LOHNES, Robert Alan, 1937–
QUANTITATIVE GEOMORPHOLOGY OF
SELECTED DRAINAGE BASINS IN
IOWA.

Iowa State University of Science and
Technology, Ph.D., 1964
Geology

QUANTITATIVE GEOMORPHOLOGY OF SELECTED DRAINAGE BASINS IN IOWA

by

Robert Alan Lohnes

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subjects: Geology
Soil Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Heads of Major Departments

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1964

TABLE OF CONTENTS

INTRODUCTION

The concepts of W. M. Davis (1909) which have dominated geomorphology for the last fifty years can be summarized by the statement: landforms result from interaction of structure, process, and stage. By structure he meant the type of rocks and their attitude. Process is any one or combination of gradational agents such as wind, rivers, waves, and ice. Both process and structure can be seen; but stage, or the evolution of the landform, must be inferred. Davis used the terms youth, maturity, and old age to designate the relative stages of landform development and defined the stages in terms of observable physiographic criteria. One could qualitatively describe the landforms and draw inferences as to their stage from this description. A critical problem, however, is that not enough is known about the time sequence to say that from youth to maturity to old age is a valid time sequence.

Although process could be observed, until recent years it has not been studied directly by examining its mechanics. One approach to the study of process is that of applying the fundamental laws of physics and mathematics to the gradational agent. For example, the principles of fluid mechanics may be applied to the open channel flow in streams. This requires that various parameters be measured and expressed in terms of mathematical relationships, and ideally should result in a quantification of geomorphic process. Unfortunately there exist too many variables for a completely rational approach, and the closest attempt is model analysis.

Another approach to the study of landform evolution is to measure various parameters of the landform itself, a refinement of the descriptive type geomorphology. Statistical tests can be made on these measurements for the purpose of finding interrelationships that may indicate significant parameters. One aspect of the latter approach is the morphometric analysis of drainage basins, or measurement of basin parameters such as area, stream length, etc. as they appear on topographic maps.

Since the degree of drainage development has traditionally been one means of differentiating glacial drift sheets of different ages, morphometric techniques are considered to be particularly applicable to the study of glaciated terrain. This study involves such an approach and has the dual purpose:

(1) of exploring the potential of morphometric studies on glaciated areas

(2) of examining the assumptions on which morphometric study is based, and proposing a rational model to isolate some of the critical parameters.

REVIEW OF LITERATURE

In the early 19th century, Playfair suggested that a relationship exists between drainage pattern and the forces of fluvial erosion, in that a river consists of a main trunk which is fed from a series of branching rivers; all valleys being adjusted to the streams which flow into them (Playfair, 1802). It therefore seems logical to conclude that measurements of the geometry of a stream system would be a valid approach to the study of drainage basin development. Over one hundred years later Horton (1945) suggested that morphological characteristics of watersheds in different geologic, vegetative, and climatic environments would have differences in the degree of their drainage system development. By measuring various basin parameters and comparing them from one area to another, one could reach some conclusions regarding controls on evolution of watersheds. In the same paper Horton defined several parameters for use in morphometric studies and proposed a system of ordering streams. In Horton's system the smallest tributary in the drainage net is a first order stream; two first order streams unite to form a second order stream; two second order streams unite to form a third order stream, etc. Horton also formulated laws of drainage composition:

(1) "Law of Stream Numbers: The numbers of streams of different orders in a given drainage basin tend closely to approximate an inverse geometric series in which the first term is unity and the ratio is

the bifurcation ratio-" Or, expressed as an equation:

$$N_o = r_b^{s-o}$$

where N_o = number of streams of order o
s = stream segments of highest order
r_b = bifurcation ratio or $\frac{N_o}{N_{o+1}}$

(2) "Law of Stream Lengths: The average lengths of streams of each of the different orders in a drainage basin tend closely to approximate a direct geometric series in which the first term is the average length of streams of the first order-" Or, expressed as an equation:

$$l_o = l_1 \, r_e^{o-1}$$

where l_o = average length of streams of order o
l_1 = average length of first order streams
r_l = stream length ratio or $\frac{l_o}{l_{o-1}}$

(3) "Law of Stream Slopes: . . . there is a fairly definite relationship between slope of the streams and stream order, which can be expressed by an inverse geometric series law." Or, expressed as an equation:

$$s = \frac{s_1}{r_s^{o-1}}$$

where s_o = average slope of streams of order o
s_1 = average length of first order streams
r_s = slope ratio or $\frac{s_o}{s_{o-1}}$

Horton concluded his paper by describing a conceptual model for the evolution of the drainage net.

In 1947 the U. S. Geological Survey used map measurements in addition to stream gaging and field measurements for hydrologic studies (Langbein and others, 1947). The hypsometric curve was introduced to drainage basin studies in this work.

A paper which did not deal directly with morphometric analyses, but has important implications in this regard, was published by Mackin (1948). Mackin clarified Davis' (1909) concept of the graded stream by defining it as a system in equilibrium. The graded stream is neither depositing nor eroding when considered over a long span of time and throughout the entire length of the stream.

Two years later morphometric evidence supported the concept that streams are systems of equilibrium when it was demonstrated that within an area of homogeneous lithology the valley side slopes are at a constant angle (Strahler, 1950). Hack (1960) expanded on the concept of equilibrium and Holmes (1964) pointed out that Hack's and Mackin's concepts are complementary.

In the last fifteen years many morphometric studies of drainage basins have been conducted. Strahler (1952a) outlined two basic approaches to quantify statements of geomorphic process and form: (1) statistical analysis of data and empirical derivation of equations and (2) through intuition based upon experience, the formulation of a simple mathematical model. The former of these two approaches to the problems of drainage pattern development has been used most extensively, and many statistical studies have been made by Strahler and his students.

The effects of lithology on the morphometeric parameters were studied by Miller (1953) and Coates (1956). Miller's study involved the folded strata of the Clinch Mountain area of

Tennessee and Virginia, whereas Coates worked on the horizontal rocks of southern Indiana. Miller demonstrated that the drainage density or total length of streams divided by the basin area, was lower in areas of dolomites and higher in the areas of shale. This could be explained by the higher infiltration capacity due to solution cavities and the greater resistance to stream corrasion in the dolomite. Coates concluded that lithology was a significant factor in such parameters as stream lengths, basin areas, perimeters, lengths of overland flow, and drainage densities. Regional relief as a control was either not present or masked by other controls, whereas length of overland flow is clearly correlated with the stage of the erosional cycle.

A study was conducted on the erosion pattern in a clay pit where quantitative data were available from maps made in 1948 and in 1952 (Schumm, 1954). This made it possible to study the effect of time on the drainage nets; and, with minor exceptions, the evolution of the drainage pattern agreed with Horton's concept. Schumm also added two laws of drainage composition:

(4) ". . . the mean drainage basin areas of streams of each order tend closely to approximate a direct geometric series in which the first term is the mean area of the first order basins." Or, expressed as an equation:

$$A_o = A_1 r_a^{\,o-1}$$

A_o = mean drainage area of streams of order o

A_1 = mean drainage area of 1st order basins

r_a = area ratio

> (5) ". . . the relationship between mean drainage basin areas of each order and mean channel lengths of each order of any drainage network is a linear function whose slope (regression coefficient) is equivalent to the area in square feet necessary on the average for the maintenance of one foot of drainage."

A very sophisticated treatment of statistical data uses the parameters of area, total length, perimeter, and available relief to postulate a growth equation to show the effect of time on these parameters (Melton, 1958).

One of the earliest quantitative studies of glacial terrain used highway profile data to show the topographic differences on the various substage drift sheets of the Wisconsin drift in Iowa (Ruhe, 1950). A more complete study, following the pattern established by Strahler's students, was attempted to contrast the Iowan and Kansan topographies (Gordon, 1960). This study was somewhat inconclusive due to the limited number of samples and the failure to use stereopairs of airphotos to aid in the definition of drainage lines.

Gray (1961) studied data from 47 watersheds in both glaciated and unglaciated portions of several states in the Midwest. By using the method of least squares, he showed that a regression line existed for a plot of basin area versus total length of all streams in a given basin.

In a recent textbook on fluvial geomorphology (Leopold, Wolman and Miller, 1964) a new graph is introduced showing the relationship of drainage density and time. Drainage densities were computed from maps of the Des Moines lobe (Ruhe, 1952) and the age of each glacial substage was plotted against the mean drainage density for that substage. These data showed a sharp increase in drainage density from about 5,000 to 20,000 years before present, and a leveling off of the curve between 20,000 and 50,000 years before present.

Some statistical studies on the evolution of drainage basins have involved the simulation of the drainage network by using random walk techniques and computers (Leopold and Langbein, 1962, Schench, 1963).

Considerable data have been collected on drainage basins and many empirical relations have been derived, but little thought has been given to the process of drainage net evolution. Most studies have attempted to select significant parameters on a statistical rather than a rational basis. This failing of quantitative geomorphology, that is, over-dependence on the empirical approach, has been pointed out by Mackin (1963). This study will put a greater emphasis on the rational approach to the quantification of drainage basins.

The assumptions that are the basis of a statistical study of morphometric parameters of drainage basins in order to determine the influence of geology on basin development are listed below.

(1) Initial surface is a horizontal, externally drained plane.

(2) Climatic and vegetational homogeneity exist between geographic areas.

(3) There is no change in climate and vegetation through-

out the time of drainage development.

(4) Geologic homogeneity exists throughout the area.

(5) In any area of uniform geology all of the basins of a given order are in the same stage of development.

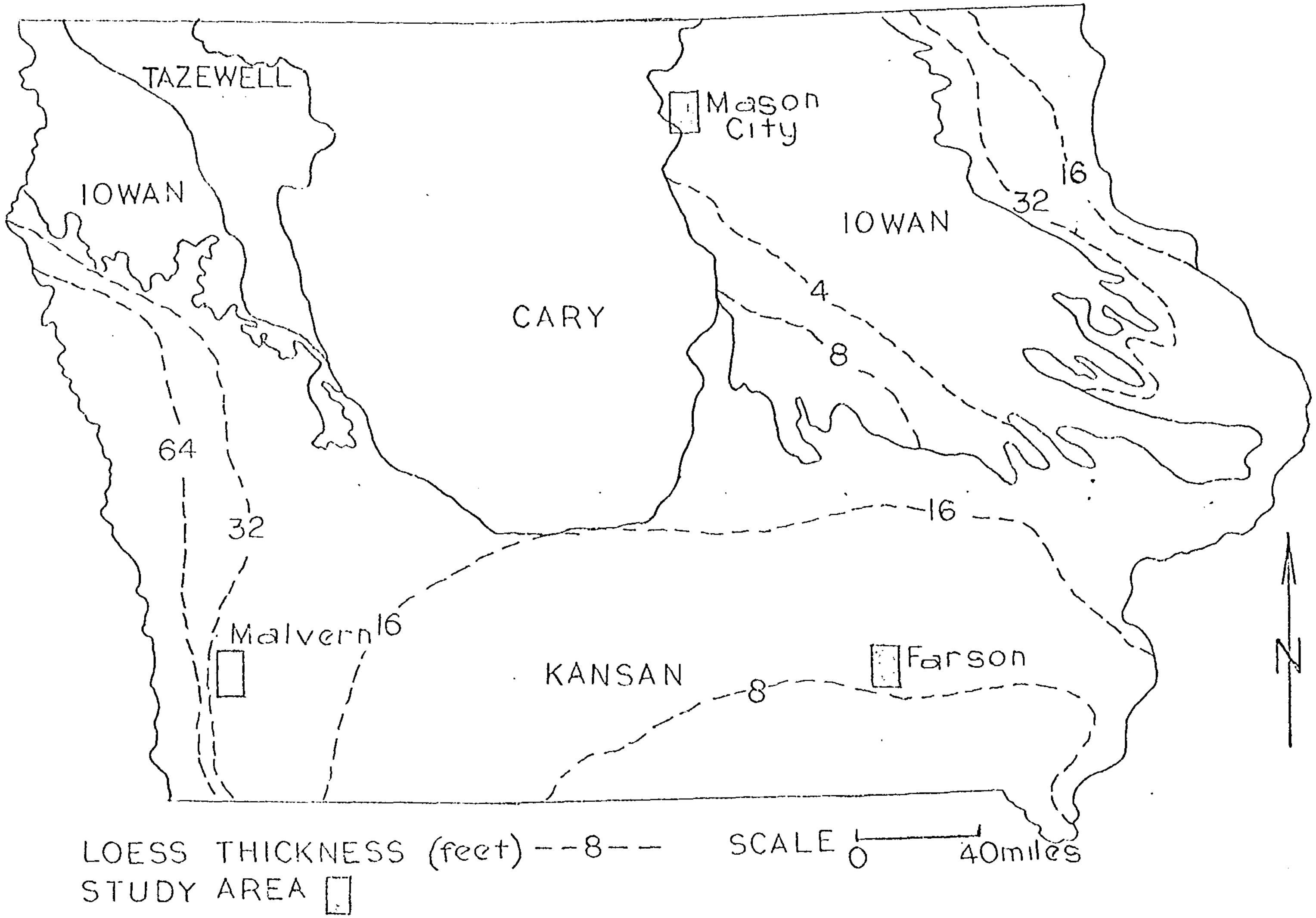
METHOD OF STUDY

Criteria for Selection of Basins

Since one purpose of this study is to quantify the differences in the degree of dissection that occurs on drift sheets of different ages, basins were selected in areas of Kansan and Iowan drift. A comparison of the erosional development occurring on different types of Pleistocene sediments was also desirable, so a third area was selected in the thick loess deposits of western Iowa. The choice of study localities was limited by the lack of modern 1:24,000 scale U.S. Geologic Survey topographic maps. However, three quadrangles were chosen; their location and relation to the Pleistocene deposits are shown on Figure 1. The Mason City Quadrangle (from 43°15'N to 43°07'30"N and from 93°15'W to 93°07'30"W) is a map of an Iowan drift area, Farson Quadrangle (from 41°07'30"N to 41°00'N and from 92°22'30"W to 92°15'W) is an area of Kansan drift, and the Malvern Quadrangle (from 41°07'30"N to 41°00'N and from 95°37'30"W to 95°30'W) is of an area of thick Wisconsin loess over Kansan drift.

On each of these maps the streams marked in blue were extended up the V-shaped contours to get a closer approximation to the drainage net as it appears in the field (Morisawa, 1957). The streams were then ordered according to the method suggested by Strahler (1957). On each map about twenty third-order basins

Figure 1. Index map showing major Pleistocene deposits (after Ruhe and Scholtes, 1959) and study areas


TAZEWELL
IOWAN
Mason City
IOWAN
CARY
4
8
16
32
16
64
32
Malvern
16
KANSAN
Farson
8
N
LOESS THICKNESS (feet) --8--
SCALE 0 40miles
STUDY AREA

were outlined. The extension of drainage lines and definition of third-order basins was done with the aid of stereoscopic pairs of airphotos. Ten basins on each map were selected at random for study. Areas were measured with a Keuffel and Esser compensating polar planimeter and linear dimensions with a Keuffel and Esser chartometer. No field work was done, so all data were obtained from map measurements.

Definition of Parameters and Method of Measurement

The measurements used in this study are similar to those used by Coates (1956) and are presented in Table 1. All the properties listed in Table 1 are measured from the channel and basin outline reduced to a horizontal plane. The dimensions for length measurements are in miles and areal measurements in square miles.

The measurements involving the third dimension are the maximum relief (H) of each third-order basin (measured in feet) and the hypsometric data. These measurements are defined and discussed thoroughly by Langbein and others (1947) and Strahler (1952b).

In addition to these previously used techniques, the data from the hypsometric analysis were used to plot area-elevation curves. That is, instead of calculating the elevation of each contour as a percentage of the total height of the basin, only the elevation was used, and rather than calculate the area between each contour and the basin perimeter as a percentage of

Table 1. Morphometric parameters used in this study

Name of parameter	Symbol	How derived	Reference for definition
1. Number of streams of each order	N_u (where) u is the order)	Counted	Horton (1945)
2. Bifurcation ratio	R_b	$R_b = \frac{N_u}{N_{u+1}}$	Horton (1945)
3. Total length of each order streams	L_u	Measured with a chartometer	Horton (1945)
4. Mean length of each order streams	$\bar{L}_u$	$\bar{L}_u = \frac{L_u}{N_u}$	Horton (1945)
5. Stream length ratio	R_l	$R_l = \frac{\bar{L}_u}{\bar{L}_{u-1}}$	Horton (1945)
6. Area of third-order basin	A_3	Measured with a planimeter	Horton (1945)
7. Total length of all streams of orders in the third order basin	L	$L = L_1 + L_2 + L_3$	Horton (1945)
8. Drainage density	D	$D = \frac{L}{A}$	Horton (1945)
9. Stream frequency	F	$F = \frac{N_u}{A_u}$	Horton (1945)
10. Basin circularity	C	$C = \frac{A_3}{\text{A or circle having same perimeter as the basin}}$	Miller (1953)

Table 1. (continued)

Name of parameter	Symbol	How derived	Reference for definition
11. Basin perimeter	P	Measured with a chartometer	Smith (1950)
12. Texture ratio	T	$T = \frac{N_1}{P}$	Smith (1950)

total basin area, the area was used. This plot gives a clearer picture of the landscape that has not been dissected.

From the data accumulated in this study it was also possible to test the validity of Horton's first two laws (1945) as they applied to glaciated terrain. This was accomplished by semilog plots of number of streams versus order of those streams, and mean length of streams against order.

Statistical Tests

Analyses of variance tests (Ostle, 1963) were run for the means of all parameters for the Farson, Mason City, and Malvern areas to see if any significant differences existed between the areas. The purpose of this test is to determine if the variability of parameters between study areas is greater than variability of parameters within study areas. The parameters which showed differences between areas were then tested by the method of Scheffe (Ostle, 1963) to see which area or areas caused the analysis of variance hypothesis of equal means to be rejected. The analysis of variance test demonstrates only that a difference does or does not exist between the three study localities. If the analysis of variance results indicate that the mean values of the study localities are different, the Scheffe test demonstrates which pair of study localities are different. The Scheffe test was made to compare the Malvern and Mason City areas where it was assumed there was the same stage of development but different lithologies. Scheffe tests were also run on the Farson and Mason City areas where there was assumed to be similar lithologies but different stages of development.

Hypsometric curves were drawn for all drainage basins. Hypsometric integrals were computed and subjected to both analysis of variance and the Scheffe test. Mean hypsometric curves were constructed graphically for each area. The data for all analysis of variance and Scheffe tests are shown in the appendix.

