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UMI®
EFFECTIVENESS OF TUTORING AID FOR
POTENTIALLY DEFICIENT STUDENTS IN A
MATHEMATICS OF FINANCE COURSE AT DRAKE UNIVERSITY

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Education

Approved:

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1950
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I. INTRODUCTION

Research techniques including comprehensive statistical treatment of data have been applied to many educational problems. Frequently, such problems are of a rather broad nature: institution-wide in scope, a comparison among organizations, or even a study within a single department of an educational unit but covering over-all policy. As has been done in the area of testing, for instance, these techniques can be applied to the much narrower field of a single subject within a department, to such problems as teaching procedures, curriculum content, and student mortality. Though the field of the problem is somewhat narrow, the need for careful methods and rather comprehensive statistical tools in conducting such a study may be great.

At Drake University the department of mathematics, which is in the Liberal Arts College, is essentially a service unit for other departments and colleges on the campus. The Commerce College requires two semesters of mathematics for all of its students, business mathematics, designated Mathematics 14, and mathematics of finance, Mathematics 15. Mathematics 14 includes a review of basic arithmetical operations with work in simple algebra involving business problems, stresses logarithms as a computational aid, and presupposes little, if any, previous high school mathematical background on the part of the student. This course is essentially preparatory for Mathematics 15, which deals with problems from the viewpoints of both the investor and the creditor in such areas as bonds, sinking funds and amortization.
Many students in the Commerce College curriculum are ill-prepared either in background training or in retention of mathematical tools and concepts. These students, as well as others, may lack aptitude for work in mathematics and many times do not appreciate the necessity for an approach, somewhat mathematical, in aiding the understanding of problems of investment and finance. Whatever the reasons, the rate of mortality of students in these courses has been high over the past few years.

On the other hand, due to the relatively simple nature of the mathematical concepts in Mathematics 14, good students are inclined to become bored with phases of the work and perhaps only in a few places are challenged by the work at all. Such problems as these naturally have been of concern to both the mathematics department and the Commerce College.

A comprehensive attack on the problems involves, first, prediction of potentially low and high students. The question of how best to accomplish this presents itself, making use of available information on students. Design of a test to aid in the predictive scheme seems desirable. Secondly, measuring effectiveness of procedures for aiding the low and the high students in their respective problems previously mentioned becomes a point of interest. Thirdly, measuring the effectiveness of the predictive scheme becomes important along with analysis of the test with a view towards its improvement.

Consideration of these problems is the purpose of this study.
II. REVIEW OF LITERATURE

Prediction of academic achievement has been a problem of interest to educators in many areas. Their specific conclusions have not been consistent in many cases, but throughout the literature there are trends with regard to thinking on the matter. Though the area of the present study is confined to a specific course in mathematics, a few of the more general studies have been cited, along with specific studies.

Of seven criterion used for predicting college success, Emme\(^1\) mentioned rank in the high school graduating class as the best single criterion. Intelligence test scores and college marks were used to improve prediction.

Thompson\(^2\) discussed the use of a regression equation for selecting A.S.T. students during World War II, which involved high school class rank, algebra pre-test, and Officer Candidate Test to give the best combination of variables.

Another study indicating the importance of some measure of high school achievement in predicting college success was that of Durflinger\(^3\). He commented that the most effective regression combinations


usually involved an intelligence test, an achievement test, and high school grade averages.

In reviewing studies over a six-year period at the University of Minnesota, Douglass\(^1\) found that combining tests with a high school grade average or with a college grade average gave the best prediction. He further found that coefficients of correlation between predictive variables and measures of scholastic success varied from year to year. On the other hand, multiple coefficients of correlation with scholastic success and combination of predictive variables were less variable from year to year. One of his conclusions was that prediction schemes should be used only within similar colleges or departments.

Among studies dealing with prediction in mathematics, Frederiksen\(^2\) discussed prediction of achievement in elementary calculus for a group of Princeton students and found the Cooperative Survey Test in Mathematics, the College Entrance Examination Board Scholastic Aptitude Test-Part M, and corrected school grades the most effective predictors. Of little or no predictive value were measures of verbal ability, number of terms of mathematics previously studied, and amount of time since high school graduation.


Kreider\textsuperscript{1} found that algebra achievement, algebra course marks, and all-college averages were reasonably effective in predicting calculus marks at Iowa State College, and that high school averages and the American Council on Education Psychological Examination (ACE) contributed somewhat to prediction effectiveness.

In a study at the University of Oregon in 1930, Douglass and Michaelson\textsuperscript{2} noted that average high school mark in all subjects was the best single factor for predicting success in college mathematics. They further concluded that prediction of success was ineffective using amount of high school training in mathematics, average high school mathematics mark, average high school mark in all subjects, rank on ACE, or any combination of these variables.

On the other hand, achievement in high school algebra correlated highest with college algebra achievement in a study by Marshall\textsuperscript{3}. Algebra aptitude and general intelligence ranked next in order of effectiveness for prediction.

