A GENERAL BOUNDARY INTEGRAL EQUATION APPROACH TO EDDY CURRENT CRACK MODELING

J. Chao, N. Nakagawa*, D. Raulerson, and J. Moulder*
United Technologies
Pratt and Whitney
P. O. Box 109600
West Palm Beach, FL 33410

*Center for Nondestructive Evaluation
Iowa State University
Ames, IA 50011

INTRODUCTION

Eddy current techniques have been widely used in the NDE inspection of aircraft engine components. Depending on the flaw characteristics and specimen composition, various EC probe designs have been employed to achieve the maximum probability of detection (POD). Traditionally, the effectiveness of a probe design for a given inspection is determined experimentally. In particular, parameters such as probe types, operating frequency, scan spacing, etc. are evaluated experimentally in terms of POD. It is obvious that this is a costly way of defining inspection parameters. A more cost-effective alternative is to evaluate the test parameters through the use of numerical simulation. This can be done by casting the entire EC inspection process in terms of a numerical model governed by a set of integral equations. By computing the solutions to the integral equations, outputs in the form of impedance changes due to flaws can be used to generate the POD. Previously, we have introduced a modified version of the Hertzian magnetic potential approach for eddy current probe design [1]-[3]. In those papers, it was shown that the formulation can be used to solve problems with arbitrary geometries including geometrical singularities such as edges and corners. In the present paper, we have modified the boundary integral equations (BIEs) formulation for computing the impedance change in the presence of ideal tight cracks. Some unique features of this model include the allowance for arbitrarily shaped air core probes and test components that include singular geometries.

BOUNDARY INTEGRAL EQUATIONS

In electromagnetism, the governing differential equations are known as Maxwell's equations. In the special case of eddy current problems, the quasistatic approximation is often made which results in the wave propagation equations in the external region becoming diffusion equations, Furthermore, if all materials in the problem domain is homogeneous, the differential equations can be transformed into a set of boundary integral equations using Green's identity. Over the years, a number of numerical techniques have been applied to solve the eddy current inspection problem. These techniques include both the finite element method and the volume integral methods [4] - [8]. However, there are many reasons why one would prefer to numerically solve the BIEs as oppose to solve the differential equations (DEs) or the volume integral equations. First, in the case of BIEs, the unknown variables are expressed in terms of equivalent source densities that exist only on the bounding surfaces of the problem geometries. In contrast, the unknowns to the Des are the actual field (potential) variables that exist everywhere in the problem domain. In the case of volume integral approach, the unknown density functions are defined volumetrically. Naturally, the number of unknowns are significantly less in the BIE governed problem than in the other two approaches. Second, since the modeled problem requires simulating the scanning process, no remeshing is needed for the BIE problem. However, remeshing is required for the DE problem as the probe changes location relative to the test component.

GOVERNING EQUATION

The general problem geometry is shown in Figure 1. In this problem, the test specimen can assume any arbitrary shape, including geometrical singularities such as edges. The model utilize a variational form of the Hertzian potential approach by defining the total magnetic fields in the region outside the specimen as

$$\vec{H} = \vec{H}^{(0)} + \nabla\phi \tag{1}$$

with the quasistatic approximation

$$\nabla^2\phi = 0 \tag{2}$$

where

$\vec{H}$ = Total magnetic fields in the air region,
$\vec{H}^{(0)}$ = incident magnetic fields in the absence of the test specimen, and
ϕ = scalar magnetic potential function.

Internal to the conductive specimen, the total magnetic field is defined as

$$\vec{H} = \vec{h} + \nabla\phi \tag{3}$$

where $\vec{h}$ is an auxiliary vector potential function. At the air/specimen interface, the normal component of the magnetic flux density and the tangential components of the magnetic fields are continuous across the boundary.

$$\left(\vec{H} - \nabla\phi\right)_t = \vec{H}_t^{(0)} = \vec{h}_t \tag{4}$$

With the quasistatic approximation, the kernel function in the air region can be assumed to be static, $\overline{G}$, while the kernel function inside the conductive specimen remains dynamic, G. Consequently, collocating on the specimen surface yields four BIEs.

