DIFFUSIVITY MEASUREMENT USING QUASI PERIODIC TEMPERATURE RESPONSE MODELING

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INTRODUCTION

Periodic heat flow methods for measurement of the diffusivity of a material have proven to be highly effective methods dating back to the work of Angstrom. Typically, these measurements have involved a determination of the phase lag between a source applied to the surface and the temperature measured at the opposite surface. When the frequency of the periodic heating is well matched to specimen, it represents perhaps the most efficient thermal diffusivity measurement technique.

For accurate phase measurement, frequency components from the transients from the start of the heat application and long term variations such as sample heat up must be filtered out. For a point detector, this is simply done by passing the output of the detector through a band pass filter. For a scanned detector, where the desire is to measure the spacial variations in the diffusivity, it is difficult to pass the output of the scanned sensor through an analog band pass filter. One approach is to digitize a long time record and post process the data to remove relevant frequency components. This is undesirable because it requires a large amount of real time data storage, which is often expensive. A second technique is to allow the specimen to come to thermal equilibrium with the environment. This is practical for thin samples with high diffusivities where the frequency is high. However, for many structures, the frequencies required for heat to be able to penetrate the structure is on the order of 1 or less Hertz. The time required for the structure to come to equilibrium may be on the order of several minutes.

To remove some of the difficulties, the thermal diffusivity can be determined from analytical solution which incorporates the transient terms. This enables an effective use of the memory required for storage of the images while still taking advantage of the periodic heating. It also allows the measurements to be performed in a shorter time period.

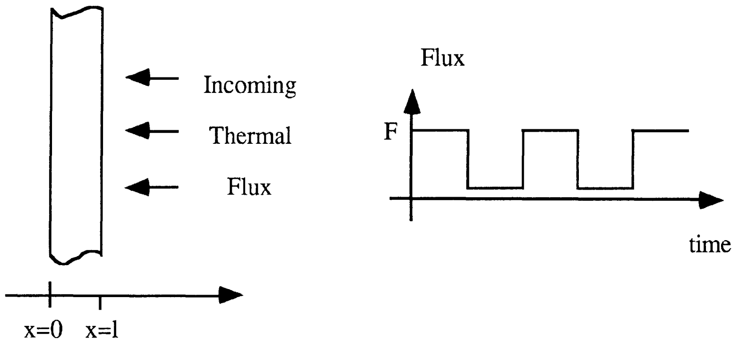

Fig. 1 Slab with incoming square pulse heat flux at the back surface

The analytical solution is used in a non linear least square routine to estimate the diffusivity of several samples divided by their thickness squared. The technique is applied to samples with varying thicknesses and diffusivity. A comparison of the results from these measurements and other measurements is presented.

THERMAL RESPONSE OF THIN PLATES

The thermal response of the slab is modeled by assuming that it is a single homogeneous layer undergoing a square pulse irradiation on its back face (Fig. 1). Convection losses are neglected. For a flux input that is evenly applied over the surface of the sample and a thin plate structure where the lateral variations are small relative to its thickness, a one dimensional solution is appropriate.

As described in Fig 1, the front surface is at x=0 and the back end at x =l.
The 1-dimensional heat equation and the boundary conditions can be written down as :

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial\theta}{\partial t} \quad \text{for } 0 \le x \le l \ , \qquad [1]$$

where θ is the temperature, α the thermal diffusivity , t the elapsed time after the beginning of the experiment and x the horizontal coordinate.

$$\text{at } x=l, \quad k\frac{\partial\theta}{\partial x} = F \quad \text{for } 2n\tau<t<(2n+1)\tau \qquad [2]$$
$$= 0 \quad \text{otherwise.}$$

where k is the thermal conductivity, F the amplitude of the incoming flux, τ the length of the heat pulse and n the number of pulses.

$$\text{at } x=0, \quad k\frac{\partial\theta}{\partial x} = 0 \qquad [3]$$

at t=0, $\theta(x,0) = 0$ (the initial temperature is set at zero).

