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## TIME-RELATED MECHANICAL BEHAVIOR OF RED OAK IN

### TENSION AND COMPRESSION PERPENDICULAR TO

# A LONGITUDINAL-RADIAL PLANE

by

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#### I. INTRODUCTION

Stress-strain relationships in materials are not independent of time, as is tacitly assumed in classical elastic theory. The existence of stress-strain-time relationships in wood were recognized at least as early as 1833 by Franz Joseph Ritter von Gerstner (51). Two common examples of such relationships are creep, the strain-time behavior under constant stress, and stress relaxation, the relationship between stress and time under constant strain. There are, however, many other types of material behavior documented which are manifestations of an interaction between stress, strain and time. The description of time-related mechanical behavior is usually classified within a branch of physics called rheology, the study of the deformation and flow of matter.

If a reliable description of the rheologic properties of wood perpendicular to the grain can be established, several common problems associated with wood and its use may be more thoroughly dealt with. For example, for stresses and strains perpendicular to the grain, most clamping problems must be associated with time-related behavior. Difficulties encountered in maintaining clamping pressure at a desired magnitude for extended periods of time are at least in part manifestations of stress relaxation. The age old problem of loosening of tools on wooden handles has recently been suggested as a

time-related stress problem (48). When wood dries and begins to shrink, rather severe and complex stress systems are imposed across the grain. Permanent deformations usually occur, which cause the wood to have built-in stresses in the dried condition. The occurrence and relief of these stresses must be subject to rheologic considerations. Stress relaxation may play a role in the loosening of nailed joints. As a final example, the established mechanical properties of wood are affected by the time-rate of testing.

This research has been conducted to devise methods for describing some aspects of the perpendicular to the grain time-related behavior of wood. Uniaxial stresses and strains perpendicular to a longitudinal-radial plane were considered, in tension and compression. A single temperature and relative humidity were employed, yielding an equilibrium moisture content of about 12 percent. Duration of each test was about 14 weeks. It is hoped that the results of this study will lead to reliable engineering criteria for predicting the stressstrain-time behavior perpendicular to the grain, at least for relatively low temperatures. No consideration is given here to the role of anatomic structure in rheologic behavior. The approach to analysis deals with macroscopic parameters, and it is assumed that wood is a homogeneous continuum.

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### II. REVIEW OF LITERATURE

Throughout this review, interest is focused on rheologic phenomena from a qualitative point of view. Species used in experiments, or values of properties obtained are not considered pertinent. It is of interest to know the general nature of time-related behaviors which have been observed and described in the literature and, where these behaviors have been described mathematically, the form of such functions.

W. Weber (52), in 1841, wrote a paper discussing what he called "elastische Nachwirkung". Todhunter and Pearson (50) translated this term as "elastic after-strain". According to the latter authors, "... it differs from elastic fore-strain in that it requires a certain duration of load; it differs from set in that if the load be removed for a certain period the after-strain disappears." Thus, as early as 1886, several rheologic phenomena were named and fairly well defined. In current literature, the elastic after-strain is called creep. Elastic fore-strain is more commonly called instantaneous elastic strain. Set, as defined by Todhunter and Pearson, is now often called flow, or secondary creep. The modern concept of set is not defined in the same fashion. Rather it is taken to mean the amount by which strain exceeds the elastic limit strain, where time is not a consideration.

Von Gerstner (51) experimented with flexure in wooden

beams, and gave some indication that he recognized a creep effect. In 1848, E. Chevandier and G. Wertheim (8) reported the results of studies of beams in flexure, in which an increase in deflection was observed with increasing time, even at relatively small loads. Numerous other experiments have been documented where wood has exhibited a pronounced timerelated behavior. The behavior has appeared in many different experimental forms. Several such forms will be discussed in the following sections. Most of the experiments were conducted in bending, and essentially all of them were conducted such that the normal stresses were parallel to the grain.

### A. Strength-Time Relations

In early efforts to obtain and tabulate the strength properties of wood, it was recognized that such properties depended, to a considerable extent, upon the time in which the tests were performed. The bending properties were thus evaluated on two bases, impact bending, where the entire duration of test was a very small fraction of a second, and static bending, where the tests were conducted at a constant deflection rate for a total time of about 4 minutes. From scanning the property tables of Markwardt and Wilson (36), it appears that the proportional limit is roughly 100 percent higher in impact bending, based on an equivalent static load, than in static bending for dry wood.

Brokaw and Foster (7) and Liska (32) reported the effect of loading at a constant rate to failure in a range of loading times from about 1/2 second up to 750 seconds. Experiments were conducted in compression parallel to the grain and in bending. Results of these works provide some rules of thumb for adjusting values of properties according to loading time. Figure 1a shows a representative relationship. Ultimate strength was related to time by a function of the form

$$P = a + b \log T$$

where P is strength, T is time, and a and b are constants. Wood (53) has reported the results of experiments in bending under constant load for a duration of greater than five years. Using these data, and those of Liska reported above, he fitted an expression of the form

$$(P - a)Tb = c$$

where c is a constant. The graph of this expression, shown in general in Figure 1b, has been published in a great many wood design and property handbooks, and is used widely as a means for establishing duration of load effects in design.

The studies discussed above represent efforts to correlate strength with time, leading to useful predicters for design purposes. The functions employed are somewhat arbitrary; a number of mathematical forms could be made to fit the





Figure 1a. The effect of rate of load application on the ultimate compressive strength for Douglas fir. Due to Liska (32)



DURATION OF MAXIMUM LOAD

Figure 1b. Relation of working stress to duration of load. Due to Wood (53)

same data. It is of interest to note that, in every case, as the time required for failure increases, strength decreases, sharply at first and then more slowly.

# B. Rate of Loading

If wood is tested at a constant strain rate, the resulting stress-strain diagram depends on the magnitude of the strain rate. Figure 2 shows a typical result from work reported by Liska (32). Higher loading rates led to a higher proportional limit and higher strength, but no measurable increase in modulus of elasticity. Other investigators have endeavored to describe the rate of loading effect mathematically (28, 58). Clearly, this behavior indicates that the effects of time must be incorporated with conventional elastic concepts for a more adequate description of the material.

### C. Hysteresis

Consider a specimen of wood loaded at a constant rate of load to a stress above the elastic limit but less than the ultimate strength of the material, and then unloaded at the same rate. Classical elastic-plastic theory admits a stressstrain diagram as shown in Figure 3a, where the area between loading and unloading curves is considered to be the energy lost in plastic deformation. The strain above the proportional limit is not recovered, and is called set. Kollman (27) has



Figure 2. Typical load-deflection curves for two matched Douglas fir flexure specimens. Due to Liska (32)



Figure 3a. A stress-strain diagram according to classical elastic-plastic theory



Figure 3b. Stress-strain diagrams for beech stressed across the grain. Due to Kollman (27)

reported the results of experiments with wood stressed perpendicular to the grain, where the stress-strain diagram deviates markedly from the classical theory. In Figure 3b, it can be seen from one of Kollman's diagrams that strain continues to increase for a time after the load has begun to drop, and that it is decreasing rapidly at the time of complete unloading. Kollman indicated that considerable recovery of strain occurred if suitable time was allowed after the test. There exists, then, an "elastic hysteresis" which must depend on the rate of loading and unloading, and therefore, on time.