GEOLOGIC SETTING

Physiography of the Areas Studied

All three of the study areas are located in the physiographic province known as the Central Lowland. The topography of the Mason City area can be described as a gently undulating surface with few streams. The small streams flow in valleys about ten feet deep, whereas the larger ones are situated in valleys which may exceed one hundred feet in depth. The interstream areas are poorly drained. Fenneman (1938) has stated that the surface is essentially as the ice left it with imperfect drainage; and, where the geographic cycle (Davis, 1909) has begun, it is in early youth.

The Farson area has been described as "rough and rugged topography" (Leonard, 1901). The major stream valleys are up to two hundred feet deep and in many localities the drainage lines are separated by broad flat divides having areas of several square miles. This has led to the conclusion that the area was once a nearly level drift plain. The origin of this flat surface has been attributed to post-glacial flattening of the Kansan drift (Shrader and Hussey, 1953). The drainage of the area is in a mature stage of development.

In the Malvern area total relief may reach two hundred feet with steep slopes and narrow divides, and appears to be influenced by ease of erosion of loess compared to glacial till.

Stratigraphy of the Areas Studied

The Iowan drift of the Mason City area is covered by a veneer of loess varying from zero to two and one-half feet in thickness. The drift itself averages ten feet thick (Kay and others, 1945) and according to Hobbs (1942) is more like imperfectly stratified sands and gravels rather than the characteristic heterogeneous mixture of clay, silt, sand, and gravel size particles that one usually associates with till in Iowa. Calvin (1896) described sands and gravels at the base of the Iowan till directly overlying the Paleozoic bedrock.

In the Farson area the loess rarely exceeds 5 or 6 feet in thickness, it is underlain by the Kansan drift averaging about 100 feet in thickness with a maximum of 200 feet (Leonard, 1901).

Wisconsin loess is the principle stratigraphic unit in the Malvern area, it varies from 30 to 60 feet in thickness (Udden, 1902). It is underlain by Kansan till which averages about 200 feet thick in southwestern Iowa (Iowa Geologic Survey, 1960).

Soils of the Areas Studied

Floyd silt loam, Clarion silt loam, and Carrington loam are the major soil types in the Mason City area. The Floyd occupies the depressions of almost imperceptible slope whereas the Clarion and the Carrington are situated on the swells or upland divides (Elwell *et al*., 1940). The soils are all essentially the same textural class.

In the Farson area the Grundy silt loam and the Clinton silt loam are the principle soils. The Grundy is developed on the gently rolling upland divides and the Clinton is developed on the stream slopes (Stevenson and Brown, 1921). There is textural similarity within the area, and between this area and the Mason City area.

Practically all of the uplands in the Malvern area are mapped as the Marshall loam. The other soil which occurs in the lower reaches of the third-order basins is the Wabash silt loam, a colluvial phase which results from the downslope movement and erosion of the upland loess into the valleys (Stevenson and Brown, 1924).

Climate and Vegetation

The climate of Iowa is generally considered to be uniform throughout the state. The ample rainfall (between 26 and 36 inches per year) and moderate temperatures provided for the luxuriant growth of prairie grasses throughout the state; however due to the high intensity of farming, only isolated areas of virgin prairie remain. Each of the study areas has been under cultivation for about one hundred years. Table 2 shows the climatic data for the weather stations nearest each study area (U.S. Department of Agriculture, 1941).

Table 2. Climatic data for the study areas

Study area	Station	Average annual precipitation (inches)	Length of growing season (days)	Av. Jan. temp. (°F)	Av. July temp. (°F)
Mason City	Mason City	30.38	148	15.2	72.8
Farson	Ottumwa	33.57	169	24.0	77.3
Malvern	Glenwood	29.75	166	24.2	77.4

PRESENTATION OF DATA

Means and Standard Deviations

The means and standard deviations of all the morphometric parameters are listed in Table 3. Data from this table were used to determine if the basins of this study conformed to Horton's (1945) law of stream numbers and law of stream length. The mean number of streams of each order for each study locality is plotted against stream order in Figure 2. All three localities conform fairly well to Horton's first law.

Means of the mean stream lengths for each study area are plotted against stream order in Figure 3. The Mason City and Malvern areas agree with the law of stream lengths, but the Farson area does not. Schumm (1954) states that Horton's second law has been questioned by Strahler, who suggested that the relationship between stream length and order might better be expressed as a power function rather than as the exponential function of Horton's geometric progression. Another reason for deviation might have been incurred by using Strahler's (1957) rather than Horton's (1945) system of stream ordering. By Strahler's system the higher order streams are bound to be shorter and the lower order streams longer than if the same stream system were ordered according to Horton's system. This relationship is shown in Figure 4. If the first order streams joined the second order streams close to the junction of the

Table 3. Morphometric data for third order basins

Locality		Malvern	Farson	Mason City
Geology		Deep Wisconsin loess	Kansan till	Iowan till
Total length of 1st order streams	$\bar{x}$	0.803	2.021	1.137
	s	0.497	1.228	0.527
	N	10	10	10
Number of 1st order streams	$\bar{x}$	10.20	22.60	5.70
	s	7.66	14.71	1.574
	N	10	10	10
Mean length of 1st order streams	$\bar{x}$	0.084	0.353	0.201
	s	0.0002	0.074	0.096
	N	10	10	10
Total length of second order streams	$\bar{x}$	0.427	0.821	0.647
	s	0.377	0.355	0.386
	N	10	10	10
Number of second order streams	$\bar{x}$	3.20	5.70	2.20
	s	1.93	2.98	0.422
	N	10	10	10
Mean length of second order streams	$\bar{x}$	0.126	0.153	0.310
	s	0.0837	0.0055	0.192
	N	10	10	10
Total length of third order streams	$\bar{x}$	0.289	0.677	0.388
	s	0.249	0.659	0.322
	N	10	10	10
Total length of all streams	$\bar{x}$	1.519	3.520	2.172
	s	1.040	2.161	1.056
	N	10	10	10
Stream length ratio, 2nd to 1st order	$\bar{x}$	1.59	0.45	0.59
	s	0.917	0.170	0.344
	N	10	10	10
Stream length ratio, 3rd to 2nd order	$\bar{x}$	5.45	5.02	0.70
	s	10.18	5.83	0.253
	N	10	10	10

Table 3. (continued)

Locality		Malvern	Farson	Mason City
Geology		Deep Wisconsin loess	Kansan till	Iowan till
Bifurcation ratio, 1st to 2nd order	$\bar{x}$	3.14	3.85	2.58
	s	0.971	1.038	0.438
	N	10	10	10
Bifurcation ratio, 2nd to 3rd order	$\bar{x}$	3.20	5.70	2.20
	s	1.93	2.98	0.422
	N	10	10	10
Perimeter of basins	$\bar{x}$	1.757	2.868	3.110
	s	0.893	1.439	1.682
	N	10	10	10
Areas of basins	$\bar{x}$	0.200	0.564	0.671
	s	0.014	0.476	0.557
	N	10	10	10
Circularity ratio	$\bar{x}$	0.76	0.83	0.77
	s	0.088	0.1414	0.105
	N	10	10	10
Drainage density	$\bar{x}$	7.764	7.119	4.062
	s	0.92	1.970	1.28
	N	10	10	10
Stream frequency	$\bar{x}$	76.66	59.21	24.25
	s	18.92	18.97	19.55
	N	10	10	10
Texture ratio T = n/p	$\bar{x}$	5.41	2.00	2.11
	s	2.17	0.412	0.748
	N	10	10	10
Maximum relief within basins	$\bar{x}$	115.5	96.5	55.0
	s	22.78	35.5	17.15
	N	10	10	10
Hypsometric integral	$\bar{x}$	0.562	0.782	0.578
	s	0.0463	.0678	0.0694
	N	10	10	10

Figure 2. Semilog plot of stream order versus number of streams for mean values of study locations (showing agreement with Horton's first law)

Figure 3. Semilog plot of stream order versus mean length of streams (in miles) for mean values of study locations (showing agreement with Horton's second law)

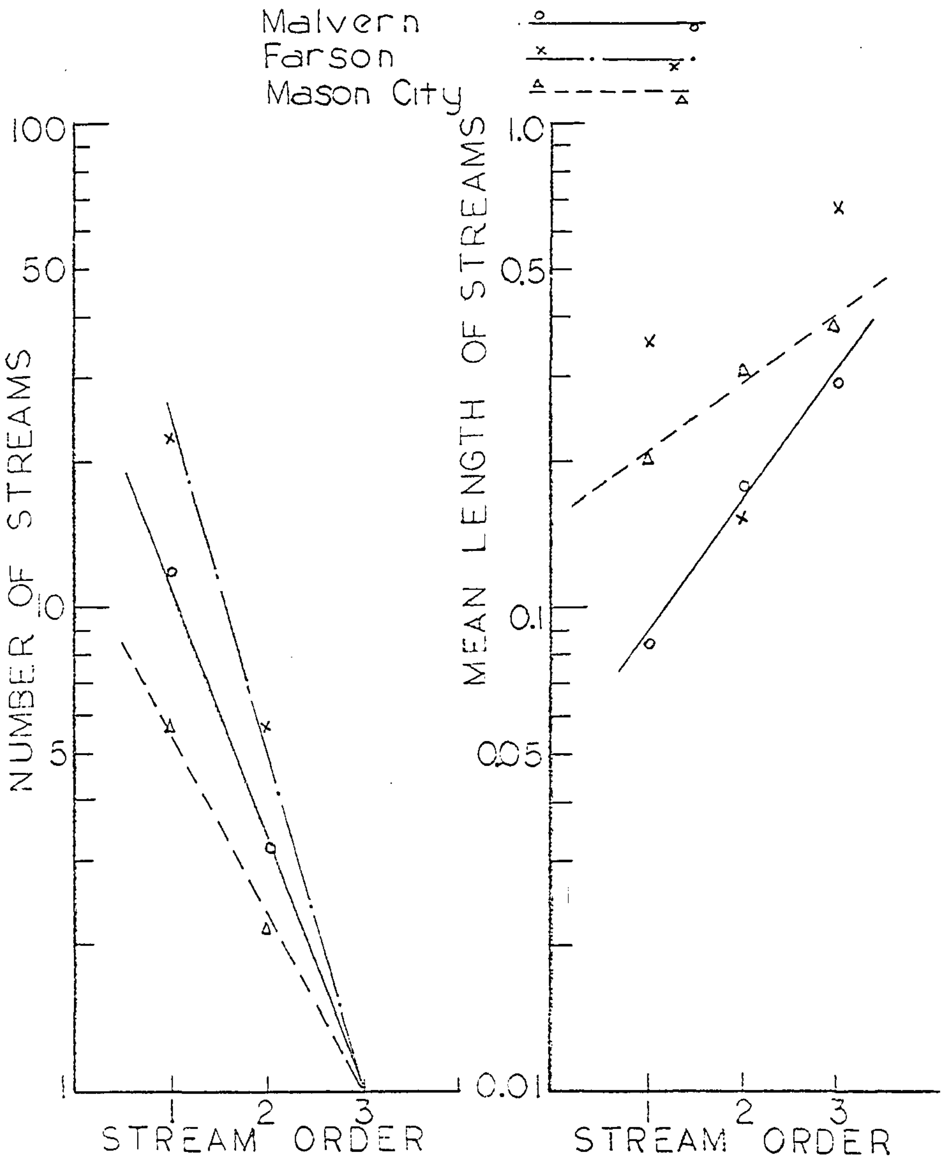
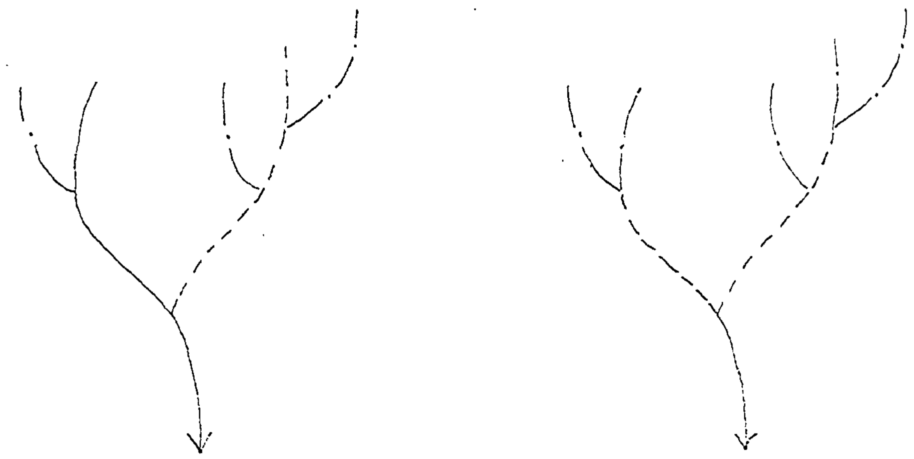

Malvern
Farson
Mason City
NUMBER OF STREAMS
100
50
10
5
1
STREAM ORDER
1
2
3
MEAN LENGTH OF STREAMS
1.0
0.5
0.1
0.05
0.01
STREAM ORDER
1
2
3

Figure 4. Methods of ordering stream systems according to Horton (1945) and Strahler (1957) (demonstrating the shorter lengths of higher order streams in Strahler's system)

Methods of Ordering Stream Systems



HORTON STRAHLER

FIRST ORDER STREAMS —— · ——

SECOND ORDER STREAMS - - - - - - -

THIRD ORDER STREAMS ————

second order stream with the third order stream, the second order streams would be much shorter and the first order streams much longer.

Analysis of Variance Results

Lengths

The difference of total lengths of third, second and first order streams between study areas is no greater than the difference within the areas according to Scheffe tests run on each property.

The mean lengths of first order streams are similar in all three study areas, but a difference does exist between the mean length of second order streams in the Malvern and Mason City areas as well as between the Farson and Mason City areas. No general pattern is obvious for either lithologic or stage of development control of the lengths of the streams of individual orders.

Coates (1956) stated that stream lengths reflected the lithologic variations of the areas studied in southern Indiana. The results of his statistical analysis only partially agree with those of this study. The only parameters which agree with Coates' results are total length of second order streams, the mean length of second order streams, the length ratio of second to first order streams and the bifurcation ratio of second to third order streams. It should be pointed out that Coates ran studies on six areas to determine the control of lithology on morphometric properties, whereas this study contrasted only two areas. Coates went no further in his study than an analysis of variance; where any one out of the six areas was greatly different from the other five, he would reject the null hypothesis. An example of this is the parameter of length ratio of third to second order streams. The analysis of variance of that parameter in this study showed an acceptance of the null hypothesis whereas Coates rejected at both levels. A study of his data shows that five areas have mean stream lengths of 2.339 to 3.969 miles but the sixth one has a value of 7.534 miles.

Dimensionless properties

Length ratios, bifurcation ratios, and numbers of streams of each order are dimensionless properties. The variance of stream length ratios within localities was as great as the variance between localities for third to second order ratios. The length ratio of second to first order streams showed a difference between the Malvern and Mason City areas.

A difference exists between the number of first and second order streams and between the bifurcation ratio of second to third order streams of all three areas. The bifurcation ratio of first order streams differs only between the Farson and Mason City areas.

The circularity ratio shows no difference between areas. This result agrees with other morphometric studies (Coates, 1956 and Miller, 1953).

Properties relating to drainage texture

The stream frequency, texture ratio, and drainage density show significant differences between the Malvern and Mason City areas; whereas the stream frequency and drainage density show a difference between the Farson and Mason City areas.

Hypsometric analysis

The hypsometric curves for the Malvern area are shown in Figure 5, the Farson area in Figure 6 and the Mason City area in Figure 7. The analysis of variance of the hypsometric integrals shows differences between the Farson and Mason City areas, but there is no difference between the Malvern and Mason City areas.

Perimeters and areas

The perimeters and areas of all three localities had a variation of values within the localities which was as great as the variation between the localities.