Kossack\textsuperscript{4} developed an initial regression equation with five

\begin{itemize}
\item\textsuperscript{1}Kreider, Orlando Clark. Factors pertaining to calculus achievement. Unpublished Ph.D. Thesis. Ames, Iowa. Iowa State College Library. 1949.
\item\textsuperscript{2}Douglass, Harold R. and Michaelson, Jessie H. The relation of high school mathematics to college marks and of other factors to college marks in mathematics. School Review. 44:615-619. 1936.
\item\textsuperscript{3}Marshall, M. V. Some factors which influence success in college algebra. The Mathematics Teacher. 32:172-174. 1939.
\item\textsuperscript{4}Kossack, C. F. Mathematics placement at the University of Oregon. The American Mathematical Monthly. 49:234-237. 1942.
\end{itemize}
In high school algebra, in college algebra, and in a standardized test of mathematical achievement, the scores are measured by the ACT, correlating more highly with success in college algebra than in college algebra alone. The use of the General Knowledge Test (GKT) in a prediction scheme for achievement in college algebra has been studied. The possible value of the American Graduate in Education in Psychology more than two years of mathematics and college mathematics achievement up to two years of mathematics and evidence suggests that the test categories are important for prediction of college achievement. A measure of test score in a continuous variable when these categories are important for college mathematics achievement. He questioned the validity of these categories in correlation analyses in order to develop a method of taking and using the test scores. These variables included placement test scores, high school mathematics course, and number of years of high school mathematics. The method was very efficient of studies involving the number of high school mathematics courses. The method was very efficient of studies involving the number of high school mathematics courses.
It was of interest to note at this point concerning Q-scores on the ACE that Barnes\(^1\) tested the effect of experience in college mathematics on the Q-score. The experiment involved students with a mathematics curriculum and those without mathematics. Not only did he conclude that the slight gain in the Q-score in favor of the mathematics group was not significant, but possible practice effect being a factor when retesting with a similar test was negligible.

Robertson\(^2\) also discussed the inclusion of the American Council on Education Psychological Examination (ACE) as one of the tests in a regression combination for use in sectioning and counseling at Iowa State College. A retention test, and a departmental test given during the tenth recitation period were the others included; however, evidence indicated the latter, alone, to be as effective as the regression equation with the combination of variables.

Although not entirely in agreement as to details, the foregoing studies indicate that among others, some measure of high school achievement, ACE, and placement tests are of value in schemes for prediction of achievement in college mathematics.

In continuing this review of literature, attention has been diverted to studies involving experimental procedures with educational

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\(^1\) Barnes, Melvin W. The relationship of the study of mathematics to Q-scores on the ACE Psychological Examination. School Science and Mathematics. 43:581-582. 1943.

\(^2\) Robertson, Fred. Some phases of the mathematics testing program at the Iowa State College. The Mathematics Teacher. 36:276-302. 1943
problems, illustrating methods that in some cases are typical of educational studies of this nature.

A four-year study of the effectiveness of classifying students into three sections of mathematics, high, middle, and low on the basis of grades in college algebra and trigonometry was reported by Taylor\(^1\) at the University of Illinois. Of particular interest was the method used to nullify effects of teaching due to a particular instructor. To accomplish this, all sections meeting at a particular hour were interchanged among the instructors involved, three times during a course. The administrative and teaching difficulties involved are obvious, not to mention possible disadvantages to the student. The experimental group consisted of students sectioned on the basis of placement tests and the control group consisted of students not sectioned. Statistical analysis consisted of a comparison of the means of the two groups on criterion test scores, with controls maintained on placement tests; and significance of gains was indicated by expressing gain as a percent of the highest mean score.

Babitz and Keys\(^2\) reported an experiment with eight classes of high school chemistry in teaching application of scientific

---


principles. One class designated experimental and one class designated for control were selected from each of two schools in both chemistry I and chemistry II. They did not state if the experimental and control groups from each school and section of chemistry were under the same instructor; consequently, it was not apparent that control was maintained to nullify effects of teaching due to different instructors. Neither was any apparent control made on ability of individual students. Control was maintained with respect to pre-test by choosing "equal" classes as determined by means on the pre-test. Statistical analysis compared means of the experimental and control groups on the achievement measure.

An experimental procedure that involved control by pairing students was described by Noll\(^1\). One psychology class under his instruction from each of two different years was involved. He maintained as nearly similar procedures as possible in teaching the two groups, except that for the experimental class, several written quizzes were given during the course work. For control, 33 students were paired between the classes on the basis of ACE and honor point ratio based on all previous work in college. Statistical analysis consisted of comparison of means and standard deviations between the two groups on achievement criterion for the 33 pairs of students.

Another experiment involving control on factors by pairing and one within the field of mathematics was that discussed by Lueck\(^1\). Experimental and control groups were matched for performance on an initial test of ability to write equations from given problems, 25 pairs of students in all. The experimental group received special instructions in problem analysis leading toward solutions. Analysis of results of the experiment consisted of a comparison of means and standard errors between the two groups. Control was also made by pairing on the basis of intelligence scores and again comparing means on the achievement criterion.

Although some of the foregoing experiments were rather carefully designed and no particular criticism can be made of the analyses as presented, it would seem that most of them should have involved use of more comprehensive statistical techniques to advantage, such as analysis of variance and covariance.

One of the more troublesome difficulties occasionally arising in the application of covariance analysis to educational problems is the matter of unequal frequencies in the subclassifications. Since this eventuality occurred in this study, attention was directed to possible methods for solution of the problem.

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Hazel\textsuperscript{1} presented an example with discrete variables using the method of fitting constants.

The general problem of disproportionality of subclass numbers in the tables of multiple classification was discussed by Snedecor and Cox\textsuperscript{2}, including a summary of the various methods available for solution of the problem with respect to analysis of variance particularly.

Tsao\textsuperscript{3} criticized the methods which deal with the unequal or disproportionate frequencies by deriving the "within" variance from original data, although other variances are derived from data adjusted for the unequal or disproportionate frequencies. Included in this rather complete discussion of the problem was an approximate method for the solution of the covariance analysis under the eventuality of unequal frequencies in the subclasses, with multiple classification, which was used in this study.