### D. Creep and Stress Relaxation

When an experiment is to be performed to obtain relationships between three variables, perhaps the most common experimental method used is to hold one of the variables constant and observe the joint behavior of the remaining two. If a constant stress is imposed on a wood specimen, and strain-time data are collected, the creep curve shown in Figure 4a may be plotted. Clouser (10) has pointed out that, in tension or bending, if the stress is high enough, then the creep curve may pass through a point of inflection, curve up rapidly, and reach a point of complete failure. Such behavior will not be studied here. If, at some time  $t_1$ , the constant stress is removed, and data collection continued, then the creep recovery



Figure 4a. Creep and creep recovery in red oak in tension perpendicular to the grain. Due to Youngs (59)



Figure 4b. Stress relaxation in red oak in tension perpendicular to the grain. Due to Youngs (59)

behavior is obtained. Leaderman (30) has documented development of mathematical theories on creep and related phenomena in filamentous materials dating from the 19th century. Nadai (39, 40) has presented some of the theories used for metals.

It is apparent from Figure 4a that the creep recovery is approaching a horizontal asymptote, and that all of the creep strain will not be recovered. This irrecoverable portion of the strain is usually called flow. It is inconceivable that a specimen could continuously develop irrecoverable strain without failure. For this reason, a precise description of flow behavior is of paramount importance.

If a constant strain is imposed on a wood specimen, and stress-time data are taken, the stress is seen to decay as shown in Figure 4b. Apparently, with wood, it does not decay to zero, but approaches an asymptote.

Many experiments have been conducted with wood to obtain some knowledge of the creep and relaxation behavior. In the following reviews, these works will be organized according to the state of stress used in the experiments.

# 1. Axial stress

Khukhryanskii (20) presented relaxation curves for wood in compression parallel to the grain which show the stress decaying to some finite asymptote. His curves indicate that the stress remains approximately proportional to strain for

all times, if the initial stress does not exceed the static proportional limit. He obtained creep recovery data from matched specimens, and here strain was proportional to creep stress for low stresses, but a large deviation from proportionality was observed at a stress equal to about the static proportional limit. It is suggested that anatomical changes must occur to cause the non-linearity, and that creep recovery can be used to identify the proportional limit. Minami (37) observed creep in tension parallel to the grain at 15 to 20 percent of the estimated ultimate strength, and what appeared to be a non-recoverable strain at 50 percent of ultimate. Dietz (13) experimented with Douglas fir in tension and compression parallel to the grain, and in bending, for durations of from 100 to 800 hours. His results indicate that creep in all cases is sufficiently low to be almost negligible if the stress is below the estimated proportional limit. However, bonded resistance wire strain gages were used. Time-related properties of gage and adhesive may have affected the results of the experiment, which is not in agreement with others in the literature. Wood, et al. (54), performed some exploratory experiments with wood in tension and compression parallel to the grain for periods of up to two years. Substantial amounts of creep, creep recovery after load removal, and stress relaxation were observed. Creep recovery was occurring 500 days after unloading, but it did not appear that the strain would

have been completely recovered if the experiment had been continued indefinitely. Creep curves were represented by a function of the form

$$c = at^{b}$$

where **e** is unit strain. Kitazawa experimented with relaxation of stress perpendicular to the grain. He employed the function

 $ae^{\sigma} = t^{b}$ 

Kitazawa suggests that the expression only works for stresses from 30 to 50 percent of the proportional limit. King (21) obtained and plotted creep versus stress level in tension parallel to the grain, and fitted an exponential to the results. In later works (22, 23) King used two straight lines to fit the same data. Kellogg (19), following the work of King, studied the relationship between creep and initial strain level. Here, too, an exponential relationship was used. Murphey (38) worked with wood in tension parallel to the grain at several stress levels, and simultaneously studied, by means of x-ray diffraction, the way in which crystallinity of the cell walls changed with increasing strain. Results indicate some of the initial strain is not recoverable.

### 2. Bending stress

A number of rheologic experiments have been conducted in bending. There are perhaps two reasons for working in bending. Most of the wood that is used structurally is used in bending. Also, small strains can be manifested in large, easily measurable deflections due to small, easily manageable loads, so that the experiment is relatively easy to perform. However, a disadvantage of at least theoretical importance also exists in a bending experiment. There is no reason to believe that rheologic behavior must be quantitatively similar in tension and compression. Then, if the stresses and strains are computed from load-deflection data, using the common engineering formulas, these stresses and strains have questionable validity.

Clouser (10) used the power function

$$\mathbf{e} = \mathbf{e}_0 + \mathrm{at}^b$$

to express creep behavior for periods up to 10 years. Here  $\mathbf{e}_0$  is initial strain, a constant. It was pointed out that the constant b appeared to be independent of stress level at least up to 60 percent of the ultimate strength in bending. Yamada, <u>et al</u>. (56), employed a linear combination of a power term and a logarithmic term to describe creep in bending. Long-term loading tests reported from Great Britian (33) indicate that for over three years, about one-half of the strain that occurs

is recoverable. Brokaw and Foster (7) reported the results of stress relaxation tests, and found little relaxation for stresses less than 50 percent of the static ultimate.

### 3. Shear Stress

Norris and Kommers (41) observed creep in plywood plates subjected to shearing stresses in the plane of the plywood. The results were fitted reasonably well by an expression derived from

$$\sigma = ae^{b} (de/dt)^{c}$$

This expression will never admit stress relaxation, however. For, if a constant strain is used, then  $d\mathbf{s}/dt = 0$  and the stress is identically zero for all times.

The various studies relating stress, strain and time discussed to this point have almost all been performed such that stresses are parallel to the grain. The mathematical models used to fit experimental data, in most cases are subject to the criticism that they permit infinite response as time approaches infinity. This behavior is unacceptable in view of the tendency for creep recovery and stress relaxation to reach finite asymptotes. The following general conclusions about wood can be drawn from the works reviewed.

a. Wood exhibits creep, creep recovery, and stress relaxation.

- b. The time-related strain phenomena probably occur at, and above, very low stress levels, if sufficiently sensitive detecting equipment is used.
- c. Stress and strain appear to be propertional at any time t, at least for suitably low stresses.
- d. In creep, all of the strain cannot be recovered if the load is removed. No precise indication is **avail**able of how much strain is irrecoverable.
- e. In stress relaxation, the stress does not relax completely.

The behaviors discussed in the preceding paragraphs are all manifestations of some general interrelation of stress, strain and time. If a general function relating the three variables were available, then it should be possible to unite the results of all time-related experiments into one unified theory. The beginnings of such a general function date back to the late 19th century, and are credited to L. Boltzmann.

E. Boltzmann's Principle and Viscoelastic Theory

A general mathematical theory has been developed to describe time-related mechanical phenomena. It has often been found particularly applicable to high polymers. A great deal has been written about viscoelasticity in recent years. Mathematical developments may be studied in the following references (1, 2, 5, 6, 14, 30, 43, 45). A number of the

sources mentioned above also contain accounts of applications of the theory to various substances. In general, the theory of viscoelasticity has the following important properties:

- a. It provides a general stress-strain-time function.
- b. It is mathematically possible to describe creep, creep recovery, and stress relaxation as related phenomena within the scope of this function.
- c. It reduces to classical elastic theory as an initial condition.