Figure 5. Hypsometric curves of Malvern area (the heavy line is the mean curve)

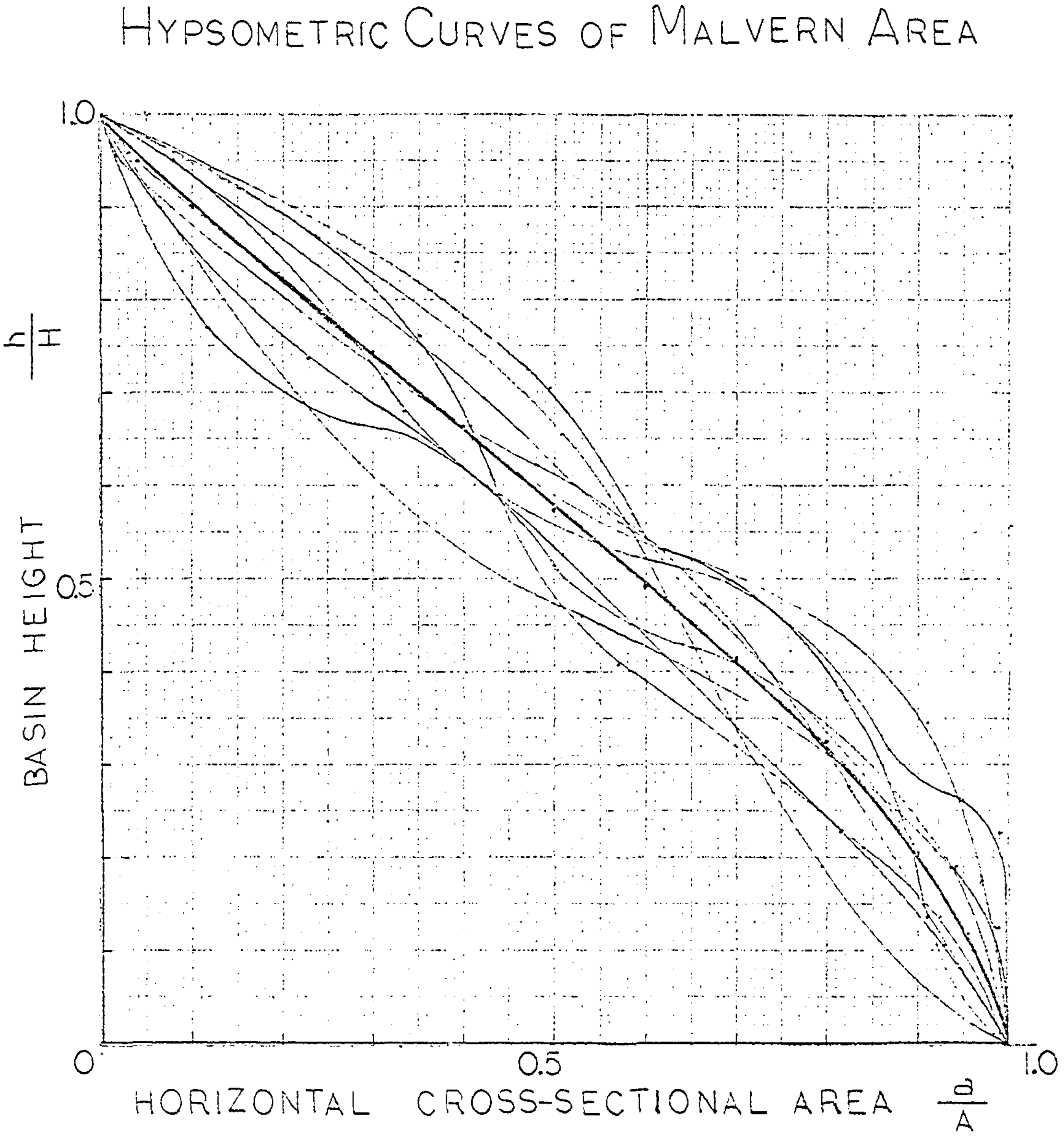

HYPSOMETRIC CURVES OF MALVERN AREA
1.0
0.5
0
BASIN HEIGHT h/H
0.5
1.0
HORIZONTAL CROSS-SECTIONAL AREA a/A

Figure 6. Hypsometric curves of Farson area (the heavy line is the mean curve)

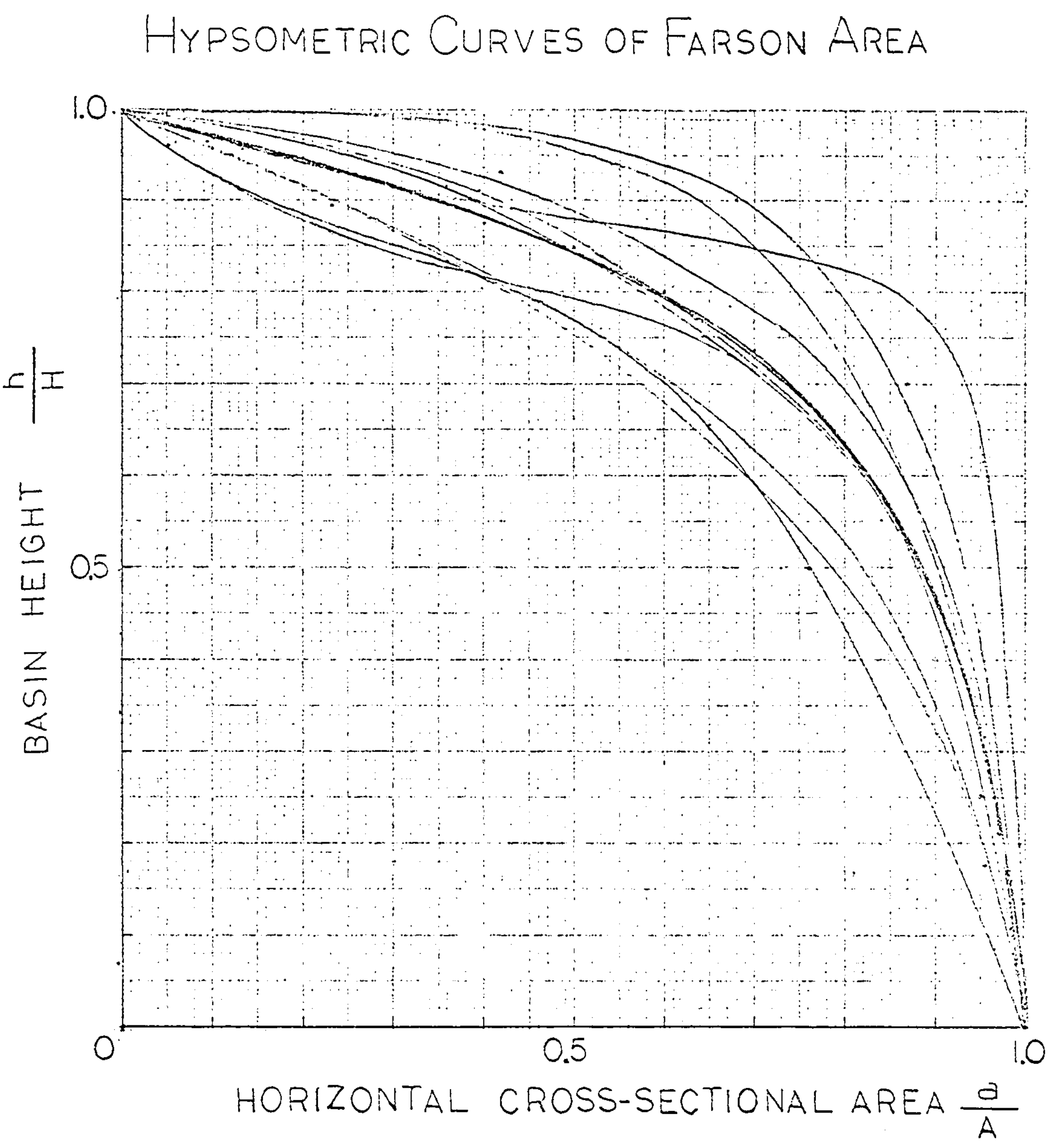
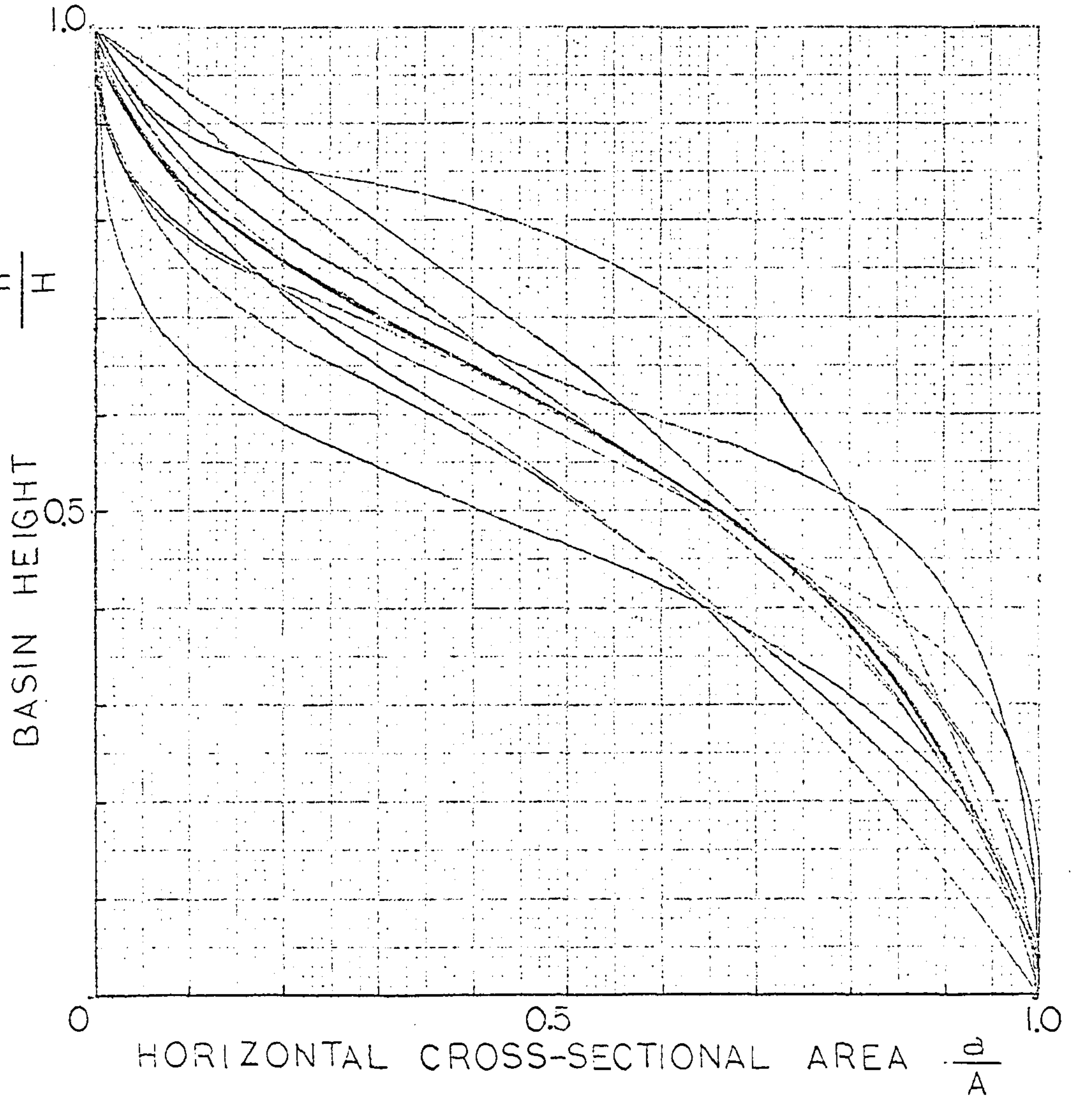

HYPSOMETRIC CURVES OF FARSON AREA
1.0
0.5
0
BASIN HEIGHT $\frac{h}{H}$
0.5
1.0
HORIZONTAL CROSS-SECTIONAL AREA $\frac{a}{A}$

Figure 7. Hypsometric curves of Mason City area (the heavy line is the mean curve)

HYPSOMETRIC CURVES OF MASON CITY AREA



DISCUSSION - STATISTICAL APPROACH

Since climate and vegetation are homogeneous in all areas, the differences in morphometric properties can be attributed to differences in lithology between the Malvern and Mason City areas. The surficial deposits of the Malvern area consist of Wisconsin loess cover several tens of feet thick over Kansan glacial drift, whereas the surficial deposits of the Mason City area are less than four feet of loess over about ten feet of Iowan drift. The ages of the dominant surficial deposits in these two areas are nearly the same.

In addition to climatic and vegetational homogeneity between the Farson and Mason City localities, there is lithologic similarity since both areas are of thin loess-covered drift. However, the drift in the Farson area is about ten times thicker than that of the Mason City locality. The drift in the Mason City locality is Iowan in age and the drift at Farson is Kansan, so differences between these areas would be due to differences in the stage of development of the drainage pattern and differences in degree of weathering.

Properties Controlled by Lithology

The morphometric properties showing significant differences between the Malvern and Mason City areas are drainage density, stream frequency, texture ratio, total maximum relief, bifurcation ratio of second to third order streams, length ratio of second to first order streams, and the number of first order streams. All of these parameters except the bifurcation ratio of second to third order streams and the number of first order streams agree with Coates' (1956) study.

Since texture ratio, drainage density, and stream frequency are measures of how thoroughly the landscape has been dissected by streams, it was expected that these parameters should show differences for the two localities. The texture ratio is the number of first order streams divided by the perimeter of the third order basin. Drainage density is the total length of all streams divided by the basin area, whereas stream frequency is the total number of all streams divided by the basin area. High values of each of these parameters indicate a greater degree of dissection.

It has been pointed out that rocks of small resistance to erosion and low infiltration capacity would tend to have high texture ratios, whereas rocks of great resistance to erosion and high infiltration capacity would tend to have low texture ratios (Smith, 1950). Loess is generally considered to be less resistant to erosion and more permeable than till. These two properties would cause opposing tendencies in the development of the drainage pattern. That is, the high infiltration capacity would tend to decrease the textural parameters of the Malvern area, but the low resistance to erosion would tend to increase them.

Table 3 shows that the values for all these parameters are higher in the loess-covered Malvern area than in the glacial drift of the Mason City area. These data suggest that the controlling factor in the development of a finer texture in the Malvern area is the more erodable nature of the loess. In other words, the low resistance of loess is more dominant than the high permeability in controlling the texture of the drainage nets.

There is no obvious explanation for the differences and/or similarities in lengths of streams of different orders. The stream length of each order stream and the length ratios are the result of the point on the trunk stream at which the tributaries join, and it seems reasonable that this would be a highly variable factor in localities of horizontal strata. Stream lengths and length ratios are not good characteristic parameters of lithology or stage of development.

Properties Controlled by Stage of Development

The properties showing differences between the Farson and Mason City areas are stream frequency, drainage density, total maximum relief, bifurcation ratio of second to third and first to second order streams, number of first and second order streams, hypsometric integrals, and mean length of second order streams.

Stream frequency and drainage density are both higher in the Farson area than in the Mason City area as shown on Table 3. The statistical tests indicate that these are significant differences. These parameters are, as mentioned above, indicators of the degree of dissection of the landscape by streams. The higher values indicate greater dissection.

In addition to lithologic properties, stage of development influences the texture of drainage patterns. Davis (1909) introduced qualitative criteria for the definition of the various stages of fluvial erosion and Strahler (1952b) has given quantitative criteria for a similar concept. As streams extend over the landscape and cut it up, the stage of erosion becomes more advanced. As streams bifurcate, more first order streams are formed; hence, the number of streams increases. As streams extend headward, their total length increases. The lower value of drainage density in the Mason City locality indicates that the streams have not extended themselves as far as possible for the given drainage area. Lower stream frequencies for the Mason City locality indicate that fewer bifurcations have occurred there than in the Farson locality.

The higher values for both morphometric properties in the Farson area are a result of the Kansan age drift of the Farson area being further along in its erosional development than the Iowan drift of the Mason City area. Glock (1932) and Johnson (1933) have concluded that available relief is a significant factor in controlling drainage texture. High available relief allows the erosion of deep valleys which favors headward erosion of streams and results in greater dissection. The data in Table 3 indicate that localities of high drainage density are also the areas of high maximum relief. The relief could also be a result of a later stage of development. As would be expected, the relief of the areas of Kansan drift is higher than in the areas of Iowan drift.

Since the texture ratio is not significantly higher in the Farson than in the Mason City area, one could conclude that parameter is not affected by stage. The total number of streams increases with time or stage of development, but the number of first order streams does not increase as rapidly. This too could be a result of Strahler's ordering system. Streams that were once first order are elevated to higher orders when younger first order streams form.

All parameters which are a measure of number of streams show significantly higher values in the Farson locality than in the Mason City locality. This is as expected, because as the drainage system would develop and become more integrated, the number of streams should increase. The Farson area of Kansan drift should have a more completely integrated drainage system than the Mason City area of Iowan drift.

Properties Showing No Differences in All Localities

The circularity ratio is similar for all three areas studied. This result is consistent with other morphometric studies (Coates, 1956 and Miller, 1953). Both previous studies found basin areas and perimeters significantly different in localities of different lithologies, although this study demonstrates they are similar in areas of different lithology.

A study of Table 3 reveals standard deviations of 0.44 and

0.55 for the Farson and Mason City localities respectively and a standard deviation of 0.01 for the Malvern locality. This is a reflection of the heterogeneity of glacial drift as compared with the homogeneity of loess. Glacial till is well known for its poor sorting, and it is possible that there are lenses of sand and gravel within the till. The standard deviation of Coates' (1956) area measurements ranged from 0.03 to 0.15. The greater lithologic heterogeneity within study localities in glaciated regions may account for the greater variability of morphometric parameters; whereas in general, bedrock materials are better sorted and hence have lower variability of morphometric parameters. A similar comparison of standard deviations of perimeter measurements in this study can be made with those of the southern Indiana study (Coates, 1956). This greater variability within areas could be a limitation to morphometric studies of glaciated localities.

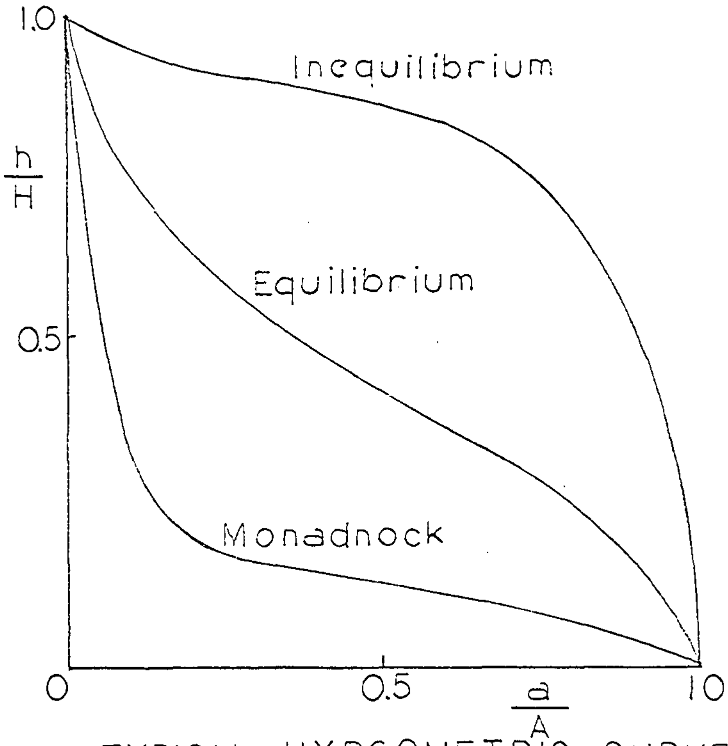
Hypsometric Analysis and Stage of Development

Strahler (1952b) has presented typical hypsometric curves for three stages in the development of a drainage basin as shown in Figure 8. He has also proposed hypsometric integrals as tentative boundaries of the stages: above 60% inequilibrium (youthful) stage, between 60% and 35% equilibrium (mature) stage, and below 35% the monadnock stage.