The first part of this review of literature contains reference to studies of prediction of college achievement in general and to mathematics achievement in particular. The second part consists of references to examples of experimental studies in various fields.


The purpose of this study was to determine if the prediction of the student's achievement in mathematics can be made from a series of objective tests. In order to answer these questions, a series of aptitude and achievement problems were administered. The problem was to determine the prediction test.

A. Developing a Prediction Test

The data of 1949 were used to determine the achievement of the student in the high school mathematics classes and a score from a test designed to predict the performance of the student in the college mathematics. The three major schemes incorporated in the achievement of a prediction test were:

1. The first problem was to develop a scheme for predicting low and high achievement in mathematics.
2. The second problem was to develop a scheme for predicting the achievement of the student.
3. The third problem was to develop a scheme for predicting the achievement of the student.

The data obtained through the administration of the tests included:

- High school mathematics achievement was rated.
- The ACT test scores resulted from testing of enrollment students.
- School Enrolling Office was selected for the study.
- ACT scores were available for the study.
- Only those for whom least scores were available on the American Council of high school students were compared for the study.
- Nearly all students enrolled in business mathematics at Drake University for the fall semester of 1949 were available for the study.

III. PREDICTING ACHIEVEMENT IN BUSINESS MATHEMATICS
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the item was indicated in order to increase the ability of the item
to differentiate between low and high students.

Whenever evidence from the preliminary work indicated an item was
valuable in differentiating between low and high students, even though
personal opinion deemed otherwise, the item was included in the new
test. This predictive test was built in four parts: Part I, estima-
tion with multiple choice response; Part II, simple arithmetic opera-
tions involving pencil work and a written answer; Part III, simple al-
gebraic equations and logarithms with response to be written; Part IV,
functional concepts with multiple choice response. A copy of the test
is included in the Appendix.

It appeared best from an administrative standpoint to give the
test to students in Mathematics 14 as early in the semester as possible;
therefore, the test was administered the second class meeting of each
of 7 sections of the course under as nearly similar conditions as pos-
sible where factors could be controlled. The same person administered
the test, a stop watch was used, and directions for procedure were as
nearly identical as possible in each section.

Scores on the test were developed in two forms: a predictive test
total score for each student referred to as $X_1$, and a partial test
score for each student not including Part III, called $X_1$. Since Part
III included work in algebra, its contribution towards prediction of
students who might be qualified to enroll in Mathematics 15 without
the course, Mathematics 14, seemed promising. On the other hand it
six weeks, and twice a week, periods, and at the end of the semester

Departmental examinations were given in Mathematics at the
need as a variable.

The part of the score, \( y \), in the prediction, Gaet was also
made college achievement; high school achievement was used as
in terms of grade, the school achievement rank was of value in predicting
students who had been presented showing that high school achievement
was as a variable in the regression equation were indicated. Since

The regression equation is of value in that direction, the
out the assumption that the I-score would

prediction of students has been advanced as a possible source of diffi-

- Poor Reading comprehension
- \( y \)-score in predicting mathematics achievement

To verify these results other studies pointed to the value of the ACT

The role and the higher achievement records, we was indicated that

The ACT \( y \)-score, \( x \), and \( I \)-score, \( x \), were chosen from among

Estimation of students in Mathematics. In

for computing purposes mentioned above, the student described in a scheme for

Using regression equation seemed the most promising method

B. Deriving a Prediction Scheme

In the theoretical background,

How students with be scored, for these students would tend to be week

Seemed likely that the correlation toward prediction of potentiality
under carefully controlled conditions. The scores from these tests were weighted 1, 1, and 2, respectively, corresponding to the weighting used by instructors in determining final marks in the course. The total weighted score was defined as a measure of achievement in the course and designated as variable $Y$. Necessary data were available for 118 of the 169 students originally enrolled in the course.

Designating $Y_e$ as the estimate of $Y$, and letting $y$, $y_e$, $x_1$, $x_2$, $x_3$, and $x_4$ represent the deviations from the respective means of the variables, the linear equation

$$Y_e = a_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 \tag{3.01}$$

yielded an estimate of $Y$ for a particular student when the respective independent variables for the student were used, under the assumption of linear relationships existing among the variables. In deviation form equation (3.01) was

$$y_e = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 \tag{3.02}$$

The best linear estimate of $y$ was determined when the sum of squares of the residuals, $\Sigma(y - y_e)^2$, was minimized, as by the least-squares method, giving the following normal equations:

$$\Sigma x_1y = b_1\Sigma x_1^2 + b_2\Sigma x_1x_2 + b_3\Sigma x_1x_3 + b_4\Sigma x_1x_4$$

$$\Sigma x_2y = b_1\Sigma x_1x_2 + b_2\Sigma x_2^2 + b_3\Sigma x_2x_3 + b_4\Sigma x_2x_4$$

$$\Sigma x_3y = b_1\Sigma x_1x_3 + b_2\Sigma x_2x_3 + b_3\Sigma x_3^2 + b_4\Sigma x_3x_4 \tag{3.03}$$

$$\Sigma x_4y = b_1\Sigma x_1x_4 + b_2\Sigma x_2x_4 + b_3\Sigma x_3x_4 + b_4\Sigma x_4^2$$

The necessary sums for the simultaneous solution of these normal
equations are shown in Table 1.