$$\int_{S_H} (-\partial_n G)\phi + (\partial_n G_o)\phi_p + G\left(\mu^{-1}B_n - h_n\right) dS = 0 \tag{5}$$

$$\int_{S_H} \left\{-\partial_n (G - G_o)\right\}\phi + G\left(\mu^{-1}B_n - h_n\right) - G_o\left(\mu_o^{-1}B_n - H_n^{(0)}\right) dS = 0 \tag{6}$$

$$\begin{aligned}\hat{n}_p \cdot \int_{S_H} &\left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} + (\nabla G_o)\left(h_n - H_n^{(0)}\right) \\ &-k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS = 0\end{aligned} \tag{7}$$

$$\hat{n}_p \times \int_{S_H} \left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} - k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS = 0 \tag{8}$$

CRACK FORMULATION

In Modeling an ideal tight crack in nonmagnetic test specimens, one can begin from the problem of a volumetric void existing in the test specimen and make appropriate changes to the governing BIEs as the void collapses to a surface breaking tight crack. Consider the void problem in cross sectional view as illustrated in Figure 2. where a void exists at the surface of the specimen. The total surface, S, of the void comprises of sub-surfaces S_1, S_2, and S_e. The corresponding BIEs for this case is shown in equations (9) through (12).

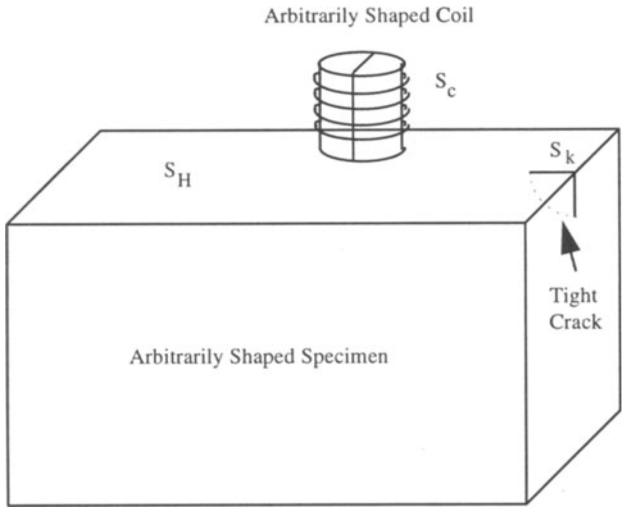

Figure 1. General Eddy Current Problem Configuration.

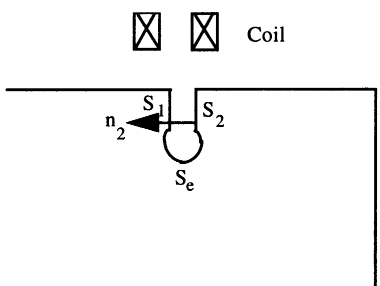

Figure 2. Eddy Current Void Problem.

$$\int_{S_H+S} (-\partial_n G)\phi + (\partial_n G_o)\phi_p + G\left(\mu^{-1}B_n - h_n\right) dS = 0 \tag{9}$$

$$\int_{S_H+S} \left\{-\partial_n (G - G_o)\right\}\phi + G\left(\mu^{-1}B_n - h_n\right) - G_o\left(\mu_o^{-1}B_n - H_n^{(0)}\right) dS = 0 \tag{10}$$

$$\begin{aligned}\hat{n}_p \cdot \int_{S_H+S} &\left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} + (\nabla G_o)\left(h_n - H_n^{(0)}\right) \\ &-k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS = 0\end{aligned} \tag{11}$$

$$\hat{n}_p \times \int_{S_H+S} \left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} - k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS = 0 \tag{12}$$

As the surface S collapses onto a single surface, S_k, to form a crack surface as shown in Figure 3, the normal vectors on the surfaces are related by

$$\hat{n}_1 = -\hat{n}_2 = \hat{n} \tag{13}$$

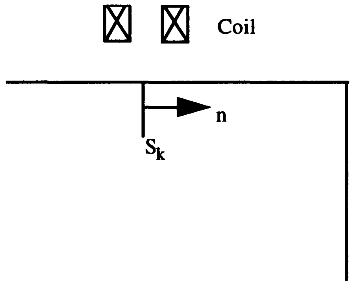

Figure 3. Eddy Current Tight Crack Problem With Collapsing of the Void.