A periodic square pulse signal can be represented as a sum of step functions shifted in time and changed in sign. The solution is found by superposing the solutions to the simpler problems with the incoming heat flux profile described as a step function (Ref. 1). The solutions are :

$$\theta_{\text{step function}}(t) = 0 \quad \text{for } t \leq 0 , \qquad [4]$$

$$\theta_{\text{step function}}(t) = \frac{2F\sqrt{\alpha t}}{k} \sum_{n=0}^{\infty} 2 \,\text{ierfc}\left(\frac{(2n+1)\,l}{2\sqrt{\alpha t}}\right) \quad \text{for } 0 \leq t \leq \frac{3.5\, l^2}{\alpha \pi^2} , \qquad [5]$$

$$\theta_{\text{step function}}(t) = \frac{Ft}{\rho c l} + \frac{Fl}{k}\left(-\frac{1}{6} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\, e^{\left(\frac{-\alpha t n^2 \pi^2}{l^2}\right)}\right) \quad \text{for } t \geq \frac{3.5\, l^2}{\alpha \pi^2} , \qquad [6]$$

where ρ and c are the density and specific heat of the material.

$$\text{Thus,} \quad \theta(t) = \sum_{n=0}^{2N-1} (-1)^n \,\theta_{\text{step function}}(t - n\tau) \qquad [7]$$

where N is the number of cycles, is the solution to the original problem.

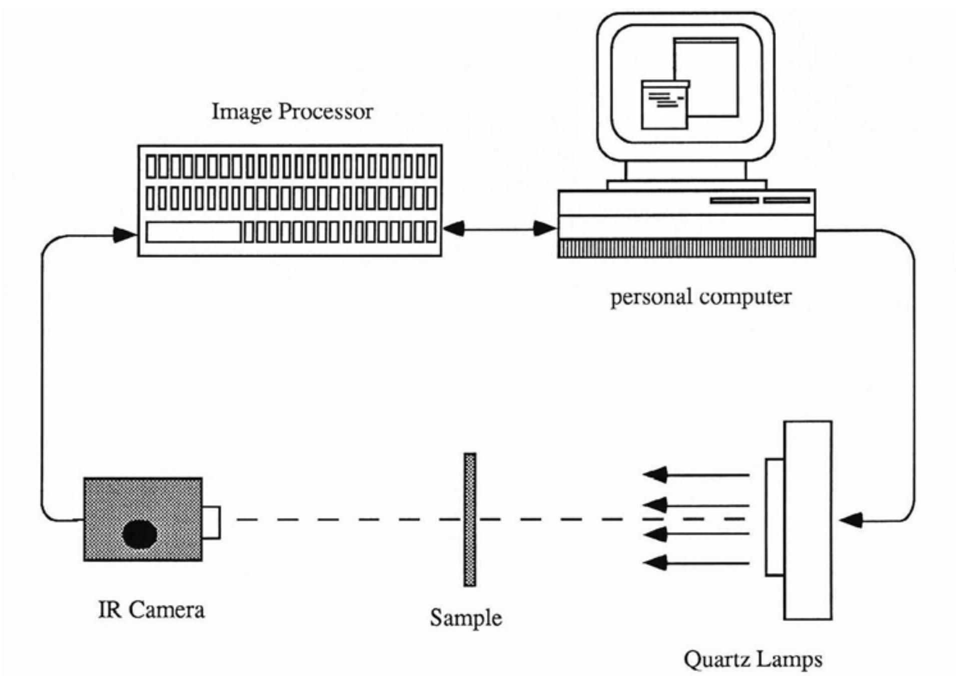

Fig. 2 Experimental setup for data acquisition.

As can be seen from equations [5] and [6], the diffusivity is most directly obtainable from the thermal response, rather, the thermal response depends on α/l^2. If two parameters are varied, such as α/l^2 and $F/\rho cl$, the thermal response as represented in equation [7] can be varied to minimize the difference between $\theta(t)$ and the measured thermal response. The criterion for minimization chosen is the minimum of the square difference between $\theta(t)$ and the measured response. This gives an estimate for the parameter α/l^2. If α is known, l can then be calculated. If l is known, it is possible to find α.