Table 1

Data for 118 Students
Enrolled in Mathematics 14

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sum</th>
<th>Variable</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1^2$</td>
<td>2,002.551</td>
<td>$x_2x_3$</td>
<td>10,471.746</td>
</tr>
<tr>
<td>$x_2^2$</td>
<td>13,533.356</td>
<td>$x_2x_4$</td>
<td>1,091.797</td>
</tr>
<tr>
<td>$x_3^2$</td>
<td>23,646.110</td>
<td>$x_3x_4$</td>
<td>1,083.788</td>
</tr>
<tr>
<td>$x_4^2$</td>
<td>78,607.331</td>
<td>$x_1y$</td>
<td>29,108.322</td>
</tr>
<tr>
<td>$y^2$</td>
<td>1,152,588.373</td>
<td>$x_2y$</td>
<td>56,162.119</td>
</tr>
<tr>
<td>$x_1x_2$</td>
<td>2,686.966</td>
<td>$x_3y$</td>
<td>61,721.916</td>
</tr>
<tr>
<td>$x_1x_3$</td>
<td>2,839.381</td>
<td>$x_4y$</td>
<td>106,063.933</td>
</tr>
<tr>
<td>$x_1x_4$</td>
<td>3,651.805</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The constants thus determined gave the required regression equation, deviation form:

$$y_e = 9.86228280x_1 - 1.59668397x_2 - .67949733x_3 - .85957776x_4 \quad (3.04)$$

Similarly, a regression equation was developed using three variables, only: partial score, $x_1$; Q-score, $x_2$; and L-score, $x_3$. The advantage of the four variable regression over this three-variable
regression without percentile rank, \( x_4 \), was tested by

\[
F = \left( R_k^2 - R_{k-1}^2 \right) / \left( \frac{1 - R_k^2}{N - k - 1} \right)
\]

(3.05)

where

\( R_k = \) multiple correlation with \( k \) variables

\( R_{k-1} = \) multiple correlation with \( k - 1 \) variables

The value of \( F \) was checked for significance with \( (1) \) and \( (N - k - 1) \) degrees of freedom.

The foregoing procedure was continued and a variable discarded whenever the advantage of retaining the variable, or as might be said, the loss due to discarding the variable, was not significant.

Table 2 gives the coefficients of regression developed for the various combinations of variables, and with these values, the figures for Table 3 may be computed. Table 3 includes multiple correlations.

### Table 2

Coeficients of Regression for Five Combinations of Variables

<table>
<thead>
<tr>
<th>Combination of Variables</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1, X_2, X_3, X_4 )</td>
<td>9.86228280</td>
<td>1.59668397</td>
<td>.67949733</td>
<td>.85957776</td>
</tr>
<tr>
<td>( X_1, X_2, X_4 )</td>
<td>10.24862289</td>
<td>2.04695186</td>
<td>.84474443</td>
<td></td>
</tr>
<tr>
<td>( X_1, X_2, X_3 )</td>
<td>11.91736227</td>
<td>1.32556041</td>
<td>.39219263</td>
<td></td>
</tr>
<tr>
<td>( X_1, X_4 )</td>
<td>13.19273439</td>
<td></td>
<td></td>
<td>.73640256</td>
</tr>
<tr>
<td>( X_2, X_4 )</td>
<td>4.04556327</td>
<td>1.29309793</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3

Analysis of Regression Data for Achievement in Mathematics $x_4$

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Multiple Correlation</th>
<th>F (Advantage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$, $X_2$, $X_3$, $X_4$</td>
<td>4</td>
<td>509,857.73</td>
<td>.665</td>
<td></td>
</tr>
<tr>
<td>Deviation</td>
<td>113</td>
<td>642,730.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_1$, $X_2$, $X_3$</td>
<td>3</td>
<td>457,891.96</td>
<td>.632</td>
<td>9.136***</td>
</tr>
<tr>
<td>Deviations Retaining $X_4$</td>
<td>114</td>
<td>694,696.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_1$, $X_2$, $X_4$</td>
<td>3</td>
<td>502,878.29</td>
<td>.661</td>
<td></td>
</tr>
<tr>
<td>Deviations Retaining $X_3$</td>
<td>114</td>
<td>649,710.08</td>
<td></td>
<td>1.277</td>
</tr>
<tr>
<td>$X_1$, $X_2$</td>
<td>2</td>
<td>462,124.11</td>
<td>.614</td>
<td>7.151***</td>
</tr>
<tr>
<td>Deviations Retaining $X_3$</td>
<td>115</td>
<td>690,464.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_2$, $X_4$</td>
<td>2</td>
<td>364,359.58</td>
<td>.562</td>
<td>24.305***</td>
</tr>
<tr>
<td>Deviations Retaining $X_2$</td>
<td>115</td>
<td>788,228.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>117</td>
<td>1,152,588.37</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** Significant at 1 per cent level

computed from the various combinations of variables and F values computed by formula (3.05) giving tests of significance of the advantages due to retaining the indicated variables.

The usefulness of the various combinations of variables in predicting achievement in Mathematics $x_4$ is indicated by the R's in Table 3. The F values for testing the advantage due to retention of
a particular variable are all highly significant, with the exception of the one for retention of \(X_3\), L-score.

Summarizing the results of this predictive phase of the study, the predictive test score, the ACE Q-score, and the high school graduating class percentile rank all seem to contribute materially to prediction of Mathematics 14 achievement. While the inclusion of the ACE L-score in the predictive scheme raised the multiple correlation slightly, the gain was of no consequence, and the additional work involved in using the variable is considerable. This may suggest either that the L-score is not a satisfactory measure of ability to read and comprehend or that the hypothesis of poor reading comprehension being a major source of difficulty for students in Mathematics 14 is untenable.
IV. MEASURING EFFECTIVENESS OF AID TO STUDENTS

At the beginning of the study, it was impossible to develop a regression equation for predicting potentially deficient or superior students in Mathematics 14 because of the lack of a criterion variable such as Y, the weighted examination score in the course. With the material that was at hand for building the predictive test previously described, the use of the test as an effective single factor for prediction of Mathematics 14 achievement seemed likely. Assuming this effectiveness, the test results were available as a basis for predicting low and high students.