It can be shown that [9] the magnetic fields on S_1 is equal to those on surface S_2 resulting in

$$\phi_1 - \phi_2 = \Delta\phi = 0 \tag{14}$$

$$B_{n1} - B_{n2} = \Delta B_n = 0 \tag{15}$$

$$h_{n1} - h_{n2} = \Delta h_n = 0 \tag{16}$$

$$H_{n1}^{(0)} - H_{n2}^{(0)} = \Delta H_n^{(0)} = 0 \tag{17}$$

and

$$\vec{H}_{t1}^{(0)} - \vec{H}_{t2}^{(0)} = \Delta\vec{H}_t^{(0)} = 0 \tag{18}$$

However, the tangential electric fields on S_1 and S_2 are not equal and results in a discontinuity at the surface. This discontinuity in current density at the surface can be represented by a scalar potential function, φ, resembling a sheet of current dipoles orientated normal to the crack surface. The relation between the tangential electric fields and the scalar dipole function is given in equation (19).

$$\vec{E}_t^1 - \vec{E}_t^2 = \Delta\vec{E}_t = -\tfrac{1}{\sigma}\nabla_t\varphi \tag{19}$$

Therefore, it can be shown that the resulting BIEs are

$$\int_{S_H} (-\partial_n G)\phi + (\partial_n G_o)\phi_p + G\left(\mu^{-1}B_n - h_n\right) dS = 0 \tag{20}$$

$$\int_{S_H} \left\{-\partial_n(G - G_o)\right\}\phi + G\left(\mu^{-1}B_n - h_n\right) - G_o\left(\mu_o^{-1}B_n - H_n^{(0)}\right) dS = 0 \tag{21}$$

$$\begin{aligned} \hat{n}_p \cdot \int_{S_H} &\left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} + (\nabla G_o)\left(h_n - H_n^{(0)}\right) \\ &-k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS + \hat{n}_p \cdot \int_{S_k} G\left(\hat{n}_q \times \nabla\varphi(q)\right) = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} \hat{n}_p \times \int_{S_H} &\left\{\nabla(G - G_o)\right\}\times\left(\hat{n}\times\vec{H}^{(0)}\right) - (\nabla G)h_n + (\nabla G_o)H_n^{(0)} - k^2 G\left(\frac{\hat{n}\times\vec{E}}{j\omega\mu} - \hat{n}_q\phi\right) dS \\ &+\hat{n}_p \times \int_{S_k} G\left(\hat{n}_q \times \nabla\varphi(q)\right) = 0 \end{aligned} \tag{23}$$

$$-j\omega\mu_h \int_{S_H} \hat{n}_p \cdot \left[(\nabla G)\times\left(\tfrac{-\hat{n}\times\bar{E}}{j\omega\mu} + \hat{n}\phi(q)\right)\right] dS - j\omega\mu_h \int_{S_k} \hat{n}_p \cdot \hat{n}_q G\varphi(q)\, dS$$

$$-\frac{1}{\sigma}\int_{S_k} \tfrac{\partial^2 G}{\partial n_p \partial n_q}\varphi(q)\, dS = j\omega\mu_h \int_{S_k} \hat{n}_p \cdot K\vec{H}_t^{(0)}(q)\, dS \tag{24}$$

where

$$K_{ij} = \delta_{ij} G + \tfrac{1}{k^2}\tfrac{\partial^2 (G-\bar{G})}{\partial x_i \partial x_j} \tag{25}$$

and σ = conductivity of the test specimen.

It be noted that a line integral over the mouth of the crack has been absorbed into the surface integral in equation (24) through the sue of Stoke's theorem. A more detailed discussion on this derivation can be found in [9].

SINGULARITY REGULARIZATION

All of the above BIEs contain kernel functions that are singular in nature. The problem associated with singular kernels are in the numerical computations of the BIEs. Typically, special care is needed to numerically evaluate the BIEs to avoid the numerical solution from diverging. The approach taken here is to reduce the singularity order by either adding or subtracting terms associated with the Taylor series expansion of the density functions. Readers are encouraged to refer to [10] for an in-depth discussion on this topic.

