or polymer gasoline. The third process corresponds to iso-octane or perhaps ethanol. The assumed octane values also correspond to alkylates, polymers, and ethanol, respectively. The assumed parameter values for process yields, octane content, and octane standards are given in Appendix B.

The results of two simulations are shown in Table 1. The right column shows the results of the market simulation without the octane constraint. The left column shows the market simulation when the quality constraint is imposed. The content of the simulations is interesting because it depicts the situation in the additives and refinery gasoline competition; the additives have more scarce qualities and higher quality attributes than the refinery qualities. Further, the average quality of the refinery is below the performance standard set in the constraint.

Now, look at how the quality standard affects the market. First, note that the price and production of two additives increases, and total additive output increases (Table 2). Hence the price and quantity of all inputs to additive production also increase. In contrast, the consumption of petroleum decreases and the production of all types of refinery gas also falls. However, the decline in output of refinery gasoline is smaller than the increase in output of gasoline additives (Table 2). Hence, the overall supply of gasoline to consumers increases and the price for all grades of consumer gasoline falls, at least in this particular example.[4]

The processing and marketing costs and returns are shown in Table 3. Three broad categories of activities are organized. Notice and the expenditures on raw materials and processing costs match intermediate product sales within one-half of one percent. Similarly, the intermediate product sales match the revenues from sales of final consumer products. Hence, the estimates show the processing and marketing system doing business at cost, which one would expect in a competitive system.

[4] One reason for the relatively small decline in refinery output is that refinery byproducts, whose prices are increasing with expansions in additive outputs offset the price declines in refinery gasolines. Further, additive output expansions are large because the input supply elasticities for non-refinery sources and additive processing are large. Actual markets may not exhibit a retail price decrease in response to regulation because byproducts are less important in refinery revenues and additive supply elasticities are relatively smaller. Nonetheless, the example demonstrates that a general equilibrium analysis may yield conclusions that are different than the conclusions that come from a firm's analysis.

Table 1
The effect of an octane standard on a hypothetical gasoline/additive sector

Variable description (example classification)		With constraint	No constraint
	NSP	669.544	672.101
Product (gasoline) consumption (Q_s)			
(Regular)	s1	48.160	47.081
(Mid)	s2	23.892	23.540
(Premium)	s3	37.928	37.664
Processed product (additive) production (Q_P)			
(MTBE)	p1	28.463	21.291
(Alkylate)	p2	10.318	19.428
(Ethanol)	p3	20.807	15.812
Factor (nat. gas/chemical) supply (Q_ℓ)			
(Butane)	ℓ1	13.437	12.744
(Butylene)	ℓ2	15.537	14.900
(Nat. gas)	ℓ3	20.955	18.967
Factor (petroleum) supply (Q_O)		172.518	167.975
Processed product (refinery) production (Q_r)			
(Cat. crack)	r1	25.169	25.878
(Coker)	r2	8.390	8.644
(Reformer)	r3	16.780	17.252
Product (gasoline) consumer price (P_s)			
	s1	15.184	15.292
	s2	15.222	15.292
	s3	15.229	15.292
Product (gasoline) supply price (marginal cost) (ϕ_i)			
	s1	13.476	15.292
	s2	13.476	15.292
	s3	13.476	15.292
F.O.B. (additive) supply price (μ_i *or* P_p)			
	p1	15.522	15.292
	p2	15.165	15.292
	p3	15.616	15.292
F.O.B. (refinery gas) supply price (θ_i *or* P_r)			
	r1	14.790	15.292
	r2	14.658	15.292
	r3	15.165	15.292
Factor (chem/nat. gas) supply price (λ_i or P_ℓ)			
	ℓ1	16.344	16.274
	ℓ2	11.277	11.245
	ℓ3	14.096	13.897
Processing margin, additive (M_{pj})			
	p1	1.712	1.532
	p2	1.258	1.486
	p3	1.520	1.396
Processing margin, petroleum (M_o)		4.299	4.413
Value of octane constraint (Ψ_i)			
	s1	0.012	0
	s2	0.012	0
	s3	0.012	0

Table 2
Distribution of refinery gasoline or additive j to consumer gasoline grade i (Z_{ji})

From component $j =$	To grade $i =$			Total		
	s1	s2	s3			
With quality constraint						
p1	8.247	8.799	11.417	28.463		
p2	0	10.318	0	10.318		
p3	8.041	0	12.766	20.807	Additives	59.588
r1	15.075	4.775	5.345	25.195		
r2	0	0	8.399	8.399		
r3	16.797	0	0	16.797	Refinery	50.391
Total	48.160	23.892	37.927	109.979		
Without quality constraint						
p1	15.567	0	5.724	21.291		
p2	14.635	0	4.792	19.427		
p3	12.827	0	2.984	15.811	Additives	56.529
r1	0	10.988	14.890	25.878		
r2	4.051	4.575	0	8.626		
r3	0	7.981	9.274	17.255	Refinery	51.759
Total	47.080	23.544	37.664	108.288		

9. Extensions

Constrained welfare maximization is still consistent with market equilibrium under a more general set of policy assumptions that apply to the fuel and additives markets.

