Statistical method and simulation on
detecting cracks in vibrothermography inspection

by

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DEDICATION

I would like to dedicate this thesis to my wife Zheng and to my son William without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance and assistance during the writing of this work.
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Vibrothermography is a nondestructive evaluation method that can be used to detect cracks in specimens and it is the main engineering technique we focused in this thesis. This study can be separated into three parts. In the first part, we develop a systematic statistical method to provide a detection algorithm to automatically analyze the data generated in Sonic IR inspections. Principal components analysis (PCA) was used for dimension reduction. Robust regression and cluster analysis are used to find the maximum studentized residual (MSD) for crack detection. The procedure proved to be both more efficient and more accurate than human inspection. A simulation tool was developed in the second part of the study by simulating background noise and the crack signal. The new simulated sonic IR movie data sets can be used to evaluate existing detection algorithm and testing and developing new algorithms. The last part of the study analyze the data from sonic IR inspections on turbine blades. Separate but similar analysis were done for two different purposes. In the first analysis, the purpose of the study was to find Sonic IR equipment settings that will provide good crack detection capability over the population of similar cracks in the particular kind of jet engine turbine blades that were inspected. In our second analysis, crack size information was added and a similar model in the first analysis was fit. Both models are random mixed effect models and are used to estimate probability of detection (POD) in certain conditions. The relationship between the POD and the crack size are calculated based on the second analysis and the confidence interval on POD estimate are studied using bootstrap.
CHAPTER 1. Introduction

1.1 Nondestructive Evaluation

Nondestructive evaluation (NDE) methods are used to characterize the state or properties of units or materials without causing any permanent physical change to the units, as described, for example, in Shull (2002), Hellier (2001) and Mix (2005). There are emerging new NDE applications in many areas in addition to the traditional applications in the aerospace and defense industries, particularly in the areas of manufacturing technology and system reliability improvement. Indeed, NDE is an important engineering research area with its own professional societies and numerous annual conferences, journals, as well as several research centers in universities and industrial laboratories, and university degree programs.

1.1.1 Vibrothermography in Nondestructive Evaluation

Vibrothermography, also known as Sonic Infrared (IR), thermoacoustics and thermosonics, is described, for example, in Henneke and Jones (1979), Reifsnider et al. (1980) and recent work by Holland (2007). It is a technique used for detecting cracks in industrial, dental, and aerospace applications. A pulse of sonic or ultrasonic energy is applied to a unit to make the unit vibrate. If a crack exists in the unit, it is expected that the faces of the crack will rub against each other, resulting in a temperature increase near the crack. An infrared camera is used to measure the temperature change in a sequence of frames over time, which we refer to as a movie. In this paper, we present methods for studying and analyzing the movie data for purposes of crack detection.
1.2 Summary of the Three Studies on Vibrothermography Testing Data

This thesis combines three statistical research studies on vibrothermography testing data. In the first study, we develop a statistical procedure to provide automatic detection of crack in vibrothermography inspections. Principal components analysis (PCA) was used for dimension reduction. The coefficients of the first principle components fitted by a linear model were then processed by a robust regression model to produce the maximum studentized residual (MSD) that are used in the crack detection rules. The procedure proved to be both efficient and accurate. It correctly identified several cracks in the movies which were not detected by human inspection. The second study developed a general simulation procedure to generate vibrothermography movie data. We show how to use the simulated data to study the properties of a vibrothermography based algorithm for detecting cracks and change of detection procedure according to possible new data. The third study describes the analysis of data from vibrothermography inspections on turbine blades. Separate but similar analysis were done for two different purposes. In both analysis, we fit statistical models with random effects to describe the crack-to-crack variability and the effect that the experimental variables have on the responses. In the first analysis, the purpose of the study was to find vibrothermography equipment settings that will provide good crack detection capability over the population of similar cracks in the particular kind of jet engine turbine blades that were inspected. Then, the fitted model was used to determine the test conditions where the probability of detection is expected to be high and probability of alarm is expected to be low. In our second analysis, crack size information was added and a similar model was fit. This model provides an estimate of POD as a function of crack size for specified test conditions.
CHAPTER 2. A Statistical Method for Crack Detection from Vibrothermography Inspection Data

ABSTRACT

Nondestructive evaluation methods are widely used in quality control processes for critical components in systems such as aircraft engines and nuclear power plants. These same methods are also used in periodic field inspection to assure that the system continues to be reliable. This paper describes a detection algorithm to automatically analyze vibrothermography sequence-of-image inspection data used to detect cracks in jet engine fan blades. Principal components analysis is used for dimension reduction. Then the fitted coefficients of the first principal component are processed by using robust regression to produce studentized residuals that are used in the crack detection rules. We also show how to quantify the probability of detection for the algorithm. The detection algorithm is both computationally efficient and accurate. It correctly identified several cracks in our experimental data that were not detected by the standard human visual inspection method.

Key Words: POD, Sonic IR, Thermosonics
2.1 Purpose of the Study

In this paper, we develop a systematic method to automatically analyze the data generated in vibrothermography inspections. The goal is to have a screening algorithm that can detect the possible presence of cracks in a movie, so that experienced evaluators only need to evaluate movies for which identification is uncertain.

2.2 Experimental Data

The vibrothermography data provided to us consisted of a movie for each of 70 specimens (60 with cracks and 10 blanks with no crack). For each movie, we were also given information on the size of the crack and whether an inspector had been able to detect the crack or not from watching an enhanced version of the movie (background of an early image subtracted out).

2.2.1 Crack Signal Signature

There were either 31 or 55 frames, ordered in time, in each of the movies that we received. Each frame contains 256 by 256 pixels. Each pixel is represented by a 12-bit integer. Figure 2.1 shows four frames (the 4th, 10th, 20th, and 30th) in Movie 02. Movie 02 has a strong signal near its center. The 4th frame shows no sign of a signal. The signal is clearly visible by the 10th frame and the signal size (the dot size) continues to grow until the last frame in the movie.

Figure 2.2 shows the temporal behavior of a grid of $3 \times 3 = 9$ pixels approximately centered on a crack signal. The paths with stronger signals are closer to the center of the square and those with weaker signals are near the corners (maximum distance from the center). The thick dark line on the top is the signal strength at the center of the $3 \times 3$ matrix. The legend in the north-west corner gives idea of which line corresponds to which point in the movie. For example, the “+” is the south-east center of 9 pixels.

Figure 2.3 illustrates the intensity change in the $Y$ direction at Frame 30, which is near the end of the energy input. A very similar shape appears in the $X$ direction. The plots show that the crack signal has a width of around 14 pixels (which is the windows size used in the
Figure 2.1 Frames (4th, 10th, 20th and 30th) from Movie 02, which has a strong crack signal around the center. The four frames show the strength of the crack signal grows with time.
Figure 2.2 Intensity of pixels as a function of time (frame number) in a grid of $3 \times 3 = 9$ pixels approximately centered on a crack signal in Movie 02. Different symbols and line types represent different locations around the crack signal, which illustrated in the upper-left corner of the plot.
scan calculation), and that the increase is almost linear from the edge of the crack signal to the center of the crack signal.

Figure 2.3 The intensity of pixels in Movie 02 around the crack signal center in Y direction of frame 30. A similar pattern exists in the X direction.

2.2.2 Noise Behavior

Figures 2.4 and 2.5 illustrate the spatial and temporal characteristics of noise in the movies and the effectiveness of subtracting out the background. Figure 2.4 shows the average intensity (averaging over all 256 × 256 pixels) versus frame number (the first row of the plots), average intensity (averaging over all frames) versus the x coordinate (the second row of the plots) and the average intensity versus the y coordinate (the third row of the plots) of Movie 08, a movie for a specimen that does not have a crack. The three plots on the left hand side reflect the raw data. The three plots on the right hand side reflect the data after subtracting out the background (taken to be frame 3). As a contrast, Figure 2.5 contains similar plots for Movie 02, which has a strong crack signal.
Figure 2.4  Average intensity by frame (first row), $x$ coordinate (second row) and $y$ coordinate (third row) of Movie 08 (does not have a crack). The left column shows plots for the original data and the right column shows plots from data after subtracting the third frame. In the two plots on the third row, the symbol “O” represents odd $y$ values and “+” represents even $y$ values to show the different trend of average intensity for odd and even $y$. 
Figure 2.5  Average intensity by frame (first row), $x$ coordinate (second row) and $y$ coordinate (third row) of Movie 02 with a crack (has a crack near the center). The left column shows plots for the original data and the right column is the same data, after subtracting the third frame, in the two plots of the third row, the symbol “O” represents odd $y$ values and “+” represents even $y$ values to show the different trend of average intensity for odd and even $y$. 
2.2.2.1 Temporal Structure of Noise

Trends and/or periodicity are clearly shown in the Figures 2.4 and 2.5. Similar trends and periodicities appear in the other movies. The temporal trends seen, for example, in Figure 2.4 appear throughout the image and are not driven by a crack signal and thus are a type of noise. The exact causes of these trends and periodicities are unknown. These noises could hide the temporal signature of a real crack.

2.2.2.2 Spatial Structure of Noise

A U-shaped pattern appears in both the intensity versus $x$ and intensity versus $y$ plots as shown in the middle and bottom left side of both Figures 2.4 and 2.5. In the intensity versus $y$ plot, there is a valley around $x = 100$ in all of the movies. This behavior can be easily identified by visually observing movies. A slightly brighter band appears around $x = 100$ for all four frames in Figure 2.1. A similar pattern appears in all of the 70 movies. Another interesting feature of the intensity appears in the third rows of Figures 2.4 and 2.5. These figures show that the odd number $y$ pixels (represented by the symbol “o”) are clearly different from the even number $y$ pixels (represented by the symbol “+”), especially for $y$ is greater than 200. This feature also appears in the other movies. As shown in the plots on the right-hand side of Figures 2.4 and 2.5, the underlying trends can be removed by subtracting out an early, representative frame. We use frame 3 for this purpose (The energy was not applied until after frame 3 was acquired).

