ULTRASONIC REFLECTION BY A PLANAR DISTRIBUTION OF SURFACE BREAKING CRACKS

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INTRODUCTION

A number of researchers have demonstrated the usefulness of surface acoustic waves (SAWs) as a means of evaluating crack size and investigating crack opening during fatigue crack growth. Rayleigh waves can be used to measure changes in depth of cracks beneath the surface in advanced engineering materials [1-3]. Recently, the crack growth behavior and the crack length distribution of a distribution of surface breaking fatigue cracks have been experimentally studied [4], and a simulation technique for the crack length distribution has been proposed.

The reflection of elastic waves by a void of arbitrary shape in an infinite homogenous isotropic solid has been analytically treated by Auld and Kino [5,6]. Ultrasonic wave reflection and transmission by an array of spherical cavities with a periodic and nonperiodic distribution have been investigated by Achenbach et al. [7,8]. Ultrasonic wave reflection and transmission by a planar distribution of cracks have been studied by Sotiropoulos and Achenbach [9]. These authors have derived general expressions for the reflection and transmission coefficients for incident longitudinal and transverse time harmonic plane waves. Thompson et al. have studied reflection from imperfect bonds, both for one-dimensional and two-dimensional planar crack distributions, based on the use of a distributed spring model [10].

The present work is aimed at developing a method to characterize the reflection of Rayleigh waves by a planar distribution of surface breaking cracks. To do that, an acoustic scattering model for a planar distribution of surface breaking cracks is derived. The numerical evaluation of the reflection coefficient using a weight function estimation method is introduced, and a generalized scattering model of Rayleigh waves for a planar distribution of surface breaking fatigue cracks is developed.

REFLECTION BY A PLANAR DISTRIBUTION OF CRACKS

Consider a plane in a homogeneous, isotropic linearly elastic solid which contains a distribution of in-plane cracks, see Figure 1. The interaction of the incident wave with the plane containing the cracks gives rise to a complicated pattern of reflected and transmitted waves. Sufficiently far from the cracked plane, at sufficiently low frequencies, it may

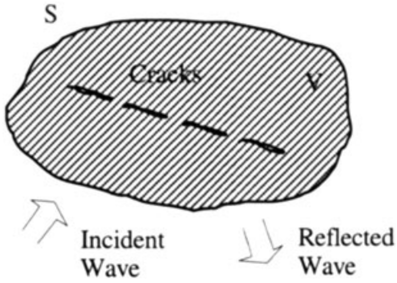

Fig 1. The schematic of a planar distribution of cracks.

however be assumed that the reflected and transmitted waves are homogeneous plane waves.

When there are no sources present within the volume, the reciprocity theorem which relates two distinct elastodynamic states, "(1)" and "(2)", of the same body can be written as

$$\int_S (t_i^{(1)}u_j^{(2)} - t_i^{(2)}u_j^{(1)})dS = 0 \tag{1}$$

where

$$S = S^e + \sum_{k=1}^{m} S_k^c \tag{2}$$

is the bounding surface which consists of the exterior bounding surface S^e of the considered volume and the surfaces S_k^c ($k = 1,2,3,\cdots,m$) of the cracks in the volume, m is the number of cracks contained in the volume, $t_i = n_i\sigma_{ij}$ are the traction components acting on the bounding surfaces, and $u_i^{(1)}$ and $u_i^{(2)}$ are correspond displacement at the surfaces.

The equation of motion, the stress-strain relation and the strain-displacement relation in the volume are

$$-\rho\omega^2 u_i = \sigma_{ij,j} \tag{3}$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

where ρ and C_{ijkl} are the density and the material constants, and ω is the angular frequency, and ε_{ij} defines the strain field.