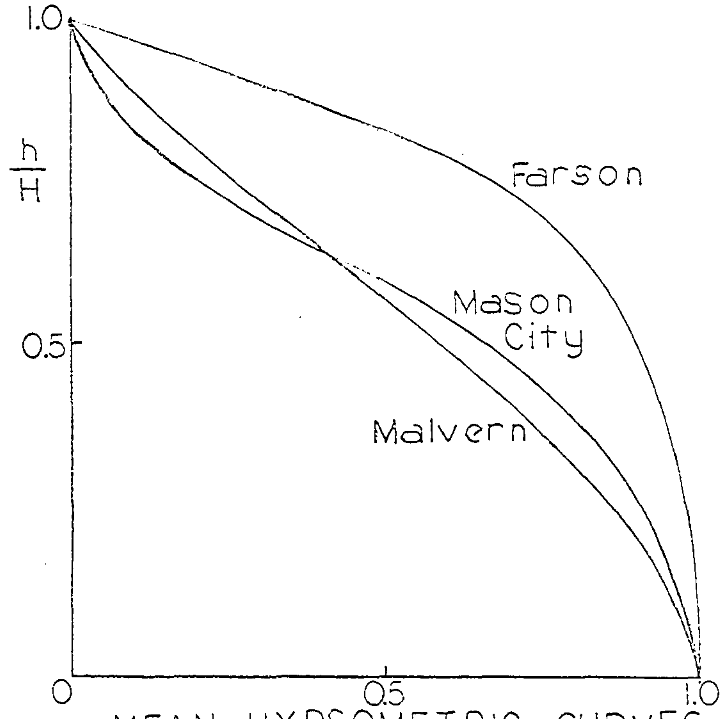
Figure 9 shows the mean hypsometric curves for each study locality. According to Strahler's classification, the Malvern

Figure 8. Typical hypsometric curves for stages of erosional development (after Strahler, 1952b)

Figure 9. Mean hypsometric curves of study localities showing stages of development (note the similarity between the curves of the Malvern and Mason City localities)



TYPICAL HYPSOMETRIC CURVES



MEAN HYPSOMETRIC CURVES

and Mason City areas are in the equilibrium stage, but the Farson area is in the inequilibrium stage. This is contrary to what is expected because the Farson area drainage has developed on Kansan drift and the Mason City area drainage on Iowan drift; therefore, the Farson drainage should be further along in the cycle than the Mason City drainage.

This anomalous relationship can be explained by a consideration of the geologic history. The other statistical evidences indicate that the Farson and Malvern localities are further along in the cycle than the Mason City locality. The drainage in the Mason City area is still rather poorly integrated so the system might be considered to be in a pre-youth or pre-inequilibrium state. That is, the region has just gone through or is still in the post-glacial flattening (Shrader and Hussey, 1953) stage and the development of the stream system is just beginning. Also, the Iowan drift plain has long been the subject of controversy and probably is poorly understood.

The drainage of the Kansan drift is still in the inequilibrium stage with deep valleys cut between the broad flat divides. Due to the greater ease of erosion of the loess in the Malvern area, the erosion cycle has proceeded at a faster rate. There have been many bifurcations of streams resulting in smaller third order basins with narrow divides giving the landscape the appearance of the equilibrium stage of the cycle. The Iowan drift is still in the pre-youth stage where isolated basins have been filled as the divides between them are eroded down. The height and area dimensions are plotted as percentages so the hypsometric curves are reduced to a common scale. The result is that the curves of the Malvern and Mason City localities give similar integrals although their erosional developments are quite different.

The differences can be seen more clearly if the same data are plotted as elevations against areas rather than as percentages of maximum basin height and percentages of total basin area. Figure 10 is a plot of area-elevation curves for the ten basins studied in the Mason City locality. Figure 11 and Figure 12 show similar plots for the Farson and Malvern localities respectively. The area-elevation curves are cumulative topographic profiles of the drainage basins. Rather than selecting cross sections and plotting linear distance against elevation, the area between the contour line and the basin perimeter is plotted against elevation.

The curves for the Farson locality show that the basins of larger areas have a broad flat segment at higher elevations. This is interpreted to represent the divides between drainage basins. The smaller area basins have curves of steeper slopes with narrow divides. This is consistent with qualitative observations of topographic maps of the Farson area.

If one interprets the flat portions of the curves as divide areas and the steep, almost vertical, portions of the curves as valley side slopes, it can be seen that the valley side slopes are almost parallel in each locality. One evidence

Figure 10. Area-elevation curves of Mason City locality (note different shape as contrasted with curves of Malvern locality)

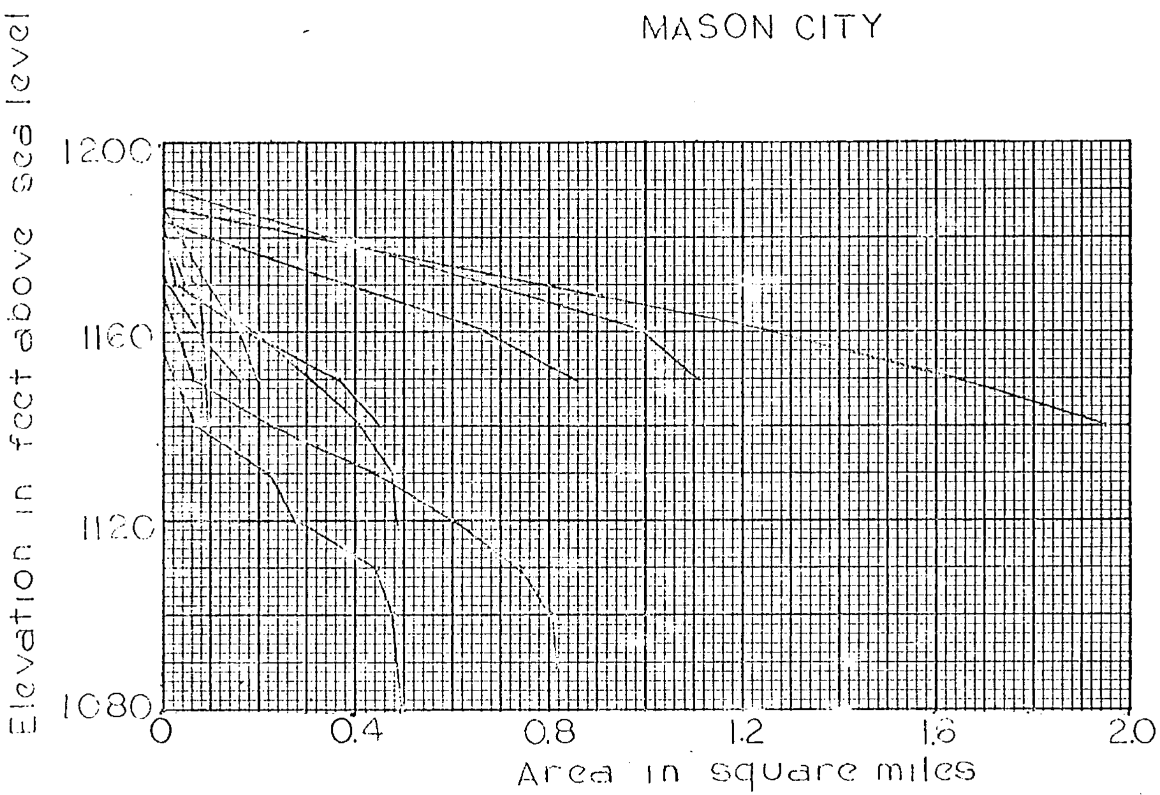
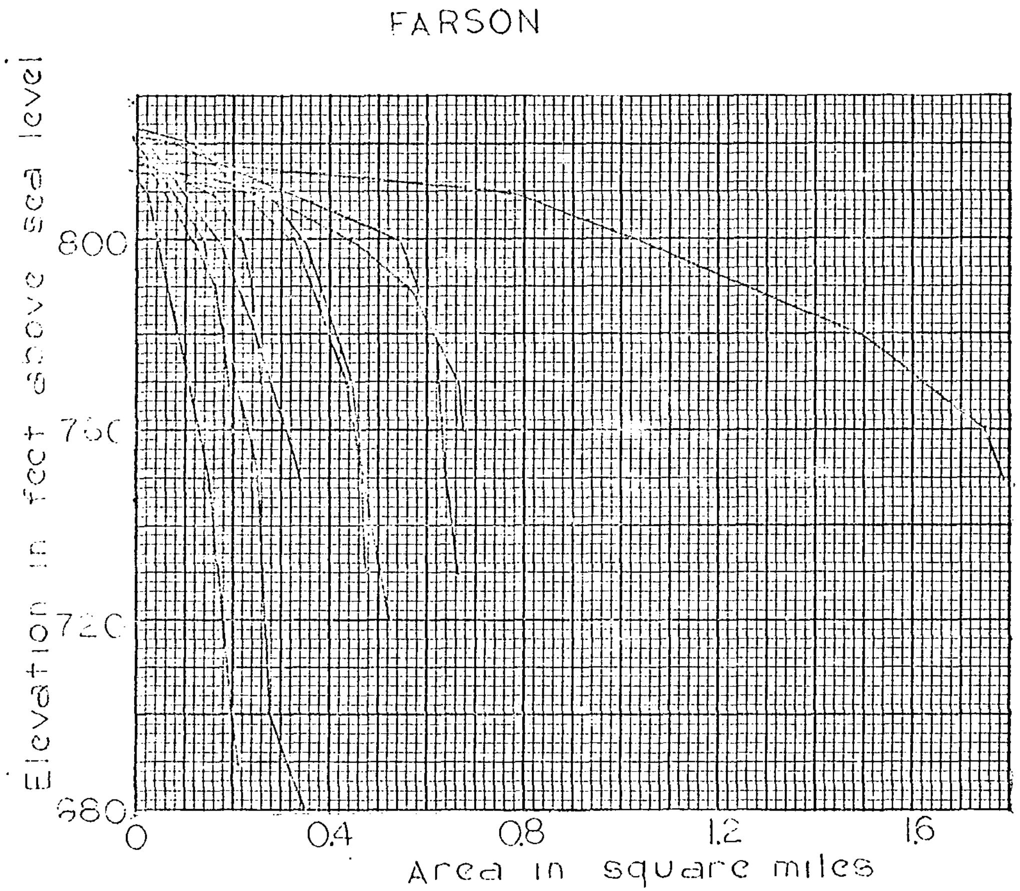
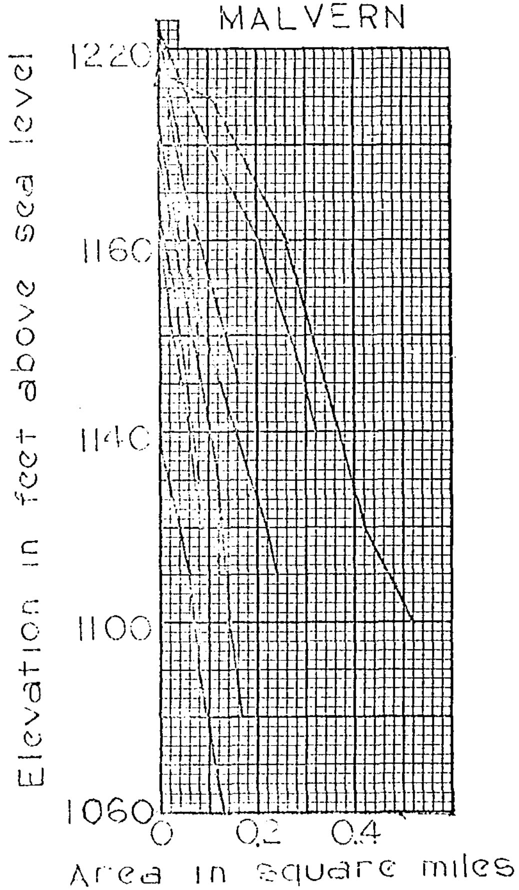

MASON CITY
Elevation in feet above sea level
1200
1160
1120
1080
0
0.4
0.8
1.2
1.6
2.0
Area in square miles

Figure 11. (left) Area-elevation curves of Farson locality

Figure 12. (right) Area-elevation curves of Malvern locality (note different shape of curves as contrasted with the Mason City locality)


FARSON
Elevation in feet above sea level
800
760
720
680
0
0.4
0.8
1.2
1.6
Area in square miles


MALVERN
Elevation in feet above sea level
1220
1160
1140
1100
1060
0
0.2
0.4
Area in square miles

for the equilibrium status of drainage basins is the similarity of valley side slopes in regions of homogeneous lithology (Strahler, 1950, Hack, 1960). One would expect the side slopes to be parallel within each of the localities of this study and the area-elevation curves affirm the prediction.

The Malvern locality curves are subparallel and show very narrow divides and long valley side slopes. It would be expected that erosional rates would be greater in areas of thick loess deposition than in areas of glacial till. The uplands would be consumed faster and the divides narrowed. This results in curves that are very different from those of the Mason City locality, even though the Mason City and Malvern deposits are essentially the same geologic age.

In the Mason City locality the area-elevation curves show that the larger basins are shallow with broad divides and the smaller basins occur at lower elevations with greater relief. The parallelism observed in the curves of the other localities is absent here. The broad flat curves are of larger areal extent than the broad curves of the Farson locality and are interpreted to represent the stage prior to valley down-cutting. The curves of the smaller basins show an upward concavity above the steep slope. This segment of the curves could represent benches resulting from the streams cutting through the glacial drift into the underlying bedrock. The cluster of curves at intermediate elevations represents stream systems that are eroding glacial drift and have reached the equilibrium configuration.

The area-elevation curves allow for an interpretation of lithology as well as stage of erosional development on an individual basis rather than considering the mean of a group of numbers. These curves provide a valuable complement to the hypsometric curves.

Conclusions

This study indicates that morphometric techniques can be applied to drainage basins in glaciated areas. The data of this study has greater variance than that for studies of bedrock due to the greater heterogeneity of glacial sediments compared with bedrock. The mean values for the streams in all three localities agree with the law of stream numbers and streams in two of the three localities agree with the law of stream lengths.

As expected, all the parameters indicating drainage texture are higher in localities of loess deposits due to the lower erosional resistance of the loess. Stream frequencies and drainage densities are greater in regions that have been subjected to erosion for longer time.

The hypsometric curves by themselves are poor indicators of stage of erosional development. The misleading appearance of the hypsometric curves of the Mason City locality may have resulted from bedrock control in some basins and the post-glacial flattening effect in others. These conclusions are suggested by the area-elevation curves.

DISCUSSION - RATIONAL APPROACH

As has been pointed out, most studies of morphometric analysis have been of the empirical type, and many data have been collected and analyzed statistically in an attempt to find the significant parameters. Horton's (1945) paper proposed a rational model for drainage net evolution, but in the collecting of data and quantitization it seems to have largely been left out of subsequent considerations of quantitative geomorphology. An attempt will be made here to expand Horton's rational model and postulate the expected variations of some of the morphometric parameters which will occur as the basin develops through time.

Models have always been necessary for a theoretical study of any phenomenon. The changing concept of the atom is a good example. The corpuscular nature of the atom gave way to the Bohr model, which in turn has been replaced by theories involving quantum mechanics. Each model had validity and was useful until new data made it necessary to replace the old model with one which better fit the observations. The only requirement that the model itself must meet is that it is internally consistent; once the model has been proposed it is tested against observations to see how closely it approaches reality. The same method will be used here. A consistent theoretical model will be proposed, then it will be tested against the data from this study as well as from Coates' (1956) study on consolidated rocks.

Champions of the statistical, empirical approach of quantitative geomorphology have said that it is the only way to solve geomorphic problems because there is too much indeterminancy in the geomorphic system. That is, there are too many variables and combinations of interdependent variables and the physical laws may be fulfilled by a variety of combinations of these interrelated factors (Leopold and Langbein, 1963). However, the understanding of any physical system initially requires that simplifying assumptions be made. As more becomes known about the system, the assumptions can be modified or replaced.

The two assumptions which underline this theory of the development of drainage basins with time are (1) Horton's (1945) concept of drainage net evolution is valid and (2) in an area of uniform lithology the drainage density is constant.

Review of Horton's Analysis

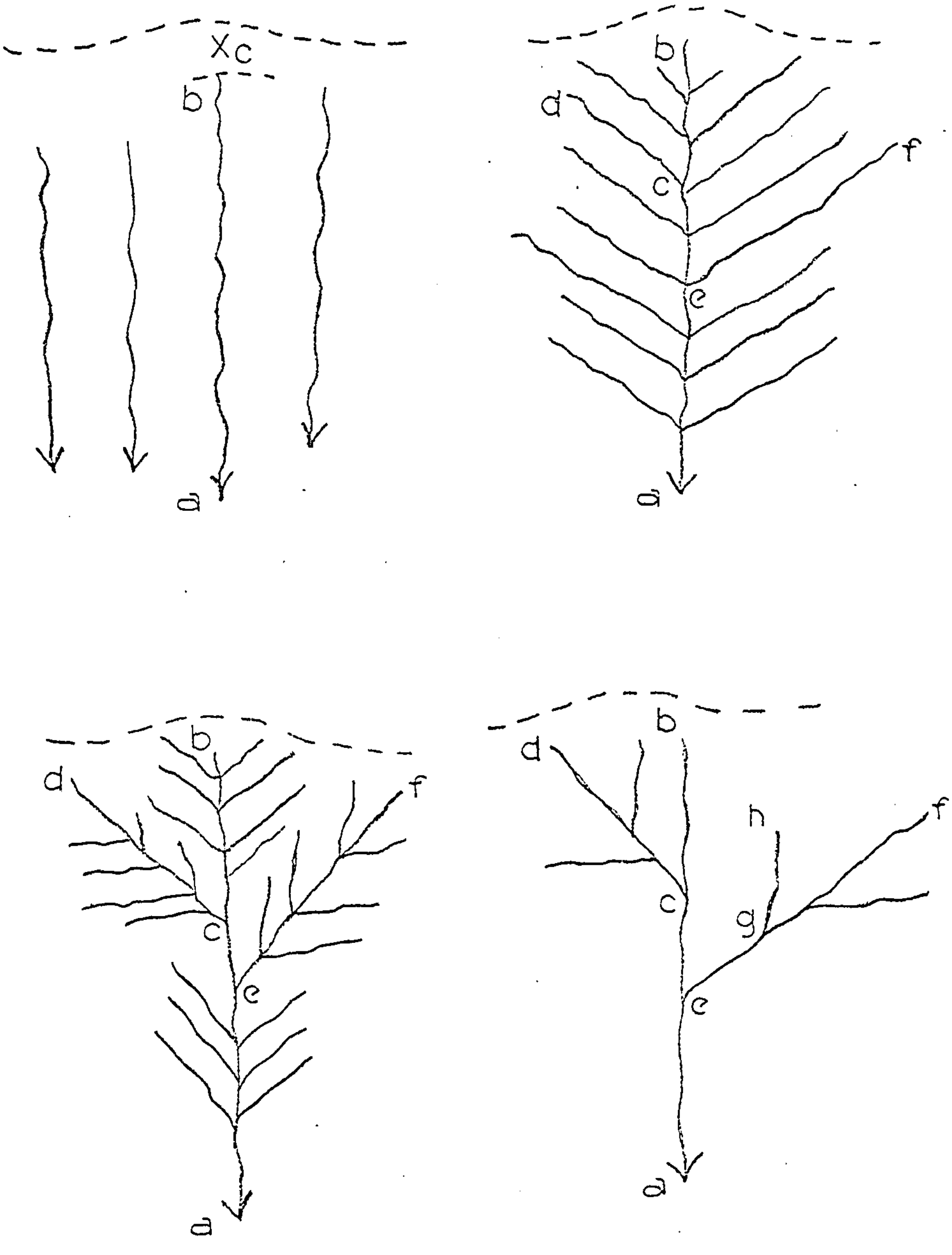
Horton's rational concept can best be understood with the aid of diagrams. Figure 13 through Figure 16 are idealized sketches of the evolution of a drainage pattern. Figure 13 represents the map view of an area recently exposed to erosion. The arrowheads point in the direction of regional slope. As precipitation falls on the area, some water will seep into the soil and the remainder will run off the surface. Near the divide the water will be in the form of sheet runoff; but as

Figure 13. Rilled surface exposed to stream erosion in early stage (after Leopold, Wolman, and Miller, 1964)

Figure 14. Dominance of master rill a-b with set of parallel tributary rills (after Leopold, Wolman and Miller, 1964)

Figure 15. Dominance of two tributary rills d-c and e-f flowing into master rill a-b (after Leopold, Wolman, and Miller, 1964)

Figure 16. Drainage net at end point of development with a-e segment of master rill as third order channel (after Leopold, Wolman, and Miller, 1964)


X c
b
a
b
d
c
f
e
a
b
d
f
c
e
a
b
d
h
f
c
g
e
a

the water accumulates down the slope and, due to the minor surface irregularities, it will be concentrated into rills. If the water has sufficient erosive energy, it will cut a channel into the landscape. At first several parallel channels will develop flowing down the initial slope. The headward extent of these rills will be limited by what Horton called the critical distance and is represented by X_c in Figure 13.