Compared with Figure 2.4, Figure 2.5 shows similar patterns: periodicity and a U-shaped trend. The peaks that appear in the plots in Figure 2.5 in the second and third rows after subtracting frame 3 (around $x = 118, y = 121$, respectively) are caused by the crack signal.

2.2.2.3 Removing the Spatial Structure of Noise by subtracting the Frame 3

The three plots in the right column of Figures 2.4 and 2.5 are the mean of intensity versus frame number $t$, versus position $x$ and versus position $y$, respectively, after subtraction of the background. Figure 2.6 shows four frames from Movie 02 after subtraction of the background.
The brighter band around $x = 100$ disappeared and the image became more homogeneous when compared with the original data in Figure 2.1.

![Fig 2.6 Frames](image)

Figure 2.6  Frames (4, 10, 20, 30) from Movie 02 after subtracting out the 3rd frame

### 2.2.2.4 Image Shifting

Image shifting happens in some of the movies. In such movies, the structure of the frame shifts in a certain manner slowly though the whole movie. Several frames from Movie 10 are shown in Figure 2.7. The cloud of noisy pixels around $(x = 220, y = 180)$ is shifting in the southwest direction by two to three pixels from beginning to the end of the movie.

### 2.3 Signal Model

#### 2.3.1 Local Signal Model

If we consider that the crack signal appears within a small region around the crack, a polynomial response surface model with up to the square terms of $r$ and $t$ such as
Figure 2.7 Frames (4, 10, 20, 30) from Movie 10. The cloud around $(x = 220, y = 180)$ shifting toward the southwest direction.
\[ S = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 r + \beta_4 r^2 + \beta_5 tr + \beta_6 tr^2 + \beta_7 t^2 r + \varepsilon. \]  

(2.1)

can be used to describe the crack signature. Here

- \( r \) denotes the distance from the center of the crack to the pixel:

\[ r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \]

- \( t \) denotes the frame number. Because the frames in the movie are collected at a constant rate, \( t \) will be proportional to the time that the frame was acquired.

- The \( \beta \)'s are the regression coefficients that will be estimated from the data.

- \( \varepsilon \) is the residual deviation for the model.

In searching for a crack signal, the model (2.1) is fitted on windows within the movie. The windows are circular and moved systematically across the image. By systematically moving the center point around the movie, all of the pixels in a movie will be covered at least once. The first three frames of the movie data are discarded because we subtract off the third frame as discussed in Section 2.2.2. The circle that is moved around has 149 pixels in each frame. The \( X \) matrix of \( n = 149 \times 28 = 4172 \) rows has the form of

\[
\begin{pmatrix}
1 & r_1 & t_1 & r_1^2 & t_1^2 & r_1 t_1 & r_1^2 t_1 \\
1 & r_2 & t_2 & r_2^2 & t_2^2 & r_2 t_2 & r_2^2 t_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & r_n & t_n & r_n^2 & t_n^2 & r_n t_n & r_n^2 t_n \\
\end{pmatrix}
\]  

(2.2)

The corresponding \( S \) vector has 4172 values. The \( S \) vector and \( X \) matrix are fitted into regression model (2.1). There are total \((256 - 2 \times 7)^2 = 58564\) regression models for each movie. Significant differences are expected to exist in the estimates of the \( \beta \)'s between a region with a crack signal and a region with only background noise. The estimates of the \( \beta \)'s could be used to decide whether a crack signal exists or not. In model (2.1), a total 8 \( \beta \) estimates would
be calculated for each spatial window location. It is, however, difficult to set a detection rule for such a large number of dimensions. A criterion having one or two dimensions is desired for the detection rule. We use principal components as a data reduction method for this purpose.

2.3.2 Dimension Reduction

Principal components analysis (PCA) is used as a data reduction method. PCA, is described, for example, by Hastie et al. (2009). PCA has found applications in fields such as face recognition and image compression, and is a common technique for finding patterns in high dimension data. It is concerned with explaining the variance-covariance structure of the data through a few linear combinations of the original explanatory variables. Its general objectives are data summarization through dimension reduction and interpretation.

2.3.3 Principal Components Regression Model

The principal components regression model based on equation (2.1) is:

\[ S = \theta_0 + \theta_1 PC_1 + \theta_2 PC_2 + \theta_3 PC_3 + \theta_4 PC_4 + \theta_5 PC_5 + \theta_6 PC_6 + \theta_7 PC_7 + \varepsilon. \] (2.3)

The variables PC\(_i\)\((i = 1, \ldots, 7)\) denote the principal components calculated from the \(X\) matrix (2.2) of variables used in the regression of model (2.1). The new PC \(X\) is in the form of

\[
\begin{pmatrix}
1 & PC_{11} & PC_{21} & PC_{31} & PC_{41} & PC_{51} & PC_{61} & PC_{71} \\
1 & PC_{12} & PC_{22} & PC_{32} & PC_{42} & PC_{52} & PC_{62} & PC_{72} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & PC_{1n} & PC_{2n} & PC_{3n} & PC_{4n} & PC_{5n} & PC_{6n} & PC_{7n}
\end{pmatrix}
\] (2.4)

where, again, \(n = 149 \times 28 = 4172\). Again, we use the similar model fitting procedure in Section 2.3 with a moving circle window. The moving circle has 149 pixels in each of the 28 frames (the first three frames are omitted).

For each circle position, the coefficients \(\theta_i(i = 1, \ldots 7)\) are estimated using ordinary least squares. The R function “prcomp” was used to calculate the principal components. The calculation is done by a singular value decomposition of the data matrix, rather than by using
the eigenvalues of the covariance matrix. This is generally the preferred method for numerical accuracy. The estimates of $\theta_i$ are used in the following analysis to define detection rules.

### 2.3.4 Principal Components Regression Results

Figure 2.8 The estimated coefficients ($\theta_{ij}; i = 1, n_x; j = 1, n_y$) for the first principal component (PC1) of Movie 02 (viewed from three different directions). Movie 02 has a strong crack signal near the center.

Figure 2.8 shows the coefficient estimates of the first principal components for Movie 02. The three plots are three different views of the same result from different angles. The plot in the north-west corner is looking from the top. The darker the color, the larger the value of the coefficients. The plots in the south-west and north-east are looking from the $X$ and $Y$ directions, respectively. The three plots in the figure indicate a positive peak around ($x = 118, y = 121$). Other movies with a crack signal show similar patterns. This indicates the coefficient for the first principal is a good indicator of crack signals. The second principal component provided no additional information for crack detection.
Figure 2.9 The coefficient estimates for the first principal component (PC1) of Movie 15, viewed from three different directions. The plot in the north-west corner is looking from the top. The darker the color, the larger the value of the coefficients. The plots in the south-west and north-east are looking from the $X$ and $Y$ directions, respectively. Movie 15 has a weak crack signal near the center of the frame square.
Figure 2.10  The estimated coefficients for the first principal component (PC1) of Movie 08 (viewed from three different directions). The specimen used to make Movie 08 does not have a crack.
Figure 2.9 shows similar plots for Movie 15. Movie 15 has a weak crack signal around the center. Comparing with the strong signals in Figure 2.8, the peak has much less contrast with background noise. Figure 2.10 shows similar plots for Movie 08, which does not have a crack. No peaks stand out in the three plots.

2.4 Comparison of Movies With and Without Cracks

2.4.1 Robust Fitting and Simple Studentized Residuals for PC1

The figures and descriptions in the Section 2.3 already show that the coefficient for the first principal component is a good candidate from which a decision rule can be made. However, large differences among similar plots for movies containing a signal make it hard to find a consistent detection rule using the row $\theta_1$. In order to make the characteristic of a crack signal easier to identify, a robust fitting is performed on the $\theta_1$ estimates and the corresponding studentized residuals are calculated as described in Huber (2009) and Maronna et al. (2006).

The reasons for using the robust regression are as follows:

- Under the usual regression model assumptions with contamination (in our case from a signal), the studentized residuals coming out of a robust regression follow approximately a standard normal distribution with 0 mean and standard deviation 1. This commonal-ity will make the results from different movies comparable. Groups of larger residuals indicate possible crack signals.

- We choose robust regression instead of ordinary least squares (OLS) because we are trying to identify outliers. The outliers will have a large impact on the result of an OLS regression and tend to hide themselves by dragging the response surface toward them. A robust regression will put less weight on outliers and that will make the outliers stand out, which is the result we are looking for.

The residuals are used to find outliers and outliers tend to indicate the existence of a crack signal. The robust regression model is:

$$\theta_1 = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 x^2 + \alpha_4 y^2 + \alpha_5 x^3 + \alpha_6 y^3 + \alpha_7 xy + \alpha_8 xy^2 + \alpha_9 x^2 y + \varepsilon. \quad (2.5)$$
Here $\theta_1$ is the coefficient of the first principal components in model (2.3) and $x$ and $y$ are the coordinates. The estimates of the coefficients of the first principal component includes $(256 - 2 \times 7)^2 = 58564$ rows. Each row has values of $x_k$, $y_k$ and $\theta_{1k}$. The $\theta_{1k}$ denotes the coefficient of first principal component of $k$th circle centered at $x_k$ and $y_k$. These 56584 rows of data are used to fit model (2.5). The studentized residuals that result from the model fitting are used for crack detection.

Figure 2.11 shows the calculation result for Movie 02 (a strong signal movie). The plots in the north-west, north-east, and south-west corners show the studentized residuals from three different angles, from the top, from the $x$ direction, and from the $y$ direction, respectively. The plot in the south-east corner shows the hills with the maximum studentized residual value MSR. A clear peak exists in the plots and the peak MSR value is 46.26. Figure 2.12 shows a similar result from Movie 15 (a weak crack signal movie) and which results in an MSR value of 6.98 (which is much smaller than the MSR value in Figure 2.11). Figure 2.13 shows the similar results for Movie 08 (no crack movie). No clear peak can be seen in the plot and all of the data in the image can be considered to be noise.