Take the state as "(1)" when there is no crack present where the elastic constants are noted as ρ and C_{ijkl}, and "(2)" when there are cracks present where the elastic constants are noted as ρ' and C'_{ijkl}, respectively. Multiplying Equation (3) in the state "(2)" by the displacement field in the state "(1)" and subtracting Equation (3) in the state "(1)" by the displacement in the state "(2)", then integrating over the volume, yields

$$\int_V (\rho - \rho')\omega^2 u_i^{(1)} u_i^{(2)} dV = \int_V (u_i^{(1)} \sigma_{ij,j}^{(2)} - u_i^{(2)} \sigma_{ij,j}^{(1)}) dV \tag{6}$$

where

$$V = V_0 + \sum_{k=1}^{m} V_k^c \tag{7}$$

consists of the summation of the crack volumes V_k^c ($k = 1,2,3,\cdots,m$) plus the non-crack volume V_0.

Following from the use of the divergence theorem and the reciprocity relation, and from the consideration that the displacement and stress fields are continuous at the surface of the cracks, the exact reflection coefficient of homogenous plane waves reflected by a planar distribution of the cracks can be derived [11] as

$$R_{21} = \frac{1}{4} j\omega \sum_{k=1}^{m} \int_{S_k^c} (t_i^{(1)} u_j^{(2)} - t_i^{(2)} u_j^{(1)}) dS \tag{8}$$

where now the fields must be known at the surface of the cracks. Note that if the cracks are vacancy $t_i = 0$ on each crack surface, then Equation (8) is

$$R_{21} = \frac{1}{4} j\omega \sum_{k=1}^{m} \int_{S_k^c} t_i^{(1)} u_j^{(2)} dS \tag{9}$$

Equation (9) is a generalized scattering model of the homogeneous plane wave reflected by a planar distribution of cracks. It can not only be used to evaluate the reflection of the longitudinal plane waves and shear waves, but most importantly, it can also be used to evaluate the reflection of surface acoustic waves as long as the condition of homogenous plane waves is hold.

REFLECTION BY A PLANAR DISTRIBUTION OF SURFACE BREAKING CRACKS

Consider the scattering of Rayleigh wave by a planar distribution of semi-elliptic surface breaking cracks in the $x-y$ plane embedded at the surface of an infinite, homogenous, isotropic and linearly elastic half space, see Figure 2. Following same argument, Equation (9) is available to be used in this case. The reflection coefficient of Rayleigh waves propagating in the z direction at normal incidence to and from the crack can then be written in the form

$$R_{21} = \frac{1}{4} j\omega \sum_{k=1}^{m} \int_{S_k} \Delta u_z^{(2)} \sigma_{zz}^{(1)} dS \tag{10}$$

where $\Delta u^{(2)}$ is the displacement jump in the perturbed SAW displacement field at the crack surface, and S_k is the side surface area of the cracks.

Usually, the superposition principle of linear elasticity can be used to compute the

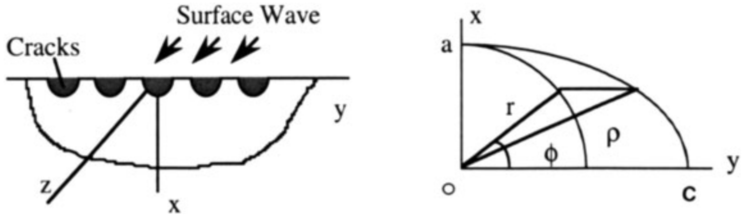

Fig 2. The schematic of a planar distribution of surface cracks and the geometric quantities of a semi-elliptic surface crack.