THERMAL DIFFUSIVITY MEASUREMENT SYSTEM

A block diagram of the experiment setup is shown in Fig. 2. A commercial infrared camera radiometrically measured the change in surface temperature as a function of time. The output of the infrared camera was digitized by an image processor. The image processor averaged the digitized images and stored the data in memory. The thermal stimuli was provided by 4 quartz tubes. The application of heat was synchronized with the digitization of the infrared image by a computer controller. The computer controlled the data acquisition by keeping a given phase of the period heating averaging into a fixed portion of memory. This minimized the amount of memory required, while maintaining a high temporal resolution.

RESULTS

To demonstrate the technique, measurements were performed on an aluminium sample. The sample geometry is shown in Fig. 3 . Three square holes were milled out at the back surface of the plate to vary the thickness as a function of position. They are, from left to right, 5.715 mm, 4.445 mm and 3.175 mm deep. A compilation of the measurements at different frequencies is given in table 1. The frequency corresponds to $1/2\tau$. The measurements represent the diffusivity as measured for the regions where no holes were present. The average value of diffusivity computed from the experiments fits the literature values for aluminium within a 7 % accuracy.

Table 1. Measurements for different frequencies. The average measured value for diffusivity is 90.5 10^{-6} m^2/s. Literature value for Aluminium averages 97.5 10^{-6} m^2/s.

Frequency (Hz)	Diffusivity ($\frac{m^2}{s}$)
1.07	97.0 10^{-6}
1.60	96.1 10^{-6}
2.13	78.5 10^{-6}

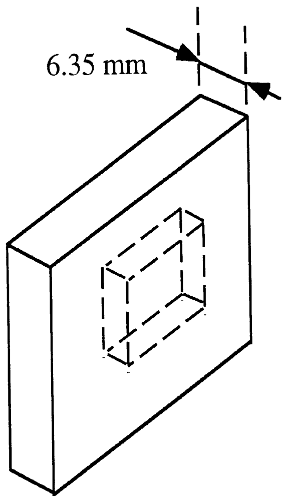

Fig. 3 Geometry of a portion of the Aluminium sample.