### 2.5 Flaw Detection Procedure Based on Cluster Analysis

MSR results from previous sections shows that a group of large MSR value tends to form (we call it hill) around the center of a crack and a hill will be a good indicator of cracks. However, cracks are not the only sources of hills. A shifting image’s MSR values will form a hill in its path way as shown in Figure 2.14. Figure 2.14 shows the result for Movie 09, which contains a weak crack signal near the center and a shifting dot at the northeast corner. The shifting dot caused the calculated MSR values formed a large hill in its pathway along with a valley (a group of extremely negative MSR values) in vicinity. One difficulty in finding the crack signals is the need to distinguish a crack signal from a shifting dot, both of which have a positive hill in the MSR plot. Real crack signals in the data only generate a positive hill. Thus a negative valley is a sign that the positive hill nearby may have been generated by a shifting dot instead of a crack signal. The task of finding a crack translates into finding a hill
Figure 2.11 Robust studentized residuals for the coefficients of PC1 in Movie 02. The plot in the north-west corner is looking from the top. The darker the color, the larger the value of the coefficients. The plots in the south-west and north-east are looking from the $X$ and $Y$ directions, respectively. The plot in the south-east is looking from the top and only values larger than 5 are plotted. Movie 02 has a strong crack signal near the center. The peak MSR value on the hill is 46.26.
Figure 2.12  Robust studentized residuals for the coefficients of PC1 in Movie 15. Movie 15 has a weak crack signal near the center. The peak MSR value on the hill is 6.98.
Figure 2.13  Robust studentized residuals for the coefficients of PC1 in Movie 08. Movie 08 was taken on a specimen that does not have a crack. The peak MSR values is 4.79
Figure 2.14 Robust studentized residuals for the coefficients of PC1 in Movie 09. Movie 09 has a weak crack signal near the center and a strong noise signal in the north-east corner.
without a nearby valley.

2.5.1 Finding Hills and Valleys Using Cluster Analysis

Cluster analysis provides methods for dividing data into groups with similarities and we use cluster analysis to find hills and valleys in the studentized residuals results. The Partitioning Around Medoids (PAM) clustering algorithm (Kaufman and Rousseeuw 1990) was used to cluster the hills and valleys seen in the studentized residual plots. The algorithm will divide the observations into \( k \) groups, the points within each group will have a high degree of similarity and the observations in different groups will be as dissimilar as possible. The algorithm requires \( k \) to be specified. To identify clearly separated hills and valleys, we define \( k \) to be the smallest integer that separates the points into clusters such that all of the clusters have a diameter smaller than the distance from the nearest cluster. We start with \( k = 2 \), then we iteratively increase \( k \) until at least one of the clusters has a diameter larger than the distance from its nearest neighbor. Then \( k - 1 \) will be the input to the PAM algorithm. If \( k = 2 \) and the two clusters are too close to each other at the beginning, \( k = 1 \) will be used in the subsequent analysis.

The detection procedure can be summarized by the following steps:

1. Mark all points which have values larger than \( C_{\text{Hill}} \).
2. Cluster the residuals with large values in step 1 into groups, which will be defined as hill(s).
3. Mark all points which have values smaller than \( C_{\text{Valley}} \).
4. Cluster the points in step 3 into groups, which will be valleys.
5. Eliminate all hill(s) that has(have) a valley within the range of \( C_{\text{Diste}} \).
6. The hill(s) will be the center of the signal(s) or no signal if no hill left.

2.5.2 Detection Thresholds

The threshold values in the above procedure are defined as:
1. $C_{\text{Hill}}$, the minimum value for a residual to be included in a hill.

2. $C_{\text{Valley}}$, the maximum value for a residual to be included in a valley.

3. $C_{\text{Dist}}$, The smallest distance between the hill and its nearest valley such that it still can be considered to be a crack signal.

These threshold values can be obtained by analyzing the distribution of MSR values in blank movies.

The distribution of MSR for blank movies is needed to choose the value of $C_{\text{Hill}}$. We obtain this distribution in the following way:

- For a movie without a crack signal, we use the maximum value of all of the studentized residuals.
- For a movie with a crack signal, the studentized residuals around the crack signal are eliminated. The maximum value of the rest of the studentized residuals will provide an MSR value. Thus each movie provides noise MSR values that will be used in the following steps.

The 70 noise MSR values were extracted from the 70 movies. Figure 2.15 is a normal probability plot of the 70 MSR noise values. The maximum of the 70 values is 4.89. A threshold value of 5 will be good enough to keep the false alarm rate very low. The MSR values for the movies with a crack signal in our data set are larger than 5 for all movies except Movie 28.

Based on the distribution of signal MSR values and the distribution of noise MSR values, we choose the following detection thresholds in order to be able to identify the hill clearly and to avoid too many false alarms:

1. $C_{\text{Hill}} = 5$ is the minimum value for a studentized residual to be included in a hill.

2. $C_{\text{Valley}} = -5$ is the maximum value for a studentized residual to be included in a valley.

The value is chosen for the same reason as that for choosing $C_{\text{Hill}}$. 
3. $C_{\text{Dist}} = 20$ pixels is the smallest distance between the hill and its nearest valley for the hill to be considered as a crack signal. A typical crack size is around 15 pixels in diameter. A hill that is fewer than 20 pixels away from a valley is probably caused by a shifting dot instead of a real crack.

### 2.5.3 Probability of Detection

The above procedure will generate a MSR value for each movie. The crack lengths of all movies were provided in the original data set. Figure 2.16 shows the plot of $\log(\text{MSR})$ versus $\log(\text{crack length})$. The linear relationship and the equal variance assumption provides good description of the data.

The data in Figure 2.16 were fitted by a left-censored linear regression model. Left-censored observations arose because the MSR values for some movies taken on specimens that have a crack signal below the noise floor. The mode can be expressed as
\[
\log(\text{MSR}) = \mu_{\log(\text{MSR})} + \varepsilon = \alpha_0 + \alpha_1 \log(L) + \varepsilon
\]

where \(\varepsilon\) has a normal distribution with standard deviation \(\sigma\). The result of fitting this model are \(\hat{\alpha}_1 = 2.81\), \(\hat{\alpha}_0 = 13.06\) and \(\hat{\sigma} = 0.67\).

The probability of detection is defined as the probability of having an MSR larger than the threshold. For a specific value of crack length \(L\), the POD can be expressed as the follows:

\[
\text{POD}(L) = \Pr(\text{MSR} > \text{MSR}_T) = 1 - \Phi_N \left( \frac{\log(\text{MSR}_T) - \mu_{\log(\text{MSR})}(L)}{\sigma} \right)
\]

This POD function can be estimated by substituting the parameter estimates into the formula. The POD estimate versus crack length is plotted in Figure 2.17. The logistic regression hit/miss fitting of POD from expert-evaluation data is also plotted as the dotted line as comparison. The improvement of POD for smaller crack sizes is seen clearly in the plot. A lower confidence bound on POD could be calculated by using either the delta method or likelihood methods.

Figure 2.16 MSR versus crack length plot
2.6 Conclusion and Future Research

This study has presented a statistical method for crack detection using thermal acoustics data. Principal components analysis is used for dimension reduction in the data processing and robust regression and cluster analysis are used to setup the detection rule. The final detection rule gives results that are better than the expert (human) detection using visual inspection of the movie after some simple signal processing. In particular, our algorithm detected cracks in movies 7, 57 and 60 that were not identified by the experts. The POD curve calculated from the distribution of the MSR has a better POD estimate for small cracks.

Possible future research directions of this research include:

- The measurement of the data used in this study stops before the temperature begins to fall. Vibrothermography data in the future are expected to include the data after the samples begin to cool down. These more complete data should increase the power of the detection and lower the false alarm rate.
• Improved crack detection methods would also be more sensitive and accurate if the various noises (e.g., trend, periodicity, and image shifting) could be eliminated or reduced by improving the inspection system.

• A simulation study (simulating more movies) would provide more insight into the principal components analysis of the movies and be used to explore possible directions to improve the detection algorithm.
CHAPTER 3. Simulation of Vibrothermography Data

ABSTRACT

Vibrothermography, also known as thermosonics and sonic infrared, is a nondestructive evaluation method that can be used to detect cracks in certain kinds of structures. The output from a thermosonic system is a sequence of images taken over time. This paper describes methods for simulating thermosonic data. We show how to use simulated data to study the properties of a thermosonic based algorithm for detecting cracks.

Key Words: POD, Sonic IR, Thermosonics, Thermal Acoustics
3.1 Introduction

3.1.1 Background

Vibrothermography, also known as Sonic Infrared (IR), thermal acoustics and thermosonics, is described, for example, in Henneke and Jones (1979), Reifsnider et al. (1980) and recent work by Holland (2007). It is a nondestructive evaluation technique used for detecting cracks in a solid unit. A pulse of sonic or ultrasonic energy is applied to a unit to make the unit vibrate. If a crack exists in the unit, it is expected that the faces of the crack will rub against each other, resulting in a temperature increase near the crack. An infrared camera is used to measure the temperature change in a sequence of frames over time, which we refer to as a movie. Gao and Meeker (2009) developed a systematic detection method to automatically analyze data generated in vibrothermography inspection. However, with only 70 data sets, many properties of the statistical procedure remain unclear. Also, there was need to evaluate the effect of possible changes to the inspection method.

3.1.2 Purpose of the Study

The method in Gao and Meeker (2009) can be described briefly in the following several steps:

1. Principal component regressions, based on an underlying polynomial spatial-temporal model, are conducted on circular windows with a radius 7 points, using all time frames remaining after subtracting out the background. Separate regressions are centered on all possible combinations of coordinates on the movie. This first step generates separate sets of coefficient estimates for the first principal component at each \((x, y)\) center point.