stress intensity factor (SIF) for many kinds of remote loading as long as the presence of the crack dose not alter the near crack load boundary condition. The SIF for the remote applied load can be computed with a system of crack surface pressure which cancels the stress distribution at the location of the crack plane in the crack free geometry. With this principle, the crack problem can be solved for the corresponding geometry with a system of the crack surface pressure which is equal to the stress distribution on the crack plane for the crack free geometry, while the work done by the boundary load on the boundaries is zero. Imagining a crack growing very slowly to the desired size, all the work of the applied load is done only by the crack surface pressure. The energy conservative principle requires that the work done by the crack surface pressure will be equal to the energy released along the crack front during the crack propagation. By using this elasto-static energy balance approach, one will get

$$\int_S \frac{K^2}{\beta E} d(\Delta S) = \frac{1}{2}\int_S \Delta u_z^{(2)} \sigma_{zz}^{(1)} dS \tag{11}$$

where ΔS is the change of the crack surface area, β is defined as $\beta = 1$ for the plane stress problem and $\beta = 1/(1-\nu^2)$ for the plane strain problem, K is the stress intensity factor of the first kind (the other two stress components of Rayleigh wave have been left out by considering that the crack depth evaluated is less than one wavelength), E is Young's modulus and ν is Poisson's ratio.

Substituting Equation (11) into Equation (10), the reflection coefficient in terms of SIF is achieved as

$$R_{21} = \frac{1}{2} j\omega \sum_{k=1}^{m} \int_{S_k} \frac{K^2}{\beta E} d(\Delta S) \tag{12}$$

Equation (12) can be written as a double integration with the integral path of a/c = constant and transformation relation of $d(\Delta S) = (c/a) a da d\phi$, i.e.,

$$R_{21} = \frac{1}{2} j\omega \sum_{k=1}^{m} \left(\frac{c_k}{a_k}\right) \int_0^{a_k} \int_0^{\pi} \frac{K^2}{\beta E} a da d\phi \tag{13}$$

Using Equation (13), the reflection coefficient of Rayleigh waves for a planar distribution of surface breaking cracks can be evaluated. It should be noted that the present approximation does not take account of the interaction effect between individual crack. Thus the validity of the present analysis is limited to crack density that is not too large.

NUMERICAL EVALUATION OF THE REFLECTION COEFFICIENT

In order to evaluate the reflection coefficient of Rayleigh waves reflected by a planar distribution of surface breaking cracks using Equation (13), the SIF of the cracks under Rayleigh wave stress field have to be solved. However, except for embedded elliptical crack in infinite body subjected to remote uniform tension, no exact solution for the crack surface displacement is available for other part-through crack configurations. Therefore, varieties of numerical method have been used to solve the problems. One of the important SIF analysis method called weight function estimation method has shown powerful usefulness in the numerical evaluation of the SIF for various crack problems, especially, for those where the stress distribution is very complex.

The weight function concept is based on the Betti-Rayleigh reciprocal theorem, so that the SIF solutions for any applied load for a configuration can be determined from the stress solution for one system of applied load as long as boundaries are not changed under the new loading. The weight function concept means that a system of geometry related functions can be found, which is independent of the applied loading, so that the SIF of any other applied load can be computed by an integration of the product of crack surface load distribution and so-called weight function.

Using the weight function estimation method, a numerical evaluation model of the reflection coefficient of Rayleigh wave reflected by a planar distribution of surface breaking cracks has been developed [12] as

$$R_{21} = \frac{1}{2} j\omega\pi \sum_{k=1}^{m} \left(\frac{c_k}{a_k}\right) \int_0^{a_k} \int_0^{\pi} \frac{\left(aM_c Y_0 \sigma_0 \sum_{i=0}^{n} C_i g_i\right)^2}{\beta E} \, da\, d\phi \tag{14}$$

where σ_0 is the stress of Rayleigh wave evaluated at the surface of the substrate in the propagation direction, C_i are the coefficients of a polynomial of the stress of a Rayleigh wave, Y_0 is the dimensionless stress intensity factor of an embedded elliptical crack in the infinite body, M_c is the configuration modification function of the cracks, and g_i is the weight function.

Using Equation (14), the reflection coefficient of a Rayleigh wave reflected by a planar distribution of the semi-elliptic surface breaking cracks can be evaluated numerically as Figures 3 and 4 shown.