In a one-dimensional case, the general stress-strain-time function for linear viscoelasticity may be written

$$\mathbf{\varepsilon} = \int_{-\infty}^{t} \mathbf{k}(t - u) \frac{d\sigma(u)}{du} du$$

where k(t) is an experimentally accessible strain response to constant stress; that is, a mathematical expression of the creep behavior. The function k(t) is an arbitrary function which fits the data. One such function commonly employed is

$$k(t) = k_0 + \sum_{i=1}^{n} k_i (1 - e^{-t/u_i}) + \frac{t}{\eta}$$

Here  $\gamma$ ,  $k_0$ ,  $k_1$  and  $u_1$  are constants. The summation index n is taken large enough to obtain any desired closeness of fit. The first term accounts for an instantaneous elastic strain when a stress is imposed. The summation term is used to describe the elastic portion of creep strain, and the last term permits description of irrecoverable strains, as a linear function of time. Of all functions mentioned thus far for describing creep behavior, only k(t) contains a time dependent term which remains finite as time approaches infinity. Materials which obey the function k(t) are said to be linear viscoelastic materials.

In an analogous development, stress relaxation is described by means of the expression

$$m(t) = m_0 + \sum_{i=1}^{m} m_i e^{-t/u_i}$$

The creep and relaxation functions are related by their Laplace transforms.

$$L[k(t)] = \frac{1}{p^2 L[m(t)]}$$

where p is the transform parameter. Alfrey (2) has shown that the above expressions are equivalent to a linear combination of terms containing stress, strain, and their time derivatives.

# F. Viscoelastic Behavior of Wood

A number of researchers in recent years have explored the possibility of using linear viscoelastic theory to describe wood behavior. These efforts have been directed primarily toward parallel to grain stresses. The series in the Section E are sufficiently powerful to fit any function having an asymptote at large t. The problem then reduces to one of checking for linearity between stress and strain, and of obtaining the fit.

Grossman and Kingston (17) performed 50 day creep and relaxation experiments in bending, and attempted to predict the results of one from the other, by means of the Laplace relationship. Using n = 3, they concluded that the relationship was correct except for the flow effect, which appeared to be nonlinear. Kingston and Clarke (24), in bending and shear, found linearity to about one-half of the ultimate strength. After a nonlinear behavior, in some cases, linearity was observed in a second stress range. Some anonymous work (44) published in Australia suggests an upper limit of linearity of about 67 percent of the ultimate strength. Pentoney (42) using a flexural vibration technique which may be developed from the viscoelastic expressions presented above, found linearity for short times and low stresses. Davidson (11) reported linearity in bending at one-half the static ultimate and for times up to about 100 minutes. Youngs (59) performed a few creep, creep recovery and stress relaxation tests on wood perpendicular to the grain for 70 hours and at from 40 to 90 percent of the ultimate strength. He found that, on the basis of his work, creep is greater in tension then compression for both the recoverable and non-recoverable component. He indicates that this observation may be due to experimental difficulty. The data were fit with a function of

the form

 $\mathbf{c} = \mathbf{c}_{\mathbf{o}} + \operatorname{at}^{\mathbf{b}}$ 

which is the same as that employed by Clouser (10, in bending. It was found that recoverable creep was linearly related to stress, but the irrecoverable creep appeared to increase more than proportionately with stress. Figure 5 shows the experimental relationship between stress and Youngs' estimate of flow.

On the basis of work reported, it appears that it may be possible to use viscoelastic theory to describe time-related mechanical behavior of wood perpendicular to the grain. The linear theory does not seem to provide an adequate means for describing flow effects.



Figure 5. The relationship between irrecoverable creep, stress and time for red oak. Due to Youngs (59)

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### III. THEORETICAL CONSIDERATIONS

In this study, it is desired to develop a means for accurately describing the stress-strain-time behavior of wood in tension and compression perpendicular to the grain, and for relatively long times. A general function will be considered adequate if it meets the following requirements.

- a. It must be possible to manipulate the function to produce all of the experimentally observable restricted behaviors. These behaviors include creep, creep recovery, stress relaxation and irrecoverable strain or flow or their equivalents.
- b. It must contain enough arbitrary constants to fit experimental results to any desired degree of accuracy.
- c. It must be well-behaved in the limits. That is, as  $t \rightarrow 0$  it must reduce to elastic theory, and as  $t \rightarrow \infty$  it must predict finite strains commensurate with physical reasoning.
- d. It must be workable in an engineering sense. This is a rather loosely defined requirement.

A complete list of the symbols used in the following considerations may be found in Appendix A.

Consider a wood specimen subjected to a uniaxial, constant stress perpendicular to the grain. If strains are

observed at several times and plotted, there results a curve having the form shown in the creep phase of Figure 4a. The second phase, creep-recovery, is that observed if the stress is removed at some time  $t_1$ . According to theory, creep may be described by the expression

$$\varepsilon(t) = \sigma \left[ D_g + \sum_{i=1}^n D_i (1 - e^{-t/\tau_i}) + \frac{t}{\tilde{\gamma}} \right]$$
(1)

Here  $D_g$  is called the glass compliance. It is the compliance associated with an instantaneous, recoverable strain obtained at the instant the stress is imposed. The adjective "glass" is used because many glass-like materials behave independent of time over a wide range of temperatures. The  $\mathtt{D}_i$  and  $\tau_i$  are constants, and the summation expression, for suitable n, can be made to describe a recoverable creep phenomenon, so long as the creep approaches some asymptote for long times. The constants  $\tau_i$  are often called retardation times, because they have units of time, and because creep rate decreases as the  $\tau_i$ increases. The last term is used to describe irrecoverable creep, or flow. It is very restricted in its behavior, because it implies a linear relation between strain and time. Diagrammatically, this implies that for very large times the creep curve must approach constant slope. The bracketed portion of 1 is called D(t), the creep compliance function or simply the creep function. If t/p is subtracted from both

sides of D(t), there results

$$D_{\mathbf{E}}(t) = D(t) - \frac{t}{\eta} = D_{\mathbf{g}} + \sum_{i=1}^{n} D_{i}(1 - e^{-t/\tau_{i}})$$
(1a)

where  $D_{E}(t)$  is called the elastic compliance. Furthermore

$$D_{e} = \lim_{t \to \infty} D_{E}(t) = D_{g} + \sum_{i=1}^{n} D_{i}$$
 (1b)

is the equilibrium elastic compliance. It is sometimes useful to suppress the glass compliance in la, and work with the delayed elastic compliance

$$D_{d}(t) = \sum_{i=1}^{n} D_{i}(1 - e^{-t/\tau_{i}}) \qquad (lc)$$

It is to be noted that D(t) is independent of stress level. It is implied that stress is proportional to strain for any time. As an initial condition this is Hooke's law, and it is commonly referred to as the linearity condition in viscoelastic theory.

If the specimen is subjected to a constant strain and stress-time data are taken, there results a curve such as Figure 4b. Then, in theory,

$$\sigma(t) = \mathbf{c} \left[ \mathbf{E}_{e} + \sum_{i=1}^{m} \mathbf{E}_{i} e^{-t/\tau_{i}} \right]$$
(2)

The  $E_i$  and  $\tau_i$  are constants. The  $\tau_i$  in 2 are not the same as the  $\tau_i$  in 1, although by convention the same symbol is usually

used. Here they are called relaxation times, and the stress decreases as the  $\tau_1$  increase. The bracketed portion of 2 is called the relaxation function E(t). It follows that

$$E_{e} = \lim_{t \to \infty} E(t)$$
 (2a)

where  $\mathbf{E}_{e}$  is called the steady state elastic modulus. It is also useful to define

$$\mathbf{E}_{g} = \lim_{t \to 0} \mathbf{E}(t) = \mathbf{E}_{e} + \sum_{i=1}^{m} \mathbf{E}_{i}$$
(2b)

 $\mathbf{E}_{g}$  is the glass modulus. It is the same as the conventional modulus of elasticity if the conventional modulus is assumed to be determined in an infinitely short experiment.