One suggested remedy for the problem of superior students in Mathematics 14 was the elimination of the requirement of the course as a prerequisite to Mathematics 15. Total scores on the predictive test ran from a low of 2 to a high of 34 points of a possible 39 points. A critical examination of the work done on the test papers with highest scores indicated little basis for designating the students involved as potentially superior students. The advisability of permitting any of the students concerned to enroll directly in Mathematics 15 on the basis of the predictive test results seemed very questionable. This phase of the study, consequently, was dropped and attention turned to aiding the potentially deficient students.

One of the sources of difficulty for the slower students in Mathematics 14 was the pressure from new topics undertaken for study in the rather wide area of elementary mathematics that must be
covered in the course. Lack of proper background in mathematics, of course, contributed to this. As a result, the decision was made to require an extra two hours of work per week of the potentially low students, along with the regular three-hour course, in special-help classes under tutors.

The two tutors chosen to do this work were senior students, majoring in mathematics, who had evinced an interest in teaching. Their work during the two hours per week, regularly scheduled, was to review students in current regular classroom topics, to give them opportunity for questions and discussion, and to provide for much work at the blackboard by the potentially low students themselves.

An experiment was designed for the purpose of determining whether or not the special tutoring was effective and covariance was used in analyzing the results of the experiment. Results would indicate an answer to the question of worthwhileness of such aid. If student-tutoring should result in favorable experience for the experimental group over the control group, assignment of a regular instructor to such work would seem even more worthwhile, and the matter of requiring potentially low students to take advantage of such aid would have firm basis.

A. Designing the Experiment

Since Part III of the predictive test included work in algebra and many of the students entering Mathematics 14 did not have this background, the partial predictive score, which did not include
Part III, was made the criterion of potential deficiency. Those students receiving a score of 10 points or below were designated potentially deficient. This gave a total of 66 students, roughly 40 per cent of the 169 enrolled in the course. Approximately 40 per cent of students in previous years had received marks of D or F in Mathematics 14.

Students were already enrolled in seven sections of the course when the predictive test was administered. The seven sections were taught by five different instructors. In order to insure randomness of selection from the sections, the potentially deficient students from all sections were listed alphabetically and numbered consecutively. By use of a table of random numbers, assignment of the 66 students was made, 33 to the experimental group, which was to receive the additional tutoring, and 33 to the control group.

Students in the control group were not told that they had been designated potentially deficient. All students, including those in the experimental group, were allowed the usual privilege of securing assistance from an instructor at any suitable time; however, a record was made by each instructor of any student availing himself of such help and the amount of time spent with him was noted. Had such time for any student in either the experimental or control group been excessive, the student would have been disqualified from the experimental study; however, since the maximum total time listed for any student was only two hours for the entire semester, this effect was not regarded as important.
The experimental group received 21 hours of tutoring assistance during the semester. A close record of attendance to the tutoring sessions was made, and any student absent more than 10 sessions was dropped from the group in the analysis of the results. Through administrative cooperation, rather constant pressure was maintained to insure regular attendance by those assigned to the experimental group.

Every effort was made with both the experimental and control groups to encourage the students so assigned to remain enrolled in the course. At least sixteen students were lost for use in the analysis of results because it was found impossible, for various reasons, to secure all necessary data. For instance, ACE scores, and percentile rank in high school graduation class, variables on which controls were maintained through use of covariance were unobtainable for these students. Despite efforts to maintain enrollment, the numbers dwindled until, at the close of the semester, only 16 students remained in the experimental group and 17 students in the control group on whom all necessary data were available.

The number of students available for analysis was likely too small to give decisive results, even though this eventuality had been anticipated and attempts made to prevent heavy loss of students. Nevertheless, the experimental part of the study has been described in detail to illustrate the type of planning and control that can be applied to the somewhat narrow but none-the-less important questions in the field of education. In this light, the following analysis is presented.
B. Analyzing Results of the Experiment

A preliminary inspection of the weighted examination scores, \( Y \), for the 16 students of the experimental group and the 17 students of the control group showed a mean value of the experimental group, 141.87, compared to a mean value of 130.11 for the control group. While this was an apparent advantage in achievement in favor of the experimental group, it was not necessarily so. Since the seven sections of Mathematics 14 from which the students were selected involved five different instructors, the possible effects due to instructors had to be nullified. It might also have been possible, despite the random selection involved, to have weighted inadvertently the experimental group with people of better ability as reflected by high school percentile rank, for instance. Thus, an analysis by use of covariance of the weighted examination scores was necessary, controlling on partial predictive score, \( Q \)-score, and high school percentile rank. Control on these particular variables was indicated by the results of the work in regression analysis.

An immediate problem in covariance analysis was the matter of unequal frequencies in the subclassifications of the experimental design. Though significant results were not expected because of the small numbers in the two groups, the covariance analysis under the condition of unequal frequency in the subclasses was developed as discussed by Tsao\(^1\) for an approximate solution to the problem.