The critical distance is the result of the hydrology and the geology of the system. In order to erode, the water in the rills must possess sufficient energy to overcome the resistance of the soil or rock material. The energy results from the mass of water and the steepness of the slope. In order to accumulate sufficient mass for a given slope, the water will travel over the surface for some distance (the critical distance) without eroding. For very long critical distances there would be no rill erosion. If the soil is resistant or very permeable, the critical distance will be high. For example, sand dunes are very permeable, so most of the moisture that falls in them infiltrates into the sediment; since only a small amount of water remains as runoff, a very long critical distance is required to accumulate sufficient water to start erosion. Generally this distance is longer than the length of slope over which water flows; therefore, there are no rill channels on most sand dunes even though rain intensity may be great.

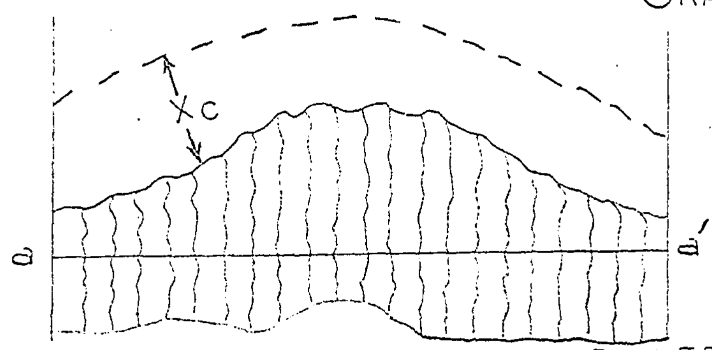
One of the parallel rills formed on the initial slope will be deeper and longer than the others, if it has had a longer length of overland flow. In Figure 13 it is represented by channel a-b. By a process of cross-grading, channel a-b will become the dominant one and will cut a valley. The process of cross-grading is shown in Figure 17. Once a second slope has been superposed upon the first, a second set of parallel rills will develop flowing down the valley side slope to channel a-b.

Again, by the process of cross-grading one channel will dominate. Figure 14 shows the parallel tributary rills; Figure 15 shows channel c-d and e-f as dominate on each side of the master channel. Both c-d and e-f develop parallel tributary rills, just as they were once one of a series of rills. The process of cross-grading continues until several tributaries dominate.

It is possible to stop the development of the net when the master rill is at any order. Again critical distance is the controlling factor. If the length of overland flow to channel h-g, as shown in Figure 16, is shorter than the critical distance, no rill erosion will occur. Figure 16 then represents the end point in the drainage net development, and further degradation of the area will be by sheetwash and not by erosion in the channels. The first order channels such as h-g will end up as the smallest streams in the system, and the master rill ends up with the reach a-e as a third order stream. Schumm's (1954) study of the Perth Amboy badlands modified this concept only slightly. He stated that bifurcation or tributary development could occur in the lower reaches of the master rill

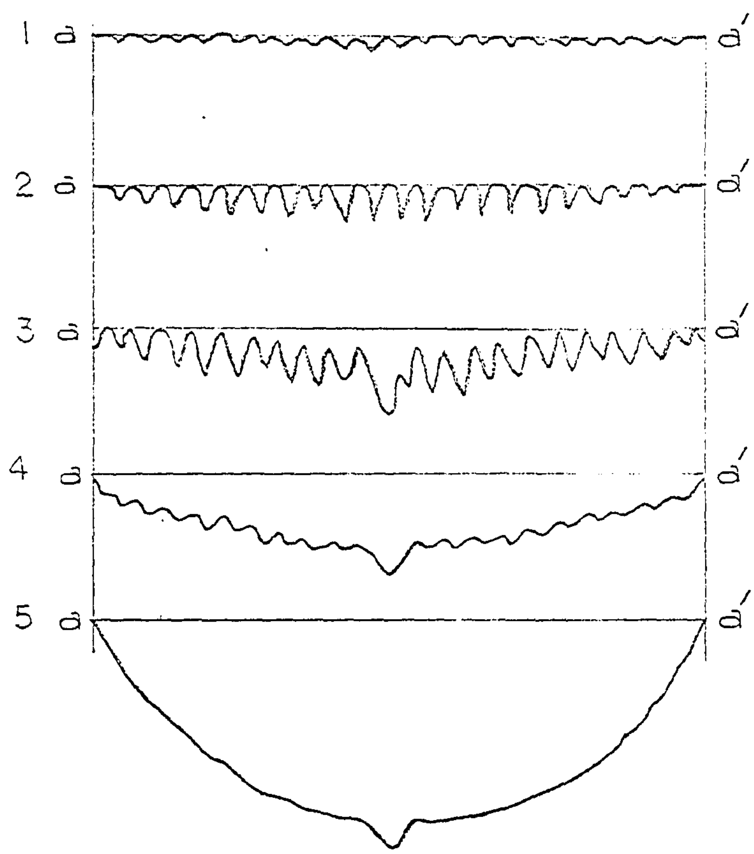
Figure 17. Development of a valley by cross-grading showing map view of original rilled surface and cross sections as successive stages (with 1, as initial surface, through 5, where master rill is dominant and has formed a valley) (after Horton, 1945)

DEVELOPMENT OF A VALLEY BY CROSS-GRADING



MAP VIEW OF ORIGINAL RILLED SURFACE

CROSS SECTIONS ALONG LINE a—a' AT SUCCESSIVE STAGES



before the master rill had reached its fullest headward extension, and that the tributary streams could shift their junctions downstream on the trunk stream.

The two main points from Horton's concept which will be applied to the model introduced here are: the drainage net development reaches an end point or stable configuration, and the area drained by the master rill and all subsequent tributaries remains constant while the total stream length increases with time.

Theoretical Development of the Drainage Density Envelope

Figure 18 through Figure 22 show streams eroding headward and developing their drainage pattern. Figure 23 is a plot of a total length of streams in first order basins versus area for the first order basins. The condition A as shown in Figure 18 would be represented by A in Figure 23 and conditions B on Figure 19 is point B on Figure 23.

In the early stage of development the master rill is a first order stream and the basin area is a constant, determined by interfluve geometry. At any point in time during the headward extension of this master rill the basin geometry can be described by a point moving upward on the vertical line I-B, Figure 23. When the master rill has eroded to its fullest headward extension, that is, it has reached its critical distance, the basin geometry can be represented by the point B.

The next step in the cycle is the development of tributary

Figure 18. (top, left) Stream eroding headward (the dashed lines in all figures represent the drainage divides)

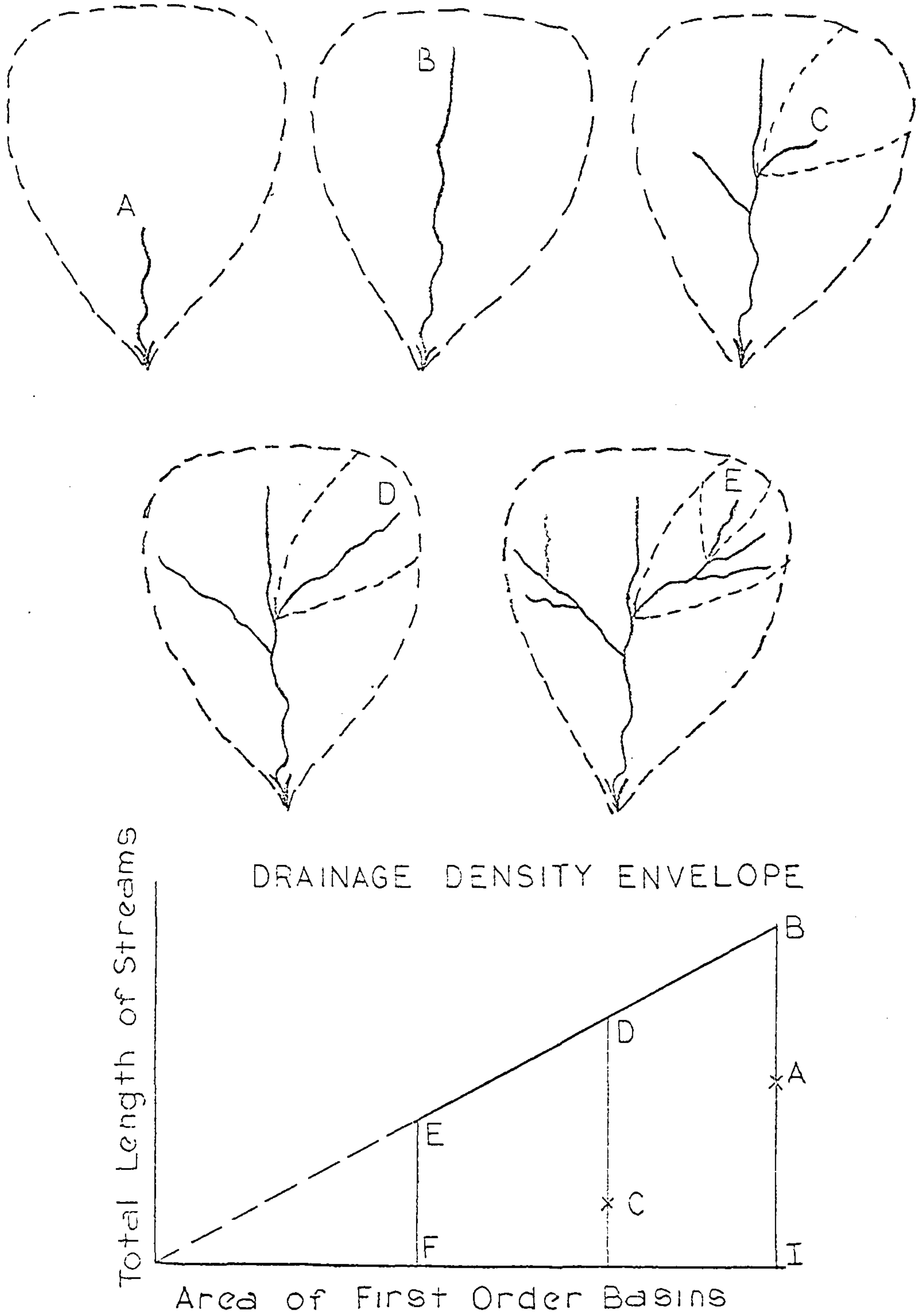
Figure 19. (top, middle) Stream extended full length

Figure 20. (top, right) First bifurcation with tributary eroding headward

Figure 21. (middle, left) Tributary stream extended full length to critical distance

Figure 22. (middle, right) Second bifurcation with basin at stable configuration

Figure 23. (bottom) Drainage density envelope showing stages represented in Figures 19 and 23 as points on area versus total length plot


A
B
C
D
E
DRAINAGE DENSITY ENVELOPE
Total Length of Streams
Area of First Order Basins
B
D
A
E
C
F
I

streams as shown in Figure 20. The master rill is now a second order stream and its tributaries first order streams; consequently the area of each first order basin is some fraction of the original area, and the coordinates for this stage in the cycle will fall on a vertical line to the left of the one for the master rill. Point C on Figure 23 represents the condition in Figure 20 and point D on Figure 23 represents the condition shown in Figure 21. Channel D has extended to its maximum length and its total length of channel is bound to be less than the total length of the master rill because the drainage area of the tributary channel is less. A second bifurcation and extension is shown in Figure 22, and the point E occurs on the length-area curve in Figure 23.

The slope of the line through points B, D and E is $\frac{L}{A}$, or the drainage density, which is thought to be a function of the material over which the stream system flows. Therefore, if the stream system is on homogeneous rock, the line E-B should be a straight line, and the extension of which would pass through the origin of the graph.

Since the smaller first order basin represents a stage further along in the drainage history of the area, the whole drainage net should be lower in elevation than at the stage represented by A or B. This idea is verified by the area-elevation curves. If the stream cuts through different layered material with time, then the slope of the line will change. A curved line indicates a gradational change, and an angular change of slope would indicate an abrupt change in lithology. A high drainage density results from a non-resistant rock of low infiltration capacity; therefore, if the slope of the line is steep, the rock could be interpreted as non-resistant rock of low infiltration capacity. A gently sloping line would represent a resistant rock of high infiltration capacity. A curve that is convex upward would indicate a gradual change from a resistant rock of high infiltration capacity above to a non-resistant rock of low infiltration capacity below. Each curve would be characteristic of a type of lithology.

The line F-E which represents the headward growth of the last bifurcation in the basin is the smallest area first order basin that can develop in that particular locality. The point E represents the drainage density condition of the basin when it has completely developed, or, one might say, the stable basin configuration. The graph then shows the limits of drainage basin development, for no basin in that locality can be smaller than the drainage area of E-F. Since the point E represents the end point of the cycle, all basins in that locality will theoretically reach that condition when sufficient time has passed.

The trapezoidal area FEBI represents a drainage density envelope. A basin can have any combination of co-ordinates that will fall within that area, but it would theoretically be impossible for a basin in a geologically homogeneous locality to have co-ordinates outside that area. The co-ordinates that lie on the line E-B represent a condition of furthest extension of stream length for a given area, and therefore are at a stage of incipient bifurcation. The points lying below the line are basins which have streams that still have the potential to extend themselves headward before another bifurcation takes place. The lines of the envelope delineate the limits of drainage net development for a given area and indicate the degree of development that the basin has attained.

Schumm's (1954) modification of drainage net evolution also can be accounted for by this model. If a bifurcation occurs before the streams have extended themselves to their greatest length, then the new stream would have a smaller basin area. The point would move to the left before it reached the envelope line.

Comparison with Field Measurements

In the development of the theoretical model the evolution of a basin in one location was followed through time. Most geomorphic features take longer to develop than a human lifetime. Therefore, in a geomorphic study one must telescope time by selecting sites each of which represents a progressive stage in the development of the feature being studied. This assumption is the basis of most geomorphic thinking and is used in Melton's (1958) theory of a growth model.

In order to have a broader frame of reference than basins developed in glaciated areas, data were used from Coates' (1956) work in addition to data from this study. Only data from third order basins were available. An analogous development may be made for third order basins as was done for first order basins in the theoretical treatment; the effect should be to translocate the origin downward and to the left along the line EB.

The areas of third order basins were plotted on the abscissa and total lengths of streams of all orders plotted on the ordinate, and the data were considered by locality. Plots are shown on Figure 24 and Figure 25.

Fairly good linear trends can be observed for most localities. Gray (1961) observed that area-stream length plots could be fitted to straight lines, but he used data from many different localities of different geologic settings, and in his empirical approach he neglected the possibility of a rational interpretation associating the geology with the drainage density and the slope of the line. Schumm (1954) used mean stream length and mean basin area and plotted straight lines for different areas. Since he plotted area on the ordinate and length on the abscissa, the slopes of these lines are the reciprocals of drainage density, which he called the constant of channel maintenance. However, since he merely collected data and fitted a regression line to it, he made no rational interpretation of this relationship beyond the fact that it must be a significant one.

Data from both this study and Coates' study (1956) demonstrate that there is a fair agreement of the theory with reality.