Other types of quality constraints and fuel sector fiscal policies are particularly relevant. Additional quality constraints, such as vapor, oxygen and benzene content give constrained welfare maximization conditions that are similar to Eqs. (6)–(15) that have been discussed, except that a sum of shadow values of quality constraints, instead of one shadow value, enter the first order conditions. Nonlinear automobile emission constraints defined in the 1990 Clean Air Act (Federal Register, 1995) also give similar equilibrium conditions, except that the partial derivative of a quality parameter with respect to a fuel type replaces the attribute concentration of the fuel type in equilibrium conditions.

Two important fiscal policies in the gasoline fuel sector are the federal excise tax on gasoline and the rebate, or blending credit, for using ethanol blends. Welfare maximization again turns out to be consistent with market equilibrium, when the net revenue that the public sector extracts from the gasoline and additives sector is subtracted from sector welfare. First, consider the excise tax. Suppose the tax on consumer gasoline of grade i is T_{si}. Then the government's revenue for grade i gasoline is $T_{si}Q_{si}$.

Second, retailers receive a rebate of S \$ for each gallon of grade i gasoline they sell that is a 10% ethanol blend. Further, a prorated subsidy is given for gasoline that contains less than a 10% blend. Suppose Z_{3i} is the quantity of ethanol blended into gasoline of grade i. Then the subsidy per gallon of grade i gasoline is $(Z_{3i}/Q_{si})(1/0.1)S$. Notice that the full subsidy is given when Z_{3i}/Q_{si} equals 0.1 and is prorated proportionately otherwise. The government expenditure, or more precisely the excise tax loss, associated with the blending credit is $(Z_{3i}/0.1Q_{si})SQ_{si}$ or $10SZ_{3i}$, which suggests that the blending credit is equivalent to a subsidy on additive 3 that is 10 times the rebate level in the gasoline market. Hence, the government's net revenue extraction from the fuel sector is the excise tax less the ethanol blending rebate, added across

Table 3
Processing and marketing costs and revenues

Purchase raw materials and pay for processing costs		
Price	Crude oil input quantity	Revenue
1	167.975	167.975
$P^*_{\rm op}$	$Q_{\rm o}$	
4.299	167.975	722.1245
$P^*_{\rm p}$	$Q_{\rm p}$	
1.172	28.463	33.359
1.258	10.318	12.980
1.52	20.807	31.627
P_ℓ	Q_ℓ	
16.344	13.437	219.614
11.277	15.537	175.211
14.096	20.955	295.382
Sum		1658.272
Sell final consumer products		
$P_{\rm s}$	$Q_{\rm s}$	Revenue
15.184	48.16	731.262
15.222	23.892	363.684
15.229	37.92	577.484
Sum		1672.429
Sell intermediate products (additives and refinery gas)		
$P_{\rm p}$	$Q_{\rm p}$	Revenue
15.222	28.463	433.2638
15.165	10.318	156.473
15.616	20.807	324.922
$P_{\rm r}$	$Q_{\rm r}$	
14.79	24.169	357.460
14.658	8.39	122.981
15.165	16.78	254.469
Sum		1649.567

gasoline grades.

$$E = \sum_{i=1}^{3} T_{si} Q_{si} - 10S \sum_{i=1}^{3} Z_{3i}. \tag{22}$$

When the revenue extraction is subtracted from the sector welfare function in the Lagrangian, the revised first order conditions give standard results for taxes and subsidies on market prices. For instance, the term T_{si} is subtracted from the LHS of the first order condition (6). This gives the standard result for an excise tax in the market, namely, that the producer price plus the excise tax (and quality adjustment) equals the consumer price. Similarly, the LHS of Eq. (10) for additive 3 now add the term $10S$.

Finally, a revised marketing margin from additive 3 to gasoline grade i that reflects the effects of the value of quality and the subsidy can be derived. Combining Eq. (6) and additive 3's Eq. (10) gives

$$P_{si} = P_{p3} - (10S - T_{si}) - \psi_i(O_6 - K_i). \tag{23}$$

Since additive 3 has the blending credit, the producer price of additive three will likely be above the consumer gasoline price by (10 times) the amount that the blending credit exceeds the excise tax. The additive quality adjustment also holds the additive price above the consumer gasoline price, as before.