SIMULATION RESULTS

The simulated eddy current problem is shown Figure 4. In this test case, a 235 turn air-core solenoid probe was used to scan a surface breaking semi-elliptical crack in a Ti-6-4 flat plate specimen. The frequency at which the scans were taken ranges from 100 Khz to 1 Mhz. The output computed by the BEM model is the change in impedance that the coil sees as it moves from no flaw region to flawed region. The dimensions of the probe and the crack are given in Table 1.

Some of the numerical results obtained from the model are plotted along with measurements taken using an Hewlett Packard impedance as shown in Figures 5 through 7. The impedance change is computed through the application of Auld's reciprocity theorem and is expressed as

$$\Delta Z = -\tfrac{1}{I^2}\int_{S_k} \vec{E}^{(0)}(q)\cdot \hat{n}(q)\varphi(q)\, dS \tag{26}$$

where $\vec{E}^{(0)}$ is the incident electric field at the crack location in the absence of the crack.

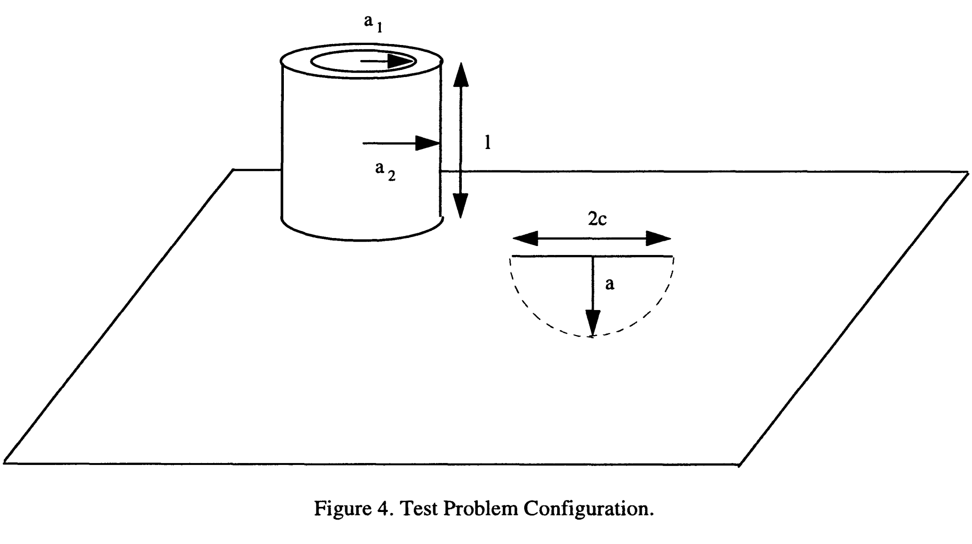

Figure 4. Test Problem Configuration.

Table 1. Problem Dimensions.

a_1	0.535 mm
a_2	1.31 mm
l	2.93 mm
2c	1.01 mm
2c/a	3
lift-off	0.72 mm
ρ	1.63×10^{-6} Ω-m

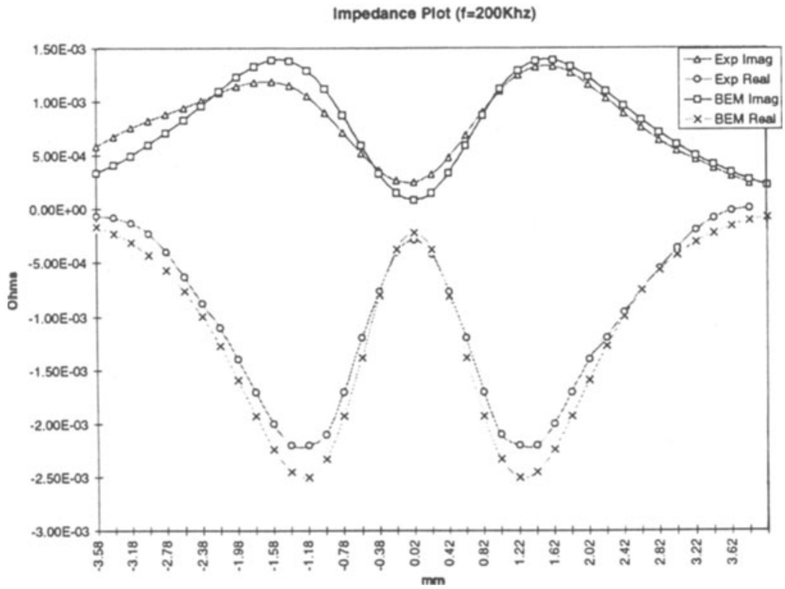

Figure 5. f=200 Khz.