The following definitions are necessary in presenting the method used:

\[ s = 1, \ldots, p, \] the number of columns
\[ i = 1, \ldots, q, \] the number of rows
\[ x_{sit} = \text{the } t\text{-th observation in the subclass } (s, i) \]
\[ \Sigma n_{si} = n_{s1} \quad (s = 1, \ldots, p), \text{ the total number of observations of row } i \]
\[ \Sigma n_{si} = n_{s1} \quad (i = 1, \ldots, q), \text{ the total number of observations of column } s \]
\[ \Sigma n_{s1} = \Sigma n_{si} = \Sigma n_{s1} = n_{s1}, \quad \text{total number of observations} \]
\[ \bar{n} = \frac{n_{s1}}{pq}, \text{ the mean number of observations in a subclass} \]
\[ \bar{x}_{s1} = \frac{\Sigma x_{sit}}{n_{s1}}, \text{ the mean of subclass } (s, i) \]
\[ \bar{x}_{s1} = \frac{\Sigma x_{s11}}{q}, \text{ the mean of column } i \]
\[ \bar{x}_{s1} = \frac{\Sigma x_{s1}}{p}, \text{ the mean of row } s \]
\[ \bar{x}_{..} = \frac{\Sigma x_{..1}}{q} = \frac{\Sigma x_{s1}}{p} = \frac{\Sigma x_{s11}}{pq}, \text{ the general mean} \]
\[ a_{s1} = \frac{n}{n_{s1}} \Sigma (x_{sit} - \bar{x}_{s1})^2 \]
\[ = \frac{n}{n_{s1}} \left[ \frac{\Sigma x_{sit}^2}{n_{s1}} - \left(\frac{\Sigma x_{sit}}{n_{s1}}\right)^2 \right], \text{ the "adjusted" sum of squares of deviations from the subclass mean for subclass } (s, i) \]
Procedure under the hypothesis of expected equal frequencies in the subclasses of the population involved the following estimates of sums of squares for different sources of variations:

(1) Column, \[ q \bar{x}_s^2 - n \bar{x}^2 = \sum_{i} (q_{si}^2 - q_{si}) \] \[ (4.01) \]

(2) Row, \[ r \bar{x}_i^2 - n \bar{x}^2 = \sum_{s} (r_{si}^2 - r_{si}) \] \[ (4.02) \]

(3) Within, \[ \sum_{s} \sum_{i} (q_{si})^2 \] \[ (4.03) \]

(4) Interaction, column by row, \[ \sum_{s} \sum_{i} (X_{sit} - \bar{x}_{si} - \bar{x}_{s} + \bar{x})^2 = \sum_{s} \sum_{i} (q_{si}^2 - q_{si}) - q \sum_{s} (\bar{x}_s - \bar{x})^2 - \] \[ + \sum_{s} (r_{si}^2 - r_{si}) - r \sum_{i} (\bar{x}_i - \bar{x})^2 + n \bar{x}^2 \] \[ (4.04) \]

Similar definitions are valid, involving another variable \( Y \) for sum of products.

\[ a'_{si} = \frac{n}{n_{si}} [X_{sit} - \bar{x}_{si}] [Y_{sit} - \bar{y}_{si}] \] \[ (4.05) \]

\[ = \frac{n}{n_{si}} [\sum_{t} X_{sit} Y_{sit} - \frac{\sum_{t} X_{sit}}{n_{si}} \frac{\sum_{t} Y_{sit}}{n_{si}} ] \]

, the "adjusted" sum of products for the sub-class \((s, i)\)
Estimates of sums of products are:

1. Column, \( \frac{p \bar{x} \bar{y}}{s} \) - \( n \bar{x} \bar{y} \) \hspace{1cm} (4.06)

2. Row, \( p \sum \bar{x} \bar{y}_i \) - \( n \bar{x} \bar{y} \) \hspace{1cm} (4.07)

3. Within, \( \sum a_i \) \hspace{1cm} (4.08)

4. Interaction, column by row,

\[
\frac{n \sum x_i y_i}{s} - \frac{p \bar{x} \bar{y}}{s} - \frac{p \sum \bar{x} \bar{y}_i}{i} + \frac{n \bar{x} \bar{y}}{}
\]

\hspace{1cm} (4.09)

For the hypothesis of expected equal frequencies in the subclasses of the population, a chi square criterion was used for a test:

\[
\chi^2 = \sum \frac{(n_{si} - \bar{n})^2}{\bar{n}}
\]

\hspace{1cm} (4.10)

with \( (pq - 1) \) degrees of freedom.

The foregoing formulas might appear somewhat formidable to a person without mathematical background who would attempt to use this approximate method. It is interesting to note the apparent significance of the various coefficients of the sums of squares involved in the left members of the equations (4.01) to (4.04), remembering the assumption of expected equal frequencies in the population.

Recalling that \( \bar{n} \) was defined as the mean number of observations in a subclass, \( \bar{n} \) obviously becomes the mean number of observations
per column, since \( q \) is defined as the number of rows involved. Then, under the estimate of sum of squares for column, the factor \( q \bar{n} \) weights each term of the sum according to the mean number of observations within each column.

In a similar fashion, in the estimate of the sum of squares for row, the factor \( p \bar{n} \) weights each term of the sum according to the mean number of observations within each row.

The factor \( \frac{\bar{n}}{n_{ai}} \), which represents a ratio between the mean number of observations within a subclass and the number within a particular subclass \((s, i)\), appears in the derivation for the estimate of the "within" sum of squares. Thus, each subclass sum of squares is weighted according to this ratio, which tends to equalize the contribution of each subclass to this variation.