REFLECTION BY A DISTRIBUTION OF SURFACE FATIGUE CRACKS

In principle, the reflection coefficient can be computed once the crack opening displacements or the SIF of surface fatigue cracks have been calculated for all cracks. However, this procedure is impractical since large numbers of cracks are dealt with. But, for most commercial metals under fatigue loading, the relation between the number of cracks and the crack length distribution may be found out. Recently, a simulation technique of the microcrack length distribution using Monte Carlo method has been introduced to predict the relation of the microcrack length distribution with the number of the cracks based on the experimental results [4].

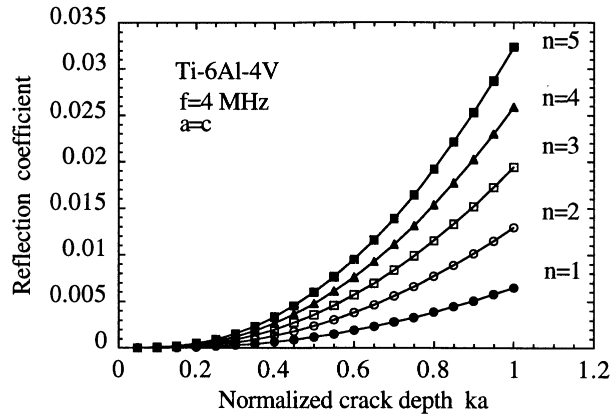

Fig 3. The reflection coefficient vs. the normalized crack depth for varying the number of cracks.

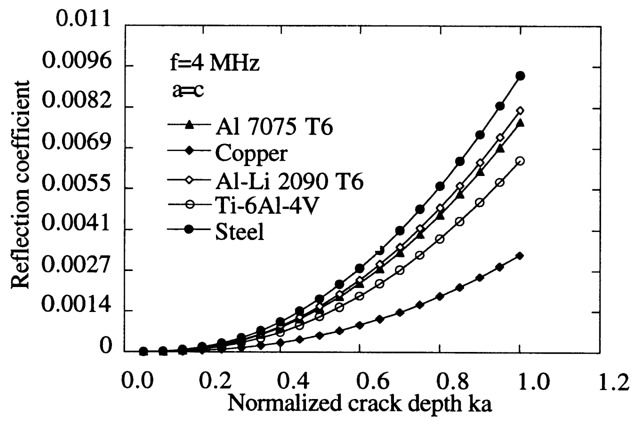

Fig 4. The reflection coefficient vs. the normalized crack depth on alternative materials.

By using this technique, the simulation relation of the number of the cracks with the crack length distribution for given materials may be generally written in the form

$$m = p(a, R, \sigma_{max}, N / N_f) \tag{15}$$

where a is the crack length, R is the ratio of cyclic stress, σ_{max} is the maximum cyclic stress in the substrate and N / N_f is the ratio of fatigue cycling number. Then a simulation scattering model of Rayleigh waves for a planar distribution of the surface breaking fatigue cracks may be expressed as

$$R_{21} = \frac{1}{2} j\omega \int_{a_{min}}^{a_{max}} p(a, R, \sigma_{max}, N / N_f)(\frac{c}{a})\{\int_0^a \int_0^{\pi} \frac{K^2}{\beta E} a da d\phi\} da \tag{16}$$

where a_{min} and a_{max} are the minimum and maximum crack length among the distribution crack lengths, respectively. By using the Equation (16), the reflection of Rayleigh waves by a planar distribution of surface breaking fatigue cracks for given materials can be simulated numerically to predict the scattering behavior of the distribution of the cracks under fatigue courses.

CONCLUSION

A scattering model for Rayleigh waves reflected by a planar distribution of surface breaking cracks has been developed. The weight function estimation method has been introduced into the model to numerically evaluate the stress intensity factors of the cracks for an incident Rayleigh wave stress field. A scattering model of Rayleigh waves for a planar distribution of surface breaking fatigue cracks has also been proposed in a generalized form so that a simulation of the reflection by the cracks can be performed as long as the crack length distribution has been achieved for given materials.

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