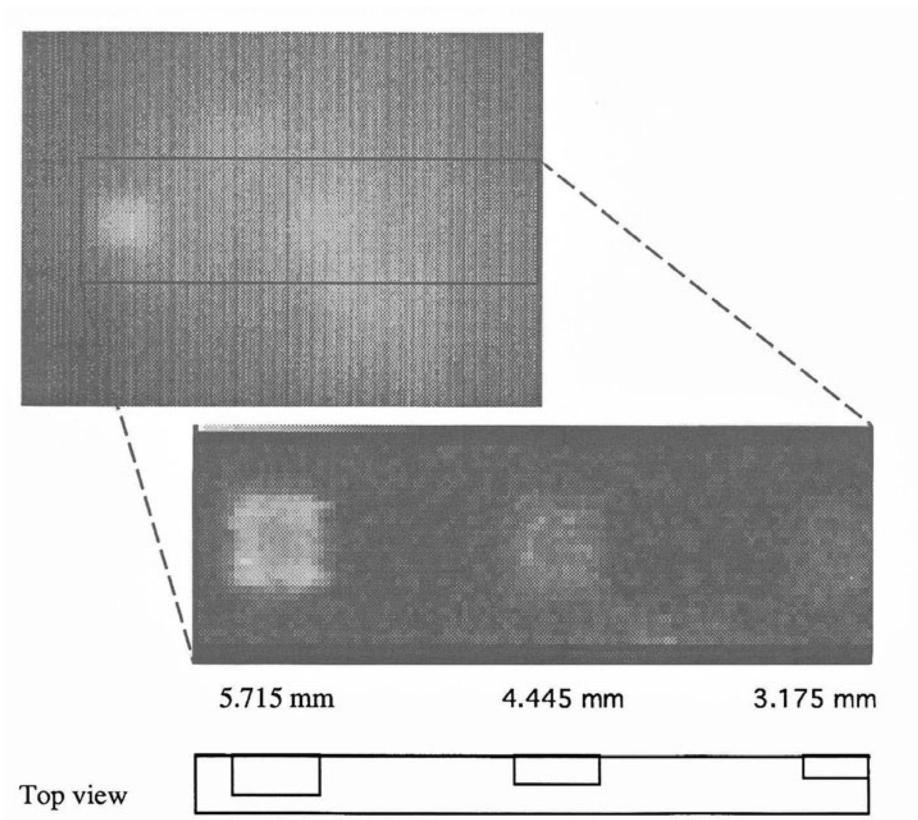

Fig. 4 Diffusivity image obtained from the thermograms of the Aluminium sample.

From the data, an image, shown in Fig. 4, was obtained for α/l^2 for the sample. Since, for this case, the diffusivity of the sample is constant , the image corresponds to $1/l^2$. For comparison, the image directly obtained from the IR imager is also shown in the figure. As can be seen, the processed image enables detection of even the shallowest hole which is not visible in the infrared image.

Measurements were also made on graphite epoxy composite samples with varying fiber volume fraction. The samples were a set of 6 simple panels (16 plies : [0/90] 4s) with fiber volume fraction 40 %, 50 %, 60 %, 65 %, 70 % and 74 %. The results obtained using the quasiperiodic method are shown in Fig. 5. Also shown are the measurements made using Angstrom's method (Ref. 2) . Both curves indicate that the diffusivity increases with the fiber volume fraction. The difference in absolute value is assumed to be due to no convection loss correction in either measurements. The high value of porosity in the samples where the fiber volume fraction is above 65 % is an explanation for the plateau. Fig. 6 shows the diffusivity images obtained from the samples. The increase in spacial variation in the diffusivity images for a fiber volume fraction of 65 % and above may correspond to the high porosity.

Finally, diffusivity measurements using the quasi periodic method were made on a Carbon-Carbon composite panel with four stiffeners at the back surface. Fig. 7 shows the geometry of the stiffeners and indicates the thicknesses.

As in Fig. 4 , the image in Fig.8 shows the variations in thicknesses of the Carbon-Carbon panel. The background grey level corresponds to a measured average diffusivity value of 0.119 cm^2/s. This demonstrates the ability of the technique to characterize complex geometries.

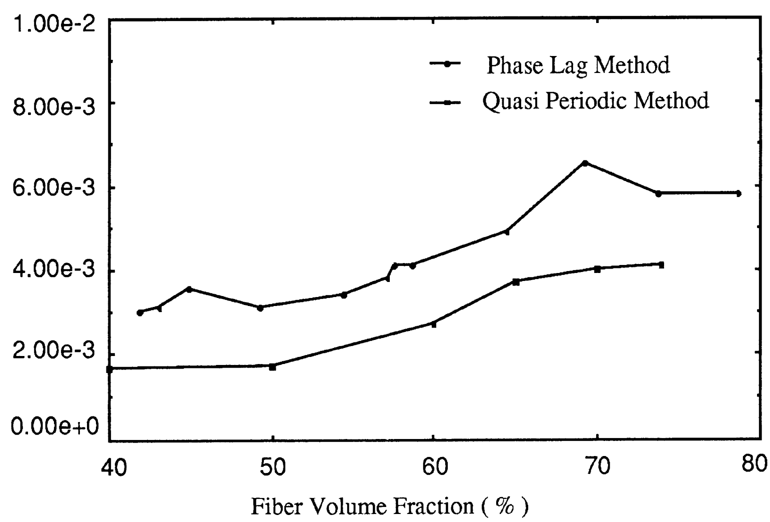

Fig. 5 Diffusivity measurements on Graphite Epoxy Composite samples as a function of the fiber volume fraction. Top curve corresponds to Angstrom method, bottom curve to Quasi periodic method. The values of diffusivity are given in cm^2/s