Formula (4.04) is readily derived under the additivity of the effects, with the first term of the left member representing the "total" sum of squares. This additivity holds only with the factor \( \frac{\bar{n}}{n_{ai}} \) again weighting the contribution to this "total" variation of the variates within each subclass, respectively.

All of these foregoing factors apparently are assumed to be the best weighting factors readily available for equalizing the numbers of observations within the respective categories of the sample, under the hypothesis of expected equal frequencies in the population subclasses.

The basic data for the 16 students in the experimental group and the 17 students in the control group are presented in Table 4. Unequal frequencies in the subclassifications are immediately apparent in the
Table 4

Basic Data for Experimental
and Control Groups

<table>
<thead>
<tr>
<th>Section</th>
<th>Experimental Variable</th>
<th>Control Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_1$</td>
<td>$X_2$</td>
</tr>
<tr>
<td>I</td>
<td>3</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>II</td>
<td>7</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>III</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>IV</td>
<td>4</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>V</td>
<td>9</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>56</td>
</tr>
<tr>
<td>VI</td>
<td>7</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>9</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
table. Using formula (4.10), the value of chi square was computed for testing the hypothesis of equal frequencies in the subclasses of the population. With $\bar{n} = 33/14$ or $2.3571$, $\chi^2 = 3.06$ with 13 degrees of freedom. The probability of a chi square value as large as this from repeated random sampling under the hypothesis was greater than 99 per cent; thus, rejection of the hypothesis was not indicated. Computations then proceeded under the hypothesis according to formulas (4.01) through (4.09). Sum of squares and products resulting from these computations are presented in Table 5.

Covariance analysis proceeded as presented by Johnson and Tsao \(^1\) with respect to the testing and disposition of interaction. A regression equation was developed for the "within" data and for "within plus interaction," using for the latter the summed data for the two sources of variation from Table 5. The sum of squares of deviations from regression was computed in each case by the formula:

$$S.S. = \Sigma y^2 - (b_1 \Sigma x_1 y + b_2 \Sigma x_2 y + b_3 \Sigma x_3 y)$$  \hspace{2cm} (4.11)

The respective sets of regression coefficients, $b_1$, $b_2$, and $b_3$ for "within" and for "within plus interaction" are given in Table 6, along with coefficients computed within various classifications which were needed for later analyses. The analysis and resulting test for the

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\(^1\) Johnson, Palmer O. and Tsao, Fei. Factorial design and covariance in the study of individual educational development. Psychometrika. 10:137-162. 1945.
Table 5

Sum of Squares and Sum of Products
Adjusted Under Equal Frequency Hypothesis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source of Variation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Methods</td>
<td>Sections</td>
<td>Interaction</td>
<td>Within</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d.f. = 1</td>
<td>d.f. = 6</td>
<td>d.f. = 6</td>
<td>d.f. = 19</td>
<td></td>
</tr>
<tr>
<td>$x_1^2$</td>
<td>1.69</td>
<td>34.41</td>
<td>31.36</td>
<td>40.53</td>
<td></td>
</tr>
<tr>
<td>$x_2^2$</td>
<td>24.24</td>
<td>752.67</td>
<td>585.29</td>
<td>989.34</td>
<td></td>
</tr>
<tr>
<td>$x_4^2$</td>
<td>46.77</td>
<td>1,003.48</td>
<td>3,053.85</td>
<td>10,629.39</td>
<td></td>
</tr>
<tr>
<td>$y^2$</td>
<td>3,872.48</td>
<td>33,979.70</td>
<td>42,764.86</td>
<td>99,648.67</td>
<td></td>
</tr>
<tr>
<td>$x_1x_2$</td>
<td>-6.40</td>
<td>-96.85</td>
<td>111.82</td>
<td>144</td>
<td></td>
</tr>
<tr>
<td>$x_1x_4$</td>
<td>8.89</td>
<td>-9.56</td>
<td>17.36</td>
<td>113.53</td>
<td></td>
</tr>
<tr>
<td>$x_2x_4$</td>
<td>-33.67</td>
<td>-159.33</td>
<td>-441.35</td>
<td>-756.51</td>
<td></td>
</tr>
<tr>
<td>$x_1y_1$</td>
<td>-80.88</td>
<td>461.55</td>
<td>795.76</td>
<td>-270.02</td>
<td></td>
</tr>
<tr>
<td>$x_2y$</td>
<td>306.35</td>
<td>1,003.09</td>
<td>4,112.62</td>
<td>-2,597.22</td>
<td></td>
</tr>
<tr>
<td>$x_4y$</td>
<td>-425.67</td>
<td>542.98</td>
<td>-2,245.88</td>
<td>-14,419.41</td>
<td></td>
</tr>
</tbody>
</table>
Table 6

Coefficients of Regression
Computed Within Various Classifications

<table>
<thead>
<tr>
<th>Source</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within</td>
<td>-2.017703</td>
<td>-3.852775</td>
<td>-1.609218</td>
</tr>
<tr>
<td>Within + Interaction (Residual)</td>
<td>11.329014</td>
<td>-.922902</td>
<td>-1.407097</td>
</tr>
<tr>
<td>Methods</td>
<td>137.223934</td>
<td>-115.090734</td>
<td>-118.039275</td>
</tr>
<tr>
<td>Methods + Residual</td>
<td>9.519049</td>
<td>-.569601</td>
<td>-1.392789</td>
</tr>
<tr>
<td>Methods + Within</td>
<td>-4.071780</td>
<td>-3.531487</td>
<td>-1.605177</td>
</tr>
<tr>
<td>Sections</td>
<td>29.217754</td>
<td>5.448921</td>
<td>1.684616</td>
</tr>
<tr>
<td>Sections + Residual</td>
<td>10.553245</td>
<td>.334791</td>
<td>-1.153992</td>
</tr>
<tr>
<td>Sections + Within</td>
<td>2.522826</td>
<td>-1.477058</td>
<td>-1.331698</td>
</tr>
</tbody>
</table>

Significance of interaction are presented in Table 7, with sum of squares under errors of estimate computed by (4.11).