2. The two dimensional coefficients of the first principal components result is fitted using a robust regression as a function of \(x\) and \(y\), position of the center of the moving circle. The studentized residuals from the robust regression of the fitting are used in the next step.
3. Hills (a collection of spatially-connected large studentized residuals) and valleys (a collection of spatially-connected small studentized residuals) are identified by using cluster analysis. The maximum studentized residual (MSR) is defined as the maximum value of the studentized residual after eliminating the value of studentized residuals of pixels near valleys. Please refer to Gao and Meeker (2009) for details of the method.

This study will evaluate the above and related procedures by simulating movie data. We show how to use simulation to study the relationship between the MSR and the crack characteristics, the formation of hills and valleys, the use of cool-down data, etc.

3.1.3 Overview

This paper develops a procedure to simulate movie data for thermosonics. We briefly describe the crack signal model in Section 2. In Section 3, we develop the procedure of the background noise simulation and signal simulation. Combining the signal and noise will generate a thermosonics movie data which can be used in evaluating crack-detection algorithms for movie data, like the one we developed in Gao and Meeker (2009). Using the movie data generating procedure in Section 3, we evaluate the effect of shifting image and adding a cool-down phase on our crack detection algorithm. Using a cool-down phase data increases the MSR value and improves POD, which is consistent with our previous thought that cool-down data will help on crack detection.

3.2 Crack Signal Characteristics

3.2.1 Illustration of Thermosonics Movies

As described in Gao and Meeker (2009), each thermosonics movie in our data set has either 31 or 55 frames. Each frame contains $256 \times 256$ pixels. Four frames in Movie 02, which has a strong crack signal $\text{MSR} = 46.26$ are shown in Figure 3.1. The crack signal increases with time (Frame No), as can be seen in the images. Figure 3.2 shows the temporal behavior of a grid of $5 \times 5 = 25$ pixels approximately centered on a crack signal. The paths with stronger signals are closer to the center of the square and those with weaker signals are near the corners.
(maximum distance from the center). The thick dark line on the top is the signal strength at the center of the $5 \times 5$ matrix. The thick line is the density average of the 25 points, which allows us to see more clearly the general shape of the crack signal up to the point where the camera was turned off. The legend in the upper-left corner of the plot gives a idea of which line according to which point in the movie. For example, the symbol “o” represent the pixel in the center of the signal and the symbol “+” represents the middle pixel of the left edge.

Figure 3.1 Frames (4, 10, 20 and 31) from Movie 02, which has a strong crack signal around the center. The 4 frames show that the strength of the crack signal grows with time.

3.2.2 Details of the Crack Signal Spatial and Temporal Characteristics

Figure 3.3 illustrates the intensity change in the $Y$ direction at Frame 30 (near the end of the energy input). A very similar shape appears in the $X$ direction. The plots show that the crack signal has a width of around 14 pixels (which is the window size used in the scan calculation), and the increase is almost linear from the edge of the crack signal to the center of the crack signal. This feature will be implemented in the crack signal simulation.
Figure 3.2 Temporal behavior of pixels in a grid of $5 \times 5 = 25$ pixels approximately centered on a crack signal in Movie 02. Different symbols represent different locations around the crack signal, which is illustrated in the upper-left corner.
Figure 3.3 Intensity of Movie 02 around the center of the image in $Y$ direction of frame 30. A similar pattern exists in the $X$ direction.
In the time domain, the crack signal initially increases rapidly and then asymptotes to a steady state level, as can be seen in Figure 3.4. Figure 3.4 shows the signal strength at the center of Movie 02. The nonlinear function in (3.1) is fitted to the data.

\[ s = A_0 + A_1 \left[ 1 - \exp \left( -\frac{t^2}{t_0^2} \right) \right]. \]  

(3.1)

Where \( t_0 \) is a constant that will control the model shape in time domain (how fast of the signal changes with time), \( A_0 \) is baseline for the noise of intensity, and \( A_1 \) will control the significance of the signal generated by the crack. The model agrees well with the data. Both the spatial and temporal features of the crack signal will be used in the following simulation of movie data.

Figure 3.4 Center pixel intensity versus time on Movie 02, which has a strong crack signal. The solid line is the fitted model in (3.1)
3.3 Thermosonics Movie Simulation

3.3.1 The Composition of a Thermosonics Movie

Let \( S(x, y, t) \) represent the pixel intensity at location \((x, y)\) and time \(t\). In our simulation model, \( f(x, y, t) \) has following components, \( x = 1, \ldots, 256 \), \( y = 1, \ldots, 256 \), \( t = 1, \ldots, n_f \) where \( n_f \) is either 31 or 55.

\[
f(x, y, t) = B(x, y, t) + T(t)G(x, y) + \varepsilon(x, y, t).
\] (3.2)

Here

- \( B(x, y, t) \) represents the background.
- \( T(t) \) represents temporal trend associated with time \( t \).
- \( G(x, y) \) is the spatial structure associated with the location \((x, y)\) of the pixels. For example, Gao and Meeker (2009) show that in general, the movies have high intensity on the edge and low intensity in the center, constant over time.
- \( \varepsilon(x, y, t) \) is the random noise.

A simulated movie should include all of the components above to make the result as realistic as possible. The following sections will explain and demonstrate our approach to simulating the movie data.

3.3.2 Crack Signal with an Exponential Model

The model

\[
S(x, y, t) = A(L) \left[ 1 - \exp \left( -\frac{t^2}{t_0^2} \right) \right] \left[ 1 - \frac{r(x, y)}{R_0} \right].
\] (3.3)

was developed based on the temporal and spatial structure of the data illustrated in Figures 3.3 and 3.4 and fitting the model in (3.1),

where
• $S(x, y, t)$ represents the simulated crack signal at position $(x, y)$ and frame $t$.

• $A(L)$ is the amplitude of the crack signal as a function of crack length $L$.

• $R_0$ is the size (radius) of the crack signal.

• $t_0$ is a constant number to control the exponential function. The larger the value, the slower the intensity change with time.

• $r(x, y)$ is the distance from the point $(x, y)$ to the crack center $(x_c, y_c)$ and it is in the form of $r(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}$. The linear ratio form is justified by Figures 3.3.

The function $A(L)$ is defined to be:

$$A(L) = \exp [\gamma_0 + \gamma_1 \log(L)].$$

The functional form and the $\gamma$ values are chosen to make the final POD plot, which will be discussed later, to be similar to the POD from the original data.

3.3.3 Simulation of the Background Noise

The components described in Section 3.3.1 can be divided into two categories: background noise and real crack signal. A general approach for the simulation of the background noise would include the simulation of random global noise, temporal trend, spatial structure, local features, temporal and spatial correlation and so on. We took a simpler but reasonable approach to the background simulation. In particular, we use a random selection one of the existing “blank” movies as the background noise. A pool of 70 blank movies was generated from the 70 movies in our data set. For the movies that do not have a crack signal, a blank movie is just the movie itself. We have 38 of such movies. For the movies that do have a crack signal, we cut out a $20 \times 20$ rectangle of pixels that had the crack signal and replaced it with a group of pixels near the crack signal, but far away enough from the crack signal that it is not affected by crack-heating.
The background simulation using the existing data has several advantages in comparison with the more general simulation approach:

- It will capture the variation and features naturally because the movies are coming from real data.

- It is much simpler. There is no need to specify complicated models associated with different noise sources. As we have seen in the previous movies, the noise features in the movies have a complicated temporal and spatial noise structure. It will be difficult to implement and justify a model that can reflect those features.

The disadvantage of using the blank movies to represent noise is that we are restricted only to the noise features that exist in the 70 movies corresponding to the particular experimental setup. A model based background simulation will have the freedom to add and study other temporal and spatial noise structures and other features that do not appear in the existing movies.

### 3.3.4 Movie Simulation with Background and Crack Signal

We randomly choose a blank movie as described in Section 3.3.3 as the background. After that, we add the signal model (3.3) using a randomly chosen crack length $L$ (from an uniform random distribution between 0.007 to 0.025). As an example, Figure 3.5 is a plot of the studentized residuals for a simulated movie with for $A = 25$, $R_0 = 7$ and $t_0 = 7$. A clear hill appears around the center $x_c = 120, y_c = 120$ in the plot with an MSR value of 39.01. The pattern is similar to outputs from the real movies with strong signals, as described in Gao and Meeker (2009).

We simulated 100 movie data sets and the crack size $L$ and corresponding MSR for each simulation were recorded. The relationship between the logarithm of MSR and the logarithm of crack size is plotted in Figure 3.6 along with a fitted linear regression line.

Using the linear relationship illustrated in Figure 3.6, we can compute probability of detection as a function crack length. The POD can be defined as the probability of an MSR value larger the threshold. For a specific value of crack length $L$, the POD can be expressed as:
Figure 3.5 The simulation result for $A = 30$, $R_0 = 7$ and $t_0 = 7$. A very clear hill appears in the result and the maximum MSR value is 46.973.
Figure 3.6 MSR versus crack length results from 100 simulations with randomly chosen movie backgrounds and a least squares line.
\[
\text{POD}(L) = \Pr(\text{MSR} > \text{MSR}_T) = 1 - \Phi\left(\frac{\log(\text{MSR}_T) - \log(\mu_{\text{MSR}}(L))}{\sigma}\right)
\]

where

\[
\log(\mu_{\text{MSR}}(L)) = \beta_0 + \beta_1 \log(L)
\]

The constant value of MSR\(_T\) is determined from the analysis of the distribution of MSR to balance the probability of detection and the probability of a false alarm. Gao and Meeker (2009) provides more details.