Figure 24. Drainage density envelopes (plotted from data obtained in this study)

Drainage Density Envelopes of Locations in This Study

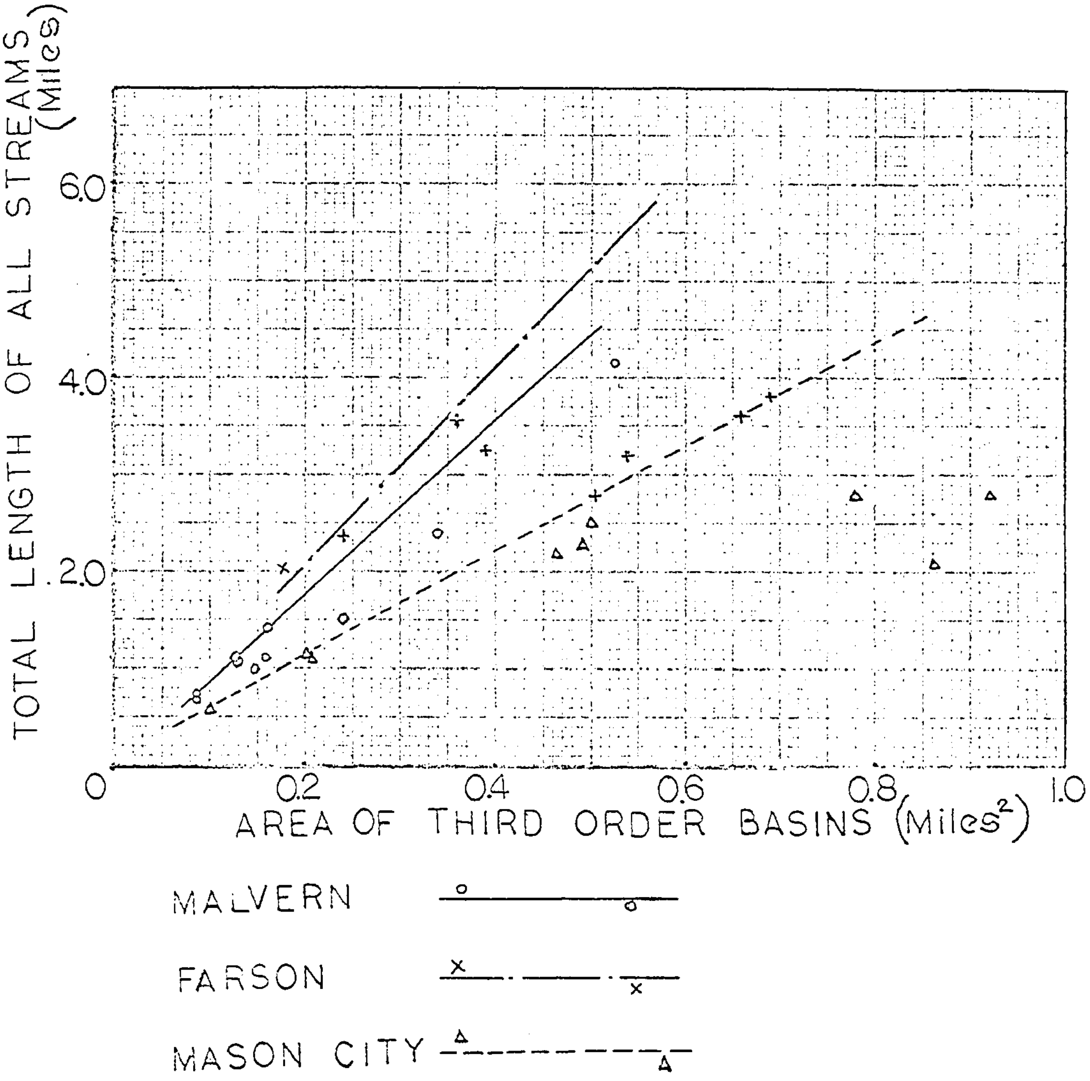
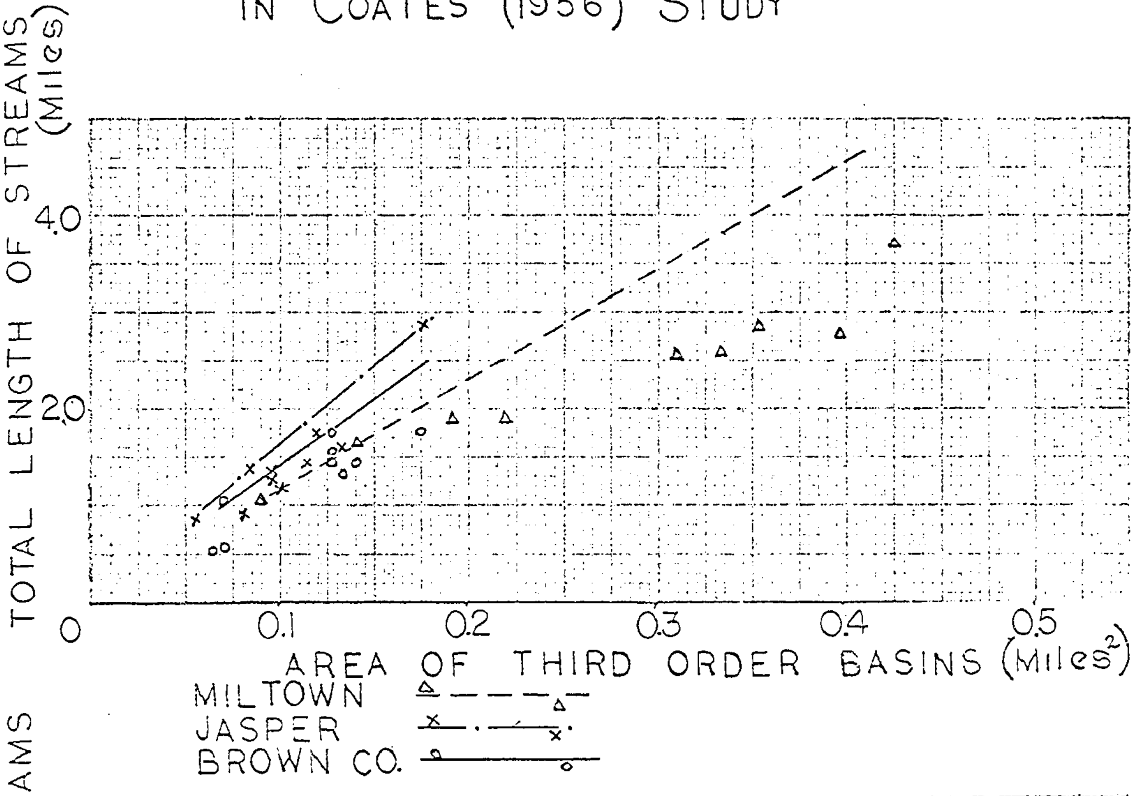
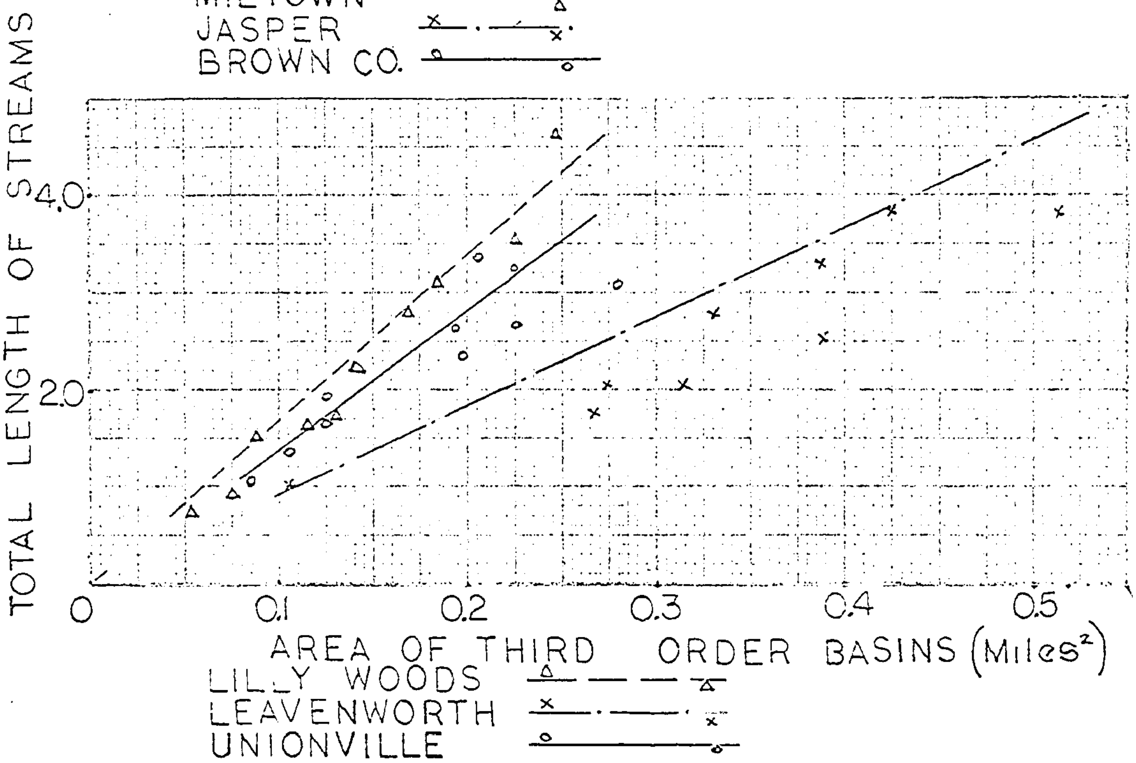


Figure 25. Drainage density envelopes (plotted from data in Coates' (1956) study)

Drainage Density Envelopes of Locations in Coates' (1956) Study





That is, the histogram in Figure 26 shows that most points are near the line, where ΔL is the vertical distance between each point and the upper line of the drainage density envelope. The highly asymmetrical appearance of the histogram indicates that the distribution of points with respect to the envelope line is not random, but that ΔL may be approaching zero.

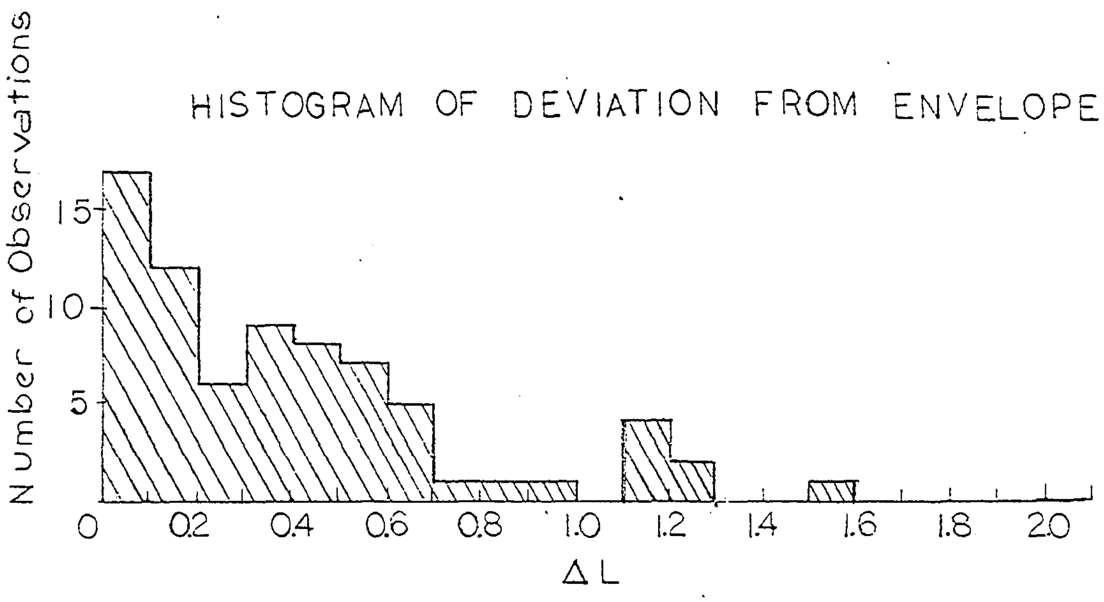
Figure 27 shows that most basins have areas between 0.1 and 0.2 square miles, and that basins in all study locations have a tendency to approach a minimum area. This strengthens the conclusion that drainage basins tend towards a stable condition, and can be used as evidence to support the concept of equilibrium conditions in drainage basins (Hack, 1960, Holmes, 1964).

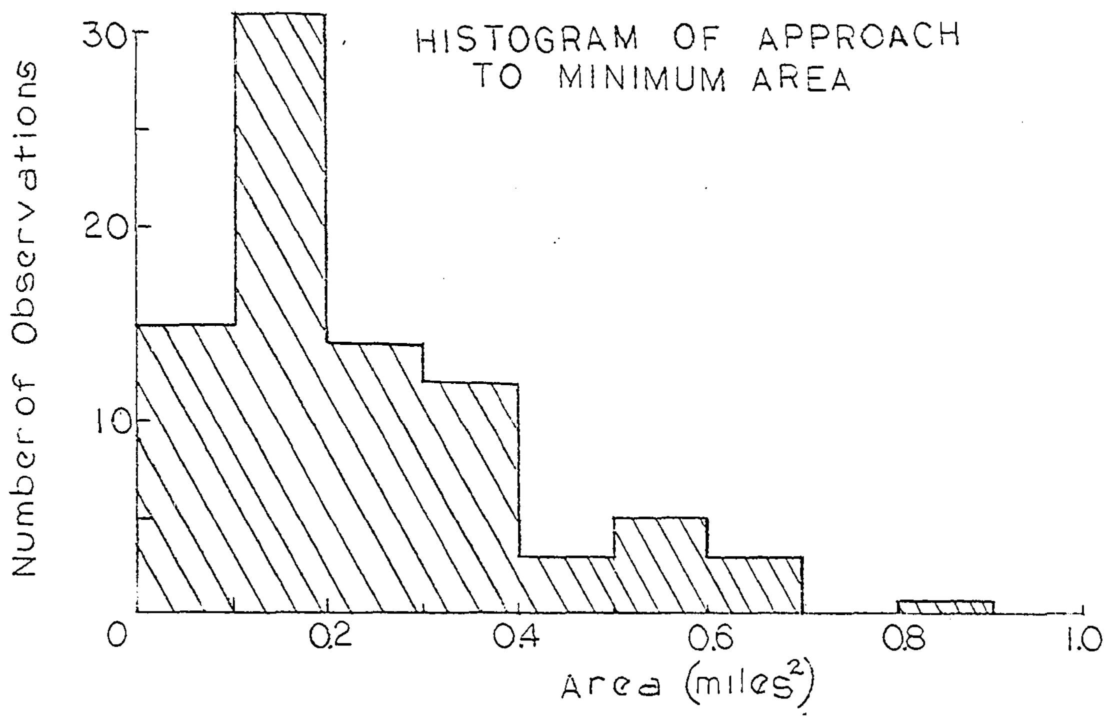
Figure 28 shows ΔL in some of the study localities is larger for larger area basins, especially in the Mason City, Farson, and Milltown localities. Because of the tendency towards the equilibrium condition and small area for basins, the large area basins probably are not as far along in their erosional development, and are not yet in equilibrium. In this early stage streams may not have extended headward as far as possible, or bifurcation may have occurred before maximum extension. The relationship of smaller basins having more of an equilibrium configuration and larger basins representing more of the inequilibrium stage is consistent with the interpretation of the area-elevation curves.

Table 4 summarizes the data shown in Figures 24 and 25,

Figure 26. Histogram of deviation from envelope with ΔL, the deviation from the envelope plotted against the number of observations (this graph shows that most of the values are close to the envelope line)

Figure 27. Histogram of approach to minimum area with area plotted against number of observations (this graph shows the majority of basins have a small area)


HISTOGRAM OF DEVIATION FROM ENVELOPE
Number of Observations
0
5
10
15
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
ΔL


HISTOGRAM OF APPROACH TO MINIMUM AREA
Number of Observations
0
10
20
30
0.2
0.4
0.6
0.8
1.0
Area (miles²)

and presents mean annual precipitation and lithologies.

Table 4. Summary of data shown in Figures 24 and 25

Study locality	Approx. annual rainfall (inches)	Dominant lithology	Slope of drainage density env.
Brown Co.	41	sandstone-silstone	14
Lilly Woods	41	sandstone-silstone	17
Unionville	41	silstone-sandstone	14
Jasper	48	sandstone & some shale	17
Leavenworth	48	sandstone limestone	9
Milltown	48	sandstone limestone	11.5
Malvern	30	loess	9
Farson	34	till	11
Mason City	30	till over limestone	6

In general all the localities from Coates' study have higher drainage density envelope slopes, but this does not necessarily indicate less resistance to erosion or smaller infiltration capacity. A more likely explanation is the climate difference; the 41 to 48 inches of precipitation in southern Indiana is much greater than the 30 to 34 inches of the study in Iowa. The greater mean annual precipitation could have caused larger drainage densities.

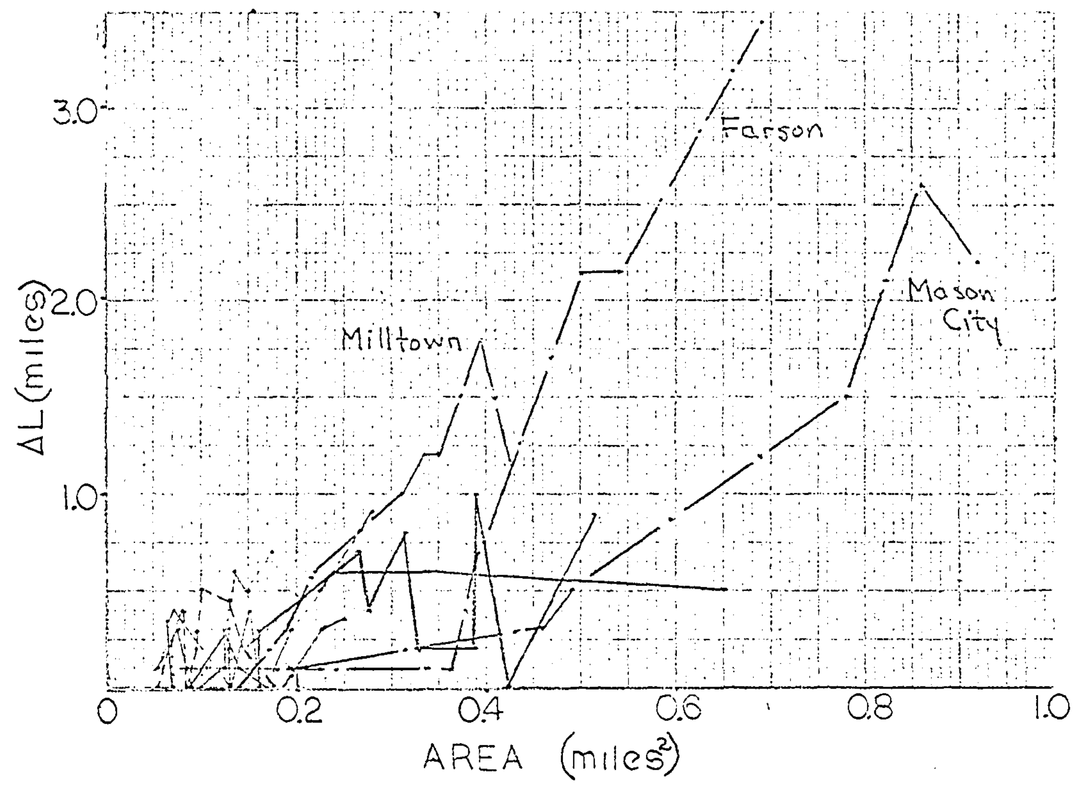
The slope of the envelope line for the Malvern area should be greater than for the Farson area if the loess of the Malvern area is truly less resistant. However, the greater infiltration capacity of the loess may have offset its smaller resistance thereby causing the slope of the envelope to be less. It is also possible that the 4 inches difference in the precipitation between these areas was sufficient to cause the slope of the upper line of the drainage density envelope to be greater in the Farson area.