10. Summary and conclusions

The markets for factors, processing and consumption in a sector of an economy are amenable to mathematical programming analysis. It was shown that markets provide the best possible outcome, in the sense of maximizing sector welfare, in the presence of the performance and environmental constraints that characterize the gasoline fuel industry. For fiscal measures such as excise taxes and ethanol blending credits, the sector welfare function must be reduced to account for the government extraction of revenue from the sector. Even then, the net welfare maximizing conditions are still the market equilibrium conditions.

Indeed, the model is general enough to apply to any sector that includes raw commodity supply, processing, and quality regulation of a blended consumer product. Other applications for product standards on protein content in the formulae feed industry, or fat content standards in consumer foods, could be based on this programming model provided that details of the processing sector are revised. An operational second best principle, that the market provides the highest welfare given the quality-constraint imposed by a product standard or government regulation, is general.

The quality-constrained welfare maximum will likely be less than the welfare maximum without the constraint. However, the standard may be justified by external environmental or safety benefits whose analysis is beyond the scope of this paper. Then regulation could improve welfare in the broadest sense, when external environmental or product safety benefits are explicit.

Nonetheless, this programming model provides a useful method for describing price and market equilibriums of value added markets in the presence of changing regulation. For instance, we shed light on the market effects of quality regulations; a standard provides a price incentive to expand output of additives and refinery gasolines with the desired attributes, and a disincentive to the output of gasoline components with less desirable attributes. As the example simulation suggests, then it is possible that a modified consumer price increase, or even a price decrease, could accompany a quality restriction when the elasticity of supply for additives is large enough to offset the declining output of refinery gasoline.

We also showed that an additive prices above the consumer product according to that additive's contribution in filling a product standard deficit. In the ethanol–gasoline example, competitive additive prices are also held above gasoline (product) prices because of the blending credit.

The programming approach is tractable for applied market analyses of changing product and environmental standards in value-added sectors. The solution process for the example was routine and results were generally sensible. Clearly, more factors, processes and products are required for realism. Also, elasticity estimates of factor supply, processing, and product demand in a particular country could improve the empirical content of simulation results. The multiple, nonlinear environmental constraints of the fuel sector are also important. But the market effects of new quality regulations can be evaluated with this programming model when limited data and no experience with a new regulation precludes econometric analysis.

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Appendix A

The supply and demand parameters are given in Table 4.

Table 4
Supply and demand parameters

Processed (oil) services supply		
$\alpha_o = 0.1$	$\beta_o = 0.025$	$P_o = 1.0$
Factor (chemical) supply		
$\alpha_{\ell 1} = 15.0$	$\beta_{\ell 1} = 0.01$	
$\alpha_{\ell 2} = 15.0$	$\beta_{\ell 2} = 0.05$	
$\alpha_{\ell 3} = 15.0$	$\beta_{\ell 3} = 0.1$	
Product (gasoline) demand		
$\alpha_{s1} = 20.0$	$\beta_{s1} = 0.1$	
$\alpha_{s2} = 20.0$	$\beta_{s2} = 0.2$	
$\alpha_{s3} = 20.0$	$\beta_{s3} = 0.125$	
Processed (additives) services supply		
$\alpha_{p1} = 1.0$	$\beta_{p1} = 0.025$	
$\alpha_{p2} = 1.0$	$\beta_{p2} = 0.025$	
$\alpha_{p3} = 1.0$	$\beta_{p3} = 0.025$	

Appendix B

The technology and quality parameters are given in Table 5.

Table 5
Technology and quality parameters

Octane standard by grade of gasoline	
$K_1 = 91$	
$K_2 = 93$	
$K_3 = 95$	
Octane content of gasoline components	
R1	70
R2	63
R3	90
P1	109
P2	90
P3	114
Refining gasoline yields per unit of oil processed	
$y_1 = 0.15$	
$y_2 = 0.05$	
$y_3 = 0.10$	

Table 5 (*continued*)

Refining gasoline yields per unit of oil processed
Refinery by product yields per unit of oil processed
$x_1 = 0.025$
$x_2 = 0.0125$
$x_3 = 0.020$

Requirement r_{ij} of input ℓi per unit output of additives type pj

	Additive		
Input	ρ_1	ρ_2	ρ_3
$\ell 1$	0.5	0.33	0.0
$\ell 2$	0.5	0.33	0.0
$\ell 3$	0.5	0.34	1.0

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