Apparently a material which exhibits a behavior described by 2 would not exhibit flow in a creep experiment. In the limit as time approaches infinity in 1, the flow strain also becomes infinite. If some sort of molecular mechanism exists which will permit this flow behavior, the same mechanism would cause complete stress relaxation. But 2 has a finite limit at infinite time, as indicated by 2a. If, in a relaxation experiment, complete relaxation does occur, then additional terms are required in 2 to account for this phenomenon.

It is possible to show that the strain response to any general stress system  $\sigma(t)$  is given by

$$\varepsilon(t) = \int_{-\infty}^{t} D(t - u) \frac{d\sigma(u)}{du} du$$
 (3)

Expression 3 is often called Boltzmann's superposition principle. The integral is a stress-history integral.<sup>1</sup> The strain at time t depends on stresses at all previous times u, on the elapsed time (t - u) and on the strain response of the material to constant stress, D(t). Implicit in the derivation of Boltzmann's principle is the assumption that irrecoverable strains are not allowed. Otherwise the function D(t)can be any functional representation of creep behavior, including la.

An analogous expression of Boltzmann's principle is

$$\sigma(t) = \int_{-\infty}^{t} \mathbf{E}(t - u) \frac{d\mathbf{e}(u)}{du} du \qquad (3a)$$

where the kernel is the stress response to constant strain. Now if 3 and 3a are both general stress-strain-time functions, independent of any flow effects, they must be equivalent. Gross (15) has shown that the elastic creep compliance  $D_{\underline{F}}(t)$ , and the relaxation modulus E(t) are related by the expression

$$L[D_{E}(t)] = \frac{1}{p^{2} L[E(t)]}$$
(4)

Although the mathematical development of 4 is perfectly rigorous, one is not assured that the Laplace transforms of

<sup>&</sup>lt;sup>1</sup>For additional information on history integrals, see, for example, (55, p. 194).

 $D_e(t)$  and E(t), or the inverse transforms, can be easily established. Ferry (14), MacLeod (34) and others have discussed approximations to these functions, but for which the transforms are relatively easy to manipulate. Bateman (4) has tabulated a great number of transforms and inverse transforms.

If a stress history on a specimen is imposed as a sequence of step functions, then 3 may be written in finite form as

$$\mathbf{e}(\mathbf{t}) = \sum_{j=1}^{q} \mathcal{D}(\mathbf{t} - u_j) \Delta \sigma(u_j)$$
(5)

Here, the stress is imposed in q steps,  $\Delta\sigma(u_j)$ , each being imposed at time  $u_j$ . This permits description of the creep recovery phenomenon, for unloading corresponds to imposing a negative stress in 5 equal to the original stress. It can be shown that, according to 1 and 5, the strain at any time after stress removal is

$$\varepsilon(t) = \sigma \left[ \frac{t_1}{\gamma} + \sum_{i=1}^{n} D_i e^{-t/\tau_i} (e^{t_1/\tau_i} - 1) \right]$$
(6)

Here,  $t_1$  is the time of stress removal. The behavior for  $t > t_1$ is creep recovery. It is of interest to look at the difference between the maximum strain achieved, at time  $t_1$ , and the strain thereafter. If this difference is called recovery strain,  $\mathbf{e}_r(t)$ , it may be expressed as

$$\mathbf{e}_{r}(t) = \sigma \left[ \mathbf{D}_{g} + \sum_{i=1}^{n} \mathbf{D}_{i}(1 - e^{-t_{1}/\tau_{i}})(1 - e^{-(t-t_{1})/\tau_{i}}) \right]$$
(7)

There are several points of interest associated with 7.  $D_g$ appears in the same fashion in 1 and 7. Therefore, the instantaneous elastic behavior is precisely the same if the stress is imposed or removed. No term expressing flow appears. This leads to an experimental means for obtaining creep behavior without irrecoverable strains. If  $t_1/\tau_1 \gg 1$  for all  $\tau_1$ , then  $\mathbf{e_r}(t)/\sigma$  is approximately the same as D(t) without flow. The validity of the inequality must be checked against any experimental results. If it holds, then it is possible to obtain delayed elastic effects from a recovery experiment. The corresponding flow effects may be obtained from the creep portion of the experiment. This is done simply by obtaining the difference between creep and recovery strain. Thus

$$\mathbf{e}_{\mathbf{f}}(\mathbf{t}) = \mathbf{e}(\mathbf{t}) - \mathbf{e}_{\mathbf{r}}(\mathbf{t}) \tag{8}$$

The compliance, D(t), and the modulus, E(t), are material properties analogous to their elastic counterparts. Here the properties cannot be expressed as constants, but as functions of time.

Much of the literature defers to a natural mathematical extension of 1 and 2. If n and m are allowed to approach infinity, it is possible to get unique expressions of the creep and relaxation behavior in the form of integral equations. Although the uniqueness property, not enjoyed by 1 and 2, is theoretically desirable, computational difficulties arise in attempting to fit experimental data, as discussed by Ferry (14). In addition, the integral forms are not considered to be usable in an engineering sense, which is one of the requirements of this work. The integral forms may, however, lead to a better understanding of the role of wood structure in rheology, if suitable research is conducted in that area.

### IV. EXPERIMENTAL INVESTIGATION

# A. Experimental Techniques

Creep experiments were used in this research for experimental verification of the viscoelastic concepts for wood perpendicular to the grain. It was felt that it is easier to experimentally meet the assumptions of the theory in creep than those of stress relaxation. In a stress relaxation study, it is necessary to impose a constant strain, and measure stress changes with time. However, any stress measuring device in the test system must necessarily behave elastically. If it is stiff, it has poor sensitivity. If it is soft, it is difficult to hold strain constant in the specimen, because the device changes strain as the stress relaxes. By comparison, in a creep experiment, a means can be devised to simply hang a weight to induce a constant stress in the specimen. It is assumed that no measurable change in the cross-section of the specimen will occur during the experiment.

The specimens were loaded so that the stress was always perpendicular to a longitudinal-radial plane. In this way, it was possible to obtain the specimens from a flat-sawn board. Red oak (<u>Quercus</u> spp.) was used for a number of reasons. Among the hardwoods, its static properties are best known. It is a species which is commercially very important. It exhibits greater seasoning problems than most species. It is available

locally for easy selection. It was desired to study the timerelated behavior independent of the effects of moisture content and time. A convenient room was used where atmospheric conditions were controlled at 80 degrees Fahrenheit and 65 percent relative humidity. This corresponds to an equilibrium moisture content of about 12 percent, based on the oven-dry weight of the wood.

Tensile and compressive behavior were studied. The test specimens used were those reported by Youngs (59). They have a 1/4 square inch least cross-section, making it possible to obtain easily measurable strains with relatively small loads. The two types of specimens are shown in Figure 6. One flat-sawn board 2 by 10 inches in cross section was obtained in the green condition. The board was sawed into wafers 1/2 inch along the grain. The wafers were numbered consecutively. They were placed on stickers, and over a period of weeks they were moved through a sequence of conditioning rooms to obtain slow, stress-free drying. At the end of the conditioning sequence they were stored in the room where the tests were to be conducted.