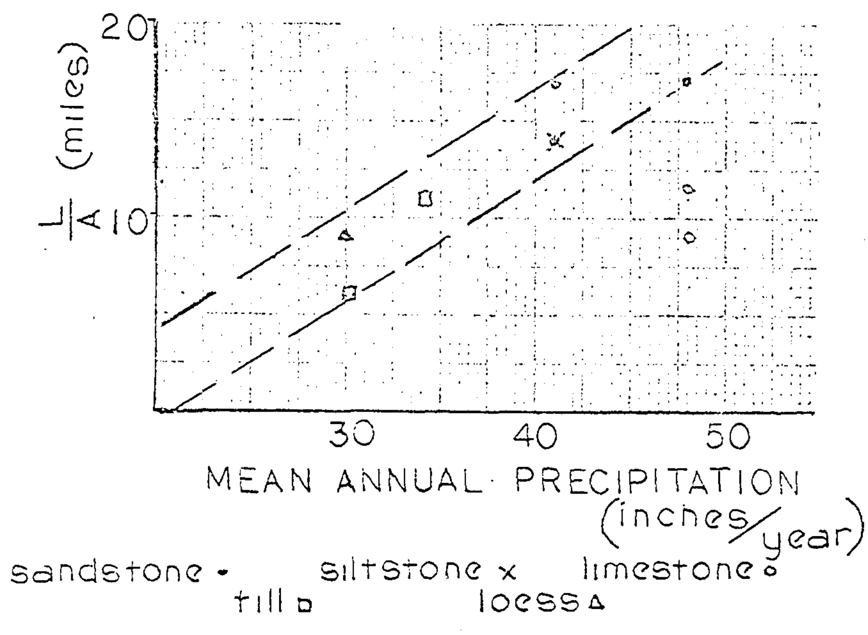
Figure 29 is an attempt to show the interrelationships between lithology, amount of precipitation and slope of the drainage density envelope. Seven of the nine localities fall within the area defined by the parallel, dashed lines. This suggests the possibility of fitting the points to a straight line and deriving the equation $\frac{L}{A} = 0.625R - 10.6 \pm 2$ for the relationship between drainage density and precipitation.

A similar graph of mean drainage density, instead of equilibrium drainage density, shows the same trend but the points have a greater scatter. The two localities which fall outside of this area are localities of limestone. Miller (1953) concluded that low drainage densities in dolomites were caused by their low resistance to corrosion and their high infiltration capacity due to solution cavities. A similar conclusion can be made for limestones. Due to higher infiltration capacity in limestones, the precipitation must be higher to get runoff sufficient to result in a drainage density equal to that of rocks of lower infiltration capacity. The horizontal dis-

Figure 28. Plot of ΔL versus basin area (note that basins of larger areas show the greater ΔL)

Figure 29. Plot of equilibrium drainage density (slope of upper limit of drainage density envelope) versus mean annual precipitation (the parallel dashed lines define the limits of the equation)


3.0
2.0
1.0
ΔL (miles)
Farson
Mason City
Milltown
0
0.2
0.4
0.6
0.8
1.0
AREA (miles²)


20
10
L/A (miles)
30
40
50
MEAN ANNUAL PRECIPITATION (inches/year)
sandstone •
siltstone x
limestone o
till □
loess △

tance from the limestone points to the lower dashed line could be thought of as an infiltration factor which would correct for the difference in runoff.

An examination of the analysis of variance data for drainage density in this study and Coates' (1956) study shows that there is a greater variation between study localities than within them. This strengthens the idea that drainage densities should be constant for an area of uniform geology.

Possible reasons that the points obtained through empirical measurements do not conform exactly to the envelope line can be listed:

(1) Some streams have not extended themselves as far as possible for the drainage area available to them. For example, the Mason City locality is thought to be in the pre-youth stage, that is, it is still undergoing some post-glacial flattening due to internal drainage. Therefore, the streams should not have extended themselves as far as possible. This is supported by the data in Figure 28, where the basins of the Mason City locality have the largest area and show the greatest deviation (ΔL) from the drainage density envelope.

(2) Variations are bound to occur within any geologic system even though it has been assumed to be homogeneous.

(3) Climatic variations could have occurred through time.

(4) Incorrect measurements could have been recorded.

CONCLUSIONS

The first part of this thesis presents a morphometric analysis of three Iowa areas: a loess area (Malvern, quadrangle), a young till plain (Mason City quadrangle), and an old till plain (Farson quadrangle). The conclusions of this part of the investigation are summarized on pages 52 and 53.

The second part of this thesis presents an analytic model for drainage basin development which suggests that as a drainage network develops, basins of a given order subdivide and thus become higher order; therefore, basins of a given order tend to become smaller. Secondly, the total length of streams in a basin tends to approach a maximum which is dictated by the final critical distance, or the distance of overland flow required for rill development. The latter should be a function of climate and lithology. Thirdly, basin subdivision is limited by the critical distance, so basins of a given order will approach a minimum area.

The graphical area enclosing all plotted points of stream length versus basin area is defined as the drainage density envelope; the upper limit of this envelope represents an equilibrium drainage density, and when extrapolated passes through the origin.

Drainage density envelope plots of data from nine locations show excellent linearity in the smaller area range, a recognizable minimum basin area, and a convergence of upper limits toward the origin. Departures from the upper limit show asymmetrical distribution, tending to approach the upper limit line. Also, departures are greater for larger basin areas, which according to the above hypothesis, should be farther from equilibrium stage; that is, more immature. The concept of the drainage density develope thus appears sufficiently confirmed, to merit further trial. Advantages of this approach are that it gives a measure of relative maturity, and allows prediction of the equilibrium drainage density, eliminating the time variable.

Evaluation of equilibrium drainage densities for the nine localities evaluated suggests a close relationship to annual rainfall, all points with exception of two limestone areas falling in the area: $\frac{L}{A} = 0.625R - 10.6 \pm 2$, where R is the mean annual rainfall and varies from 30 to 48 inches, L is total length of streams, and A is third-order basin area. The two limestone areas have considerably lower equilibrium drainage densities, indicating infiltration of about one-third of the rainfall, if the equation is correct. Except for this divergence, the influence of lithology on equilibrium drainage density is not readily apparent from the data; perhaps this influence is a minimum as a drainage network approaches equilibrium.

EPILOGUE

Advantages of the Theoretical Approach

The main value of this type of approach for the comparison of drainage basins is that it takes into account the areal geology which can be lost in purely statistical treatments. The linear relationship between basin area and length has been observed by others (Gray, 1961, Schumm, 1954), but no one has tried to put it on a rational basis. The idea of drainage texture has long been a part of geomorphic thinking and this approach unites the quantitative and qualitative aspects of drainage net development.

The drainage density envelope accounts for some of the indeterminancy that is not obvious from a study of drainage density statistics. That is, it shows some limitations of the statistical analysis by considering the basins on an individual basis. The drainage density envelope shows the relative stages of development of the individual basins, and can be used as an indirect estimator of critical distance by showing the minimum size basin that can develop in that locality.

Discussion of Assumptions

The assumption that the original surface was nearly a flat plane is valid for continental glaciated terrain where the process of post-glacial flattening occurs.

Climatic and vegetational homogeneity between areas may exist on a large scale, but not enough data have been accumulated to know the effect of local variations on morphometric analysis. Another big problem is the determination of any climatic variation through time. Within the Pleistocene the localities of this study were subjected to such variations during glacial, as opposed to interglacial stages, the climate, and therefore the vegetation would be different. One possible answer to this criticism is that by studying lower order basins, one is studying relatively young basins which would have developed in the fairly recent past. Of course, there are no data available on the rate of development of third order basins.

The assumption of geologic homogeneity is probably weak in any study of a glaciated area due to the inherent variability of the drift. The evidence for this is that standard deviations obtained in this study are higher than standard deviations obtained in morphometric studies of drainage basins developed on bedrock.

The assumption of lithologic homogeneity is not valid in this particular study if the area-elevation curves have been properly evaluated. The interpretation of two of those curves was that some basins in the Mason City area contained streams which had incised through the till to bedrock.

The drainage density envelopes which were constructed for the Farson and Mason City localities indicate that there is no lithologic homogeneity between these two areas. This could result from the streams in the Mason City locality not having reached their fullest extent, and so the drainage density line constructed for the Mason City area might not be valid. All the Mason City points are within the Farson envelope. Another possibility is that the Mason City area is the proper one for both localities and the points of the Farson locality above the envelope line are due to inhomogeneities within the Farson locality. There are three Farson points which coincide with the Mason City envelope which is evidence in support of this second possibility. The third possibility is that there is a lithologic variation between the two drifts as should be, due to differences of weathering. Field work should accompany all morphometric studies so that more complete information on lithologies might be known.

Another complication of the assumption of lithologic homogeneity occurs with a consideration of the Malvern area. How was the loess deposited? If it was aeolian, was it deposited rapidly so that the drainage developed on a fresh surface, or was it deposited as the drainage lines were developing? If the latter of these two possibilities occurred, then lithology has in a sense varied with time. Dahl (1961) has suggested that the loess adjacent to the Missouri River is water-laid. If the loess were water-lain over some unit of time, it would represent a subaerial aggradational surface on which some drainage lines would be present at all times during the development of the surface.

By considering basins on an individual basis rather than as a statistical mean, one can see that at any point in time the basins within a locality will be in varying stages of development. This will be a function of the basin's position in the overall drainage net. Some third order basins will drain directly into streams of greater than fourth order, whereas other third order basins will have a more normal position of draining into fourth order streams. This factor can cause a great scatter in the morphometric values making it difficult to compare the mean values of several localities.

Suggestions for Further Study

In any future morphometric study of glaciated terrain, field work should accompany map analysis in order to verify the assumptions of geologic homogeneity. Until large amounts of data have been accumulated to determine the degree of variability to be expected in glaciated areas, it will be necessary to do field work.

It is also urged that further investigations use a rational approach to the problem rather than the random collection of data.

The drainage density envelope suggests the desirability of model studies, for by this means alone can one have sufficient control over the evolution of the drainage net to be able to make valid quantitative predictions.

LITERATURE CITED

Calvin, S.
1896 Geology of Cerro Gordo County. Iowa Geological Survey Report 7: 117-197.

Coates, D. R.
1956 Quantitative geomorphology of small drainage basins in southern Indiana. U. S. Office of Naval Research Project NR389-042, Technical Report 10.

Dahl, A. R.
1961 Missouri River studies: alluvial morphology and Quaternary history. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology.

Davis, W. M.
1909 Geographical essays. New York, N. Y., Ginn and Co.

Elwell, J. A., Dean, H. L., Foster, R., and Tigges, E. W.
1940 Cerro Gordo County Iowa. U. S. Department of Agriculture, Bureau of Plant Industry, Soil Survey, Series 1935, No. 13: 1-68.

Fenneman, N. M.
1938 Physiography of Eastern United States. New York, N. Y., McGraw-Hill Book Co., Inc.

Glock, W. S.
1932 Available relief as a factor of control in the profile of a land form. Journal of Geology 40: 77-83.

Gordon, D. L.
1960 Morphometric study of selected Iowa drainage basins. Unpublished M.S. thesis. Iowa City, Iowa, Library, State University of Iowa.

Gray, D. M.
1961 Interrelationships of watershed characteristics. Journal of Geophysical Research 66: 1215-1223.

Hack, J. T.
1960 Interpretation of erosional topography in humid temperate regions. American Journal of Science 258-A: 80-97.

Hobbs, W. H.
1942 "Iowan till" of Iowa, an outwash from the Des Moines lobe of the late Wisconsin glaciation. (Abstract) Geological Society of America Bulletin 53: 1849.

Holmes, C.
1964 Equilibrium in humid-climate physiographic processes. American Journal of Science 262: 436-445.

Horton, R. E.
1945 Erosional development of streams and their drainage basins, hydrophysical approach to quantitative morphology. Geological Society of America Bulletin 56: 275-370.

Iowa Geological Survey
1960 Highway construction materials from the consolidated rocks of southwestern Iowa. Iowa Highway Research Board Bulletin 15.

Johnson, D. W.
1933 Available relief and texture of topography. Journal of Geology 41: 293-305.

Kay, G. F. and others
ca. 1945 The Pleistocene geology of Iowa. Iowa City, Iowa, Iowa Geological Survey.

Langbein, W. B. and others
1947 Topographic characteristics of drainage basins. U. S. Geological Survey Water Supply Paper 968-C.

Leonard, A. G.
1901 Geology of Wapello County. Iowa Geological Survey Report 12: 439-499.

Leopold, L. B. and Langbein, W. B.
1962 The concept of entropy in landscape evolution. U. S. Geological Survey Professional Paper 500-A.

________ and ________
1963 Association and indeterminancy in geomorphology. In Albritton, C. C., Jr., ed. The fabric of geology. pp. 184-192. Reading, Massachusetts, Addison Wesley Publishing Co., Inc.

________, Wolman, M. G., and Miller, J. P.
1964 Fluvial processes in geomorphology. San Francisco, California, W. H. Freeman and Co.

Mackin, J. H.
1948 Concept of the graded river. Geological Society of America Bulletin 59: 463-511.

________, 1963 Rational and empirical methods of investigation in geology. In Albritton, C. C., Jr., ed. The fabric of geology. pp. 135-163. Reading, Massachusetts, Addison Wesley Publishing Co., Inc.

Melton, M. A.
1958 Geometric properties of mature drainage systems and their representation in an E_4 phase space. Journal of Geology 66: 35-54.

Miller, V. C.
1953 A quantitative geomorphic study of drainage basin characteristics in the Clinch Mountain Area Virginia and Tennessee. U. S. Office of Naval Research Project NR389-042, Technical Report 3.

Morisawa, M.
1957 Accuracy of determination of stream lengths from topographic maps. American Geophysical Union Transactions 38: 86-88.

Ostle, B.
1963 Statistics in research. 2nd ed. Ames, Iowa, The Iowa State University Press.

Playfair, J.
1802 Illustrations of the Huttonian theory of the earth. Edinburgh, Scotland, W. Creech.

Ruhe, R. V.
1950 Graphical analysis of drift topographies. American Journal of Science 248: 435-443.

________ 1952 Topographic characteristics of the Des Moines lobe. American Journal of Science 250: 46-56.

________, and Scholtes, W. H.
1959 Important elements in the classification of the Wisconsin glacial stage (Iowa) -- a discussion. Journal of Geology 67: 585-598.

Schench, H., Jr.
1963 Simulation of drainage basin networks with a digital computer. Journal of Geophysical Research 68: 5939-5745.

Schumm, S. A.
1954 Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey. U. S. Office of Naval Research Project NR389-042, Technical Report 8.

Shrader, W. D. and Hussey, K. M.
1953 Evolution of the level interfluvial divides on the Kansan till plains in Iowa and Missouri. Iowa Academy of Science Proceedings 60: 408-413.

Smith, K. G.
1950 Standards for grading texture of erosional topography. American Journal of Science 248: 655-668.

Stevenson, W. H. and Brown, P. E.
1921 Soil survey of Iowa - Wapello County. Iowa Agricultural Experiment Station Soil Survey Report 18.

______ and ______
1924 Soil survey of Iowa - Mills County. Iowa Agricultural Experiment Station Soil Survey Report 33.

Strahler, A. N.
1950 Equilibrium theory of erosional slopes approached by frequency distribution analysis. American Journal of Science 248: 673-696.

______ 1952a Dynamic basis of geomorphology. Geological Society of American Bulletin 63: 923-938.

______ 1952b Hypsometric (area-altitude) analysis of erosional topography. Geological Society of America Bulletin 63: 1117-1142.

______ 1957 Quantitative analysis of watershed geomorphology. American Geophysical Union Transactions 38: 913-920.

Udden, J. A.
1902 Geology of Mills and Fremont Counties. Iowa Geologic Survey Report 13: 123-185.

U. S. Department of Agriculture
1941 Climate and man: Yearbook of Agriculture. Washington, D. C., U. S. Government Printing Office.

ACKNOWLEDGMENTS

The author would like to acknowledge Dr. K. M. Hussey, Dr. John Lemish, and Dr. C. J. Roy for reading the original manuscript and making valuable criticisms and contributions.

Special acknowledgment is given to Dr. D. K. Hotchkiss of the Department of Statistics for helping set up the statistical tests and to Dr. R. L. Handy who spent many hours with the author in discussions on the rational approach to quantitative geomorphology. Dr. Handy contributed much in the way of constructive criticism, encouragement, and ideas.

Thanks are given to my wife, Jane, who typed the original manuscript and helped assemble the final manuscript.