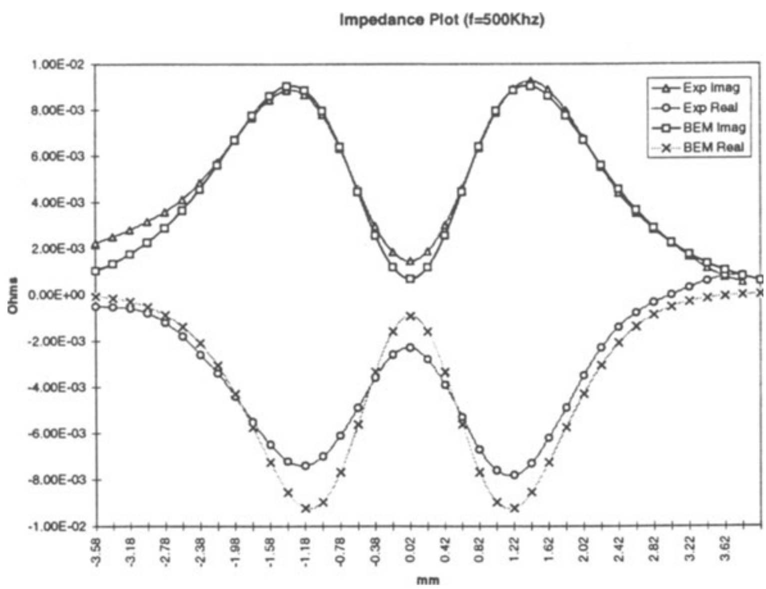

Figure 6. f=500 Khz.

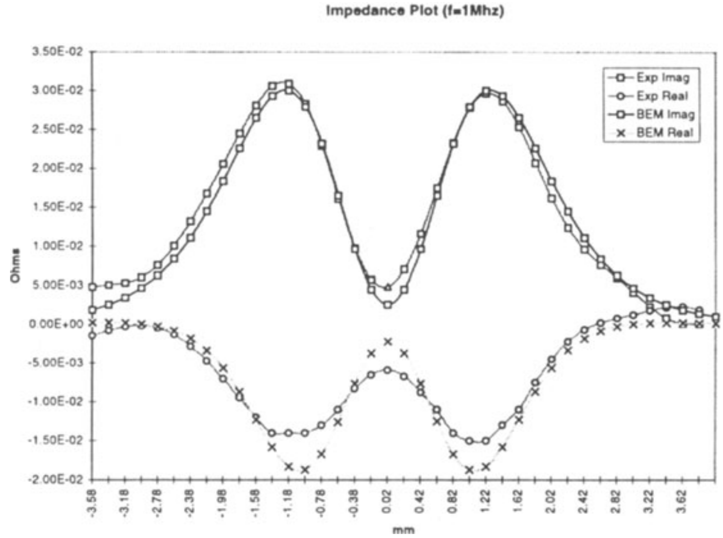

Figure 7. f=1 Mhz.

CONCLUSION

A boundary integral equation based formulation has been developed for solving general eddy current inspection problems in the presence of tight cracks and arbitrarily shaped test specimens. At present, the formulation is valid for non-magnetic test specimens. However, some modifications to the crack interface conditions can be made to include the ferromagnetic case, As shown in the test simulations, the impedance outputs predicted by the model are in excellent agreement with experimental measurements. Some minor discrepancies seen in the comparisons can be attributed to uncertainties in the exact dimensions of the probe, specimen, and flaw. For future development, the model capability will be expanded to include general shaped ferrite core probes and ferromagnetic test specimens.

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