The wafers were inspected after conditioning, and a few were culled due to growth irregularities. About every third specimen was segregated from the batch and stored for a separate study. Of those remaining, 16 evenly spaced wafers were designated for control specimens. The remainder were



Figure 6. Sketches of the specimen used for testing, with stress and strain perpendicular to the grain
assigned for creep tests. The specimens were machined, and again stored on stickers in the test room. The control specimens, eight for each stress state, were tested at a constant rate of strain to failure. The average of eight strength values were used as an estimate of the static ultimate strength of the wood.

In order to test linearity, it was decided to conduct tests at 20, 30, 40, 50, and 60 percent of the estimated ultimate strength of the wood. For an estimate of the variation between specimens tested alike, two replications were made. In addition, a single exploratory replication was conducted at 10 and 80 percent of the estimated strength and for both states of stress.

It was necessary to devise a strain detecting device with a resolution of about 100 micro-inches per inch, and which was not itself subject to any time-related behavior. A special optical gage was fabricated and is shown mounted on a compression specimen in Figure 7. It has a 1-inch gage length, a range of about 0.06 inches, and the desired sensitivity. The gage is made of aluminum, and fastens to the specimen by set screws top and bottom. Properly adjusted, the holes made by these screws are no larger in diameter than the vessels in the oak. The front is removable for attachment to the tension specimen. The lower mirror remains fixed during tests, and the upper mirror rotates about a horizontal axis according to

Figure 7. Optical strain gage used in creep experiments

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the amount of strain that occurs. The gage has a multiplication ratio of about 367, when used with a scale 24.00 inches from the axis of rotation of the upper mirror.

An ordinary 35 millimeter slide projector, containing a slide with two suitably spaced cross-hairs, and a scale were used for reading the strains. This assembly is shown in Figure 8. The scale was pasted on a circular arc of wood, designed to have the proper radius of curvature for linear strain readings from the strain gage. The cross-hairs were projected onto the gage mirrors and the reflected images observed on the scale. Differences between fixed and movable readings at different times then represent strains, when divided by the multiplication ratio.

Loads were imposed by hanging a bucket of lead shot on a lever system. It was necessary to hang the weight as quickly as possible in order to estimate the instantaneous elastic strain, but not fast enough to cause noticeable dynamic effects. Several levers were available so that the maximum weight employed in any test was 33 pounds. Although atmospheric conditions were controlled in the test room, it was felt that considerable traffic in and out during the day would cause the ambient conditions to vary. Therefore, the specimens were tested inside plywood boxes to buffer this effect. A glass pane in the front permitted reading of the strain gage. Figure 9 shows the tension apparatus. The entire system was

Figure 8. Projector and scale for reading strain gages

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Figure 9. Tension specimen and test apparatus used for creep experiments. During tests, the door is screwed firmly in place

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mounted on foam rubber for vibration isolation. Sufficient pin-end connections were used in the systems to ensure uniaxial loading.

The tension specimens were gripped by bolting heavy steel friction plates on the ends. Electric time clocks with limit switches were used on the tension tests, so that if destructive failure occurred, the time would be recorded. Sufficient apparatus was constructed to conduct four tension tests at a time.

A typical compression apparatus is shown in Figure 10. A cage-type specimen jig, similar to that described by Youngs (59), was used to change a hanging load to a compressive stress. For the compressive tests, no time clocks were used, but the date and time of day were recorded, along with strains. Four compression tests were conducted simultaneously.

The experiments were somewhat exploratory in terms of attempting to establish a reasonable duration of test. Therefore, the first tests, at 20, 40, 60 and 80 percent of estimated ultimate strength, were watched carefully as they progressed. In some cases they were continued for up to 2700 hours, in order to establish trends in behavior. Later tests were standardized to about 1700 hours of creep and 700 hours of creep recovery. At the beginning of the experiments strain-time data were recorded for times of 0, 1, 2, 3, 4, 5, 8, 10, 15, 20, 30, 50, 100 and 300 minutes. One reading was

Figure 10. Compression specimen and test apparatus used for creep experiments. During tests, the door is screwed firmly in place

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taken each day for the next three days, and one reading about every two or three days thereafter.

## B. Analysis of Results

The data were analyzed in terms of compliance, the ratio of creep strain to stress. If the stress-strain behavior for the material is linear, the compliance is independent of the stress level at which it was obtained. It is possible, then, to compare the variation in compliance obtained between stress levels with the variation for specimens tested at similar stress levels. By the nature of the theoretical concepts discussed in the previous section, it is necessary to establish the glass compliance, the delayed elastic compliance and the compliance associated with flow in that order.

## 1. Glass compliance

According to Equation 7, the instantaneous elastic behavior observed upon loading and upon unloading are equivalent. Each creep and creep recovery test yields two estimates of the glass compliance property,  $D_g$ , from the same specimen.

Strain changes quite rapidly with time during the first few seconds of a creep experiment. It is difficult to obtain a strain reading at precisely the instant that load is in place. Therefore, considerable variation was expected, and occurred.

Strain was regressed on stress by the method of least squares. The slope of the regression equation is  $D_g$ . Figure 11 shows the regressions obtained in tension and compression. The line for compression is based upon 20 data points. The data at 80 percent of ultimate stress were rejected because the test exceeded the limits of the strain gage. One test in compression at the 60 percent stress level was culled due to experimental difficulties at the time of unloading. The regression for tension is based on 24 data points.

## 2. Delayed elastic compliance

In order to extract the delayed elastic compliance from the data, it is necessary to know the glass compliance. It was assumed that, for any specimen, the glass compliance obtained from the regression of Figure 11 was a better estimate of the true glass compliance than either value obtained from the test of that specimen. This was done because of the large experimental error expected in obtaining any single value of  $D_g$ . Put another way, it was assumed that variation in the experiment was of greater consequence than variation in the material.

According to expression 7, recovery strain yields a good approximation for elastic compliance  $D_E(t)$ , subject to the restriction that  $t_1/\tau_1 \gg 1$ . Expression la was fit to recovery data, using Prony's method, as documented by Hildebrand (18).



Figure 11. The regression of instantaneous strain upon stress. All data at a particular stress level were averaged for plotting purposes

Appendix B gives a discussion of Prony's method, as applied in this study. Table 1 gives the ratio  $t_1/\tau_1$  for all specimens tested.

It was found that la fit the data reasonably well if n was taken to be two. Then four constants, in addition to Dg, were evaluated by Prony's method. No one curve so obtained was congruent with any other. It was desired to determine if each curve was an estimate of the same property, as is assumed in linear viscoelastic theory, or if a difference in the curves existed between stress levels. Where possible, constants were treated in an analysis of variance in a completely randomized design. The hypothesis tested was that a particular constant was the same for all stress levels. Data at the 10 and 80 percent stress levels, and at the 60 percent stress level in compression only, were not used in the analyses due to lack of replication.

It is shown in Appendix B that a set of constants  $\zeta_1$  are equivalent to the time constants  $\tau_1$ .  $\zeta_1$ , and therefore  $\tau_1$ , was found to be significant at the 1 percent level in compression. The complete set of  $\zeta_1$  for all compression tests is tabulated in Table 2. It is apparent that the values for the 20 percent stress levels are markedly lower than at any other stress level. In tension  $\zeta_1$  was not significantly different between stress levels. The average values of  $\zeta_1$ , and the Fvalues, are given in Table 3.