Figure 3.7 shows the POD as a function of crack length based on the simulation data.
3.4 Simulation of a Shifting Image

3.4.1 The Impact of a Shifting Image on the Calculation of MSR

Among the 70 movies in our data set, some had an image shifting problem. When a bright dot (that is not caused by a crack) exists in a movie and shifts in a certain direction, it will generate groups of large positive and negative studentized residuals. For example, Figure 3.8 shows parts of the four frames in Movie 09. A dot is moving in the south-west direction. The studentized residual calculation result is shown in Figure 3.9. A clear hill and a clear valley are formed near the position of the moving dot. Such image shifting can confuse the procedure to calculate the coefficients of the principal components, because the algorithm will see an area of pixels (near the south-west edge of the dot) with intensity increasing in time, which is similar to a crack signal. In the north-east side of the moving dot, the intensity will appear to decrease with time, which can be used to distinguish the moving dot with a real crack signal. The studentized residual result of Movie 09 is shown in Figure 3.9. The increase of intensity is caused by the dark dot (with high intensity) shifting to the smaller $X$ range. The following section will use simulations to study the effect on MSR.

3.4.2 Shifting Image Simulation Based on Movie 66

In this section, we will simulate moving dots to study the relationship between the moving dots and the corresponding studentized residuals generated from them. We use Movie 66 as the background for this simulation because it is a movie based on a blank specimen and because it has a relatively simple noise characteristics. Several properties of a shifting dot we want to study are:

- Center intensity of the dot
- The size of the dot
- The shifting velocity of the dot
Figure 3.8 The moving dot in Movie 09. The dot appears in the whole movie moving several pixels in the south-west direction per frame.
Figure 3.9 Robust studentized residuals for Coefficients of PC1 in Movie 09. Movie 09 has a weak crack signal near the center and a strong noise signal in the north-east corner.
For our simulation, the choice of these properties will be based on the shifting dot in Movie 09. The simulation will use Movie 66 as the background and add a simulated dot on top of it. The intensity of the dot will be based on the following model:

\[
I(x, y, t) = B(x, y, t) + \frac{R_d - d(x, y)}{R_d} \times I_d(t),
\]

(3.5)

where

- \(B(x, y, t)\) is the background based on Movie 66.
- \(R_d\) is the radius of the dot.
- \(x, y\) are the coordinates of a pixel
- \(d(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}\) is the distance from the center of the dot \((x_c, y_c)\) to the position \((x, y)\).
- \(I_d(t)\) is the intensity at the center of the dot at time \(t\).

Figure 3.10 is the result for \(R_d = 4\) and \(I_d = 100\) (the \(I_d\) in Movie 09 is approximately 200). We see a pattern similar to that in Figure 3.9. The peak value for the hill is 20.50 and the peak value for the valley is 20.23. In the simulation, the valley peak and hill peak are near to each other in the vicinity of the shifting dots. That again confirms that shift dots can be identified by hills with a nearby valley.

3.4.3 Explanation of the Formation of the Hill and Valley Shapes in a Shifting Image

Figure 3.10 shows that the hill forms in the moving direction (the simulated dot moving to the south-west) and the valley forms against the moving direction. A large MSR will appear around the region with increasing intensity over time. The intensity of the pixels on the path of a moving dot is increasing with time because the dot with large intensity values is replacing the lower intensity values. Large studentized residuals will appear around the pixels with the increasing pattern and form a hill much as the other crack signals do. On the other
Figure 3.10  Studentized residual plot for a simulated moving dot based on model (3.5) with $I = 100$. Both a hill and a valley are formed near the moving dot with a maximum of 20.238 and a minimum of -20.504.
hand, the intensity of the pixels against the direction of the shifting image is decreasing with
time since the dot with a large intensity is moving away and are replaced by the background
noise. A valley appears around those pixels as the studentized residuals capture the decreasing
behaviors. The hills and valleys formed by the moving dots are close to each other because
the dot does not move very far. The feature of a negative valley can be used to distinguish the
moving dot hill from a hill from a real crack signal, which appears alone without a valley.

3.5 Evaluation of Crack Signals with Cool-Down Phase

The experimental data we received stopped taking IR images before the sample was allowed
to cool down. We expect that having data extending into the cool down phase would improve
detection capability. The following simulation will add cool-down data after warm-up data and
the compute the distribution of MSRs to compare with the result with only warm-up phase.

3.5.1 Simulation of a Cool-Down Phase

The current movie data has 31 frames. We simulate a collection of 100 movies and in
each we replace the last 14 frames with a simulated cool-down signal. The warm-up phase
will continue to use model (3.3). We design the cool-down phase using a similar exponential
function with the opposite sign in the exponential function associated with time \( t \):

\[
S(x, y, t) = A(L) \left[ \exp \left( -\frac{t^2}{t^2_c} \right) \right] \left[ \frac{R_0 - \sqrt{(x-x_c)^2 + (y-y_c)^2}}{R_0} \right]. \tag{3.6}
\]

The simulation assumes that the vibrational process (energy input) will stop on frame 17
and that the sample will cool down starting from frame 18. A plot of the intensity in the
center of the crack signal is shown in Figure 3.11. The left side (warm-up phase) of the plot
is similar to the Movie 02 plot in Figure 3.4. The right side of the plot in Figure 3.11 shows
simulated data from the cool-down model (3.6) using the same value of \( t_0 \) and \( R_0 \).
Figure 3.11  The simulated intensity at the center of a crack signal based on model (3.3) and (3.6) with $A = 10$ with background from Movie 08.
3.5.2 Modification of the Detection Procedure

Intuitively, for a same crack specimen, a data collection with a cool-down phase contains more information and characteristics of the crack and should to be detected easier than a data collection with only a warm-up phase. The procedure developed in Gao and Meeker (2009) uses estimates of regression coefficients as the indicator of a crack signal. In order to apply a similar procedure with the two phase data, we modify the original procedure to detect cool-down phase as well as warm-up phase.

Suppose that energy input stops at $N_s = 17$. There is generally a time delay after energy is turned off and the intensity decreases. The new modified procedure will treat the data differently after frames 17. We do this by defining modified intensity values that will transform the non-monotone pattern in Figure 11 into a pattern that would, except for noise, be monotonically increasing. In particular, we define

$$I'(x, y, t) = \begin{cases} 
I(x, y, t) & t \leq 17 \\
2I_h(x, y) - I(x, y, t) & t > 17 
\end{cases}$$

where $I_h(x, y)$ is slightly smoothed version of $I$ which is calculated by average the intensity of pixels in a circle of radius for all $x, y$ combinations in the frame. Then a detection procedure is applied to the new intensity data $I'(x, y, t)$, which are expected to be increasing from the beginning frame until frame 31 for a two phase data as illustrated in Figure 3.12. When $I(x, y, t)$ is coming from a blank movie with only noises, the moving average function $I_h(x, y)$ will be close to the mean and the output of $I'(x, y, t)$ will be close to noise. That will make the false alarm rate in both original and modified procedure similar. The lognormal probability plot for the noise MSR of the modified procedure is in Figure 3.13 along with the original noise MSR. The solid points are for the modified procedure noise MSR. Figure 3.13 shows that the modified procedure generates a noise MSR with a distribution that is close to the original procedure. Thus the false alarm rate similar in both procedures and the POD curve are comparable.
Figure 3.12 The simulated intensity $I'(x, y, t)$ at the center of a crack signal based on model (3.3) and (3.6) with $A = 10$ with background from Movie 08. The solid dots represent $I'(x, y, t)$ and the open dots are the original $I(x, y, t)$ values before flipping.
Figure 3.13  Noise MSR for both original procedure and the modified procedure. The circle is for original and the solid point is for modified.
3.5.3 Comparison of Warm-up/Cool-down Data with Warm-up-only Data

The result in Figure 3.14 shows larger MSR values for the same amplitude of the crack signal. This suggests that the cool-down phase will help to identify crack signals. The POD is again calculated with (3.4). The POD plot comparing of the two phases data (warm-up/cool down) and one phase data (warm-up only) is in Figure 3.15. Again, the data with both warm-up and cool-down phases shows better POD as expected.

Figure 3.14 Comparison of the MSR versus crack length relationship between two phases (warm-up and cool down represented by symbol of empty circle) and one phase (warm-up represented by symbol of solid circle) only data. The two phases data clearly shows a trend larger MSR for the same value of crack length.

3.6 Concluding Remarks and Areas for Future Research

This study develops a tool to simulate thermosonic movie data and shows how to apply it to evaluate the detection algorithm developed in Gao and Meeker (2009). We developed a
Figure 3.15  Probability of detection plot of comparison between two phases (warm-up/cool down) and one phase (warm-up only) data. The two phases data shows a generally larger POD.
model that allows Thermosonics data to be simulated from an existing crack signal by changing the properties like amplitude and size. The tool can also simulate the crack signal based on a mathematical model, using specified parameters as input. We used the simulation tool to evaluate the improvement in the distribution of MSR that could be obtained by using cool-down data in addition to warm-up information in Thermosonics testing.

In future work, we plan to extend the simulation to generate background noise simulation with temporal and spatial correlation which would allow simulation for a more general class of inspection situations. A more general background noise simulator will help study of the effect of temporal and spatial structure of the noise on MSR and POD.
CHAPTER 4. Detecting Cracks in Fan Blades Using Vibrothermography

ABSTRACT

This paper describes the analysis of data from vibrothermography inspections on turbine blades. Separate but similar analysis were done for two different purposes. In both analysis, we fit statistical models with random effects to describe the crack-to-crack variability and the effect that the experimental variables have on the responses. In the first analysis, the purpose of the study was to find vibrothermography equipment settings that will provide good crack detection capability over the population of similar cracks in the particular kind of jet engine turbine blades that were inspected. Then, the fitted model was used to determine the test conditions where the probability of detection is expected to be high and probability of alarm is expected to be low. In our second analysis, crack size information was added and a similar model was fit. This model provides an estimate of POD as a function of crack size for specified test conditions. Key Words: POD, random effects, thermal acoustics, sonic IR, thermosonics
4.1 Introduction

4.1.1 Purpose of the Study

The purpose of the study is to find vibrothermography equipment settings that will provide good crack detection capability over the population of cracks that could exist in jet engine turbine blades and to estimate the POD for specified test conditions as a function of crack size. We fit statistical models to describe the crack detection data taken on a group of turbine blades. Then we use the model to determine the test conditions where the probability of detection is expected to be high and probability of alarm is expected to be low. We fit a similar model that includes crack size as an explanatory variable and use this model to quantify Probability of Detection (POD) as a function of crack size.