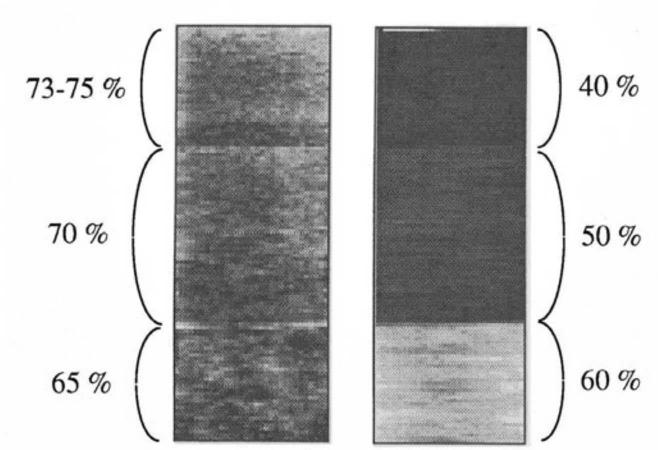

Fig.6 Diffusivity images of Graphite Epoxy Composite samples. The grey scale look up tables used in the two images are linear but different. Due to the small differences in diffusivity values for the last three samples (65 %, 70 %, 74 % in fiber volume fraction) the contrast has been linearly stretched for a better visualization.

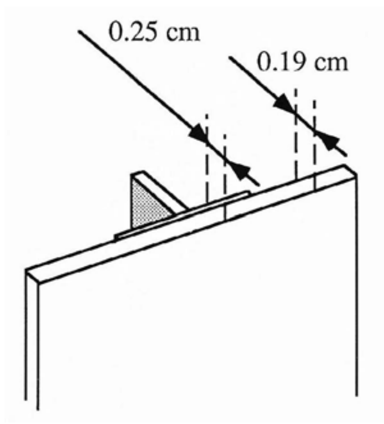

Fig.7 Detail of a stiffener on the Carbon-Carbon panel.

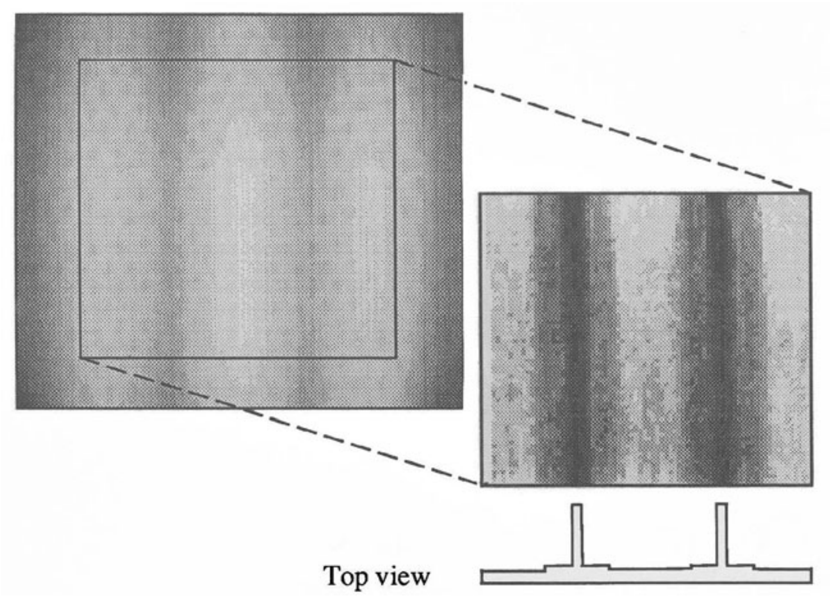

Fig. 8 Thermogram and diffusivity image of the Carbon-Carbon composite panel.

CONCLUSIONS

A model for through-the-thickness diffusivity imaging was applied to Aluminium and composite materials. This model, by taking into account the transient terms in the temperature response, allows quick measurements. The results obtained using the quasi periodic method are found in good agreement with either the literature values or with the diffusivity values obtained by other experimental methods.

REFERENCES

1. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed. , edited by Oxford Science Publications, p. 112 .
2. *Quantitative Thermal Diffusivity Measurements on Composite Fiber Volume Fraction Samples*,by Joe Zalameda and William P. Winfree, NASA Langley Research Center, MS 231, Hampton, VA 23665, presented at QNDE 92.