Tension			-	Co	Compression_		
Specimen	tl (hours)	t <sub>l</sub> /T <sub>i</sub>	iª	Specimen	t <sub>l</sub> (hours)	t <sub>l</sub> /Ti	
T-1-3	1871	60.0 84.1	1 2	C-4-1-Rep	2014	11.6 314.6	
T-1-2	1657	9.5 45.2	1 2	0-2-1	2152	120.2 164.3	
T-1-1	1867	22.0 121.4	1 2	0-1-2	1990	199.0 2 <b>39.8</b>	
T-3-l-Rep	1795	68.2 371.8	1 2	C-1-3	1656	14.4 80.4	
T-3-3	1678	1 <b>3.6</b> 71.2	1 2	0-2-3	2059	22.1 92.7	
T-2-1	1 <b>84</b> 5	35 <b>.3</b> 52 <b>.</b> 1	1 2	C-1-1	2153	11.0 129.7	
T-2-2	1658	7.6 123.8	1 2	C-2-2	16 <b>81</b>	15.1 27.2	
T-2-3	1870	16.4 161.8	1 2	0-4-3	1846	1 <b>8.6</b> 95.2	
T-4-3	1677	18.4 160.9	1 2	0-3-3	1845	16.8 121.4	
T-3-2	2507	12.3 142.2	1 2	0-3-2	1655	9.9 140.2	
T-4-2	1796	19.7 101.9	1 2	C-4-2	169	1.6 14.2	
T-4-1	1704	7.7 145.0	1 2				

Table 1. Ratios  $t_1/\tau_i$  for checking validity of use of recovery data

<sup>a</sup>In fitting expression la to the data, n was taken to be 2, so that i takes on the successive values 1, 2.

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Renli.	Stress level (percent)								
cation	10	20	<b>3</b> 0	40	50	60	80		
1	0.9443	0.5726	0.9170	0.949 <b>8</b>	0.9041	0.9418	0.9096		
2		0.3696	0.9071	0.9141	0.9 <b>13</b> 1				

Table 2. Time constants  $\boldsymbol{\zeta}_1^{a}$  for compression tests

<sup>a</sup>To obtain the time constant  $\boldsymbol{\zeta}$ , time is measured in tens of hours.

It was not possible to test the second time constant,  $\zeta_2$ , because it often had a negative value. Details of the difficulties encountered are given in Appendix B. Table 4 gives the magnitudes of  $\zeta_2$  obtained. It can be seen in Table 4 that considerable variation exists in  $\zeta_2$  among and within stress levels.

The delayed equilibrium elastic compliance  $D_e-D_g$  was tested. The average values are given in Table 3, along with F-ratios, and corresponding critical values of F.

# 3. Flow compliance

According to expression 8, irrecoverable strain, or flow, can be obtained by subtracting recovery strains from creep strains at any choice of times, subject again to the restriction that  $t_1/\tau_1 \gg 1$ . If 8 is divided by  $\sigma$ , an expression for

	Tension			Compression		
	Average value	Fa	Level of <sup>b</sup> significance (percent)	Average value	Fc	Level of significance (percent)
ζι	0.888	0.60	>20	0.806	18.7	< 1
$\mathbf{D}_{e} - \mathbf{D}_{g}^{d}$	2.978	1.34	>20	3.108	3.57	>10

<sup>a</sup>The ratio has 4 and 5 degrees of freedom.

<sup>b</sup>The level of significance is the probability of obtaining by chance a sufficiently large calculated F to reject the hypothesis when, in fact, the hypothesis is true.

<sup>C</sup>The ratio has 3 and 4 degrees of freedom.

<sup>d</sup> These constants have units  $10^{-6}$  in.<sup>2</sup>/lb.

Stress level (percent)	Tension	Compression	
10	-0,6380	-0.2105	
20	0,7613 -0.5221	-0.4656 -0.2995	
30	0.1265 -0.6540	0.6158 -0.6 <b>36</b> 9	
40	-0.7536 -0.4743	0.5475 0. <b>31</b> 29	
50	-0.4208 0.3835	-0.59 <b>8</b> 4 0.5177	
60	0.5668 -0.5670	0.4294	
80	0.4270	0.4334	

Table 4. Time constants  $\zeta_2^a$  obtained from Prony's method

<sup>a</sup>The constants have units  $\zeta^{1/\text{tens}}$  of hours.

 $D_{f}(t)$ , the flow compliance is obtained.

A curve was drawn through the data points for creep in each experiment. Figure 12 shows such a curve for specimen C-2-1, a typical creep compliance curve. At intervals ranging from 10 hours to 200 hours, the ordinates of the corresponding recovery curve obtained by Prony's method were subtracted from the creep compliance curve. The lower curve in Figure 12 is an example of a flow compliance curve obtained by subtraction. Now it is apparent that flow here is not linear in time, as



Figure 12. A representative creep compliance curve and computed flow compliance curve

given in 1. However, it was found that if the flow curves were plotted on logarithmic paper, in many instances they plotted as essentially straight lines. Figure 13 shows such a plot, again for specimen C-2-1. Then, apparently, at least within the time range of these experiments, flow can be expressed in the form

$$D_{f}(t) = at^{b}$$
(9)

In compression, 9 appeared to fit the flow quite well, particularly at the higher stress levels. In tension, much of the flow data was very erratic, and showed no trend of any kind. Where the tension data were reasonably consistent, 9 seemed to fit adequately. The constants for compression were tested in the same statistical fashion as before. The results are given in Table 5. No statistical test could be performed on the tension flow data, because the few results that were reasonably well-behaved did not provide sufficient replication.

The flow curves obtained by using average constants are plotted in Figure 14.

If la is substituted into 4, the transform manipulations that will, in theory, permit solution for E(t) are unknown. However, MacLeod (34) has suggested that if the logarithmic plot of  $D_E(t)$  versus t is a straight line, at least for specified ranges of t, then approximately

$$D_{\mathbf{E}}(t) = at^{\mathbf{D}} \tag{10}$$



Figure 13. Logarithmic plot of flow curve from Figure 12



Figure 14. Average flow compliance curves

	Tension	Compression			
Constant	Average value <sup>a</sup>	Average value <sup>b</sup>	₽ <sup>C</sup>	Level of significance (percent)	
a <sup>d</sup>	0.444	1.040	0 <b>.</b> 7 <b>3</b>	>20	
be	0.541	0.520	0.40	>20	

Table 5. Average constants and F-ratios for flow compliance

<sup>a</sup>Based on 1 test at 30, 40 and 50 percent of ultimate strength, and 2 tests at 60 percent.

<sup>b</sup>Based on tests at 20, 30, 40, and 50 percent of ultimate strength.

<sup>c</sup>The ratio has 3 and 4 degrees of freedom.

<sup>d</sup>The constant a has units of  $10^{-6}$  in.<sup>2</sup>/lb.

<sup>e</sup>The constant b is dimensionless. t is measured in tens of hours.

The transforms are established for 10, and yield

$$E(t) = \frac{\sin b\pi}{b\pi D_{\rm E}(t)}$$
(11)

Here b is the slope of the logarithmic plot. It is obvious that when  $D_{\rm E}(t)$  approaches its asymptote, b approaches zero, so that for all very long times the relaxation modulus and elastic compliance functions are reciprocals.

Figure 15 shows the logarithmic plot of the compression recovery data. It appears that three straight lines describe the behavior quite well above ten hours. Figure 16 is a plot



Figure 15. Logarithmic plot of compression elastic compliance function,  $D_E(t)$ , obtained from recovery at 30, 40, and 50 percent stress level



Figure 16. Relaxation function computed from compression elastic compliance. Based on 30, 40, and 50 percent stress levels

of E(t) obtained from Figure 15. It appears that very little error would be introduced if it was assumed that E(t) and  $D_E(t)$  are reciprocals.