4.1.2 Overview

Section 4.2 describes the experimental setup with data description. Section 4.3 builds a mixed effect model for the response variable. Probability of detection are studied in Section 4.4 and the alarm condition are studied in section 4.5. Based on the result from Sections 4.4 and 4.5, a procedure to choose a good test condition is discussed in Section 4.6. Section 4.7 adds crack size information into the mixed effect model and the corresponding probability of detection methods are studied in Section 4.8. Also included in Section 4.8 are the confidence interval calculation and an evaluation of the coverage probability of the confidence intervals.

4.2 Experimental Setup

The remainder of this paper is organized as follows. Figure 4.1 shows a picture of a turbine blade used in this study. The test matrix consisted of experimental factors vibration amplitude, pulse length, and trigger force, each at three levels, as shown in Table 4.1. This test matrix was used to inspect 10 cracks in seven blades (two cracks were tested on Blade 4 and three cracks were tested on Blade 7). There were 32 tests (all 27 combinations of the $3^3$ full factorial plus one additional test at 10-83-10, two additional tests at 35-150-35, and two
additional tests at 60-217-60) for each crack. The response was maximum contrast, defined as the difference between peak value at the crack and background (3 × 3 pixel area 15 pixels to the left of the peak), were manually extracted. The responses (maximum contrasts) from the three replications in the same test were averaged before being used in model fitting. Table 4.2 gives sample data (two tests result) from the crack in Blade 1.

Table 4.1 Test Factor Levels

<table>
<thead>
<tr>
<th>Level</th>
<th>Vibration Amplitude</th>
<th>Pulse Length</th>
<th>Trigger Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>10</td>
<td>83</td>
<td>10</td>
</tr>
<tr>
<td>Medium</td>
<td>35</td>
<td>150</td>
<td>35</td>
</tr>
<tr>
<td>High</td>
<td>60</td>
<td>217</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 4.2 Sample Test Data of the Crack in Blade 1, which include the test conditions and response results

<table>
<thead>
<tr>
<th>Vibration Amplitude</th>
<th>Pulse Length</th>
<th>Trigger Force</th>
<th>Maximum Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>150</td>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>35</td>
<td>150</td>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>35</td>
<td>150</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>60</td>
<td>217</td>
<td>35</td>
<td>117</td>
</tr>
<tr>
<td>60</td>
<td>217</td>
<td>35</td>
<td>122</td>
</tr>
<tr>
<td>60</td>
<td>217</td>
<td>35</td>
<td>109</td>
</tr>
</tbody>
</table>

Figure 4.1 Air Force jet engine turbine blade, approximately 5.5 inches long
4.3 Analysis of Signal Response Data

4.3.1 Transformation and Fixed Effects Linear Model

Different model forms and transformation of variables were tried to find a model that describes the relationship between the response and the test conditions. A linear model with a log transformation on the response and the explanatory variables was chosen to describe the each individual crack. The linear model for the maximum contrast response (denoted by $m$) can be expressed in:

$$
\log(m + 1) = \beta_0 + \beta_1 \log(v) + \beta_2 \log(p) + \beta_3 \log(r) + \\
\beta_4 \log(v) \log(r) + \beta_5 \log(v) \log(p) + \beta_6 \log(p) \log(r) + \\
\beta_7 \log(v)^2 + \beta_8 \log(p)^2 + \beta_9 \log(r)^2 + \epsilon
$$

where $v$ represents Vibration Amplitude, $p$ represents Pulse Length, and $r$ represents Trigger Force. The function `lm` in R (R Development Core Team 2008) was used to fit the model using ordinary least squares. The estimates of the first four coefficients are listed in Table 4.3. The large crack to crack variability in these coefficients suggested a random effect model in which some of the parameters are random.

Table 4.3 Fitting Result of Model (4.4) for Individual Cracks

<table>
<thead>
<tr>
<th>Crack</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.580</td>
<td>-7.651</td>
<td>2.068</td>
<td>5.428</td>
</tr>
<tr>
<td>2</td>
<td>-20.505</td>
<td>-4.909</td>
<td>8.423</td>
<td>0.287</td>
</tr>
<tr>
<td>3</td>
<td>38.022</td>
<td>-5.006</td>
<td>-11.639</td>
<td>-4.157</td>
</tr>
<tr>
<td>4</td>
<td>12.915</td>
<td>-3.180</td>
<td>-4.118</td>
<td>-1.793</td>
</tr>
<tr>
<td>5</td>
<td>-27.055</td>
<td>-7.914</td>
<td>15.356</td>
<td>-1.762</td>
</tr>
<tr>
<td>6</td>
<td>6.967</td>
<td>0.326</td>
<td>-5.131</td>
<td>-2.438</td>
</tr>
<tr>
<td>7</td>
<td>11.945</td>
<td>-3.160</td>
<td>-3.948</td>
<td>-0.155</td>
</tr>
<tr>
<td>8</td>
<td>-37.345</td>
<td>-0.495</td>
<td>10.730</td>
<td>6.463</td>
</tr>
<tr>
<td>9</td>
<td>4.654</td>
<td>-1.119</td>
<td>-3.567</td>
<td>0.153</td>
</tr>
<tr>
<td>10</td>
<td>-179.016</td>
<td>1.227</td>
<td>59.186</td>
<td>23.576</td>
</tr>
</tbody>
</table>
4.3.2 Full Mixed Effects Model

In order to describe a larger population of cracks using one model, we use a linear mixed effects model (described, for example, in Venables and Ripley 2002 and Pinheiro and Bates 2000) in which the intercept and main effect (linear) model terms are modeled as random effects to describe the crack-to-crack variability in the response. We assume that the 10 cracks in the study represent a random sample from that population of cracks. The final model for the maximum contrast is:

\[
\log(m+1) = \beta_{0,b} + \beta_{1,b} \log(v) + \beta_{2,b} \log(p) + \beta_{3,b} \log(r) + \\
\beta_4 \log(v) \log(r) + \beta_5 \log(v) \log(p) + \beta_6 \log(p) \log(r) + \\
\beta_7 \log(v)^2 + \beta_8 \log(p)^2 + \beta_9 \log(r)^2 + \varepsilon.
\]  

(4.2)

The random error \(\varepsilon\) is assumed to have a normal distribution with mean 0 and variance \(\sigma_\varepsilon\). For each crack, the coefficients \((\beta_{0,b}, \beta_{1,b}, \beta_{2,b}, \beta_{3,b})\) are assumed to have a multivariate normal distribution independent of \(\varepsilon\), with mean \(\mu_{\beta_b} = (\beta_0, \beta_1, \beta_2, \beta_3)\) and covariance matrix \(\Sigma_{\beta_b}\), where \(\mu_{\beta_b}\) is a vector and \(\Sigma_{\beta_b}\) is a 4 \times 4 matrix. These model assumptions can be stated succinctly as

\[\varepsilon \sim \text{Normal}(0, \sigma_\varepsilon).\]

and

\[
\begin{pmatrix}
\beta_{0,b} \\
\beta_{1,b} \\
\beta_{2,b} \\
\beta_{3,b}
\end{pmatrix}
\sim \text{Multinormal}
\begin{pmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{pmatrix}, \Sigma_{\beta_b}.
\]

The parameter estimates, which are calculated by function `lme` (in package `nlme`) in R (R Development Core Team 2008), are given in the Table 4.4 and the matrix elements estimates are given in Table 4.5.
Table 4.4  Fixed effect estimates of the full mixed-effect model

<table>
<thead>
<tr>
<th>$\beta$s</th>
<th>Value</th>
<th>Std.Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>-7.843</td>
<td>11.620</td>
<td>-0.675</td>
<td>0.500</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-3.580</td>
<td>1.272</td>
<td>-2.814</td>
<td>0.005</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>2.893</td>
<td>4.672</td>
<td>0.619</td>
<td>0.536</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.137</td>
<td>1.427</td>
<td>0.797</td>
<td>0.426</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.671</td>
<td>0.166</td>
<td>4.034</td>
<td>0.000</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.092</td>
<td>0.086</td>
<td>1.061</td>
<td>0.290</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>-0.299</td>
<td>0.173</td>
<td>-1.729</td>
<td>0.085</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>0.214</td>
<td>0.152</td>
<td>1.409</td>
<td>0.160</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>-0.265</td>
<td>0.472</td>
<td>-0.561</td>
<td>0.575</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>0.129</td>
<td>0.163</td>
<td>0.791</td>
<td>0.429</td>
</tr>
</tbody>
</table>

Table 4.5  Covariance matrix of the random effect model

<table>
<thead>
<tr>
<th>$\Sigma_{\beta}$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>19.073</td>
<td>1.004</td>
<td>-3.688</td>
<td>-2.120</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1.004</td>
<td>0.117</td>
<td>-0.215</td>
<td>-0.126</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-3.688</td>
<td>-0.215</td>
<td>0.721</td>
<td>0.421</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-2.120</td>
<td>-0.126</td>
<td>0.421</td>
<td>0.281</td>
</tr>
</tbody>
</table>
Table 4.6 Fixed effect estimates for the reduced mixed-effect model

<table>
<thead>
<tr>
<th>βs</th>
<th>Value</th>
<th>Std.Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₀</td>
<td>-2.974</td>
<td>0.327</td>
<td>-9.085</td>
<td>0.000</td>
</tr>
<tr>
<td>β₁</td>
<td>-1.248</td>
<td>0.442</td>
<td>-2.822</td>
<td>0.005</td>
</tr>
<tr>
<td>β₅</td>
<td>0.521</td>
<td>0.076</td>
<td>6.858</td>
<td>0.000</td>
</tr>
<tr>
<td>β₉</td>
<td>0.139</td>
<td>0.026</td>
<td>5.356</td>
<td>0.000</td>
</tr>
</tbody>
</table>

4.3.3 Reduced Mixed Effect Model

We reduced the full model in (4.2) based on Akaike’s information criterion (AIC) (Akaike 1974) and the $t$ ratios of the regression coefficients. The final model that we used is:

$$
\begin{align*}
\log(m + 1) &= β₀, b + β₁, b \log(v) + β₂, b \log(p) + β₃, b \log(r) + \\
&\quad β₅ \log(v) \log(p) + β₉ \log(r)^2 + ε.
\end{align*}

(4.3)

We still assumed that $(β₀, b, β₁, b, β₂, b, β₃, b)$ has a multivariate normal distribution independent of $ε$, but with fixed 0 mean for $β₂, b$ and $β₃, b$, i.e., $μ_β = (β₀, β₁, 0, 0)$. The reduced model sets $β₅$, $β₇$ and $β₈$ to be 0. The results for fitting the reduced model are listed in Table 4.6. The covariance matrix results are given in Table 4.7.