APPENDIX

Table 5. Total length of first order streams

	Malvern	Farson	Mason City		Analysis of variance: means	Analysis of variance: within
	0.466	2.122	1.270	Sum of squares	7.92	18.30
	0.569	2.009	1.524			
	0.652	1.327	0.580			
	0.417	1.895	0.948	d.f.	2	27
	0.826	1.800	2.028			
	1.270	1.724	1.315	F-ratio	5.84	
	2.008	2.225	1.251			
	0.360	0.720	0.542	$F_{.95}$	3.35	
	0.739	5.249	1.535			
	0.720	1.137	0.379	$F_{.99}$	5.49	
Σx	8.027	20.208	11.372			
$\bar{x}$	0.803	2.021	1.137	Decision:	Reject @ 0.05	
Σx^2	8.661	54.417	15.93		Reject @ 0.01	
$\frac{(\Sigma x)^2}{N}$	6.44	40.84	12.93			
s^2	0.247	1.509	0.278			
s	0.497	1.220	0.527			

Table 6. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 148.38$	@ .05	6.7	13.56	90.85
$C_3 = (T_1 - T_3)^2 = 11.19$	@ .01	10.98	13.56	148.89

Decision: C_1 Reject @ .05 accept @ .01

C_3 Accept

Table 7. Number of first order streams

	Malvern	Farson	Mason City		Analysis of variance: means	Analysis of variance: within
	2.646	5.00	2.236	Sum of		
	2.646	5.196	2.646	squares	25.247	27.523
	2.236	3.873	2.236			
	2.236	4.472	2.449	d.f.	2	27
	3.00	4.00	2.236			
	4.00	5.385	2.828	Mean		
	5.477	4.359	2.828	square	12.624	1.019
	2.449	2.676	2.00			
	2.646	7.681	2.236	F-ratio	12.389	
	3.162	3.00	2.00			
Σx	30.498	45.642	23.695	$F_{.95}$	3.35	
$\bar{x}$	3.050	4.564	2.370			
Σx^2	102.00	226.00	57.00	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	93.013	208.319	56.145	Decision:	Reject @ .05	
s^2	58.40	215.60	2.473		Reject @ .01	
s	7.66	14.71	1.574			

Table 8. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 229.34$	@ .05	6.7	20.38	136.54
$C_3 = (T_1 - T_3)^2 = 2025$	@ .01	10.98	20.38	223.77

Decision: C_1 Reject @ .05 Reject @ .01

C_3 Reject @ .05 Reject @ .01

Table 9. Mean length of first order streams

	Malvern	Farson	Mason City	Analysis of variance	means		within
	0.067	0.303	0.254	Sum of squares	0.365		0.137
	0.081	0.402	0.218				
	0.130	0.442	0.116				
	0.083	0.271	0.158	d.f.	2		27
	0.092	0.360	0.406				
	0.079	0.287	0.164	Mean square	0.188		0.057
	0.067	0.445	0.156				
	0.060	0.240	0.136				
	0.106	0.404	0.307	F-ratio		3.30	
	0.072	0.379	0.095				
Σx	0.837	3.533	2.010	$F_{.95}$		3.35	
$\bar{x}$	0.084	0.353	0.201				
Σx^2	0.074	1.297	0.488	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	0.070	1.248	0.404	Decision:	Accept		
s^2	0.0004	0.0054	0.0093				
s	0.0002	0.074	0.096				

Table 10. Total length of second order streams

	Malvern	Farson	Mason City		Analysis of variance		
					means		within
	0.569	1.020	0.758	Sum of			
	0.417	0.633	0.303	squares	0.781		3.758
	0.303	0.580	0.531				
	0.171	0.542	1.201	d.f.	2		27
	0.477	1.061	1.270				
	0.569	1.137	0.663	Mean			
	1.364	0.815	0.796	square	0.391		0.139
	0.209	0.550	0.353				
	0.171	1.516	0.569	F-ratio		2.813	
	0.019	0.360	0.027				
Σx	4.269	8.214	6.471	$F_{.95}$		3.35	
$\bar{x}$	0.427	0.821	0.647				
Σx^2	3.104	7.884	5.526	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	1.822	6.747	4.187	Decision:	Accept		
s^2	0.142	0.1263	0.149				
s	0.377	0.355	0.386				

Table 11. Bifurcation ratio of second to third order streams and number of second order streams

	Malvern	Farson	Mason City		Analysis of variance: means	within
	1.732	2.646	1.414	Sum of		
	1.732	2.236	1.414	squares	3.262	5.588
	1.414	1.732	1.414			
	1.414	2.646	1.414	d.f.	2	27
	1.414	2.236	1.732			
	2.236	2.449	1.732	Mean		
	2.828	2.236	1.414	square	1.631	0.207
	1.414	1.732	1.414			
	1.732	3.606	1.414	F-ratio	7.879	
	1.414	1.732	1.414			
Σx	17.330	23.251	14.776	$F_{.95}$	3.35	
$\bar{x}$	1.733	2.325	1.478			
Σx^2	32.00	57.00	22.00	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	29.518	54.061	21.833	Decision:	Reject @ .05	
s^2	3.73	8.90	0.178		Reject @ .01	
s	1.93	2.98	0.422			

Table 12. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 35.058$	@ .05	6.7	4.14	27.74
$C_3 = (T_1 - T_3)^2 = 71.826$	@ .01	10.98	4.14	45.46

Decision: C_1 Reject @ .05 Accept @ .01

C_3 Reject @ .05 Reject @ .01

Table 13. Mean length of second order streams

	Malvern	Farson	Mason City		Analysis of variance mean	within
	0.190	0.146	0.379	Sum of squares	0.144	0.393
	0.139	0.127	0.152			
	0.152	0.193	0.266			
	0.086	0.077	0.601	d.f.	2	27
	0.239	0.212	0.635			
	0.114	0.190	0.331	Mean square	0.072	0.0145
	0.171	0.163	0.265			
	0.105	0.183	0.177			
	0.057	0.117	0.285	F-ratio	4.96	
	0.010	0.120	0.013			
Σx	1.263	1.528	3.104	Decision:	Reject @ 0.05	
$\bar{x}$	0.126	0.153	0.310			
Σx^2	0.216	0.237	1.294		Accept @ 0.01	
$\frac{(\Sigma x)^2}{N}$	0.156	0.234	0.964			
s^2	0.007	0.003	0.037			
s	0.0837	0.0055	0.192			

Table 14. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 2.48$	@ .05	6.70	0.29	1.94
$C_3 = (T_1 - T_3)^2 = 3.389$	@ .01	10.98	0.29	3.18

Decision: C_1 Reject @ .05 Accept @ .01

C_3 Reject @ .05 Reject @ .01

Table 15. Total length of third order streams

	Malvern	Farson	Mason City	Analysis of variance	mean		within
	0.066	0.474	0.190	Sum of squares	0.813		5.403
	0.550	0.569	0.398				
	0.057	0.531	0.038				
	0.171	0.379	0.739	d.f.	2		27
	0.114	0.409	0.826				
	0.531	0.682	0.872	Mean square	0.406		0.200
	0.777	0.780	0.443				
	0.114	0.220	0.220				
	0.171	2.483	0.114	F-ratio		2.03	
	0.341	0.246	0.038				
Σx	2.892	6.773	3.878	$F_{.95}$		3.35	
$\bar{x}$	0.289	0.677	0.388				
Σx^2	1.397	8.489	2.444	$F_{.99}$		5.99	
$\frac{(\Sigma x)^2}{N}$	0.836	4.587	1.504	Decision:	Accept		
s^2	0.062	0.434	0.104				
s	0.249	0.659	0.322				

Table 16. Total length of all streams of all orders

	Malvern	Farson	Mason City		Analysis of variance mean		within
	1.101	3.616	2.218	Sum of squares	20.82		61.79
	1.536	3.211	2.225				
	1.012	2.438	1.149				
	0.759	2.816	2.888	d.f.	2		27
	1.417	3.270	4.124				
	2.370	3.543	2.850	Mean square	10.41		2.289
	4.149	3.820	2.490				
	0.683	1.490	1.115				
	1.081	9.248	2.218	F-ratio		4.548	
	1.080	1.743	0.444				
Σx	15.188	35.195	21.721	$F_{.95}$		3.35	
$\bar{x}$	1.519	3.520	2.172				
Σx^2	32.819	165.881	57.221	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	23.07	123.87	47.18	Decision:		Reject @ 0.05	
s^2	1.082	4.668	1.116			Accept @ 0.01	
s	1.040	2.161	1.056				

Table 17. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 181.55$	@ .05	6.70	45.78	306.72
$C_3 = (T_1 - T_3)^2 = 42.680$	@ .01	10.93	45.78	502.66

Decision: C_1 Accept

C_3 Accept

Table 18. Stream length ratio of second order streams to first order

	Malvern	Farson	Mason City		Analysis of variance: means		Analysis of variance: within
	2.84	0.48	0.60	Sum of squares	7.78		8.89
	1.71	0.32	0.20				
	1.17	0.44	0.92				
	1.04	0.28	1.27	d.f.	2		27
	2.60	0.59	0.63				
	1.44	0.66	0.50	Mean square	3.89		0.329
	2.70	0.37	0.64				
	1.75	0.76	0.65				
	0.54	0.29	0.37	F-ratio		11.237	
	0.14	0.32	0.07				
Σx	15.93	4.51	5.85	$F_{.95}$		3.35	
$\bar{x}$	1.59	0.45	0.59				
Σx^2	32.938	2.290	4.48	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	25.37	2.03	3.42	Decision:	Reject @ .01		
s^2	0.841	0.029	0.118		Reject @ .05		
s	0.917	0.170	0.344				

Table 19. Scheffe test

			A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 =$	1.80	@ .05	6.70	6.58	43.95
$C_3 = (T_1 - T_3)^2 =$	101.606	@ .01	10.98	6.58	72.24

Decision: C_1 Accept

C_3 Reject @ .05 Reject @ .01

Table 20. Stream length ratio of third order streams to second order streams

	Malvern	Farson	Mason City		Analysis of variance means		within
	0.35	3.25	0.25	Sum of squares	137.69		1238.83
	3.96	4.48	1.31				
	0.38	2.75	0.07	d.f.	2		27
	1.99	4.92	0.62				
	0.48	1.93	0.65	Mean square	68.85		45.88
	4.66	3.59	1.32				
	4.54	4.79	0.57	F-ratio		1.50	
	1.09	1.20	0.62				
	3.00	21.22	0.20	$F_{.95}$		3.35	
	34.00	2.05	1.41				
Σx	54.45	50.18	7.02	$F_{.99}$		5.99	
$\bar{x}$	5.45	5.02	0.70				
Σx^2	1228.65	557.890	5.502				
$\frac{(\Sigma x)^2}{N}$	296.48	251.80	4.93	Decision:	Accept		
s^2	103.57	34.01	0.064				
s	10.18	5.83	0.253				

Table 21. Bifurcation ratio, first order streams to second order streams

	Malvern	Farson	Mason City		Analysis of variance means		within
	2.33	3.57	2.50	Sum of squares	8.09		19.92
	2.33	5.40	3.50				
	2.50	5.00	2.50	d.f.	2		27
	4.50	2.85	3.00				
	3.20	3.20	2.50	Mean square	4.05		0.7378
	3.75	4.83	2.66				
	3.00	3.80	2.67				
	2.33	2.33	2.00				
	5.00	2.54	2.50	F-ratio		5.49	
	2.50	3.00	2.00				
Σx	31.44	38.52	25.83	$F_{.95}$		3.35	
$\bar{x}$	3.14	3.85	2.58				
Σx^2	107.34	158.08	68.45	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	98.85	148.38	66.72				
				Decision:	Reject @ .05		
s^2	0.943	1.078	0.192				
s	0.971	1.038	0.438		Reject @ .01		

Table 22. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 161.03$	@ .05	6.70	14.76	98.89
$C_3 = (T_1 - T_3)^2 = 31.47$	@ .01	10.98	14.76	162.07

Decision: C_1 Reject @ .05 Accept @ .01

C_3 Accept

Table 23. Perimeter of third order basins

	Malvern	Farson	Mason City
	1.327	2.987	2.532
	2.039	2.937	2.774
	1.535	1.876	1.687
	1.205	2.767	4.332
	1.706	2.293	6.405
	2.509	2.293	3.676
	2.778	3.108	2.672
	1.137	2.085	1.906
	1.448	6.708	3.695
	1.838	1.630	1.421
Σx	17.567	28.684	31.100
$\bar{x}$	1.757	2.868	3.11
$\Sigma \bar{x}^2$	38.059	100.901	122.168
$\frac{(\Sigma x)^2}{N}$	30.89	82.27	96.72
s^2	0.797	2.070	2.828
s	0.893	1.439	1.682

	Analysis of variance: means	within
Sum of squares	17.11	51.248
d.f.	2	27
Mean square	8.56	1.898
F-ratio	4.51	
$F_{.95}$	3.35	
$F_{.99}$	5.40	
Decision:	Reject @ .05	
	Accept @ .01	

Table 24. Scheffe test

			A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 =$	5.837	@ .05	6.70	37.96	254.33
$C_3 = (T_1 - T_3)^2 =$	182.899	@ .01	10.98	37.96	416.80

Decision: C_1 Accept

C_3 Accept

Table 25. Area of third order basins

	Malvern	Farson	Mason City		Analysis of variance means		within
	0.127	0.662	0.464	Sum of			
	0.242	0.539	0.487	squares	1.218		5.007
	0.144	0.239	0.199				
	0.086	0.504	1.149	d.f.	2		27
	0.164	0.389	1.956				
	0.342	0.360	0.789	Mean			
	0.524	0.688	0.501	square	0.609		0.185
	0.084	0.230	0.210				
	0.130	1.826	0.858	F-ratio		3.29	
	0.158	0.202	0.101				
Σx	2.001	5.639	6.714	$F_{.95}$		3.35	
$\bar{x}$	0.200	0.564	0.671				
Σx^2	0.571	5.222	7.302	$F_{.99}$		5.49	
$\frac{(\Sigma x)^2}{N}$	0.400	3.180	4.508	Decision:		Accept	
s^2	0.019	0.227	0.310				
s	0.014	0.476	0.557				

Table 26. Circularity ratio

	Malvern	Farson	Mason City		Analysis of variance: means	Analysis of variance: within
	0.91	0.95	0.91	Sum of		
	0.73	0.79	0.79	squares	0.02	0.35
	0.77	0.87	0.88			
	0.75	0.83	0.76	d.f.	2	27
	0.71	0.93	0.56			
	0.68	0.86	0.75	Mean		
	0.85	0.90	0.88	square	0.01	0.013
	0.82	0.67	0.73			
	0.78	0.51	0.81	F-ratio	0.769	
	0.59	0.96	0.63			
Σx	1.59	8.27	7.71	$F_{.95}$	3.35	
$\bar{x}$	0.76	0.83	0.77			
Σx^2	8.83	7.02	6.04	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	5.76	6.84	5.94	Decision:	Accept	
s^2	0.0078	0.02	0.011			
s	0.088	0.1414	0.105			

Table 27. Drainage density

	Malvern	Farson	Mason City	Analysis of variance	means	within
	8.669	5.46	4.78	Sum of squares	78.20	56.726
	6.347	5.96	4.57			
	7.027	10.20	5.77			
	8.825	5.59	2.51	d.f.	2	27
	8.640	8.41	2.11			
	6.929	9.84	3.61	Mean square	39.10	2.101
	7.917	5.55	4.97			
	8.130	6.48	5.31			
	8.315	5.07	2.59	F-ratio	18.61	
	6.84	8.63	4.40			
Σx	77.639	71.19	40.62	$F_{.95}$	3.35	
$\bar{x}$	7.764	7.119	4.062			
Σx^2	610.38	541.150	179.776	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	602.78	506.80	165.00	Decision:	Reject @ .05	
s^2	0.84	3.88	1.64		Reject @ .01	
s	0.92	1.97	1.28			

Table 28. Scheffe test

			A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 =$	934.5	@ .05	6.70	42.02	281.5
$C_3 = (T_1 - T_3)^2 =$	1,370.41	@ .01	10.98	42.02	461.38

Decision: C_1 Reject @ .05 Reject @ .01

C_3 Reject @ .05 Reject @ .01

Table 29. Stream frequency

	Malvern	Farson	Mason City		Analysis of variance means	within
	86.61	49.85	17.24	Sum of squares	14235.06	9655.78
	45.46	61.23	20.53			
	55.56	79.50	40.20			
	93.02	55.56	7.83	d.f.	2	27
	73.17	56.56	4.09			
	64.33	100.00	15.21	Mean square	7117.53	357.62
	74.43	36.34	23.95			
	107.14	47.83	34.83			
	84.62	39.98	9.32	F-ratio	19.90	
	82.28	64.36	69.31			
Σx	766.62	591.21	242.51	$F_{.95}$	3.35	
$\bar{x}$	76.66	59.12	24.25			
Σx^2	61,749.08	38189.25	9322.11	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	58770.62	34952.93	5881.11	Decision:	Reject @ .05	
s^2	330.94	359.92	382.33		Reject @ .01	
s	18.92	18.97	19.55			

Table 30. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 121{,}591.69$	@ .05	6.70	7152.4	47,921.08
$C_3 = (T_1 - T_3)^2 = 274{,}691.29$	@ .01	10.98	7152.4	78,533.35

Decision: C_1 Reject @ .05 Reject @ .01

C_3 Reject @ .05 Reject @ .01

Table 31. Texture ratio

	Malvern	Farson	Mason City		Analysis of variance means	within
	5.28	2.34	1.98	Sum of squares	75.30	47.14
	3.43	1.70	2.52			
	3.26	1.60	2.96			
	4.15	2.53	1.39	d.f.	2	27
	5.28	2.18	0.78			
	6.38	2.62	2.18	Mean square	37.65	1.746
	10.80	1.61	2.99			
	5.28	1.44	2.10			
	4.83	2.14	1.35	F-ratio	21.564	
	5.44	1.84	2.82			
Σx	54.13	20.00	21.07	$F_{.95}$	3.35	
$\bar{x}$	5.41	2.00	2.11			
Σx^2	333.52	41.57	49.45	$F_{.99}$	5.49	
$\frac{(\Sigma x)^2}{N}$	293.00	40.00	44.40	Decision:	Reject @ .05	
s^2	4.50	0.17	0.56		Reject @ .01	
s	2.17	0.412	0.748			

Table 32. Scheffe test

			A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 =$	1.14	@ .05	6.70	34.92	233.96
$C_3 = (T_1 - T_3)^2 =$	929.64	@ .01	10.98	34.92	383.42

Decision: C_1 Accept

C_3 Reject @ .05 Reject @ .01

Table 33. Maximum relief within third order basins

	Malvern	Farson	Mason City		Analysis of variance means	within
	95	95	50	Sum of squares	19145	18675
	145	115	70			
	110	135	30	d.f.	2	27
	125	100	50			
	95	130	60	Mean square	9573	691.67
	95	150	80			
	130	60	80	F-ratio	13.84	
	85	50	40			
	125	70	40	$F_{.95}$	3.35	
	150	60	50			
Σx	1155	965	550	$F_{.99}$	5.49	
$\bar{x}$	115.5	96.5	55.0			
Σx^2	138075.0	104475.0	32900.0			
$\frac{(\Sigma x)^2}{N}$	133402.5	93122.5	30250.0	Decision:	Reject @ .05	
s^2	519	1261	294		@ .01	
s	22.78	35.5	17.15			

Table 34. Scheffe test

	A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 172,225$ @ .05	6.70	13833	92681.1
$C_3 = (T_1 - T_3)^2 = 366,025$ @ .01	10.98	13833	151886.34

Decision: C_1 Reject @ .05 Reject @ .01

C_3 Reject @ .05 Reject @ .01

Table 35. Hypsometric integrae

	Malvern	Farson	Mason City
	0.566	0.805	0.451
	0.513	0.850	0.549
	0.478	0.717	0.483
	0.590	0.740	0.565
	0.514	0.710	0.510
	0.579	0.728	0.534
	0.576	0.800	0.624
	0.622	0.884	0.620
	0.615	0.805	0.580
	0.570	0.780	0.661
Σx	5.623	7.819	5.577
$\bar{x}$	0.562	0.782	0.578
Σx^2	3.181	6.144	3.150
$\frac{(\Sigma x)^2}{N}$	3.162	6.114	3.110
s^2	0.0022	0.0046	0.0048
s	0.0851	0.0708	0.0688

	Analysis of variance means	within
Sum of squares	0.329	0.089
d.f.	2	27
Mean square	0.165	0.0033
F-ratio	6.35	
$F_{.95}$	3.35	
$F_{.99}$	5.49	

Decision: Reject @ 0.05

Reject @ 0.01

Table 36. Scheffe test

		A^2	$\hat{V}_{ci}$	$A^2 x \hat{V}_{ci}$
$C_1 = (T_2 - T_3)^2 = 5.026$	@ .05	6.70	0.0658	0.441
$C_3 = (T_1 - T_3)^2 = 0.0025$	@ .01	10.98	0.0658	0.7275

Decision: C_1 Reject @ .05 Reject @ .01

C_3 Accept