#### V. DISCUSSION

The glass compliance appears to be quite independent of stress level, on the basis of the regression plotted in Figure 11. The plotted averages indicate a straight line relationship between strain and stress, even at stresses as high as 80 percent of the estimated strength. There appears to be very little difference between glass compliance in tension and compression. It would be possible to perform a statistical test for congruence of the two lines in Figure 11. This was not done. It was felt that the method of obtaining instantaneous strains permitted a large amount of variation, so that it would be difficult to show a significant difference between the two lines even if a difference did exist. However, when determining modulus of elasticity for a single species in a standard test, coefficients of variation of greater than 20 percent are not uncommon. In the face of such large variation in a relatively heterogeneous material, it seems quite practical to assume that there is no difference between the two lines as shown. Then it is possible to pool the data for a new estimate of the glass compliance. In this case,  $D_g = 6.40$ x  $10^{-6}$  square inches per pound for tension and compression.

The requirement that  $t_1/\tau_1 >> 1$  was met for all but the 80 percent stress level in compression. For that case it was necessary to stop the test in a very short time because the

range of the strain gage was too short. It can readily be shown that if the ratio  $t_1/\tau_1$  is seven or greater, there is no theoretical error to four significant digits in using recovery data to approximate elastic compliance. In this experiment several of the ratios turned out to be just a little greater than seven. It does not necessarily follow that these tests were conducted for the length of time necessary to meet the requirement. Expression 1 has no unique set of constants for a given set of test data. A unique set of constants is acquired only under some fixed criterion for establishing those constants. In this case, the criterion was Prony's method. If a different method had been devised, the set of ratios in Table 1 would likely have been quite different. It does appear, on the basis of this experiment, that for oak at 80 degrees Fahrenheit and 12 percent moisture content, and where Prony's method will be used, about 1600 hours of creep testing prior to a creep recovery experiment are needed.

It is not clear why the time constant  $\zeta_1$  was markedly different at the 20 percent stress level in compression than elsewhere. It may involve some mechanism undergoing change in the material, or it may involve undetected experimental difficulty. Certainly there is a need for further experimentation in that region of stress.

The usual interpretation of the time constants is that they represent mechanisms in the material which prevent the

elastic strain from occurring entirely instantaneously. A material will, in general, have an infinity of such mechanisms continuously distributed. If a finite set of terms such as expression 1 is used to describe the behavior, the distribution is approximated by a set of lumped constants. The  $\tau$ 's are time constants associated with the attenuation of contributions to compliance. The contributions to compliance are the corresponding D's. When the t becomes about seven times larger than a particular  $\tau$ , the corresponding D is no longer attenuated and contributes fully to total compliance. Thus a relatively small  $\tau$  is called a "fast" retardation time, because only a short time is required for the attenuation to vanish.

It is not possible, within the concepts above, to explain the physical meaning of the negative time constants  $\mu$ , or **(**. In every case, the negative value was associated with the smaller of the two retardation times. It seems probable that the negative values are associated with scatter in the data, and the inadequacy of the fit by eye of the creep recovery data. This may cause particular difficulty in the early part of the recovery experiment, when the retarded elastic strains are small, and the faster retardation constant is in the process of becoming established.

On the basis of this experiment, there is an indication that the delayed elastic behavior is described by a linear

stress-strain relation. It may also be observed in Table 3 that the slow time constant, and the delayed elastic compliance have about the same magnitude in tension and compression. It should be pointed out that the variation in the results was large. The inability to find a significant difference in the constants between stress levels may be because no difference existed, or it may be because the experiment was not sufficiently precise.

Flow compliance was found by subtracting the regression estimate of glass compliance and the Prony estimate of delayed elastic compliance from creep compliance. This estimate of flow then is a catchall for all the accumulated error in the preceding estimates. It is not surprising that it was erratic in some cases. Expression 9 functioned quite well in fitting the compression data. There appears to be very little probability that flow compliance is dependent upon stress level. This is not in accord with the results reported by Youngs (see Figure 5). On the basis of the limited results in this study, flow may be twice as great in compression as tension at equivalent times. The results of work by Youngs (59) indicated that creep is greater in tension than compression for red oak. These findings conflict with the results of this study. For it is indicated here that the elastic components of creep are about the same in tension and compression, and flow is much greater in compression. Youngs indicates his results are

suspect due to experimental difficulties.

It is of interest to note the relative magnitudes of the components of compliance in a general sort of way. Equilibrium retarded elastic compliance is about half that of glass compliance. It is not possible to compare flow compliance unless a specific time is mentioned, because it appears never to reach an equilibrium value. Using the average flow curves of Figure 14, flow compliance in compression is about equal to the equilibrium elastic compliance at 650 hours. The tensile flow compliance is about half as much at the same time.

Because some time constants were negative, it was not possible to find an average delayed elastic compliance curve, as it was for flow compliance. It is possible, however, to average the ordinates of the recovery curves at a selected set of times and obtain an average curve. This should only be done if it appears certain that all recovery curves are estimates of the same delayed elastic behavior.

If Prony's method for fitting recovery data is to be used, some suggestions can be recommended on the basis of this experiment. It is desirable to perform the computations using ordinate values for the same equally spaced values of the abscissa. It is particularly desirable to use a great many points if the compliance reaches equilibrium rather rapidly; that is, if all retardation times are relatively "fast". Probably increments of ten hours are sufficient. This will

cause need for large numbers of computations. However, the entire method seems amenable to digital computer solution.

A strain detecting device that will drive a recorder would be of great value. The human error in performing a curve fit by eye would be removed if recording could be done automatically at the desired increments of time. If there is a fast retardation time associated with the first few hours, it would be helpful to record continuously during that time. This may also make it simpler to detect the strain at the instant the load is applied. Greater resolution is needed in the strain gage if variation is to be reduced. Although oak is one of the stiffer woods, it can be expected that it will exhibit greater stiffness in the other two principal directions. Thus, to reduce errors, it would be desirable to increase the sensitivity of the gages by at least a factor of ten.

It is indicated that the duration of the creep experiment should be about 1600 hours, prior to creep recovery. This may differ for other species, for other directions in oak, at other temperature and equilibrium moisture content conditions, and for a more precise experiment. It is unlikely that large sample sizes can ever be used in experiments which are so expensive in terms of time and equipment. It is not possible to predict a good sample size on the basis of this study, because it is felt that much of the variation can be reduced by

improving upon the experimental method. It is probable that a refined experiment along the same lines as this one could give a good indication of necessary sample size.

There are a great many closely related problems of interest. If flow is capable of being described by Equation 9, then it becomes infinite as time becomes infinite. In tension this probably indicates destructive failure of the specimen. In compression, it hints of continuous densification of the material. This seems an unlikely consequence. Thus the entire realm of very long times warrants exploration. Expressions 4 and 11 indicate relaxation behavior can be computed if creep behavior is known. However, this can only be finally proven by experiment. Also, the role of flow in the relation between creep and relaxation is not entirely clear. Eventually the effect of varying temperature and moisture conditions must be studied, as well as the effect of chemical treatments the wood may undergo. A scheme for accelerated testing could be of great importance in establishing rheologic properties for a variety of species and conditions. In a somewhat more mathematical vein, the effect of combined stresses might well be considered.