The model assumptions can be stated succinctly as $ε \sim \text{Normal}(0, σε)$ and

$$
\begin{pmatrix}
β₀, b \\
β₁, b \\
β₂, b \\
β₃, b
\end{pmatrix}
\sim \text{Multinormal}

\begin{pmatrix}
β₀ \\
β₁ \\
0 \\
0
\end{pmatrix}, Σ_β$
Table 4.7  Covariance matrix for the random effects for the reduced mixed effect model

<table>
<thead>
<tr>
<th>$\Sigma_{b}$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>18.079</td>
<td>0.935</td>
<td>-3.482</td>
<td>-2.034</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.935</td>
<td>0.109</td>
<td>-0.199</td>
<td>-0.119</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-3.482</td>
<td>-0.199</td>
<td>0.678</td>
<td>0.403</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-2.034</td>
<td>-0.119</td>
<td>0.403</td>
<td>0.273</td>
</tr>
</tbody>
</table>

4.4 Probability of Detection for a Population of Cracks

4.4.1 Distribution of Response Variable

Under the reduced mixed effect model (4.3), for given fixed values of $v$, $p$ and $r$, the response variable $m + 1$ has a normal distribution with mean

$$
\mu(v, p, r) = \beta_{0,b} + \beta_{1,b} \log(v) + \beta_{2,b} \log(p) + \beta_{3,b} \log(r) + \beta_5 \log(v) \log(p) + \beta_9 \log(r)^2
$$

and variance

$$
\sigma^2(v, p, r) = X_l^T \Sigma_b X_l + \sigma^2_e. \tag{4.5}
$$

Here $X_l = (1, \log(v), \log(p), \log(r))^T$ where superscript T indicates vector transpose. Equation (4.5) will provide basis for estimating probability of detection (POD) for the population of potential cracks as a function of the experimental factors.

4.4.2 Calculation of Probability of Detection

Probability of detection (POD) is described in detail in MIL-HDBK-1823A (2009) and Georgiou (2006). In our application, POD is the probability that $m$ from a thermal image sequence exceeds a threshold $m_T$ for a crack taken at random from the population of cracks. This probability can be expressed as

$$
POD(m, v, p, r) = \Pr(m > m_T) = 1 - \Phi \left( \frac{\log(m_T + 1) - \mu(v, p, r)}{\sigma} \right) \tag{4.6}
$$
where $\Phi(x)$ is the standard normal cumulative distribution function distribution. We can estimate the POD by evaluating equation (4.6) using the estimates of the model parameters in Table 4.4.

The threshold was chosen as $m_T = 10$. This value was chosen to balance sensitivity with the probability of generating false positives. Figure 4.2 displays the output of our mixed-effects model for the POD. The darker the color, the lower the POD value.

Figure 4.2 POD plot for pulse length = 217. The lighter the color, the higher of the POD.

4.5 Analysis of the Alarm Data

Some tests ended with an alarm, usually caused by a lock-up of the welder/sample system when the horn can no longer vibrate properly. This is caused by protective feedback loop that terminates power to the welder to prevent damage to the piezoelectric element. Alarm
incidents are undesirable because no useful information is obtained from the test. We would like to find test conditions that will also result in a small probability of an alarm. When there was an alarm, the time to alarm was recorded. When there was no alarm, the alarm time was taken to be right censored at the length of the pulse because the unknown alarm time is greater than the pulse length. We fit a log normal regression model to the censored time-to-alarm data to assess the probability of alarm as a function of the test conditions.

Let $T_A$ denote the time of an alarm. A probability plot of the censored data suggested that $T_A$ can be described by a lognormal distribution so that

$$\text{POA} = \Pr (T_A \leq p) = \Phi_{\text{nor}} \left( \frac{\log(p) - \mu_a}{\sigma_a} \right),$$

(4.7)

Here $\mu_a = \beta_0^a + \beta_1^a v + \beta_2^a r$. Higher order terms were not statistically significant. The restricted maximum likelihood (REML) estimates of the parameters are $\hat{\beta}_0^a = 6.027$, $\hat{\beta}_1^a = -0.0188$, $\hat{\beta}_2^a = 0.00531$, $\hat{\sigma}_a = 0.1625$. Figure 4.3 depicts estimates of POA for pulse length equal to 217ms.

### 4.6 Choosing Good Test Conditions

A choice of good test conditions needs to fulfill two requirements: a large response for the crack and a small chance of triggering an alarm that would terminate the test and yield no information. Figure 4.4 combines the information in the POD plot in Figure 4.2 and the POA plot in Figure 4.3, showing the test conditions where the probability of detection is expected to be high and probability of alarm is expected to be low. Such overlay threshold plots require specification of acceptable POD and PFA thresholds and we chose those to be POD $> 0.90$ and POA $< 0.05$. The south-east region of Figure 4.4 meets both conditions, suggesting the test conditions to be used in future tests.

### 4.7 Modeling Response as a Function of Effective Crack Size

In this section, we will build a statistical model to describe signal amplitude as a function of crack length. This model can then be used to estimate POD as a function of crack length.
4.7.1 Crack Sizes

Table 4.8 lists the crack size information that was obtained by using a method known as acetate replication, a nondestructive technique. Apparent length is the portion of the total wrap-around crack length visible from the same side of the airfoil as the IR camera. Edge length is the portion of the crack length that wraps around the edge of the airfoil but is not visible from either side of airfoil. Opposite apparent length is the portion of the crack that would be visible from the opposite side of the airfoil. Especially because there were zeros for some of these reported sizes, we use a convex combination of these recorded sizes to define an effective crack length. We then treat this variable as a fixed effect in the regression model.
Figure 4.4 Overlay threshold plot for pulse length = 217. The South-east corner of the plot satisfied both criteria (i.e., POD > 0.90 and POA < 0.05).

Table 4.8 Size Information for the 10 Cracks

<table>
<thead>
<tr>
<th>Crack</th>
<th>Apparent</th>
<th>Opposite Apparent</th>
<th>Edge</th>
<th>Total</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04</td>
<td>0</td>
<td>0.0039</td>
<td>0.0439</td>
<td>0.0046</td>
</tr>
<tr>
<td>2</td>
<td>0.1058</td>
<td>0.0304</td>
<td>0.021</td>
<td>0.1572</td>
<td>0.0226</td>
</tr>
<tr>
<td>3</td>
<td>0.2903</td>
<td>0.228</td>
<td>0.0153</td>
<td>0.5336</td>
<td>0.0204</td>
</tr>
<tr>
<td>4A</td>
<td>0.0793</td>
<td>0.1875</td>
<td>0.0313</td>
<td>0.2981</td>
<td>0.0322</td>
</tr>
<tr>
<td>4B</td>
<td>0.2907</td>
<td>0</td>
<td>0</td>
<td>0.2907</td>
<td>0.0054</td>
</tr>
<tr>
<td>5</td>
<td>0.0935</td>
<td>0.1129</td>
<td>0.0163</td>
<td>0.2227</td>
<td>0.0177</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.0718</td>
<td>0.0066</td>
<td>0.0784</td>
<td>0.0065</td>
</tr>
<tr>
<td>7A</td>
<td>0.2781</td>
<td>0.2449</td>
<td>0.0156</td>
<td>0.5386</td>
<td>0.0205</td>
</tr>
<tr>
<td>7B ( &amp; C)</td>
<td>0.0962</td>
<td>0.1321</td>
<td>0.01</td>
<td>0.2383</td>
<td>0.0116</td>
</tr>
<tr>
<td>7D</td>
<td>0.1039</td>
<td>0.0503</td>
<td>0.0126</td>
<td>0.1668</td>
<td>0.0143</td>
</tr>
</tbody>
</table>
4.7.2 Full Mixed Effect Model With Crack Size Information

The full mixed effects model that includes crack length, shown in (4.8), is similar to model (4.2).

\[
\log(m + 1) = \beta_0 + \beta_1 \log(v) + \beta_2 \log(p) + \beta_3 \log(r) + \\
\beta_4 \log(v) \log(p) + \beta_5 \log(v) \log(r) + \beta_6 \log(p) \log(r) + \\
\beta_7 \log(v)^2 + \beta_8 \log(p)^2 + \beta_9 \log(r)^2 + \beta_{10} \log(L_N) + \varepsilon. 
\]  

(4.8)