### VI. SUMMARY AND CONCLUSIONS

This research was done to develop a means for describing the perpendicular-to-grain rheologic behavior of wood in direct stress. Twenty-three red oak specimens were tested under constant load and the strain-time behavior observed. Tests were conducted in tension and compression perpendicular to a longitudinal-radial plane, and at 10, 20, 30, 40, 50, 60 and 80 percent of the estimated static ultimate strength. Strains were measured to the nearest 100 micro-inches per inch. The tests were conducted under controlled conditions of 80 degrees Fahrenheit and an equilibrium moisture content of 12 percent. The duration of the tests was about 10 weeks of creep and 4 weeks of creep recovery.

The creep compliance was described by the function

$$D(t) = D_g + \sum_{i=1}^{2} D_i(1 - e^{-t/\tau_i}) + at^b$$

The first term on the righthand side of the equation represents the instantaneous elastic effect, the second and third terms represent the delayed elastic component, and the fourth term was used to describe the irrecoverable effect. All terms were found to be independent of stress level. The elastic effects were of the same magnitude in tension and compression. Flow compliance was twice as large in compression as tension at any time. In general, glass compliance was about twice

equilibrium retarded elastic compliance. Average flow in compression was equal to glass compliance at about 650 hours.

The experiment consisted of two replications in order to obtain some small measure of the variation involved. Variation was found to be quite large. This is usually to be anticipated with a relatively heterogeneous material such as wood. It is suspected that some of the variation can be removed by refinements in the experiment. The delayed elastic portion of the function was fit to recovery data using Prony's method. The fast retardation time often was slightly negative, which is physically absurd. This was accredited to inaccuracies in the experiment and the fitting technique in the very early times.

On the basis of this work, it appears that red oak behaves as a viscoelastic material which is linear in stress and strain perpendicular to the plane considered, at least up to 60 percent of the static ultimate strength. This should make it possible to describe elastic behavior, creep, creep recovery and stress relaxation all within the framework of a single mathematical theory. The few terms of the creep compliance function are quite simple.

It is desirable to develop improved experimental and computational techniques to reduce variation. Considerable additional information is needed on the nature of flow.
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# IX. APPENDIX A

A. List of Symbols

Wherever pertinent, the symbols used are those recommended by the Society of Rheology (31).

a, b, c	=	constants
$\mathtt{D}_{g}$	=	glass compliance
D(t)	Ξ	creep compliance function
D <sub>E</sub> (t)	Ξ	elastic compliance
D <sub>e</sub> (t)	=	equilibrium elastic compliance
D <sub>d</sub> (t)	Ξ	delayed elastic compliance
D <sub>f</sub> (t)	=	flow compliance
D <sub>i</sub>	=	compliance constants
Eg	=	glass modulus
E(t)	Ξ	relaxation modulus function
Ei	=	modulus constants
k	Ξ	general compliance variable
m	Ξ	general modulus variable
q	Ξ	Laplace transform parameter
t	=	time
u	Ξ	dummy time variable
e	=	unit strain
σ	=	unit stress
η	=	flow constant
۳ <sub>i</sub>	=	retardation and relaxation constants

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### X. APPENDIX B

## A. Fitting the Elastic Compliance Function

It is desired to fit the elastic compliance function

$$D_{E}(t) = D_{g} + \sum_{i=1}^{n} D_{i}(1 - e^{-t/\tau_{i}})$$
 (1)

to creep recovery data, and to compare the constants  $D_i$  and  $\tau_i$  for specimens tested at the same stress level, and for specimens tested at different stress levels.<sup>2</sup>  $D_g$  is assumed known. Values of  $D_E(t)$  are available from experiment over a range of t. The method for obtaining the constants in 1 is called Prony's method. It is described by Hildebrand (18).

Let 1 be written in the form

$$f(t) = \sum_{i=1}^{n} D_{i} \mu_{i}^{t}$$
(2)

Then the following transformation equations hold

$$f(t) = D_g + \sum_{i=1}^{n} D_i - D_E(t)$$

$$\mu_i = e^{-1/\tau_i}$$
(3)

Note that the first of 3 is simply the equilibrium elastic compliance minus the experimental data.

<sup>&</sup>lt;sup>2</sup>The numbering system for equations here is independent of that used in the text.

Let f(t) be obtained from the data at N equally spaced points  $t = 0, 1, 2, \dots, N-1$ . Then, for 2 to hold at these values of t, the following set must be satisfied.

This set is solvable for the D's in terms of the  $\mu$ 's by least squares if N > n.

Now, let  $\mu_1, \mu_2, \cdots, \mu_n$  be the roots of

$$\mu^{n} - \alpha_{1}\mu^{n-1} - \alpha_{2}\mu^{n-2} - \cdots - \alpha_{n-1}\mu - \alpha_{n} = 0$$
 (5)

Hildebrand shows that the a's must then satisfy the set

The set 6 is solvable for the  $\alpha$ 's by least squares if N > 2n.

In this study, n was taken to be two. This seemed to yield a fair fit to the experimental data. Several values of N were employed, depending on how well behaved the data appeared. N was usually about 10. Actual times were coded by dividing by a suitable constant so that t took on the successive integral values 0, 1, 2, ..., N-1. The time increment was chosen so that f(N-1) corresponded to the equilibrium elastic compliance. A curve was drawn by eye through the plotted experimental data. Ordinates were taken from this curve, and f(t) computed with the use of 3.

The set 6 was solved for the a's by least squares. The roots of 5 were then determined from the quadratic formula. Here the advantage in taking n no greater than two becomes apparent. For if n = 3, 5 is a cubic equation which is somewhat difficult to solve. As n takes on larger values, 5 becomes increasingly more difficult to solve for the roots. After the  $\mu$ 's were established, the set 4 was solved for the D's by least squares. Figure 17 shows some recovery data, and a plot of the curve established by the method described.

It is expected that the  $\tau_1$  are positive real constants. However, when fitting 2 to the data, in some cases a  $\mu_1$  was real and negative. Then the term  $\mu_1^t$  is real only when t is an integer. Hildebrand suggests that a suitable interpolating function which is real for all values of t is given by

$$|\mu_{i}|^{t} \cos \pi t$$

or

$$e^{t \ln |\mu_i|} \cos \pi t$$

If constants are to be compared statistically within and between stress levels, it is not necessary that the  $\tau_i$  actually

(7)

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Figure 17. Experimental recovery data, with curve fit by Prony's method with n = 2

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be found. It is sufficient to compare the constants  $\mu_i$ . It is necessary, however, that the  $\mu_i$  to be compared all be based on the same units of time. If T represents time in hours, then a scale factor m may be chosen in 2 so that

$$t = mT$$
$$\mu_{i}^{t} = \mu_{i}^{mT}$$
(8)

It was mentioned previously that t must take on integral values, thus fixing the nature of m. Now it may be desirable to express 2 in terms of some other time interval, call it nT. Then it is required that

$$\mu_{i}^{mT} = \zeta_{i}^{nT}$$

$$\zeta_{i} = \mu_{i}^{m/n}$$
(9)

or

Expression 9 makes it possible to find constants  $\zeta_1$  based on a single time interval for purposes of comparison, even if several different scale factors were used in fitting the several curves with 2. This is only possible, however, for the case where the  $P_1$  are all real and positive. If some value of P is real and negative, then the term containing it undergoes a sign change each time t changes by unity. Figure 18 shows the sort of behavior that describes such a term. It is absurd to make any comparison of a real positive P with a real and negative P, because the nature of their contribution to compliance is entirely different.



Figure 18. Behavior of a single normalized term of the elastic compliance function

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