Here \( L_N = \alpha L_a + (1 - \alpha)L_e \) is the effective crack length and \( \alpha = 0.0185 \) was chosen to maximize the likelihood. That is, a sequence of models was fit using different values of \( \alpha \) between 0 and 1. The value of \( \alpha \) that gave the largest value of the likelihood was \( \alpha = 0.0185 \). The fixed effect parameter estimates of model (4.8) are given in Table 4.9.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>Value</th>
<th>Std.Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>-3.095</td>
<td>11.765</td>
<td>-0.263</td>
<td>0.793</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-3.517</td>
<td>1.283</td>
<td>-2.741</td>
<td>0.007</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.874</td>
<td>4.713</td>
<td>0.610</td>
<td>0.543</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.185</td>
<td>4.713</td>
<td>0.610</td>
<td>0.543</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>0.662</td>
<td>0.168</td>
<td>3.944</td>
<td>0.000</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>0.087</td>
<td>0.153</td>
<td>3.944</td>
<td>0.000</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>-0.300</td>
<td>0.175</td>
<td>-1.716</td>
<td>0.088</td>
</tr>
<tr>
<td>( \beta_7 )</td>
<td>0.213</td>
<td>0.153</td>
<td>1.391</td>
<td>0.165</td>
</tr>
<tr>
<td>( \beta_8 )</td>
<td>-0.260</td>
<td>0.147</td>
<td>-1.716</td>
<td>0.088</td>
</tr>
<tr>
<td>( \beta_9 )</td>
<td>0.124</td>
<td>0.164</td>
<td>0.755</td>
<td>0.451</td>
</tr>
<tr>
<td>( \beta_{10} )</td>
<td>1.122</td>
<td>0.187</td>
<td>6.006</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The corresponding estimates of the variances and covariance of the random effect terms are given in Table 4.10.
Table 4.10 Covariance matrix of random effect of full model with size

<table>
<thead>
<tr>
<th>$\Sigma_{\beta b}$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>21.158</td>
<td>0.929</td>
<td>-3.797</td>
<td>-2.336</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.929</td>
<td>0.114</td>
<td>-0.206</td>
<td>-0.128</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-3.797</td>
<td>-0.206</td>
<td>0.705</td>
<td>0.434</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-2.336</td>
<td>-0.128</td>
<td>0.434</td>
<td>0.270</td>
</tr>
</tbody>
</table>

4.7.3 Reduced Mixed Effect Model with Crack Size Information

Similar to what we did in Section 4.3.3, the full model can be reduced to a smaller model based on AIC. The model suggested by this criterion is:

$$
\log(m + 1) = \beta_{0,b} + \beta_{1,b} \log(v) + \beta_{2,b} \log(p) + \beta_{3,b} \log(r) +
\beta_5 \log(v) \log(r) + \beta_6 \log(p) \log(r) + \beta_9 \log(r)^2 + \beta_{10} \log(L_N) + \varepsilon.
$$

(4.9)

We again assume that $(\beta_{0,b}, \beta_{1,b}, \beta_{2,b}, \beta_{3,b})$ has a multivariate normal distribution independent of $\varepsilon$, but with fixed 0 mean for $\beta_{2,b}$ and $\beta_{3,b}$ (i.e., $\mu_{\beta b} = (\beta_0, \beta_1, 0, 0)$). The results for fitting the reduced model are given in Table 4.11 and the covariance matrix results are given in Table 4.12.

Table 4.11 Fixed effect estimates of the reduced mixed-effect model with $m$ as a function of crack size

<table>
<thead>
<tr>
<th>$\beta$s</th>
<th>Value</th>
<th>Std.Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>3.010</td>
<td>1.145</td>
<td>2.628</td>
<td>0.009</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-1.774</td>
<td>0.564</td>
<td>-3.144</td>
<td>0.002</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.637</td>
<td>0.108</td>
<td>5.874</td>
<td>0.000</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>-0.164</td>
<td>0.107</td>
<td>-1.531</td>
<td>0.127</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>0.253</td>
<td>0.079</td>
<td>3.192</td>
<td>0.002</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>1.127</td>
<td>0.186</td>
<td>6.048</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 4.12  Estimated covariance matrix of random effect for the reduced model with $m$ as a function of size

<table>
<thead>
<tr>
<th>$\Sigma_{\beta_b}$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>20.302</td>
<td>0.884</td>
<td>-3.633</td>
<td>-2.257</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.884</td>
<td>0.109</td>
<td>-0.197</td>
<td>-0.123</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-3.633</td>
<td>-0.197</td>
<td>0.674</td>
<td>0.419</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-2.257</td>
<td>-0.123</td>
<td>0.419</td>
<td>0.263</td>
</tr>
</tbody>
</table>

4.8  POD as a Function of Effective Crack Size

4.8.1  POD versus Crack Size

It is necessary to estimate POD as a function of crack size for purposes such as planning periodic inspections. In this section, we are going to use model (4.9) to recalculate the POD function. In contrast to equation (4.6), POD will now also be a function of effective crack size and can be expressed as:

$$\text{POD}(m, v, p, r, l) = \Pr(m > m_T) = 1 - \Phi \left( \frac{\log(m_T + 1) - \mu(v, p, r, l)}{\sigma} \right).$$  \hspace{1cm} (4.10)$$

An estimate of POD as a function of effective crack size can be computed by substituting the model estimates from Tables 4.11 and 4.12 into model (4.9). Figure 4.5 shows the POD estimate as a function of effective crack length for different test conditions (Vibration Amplitude, Pulse Length and Trigger Force, respectively). These particular conditions were chosen to provide examples of different POD values. The inside tic marks at the bottom of the plot show the measured effective sizes of the cracks in the data set. The POD estimates to the right of effective size 0.03 involve extrapolation. Both of the full model estimates and reduced model estimates are shown in the plot indicating little difference between the two models when estimating POD. In the following sections we use the reduced model for its simplicity.

4.8.2  Bootstrap Confidence Interval of POD Estimation

The complicated model fitting procedure and calculation of POD make the direct computation of a confidence interval of POD difficult. Bootstrap provides a convenient alternative
Figure 4.5 POD as a function of effective crack length. The solid line is for the reduced model and the dotted line is for the full model. The three test condition parameters (Vibration Amplitude, Pulse Length and Trigger Force) are indicated in the parentheses near the lines.
for such complicated procedures as described in Efron and Tibshirani (1994). The boot-
strap procedures that we used are fully parametric. That is, we simulate bootstrap samples
from our assumed parametric model and the REML estimates (Bartlett (1937)). Figure 4.6
shows the result for both the percentile and the BCA (bias-corrected accelerated) pointwise
nonparametric bootstrap confidence intervals for POD versus effective crack length in test
condition(35,150,35). 5000 bootstrap samples were used in the calculation.

Figure 4.6 95% confidence intervals based on the the percentile and the
BCA bootstrap methods for test condition (Vibration Amplitude=35, Pulse Length=150 and Trigger Force=35). The solid
line in the middle is the POD estimate from the mixed effect
model. The dotted lines are the pointwise confidence intervals,
where the large dotted line is for the BCA method and the small
dotted line is for the percentile method.
4.8.3 Coverage Probabilities of the Percentile and the BCA Confidence Interval

Although bootstrap procedures are expected to provide actual coverage probabilities that are closer to nominal than simpler methods such as the Wald’s approximate method, the actual coverage probability, in general, will only be approximate. We used simulations to compare the coverage probabilities of the two bootstrap confidence interval calculation methods: the sample percentile method and the more refined the BCA method using 2000 new data sets were simulated from the fitting result of the reduced model and the percentile and the BCA bootstrap methods were used to calculate confidence intervals. The observed coverage probability is defined as

\[
\text{Coverage} = \frac{N_I}{N} \quad (4.11)
\]

where \(N_I\) denotes the number of confidence intervals that include the true value of POD (which is calculated in the model from which the data sets are simulated) and \(N\) is the total number simulated data sets. The simulation results of the coverage probability are shown in Figures 4.7, for data sets with 10, 20 and 40 experimental units with three different symbols. The solid lines are for the BCA method and the dotted lines are for the percentile method. The BCA shows closer to nominal converge probabilities than the percentile method, as expected. The coverage probability of both the BCA method and the percentile method become closer to nominal when the sample size increases. The BCA method with 40 experimental units is the closest to nominal among all six evaluations. When sample size is small, (i.e. ten), the coverage probabilities are far away from nominal for larger cracks. As the sample size increases, the difference in coverage between small cracks and large cracks becomes smaller.

4.9 Conclusion and Future Work

This paper has shown how to find good vibrothermography test conditions for detecting cracks in fan blades and how to estimate POD as a function of the effective size of a crack. In future work we will:
Figure 4.7 Coverage probability of POD Confidence Intervals calculated by the BCA and the percentile bootstrap method for 10, 20 and 40 crack samples. The solid line is by the BCA method and the dotted line is by the percentile method. For the range of the crack size of our sample, the BCA method has better coverage than the percentile method.
• Investigate methods for quantifying and displaying uncertainty in the overlay threshold plots like Figure 4.2.

• Investigate the use of other response variables in an attempt to use more completely information in the sequence of image produced by a vibrothermography test system.

• Investigate more details of Bayesian method approach.
CHAPTER 5. Conclusion and Future Work

The Sonic IR movie study has presented a statistical method for crack detection in thermal acoustics experiment data. Principal components analysis is used for dimension reduction in the data processing and robust regression and cluster analysis are used to setup the detection rule. The final detection rule gives results that are better than the human inspection using visual inspection of the movie after some simple signal processing. The POD curve shows a better POD estimate for small cracks. The simulation tool for Sonic IR movie data developed in the second part shows how to apply it to evaluate the detection algorithm developed in Gao and Meeker (2009). Amplitude and size of the crack and the signature of the background noise can be simulated using the tool. We used the simulation tool to evaluate the improvement in the distribution of MSD that could be obtained by using cool-down data in addition to warm-up information in Sonic IR testing. We plan to extend the simulation to generate background noise simulation with temporal and spatial correlation which would allow simulation for a more general class of inspection situations. The blade data analysis has shown how to find good Sonic IR test conditions for detecting cracks in fan blades and how to estimate POD as a function of the effective size of a crack. In future work we will investigate methods for quantifying and displaying uncertainty in the overlay threshold. The current data we have have only one data point for each movie, more information on the movie will help us to develope a better procedure. More sofisticate Baysian method may help on estimation of POD and confidence intervals.
Bibliography