Since the value of $F$ for the test of interaction was not significant, the interaction was combined with "within" for the error or residual data, against which to test main effects, as suggested by Johnson and Tsao\(^1\). Following the previously outlined procedure and

\(^1\)Ibid.
Table 7
Analysis of Covariance
for Testing Interaction

<table>
<thead>
<tr>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction</td>
<td>6</td>
<td>42,764.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td>19</td>
<td>99,648.67</td>
<td>16</td>
<td>65,893.37</td>
<td>4,118.34</td>
</tr>
<tr>
<td>Interaction + Within</td>
<td>25</td>
<td>142,413.53</td>
<td>22</td>
<td>114,406.30</td>
<td></td>
</tr>
<tr>
<td>Adjusted: Interaction</td>
<td></td>
<td></td>
<td>6</td>
<td>48,512.93</td>
<td>8,085.49</td>
</tr>
</tbody>
</table>

\[ F = \frac{8,085.49}{4,118.34} = 1.963 \]

again using equation (4.11) for computing the necessary sums of squares for errors of estimate gave results for the covariance analyses in Table 8 and Table 9. Table 8 includes a test of significance of the effects of methods, and Table 9 includes a test of significance of the effects of sections. Neither value of F is significant.
Table 8
Analysis of Covariance
for Testing Methods Against Residual

<table>
<thead>
<tr>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>1</td>
<td>3,872.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>25</td>
<td>142,413.53</td>
<td>22</td>
<td>114,406.30</td>
<td>5,200.29</td>
</tr>
<tr>
<td>Method + Residual</td>
<td>26</td>
<td>146,286.01</td>
<td>23</td>
<td>119,284.94</td>
<td></td>
</tr>
<tr>
<td>Adjusted: Methods</td>
<td>1</td>
<td>4,878.84</td>
<td></td>
<td>4,878.64</td>
<td></td>
</tr>
</tbody>
</table>

F < 1

Table 9
Analysis of Covariance
for Testing Sections Against Residual

<table>
<thead>
<tr>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sections</td>
<td>6</td>
<td>3,872.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>25</td>
<td>142,413.53</td>
<td>22</td>
<td>114,406.30</td>
<td>5,200.29</td>
</tr>
<tr>
<td>Sections + Residual</td>
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<td>146,286.01</td>
<td>28</td>
<td>146,525.93</td>
<td></td>
</tr>
<tr>
<td>Adjusted: Sections</td>
<td>6</td>
<td>32,119.63</td>
<td></td>
<td>5,353.27</td>
<td></td>
</tr>
</tbody>
</table>

F = 1.029
For the more common test of the main effects, following the foregoing procedure for securing the sum of squares for errors of estimate, the covariance analyses in Table 10 and Table 11 for methods and sections, respectively, are provided. In these cases, the "within" and

Table 10
Analysis of Covariance
for Testing Methods Against Within

<table>
<thead>
<tr>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>Errors of Estimate</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>1</td>
<td>3,872.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td>19</td>
<td>99,648.67</td>
<td>16 65,893.37 4,118.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method + Within</td>
<td>20</td>
<td>103,521.15</td>
<td>17 70,173.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted: Methods</td>
<td>1</td>
<td>4,279.83  4,279.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

"interaction" were not combined as the basis for the F test; rather, the "within" sum of squares provided the experimental error estimate. As before, neither of the values of F for this procedure was significant.

Since the value of F in the tests of section effects was not significant, no difference in teaching effectiveness among instructors could be demonstrated. Although such information might have been of incidental interest in the study, the important thing was to have the
possible effects of variation in teaching segregated as has been done in the analysis. With the non-significant F value for the test of tutoring aid, the apparent advantage of the experimental group over the control group with respect to Mathematics achievement has disappeared. Despite the lack of statistical significance in the difference between the groups receiving and not receiving tutoring aid, any conclusion of ineffectiveness of such aid must be withheld pending an experiment with larger groups than were available in this study.

Table II
Analysis of Covariance
for Testing Sections Against Within

<table>
<thead>
<tr>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>Source</th>
<th>D.F.</th>
<th>S.S.</th>
<th>H.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sections</td>
<td>6</td>
<td>33,979.70</td>
<td>Errors of Estimate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td>19</td>
<td>99,648.67</td>
<td></td>
<td>16</td>
<td>65,893.37</td>
<td>4,118.34</td>
</tr>
<tr>
<td>Sections + Within</td>
<td>25</td>
<td>133,628.37</td>
<td></td>
<td>22</td>
<td>112,311.33</td>
<td></td>
</tr>
<tr>
<td>Adjusted: Sections</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>46,417.96</td>
<td>7,736.33</td>
</tr>
</tbody>
</table>

F = 1.879

By way of summary, two points might be emphasized: first, the experiment was designed and procedures planned and controlled so that statistical analysis would be possible. Such planning is usually administratively possible, where full advantage of available statistical
techniques is secured. For instance, it would have been administra-
tively difficult to have maintained the experiment under one instruc-
tor and even then guarantee of similar learning environment between
sections would have been difficult. Likewise, matching of students
with respect to ability and background as reflected by ACE scores,
predictive test scores, and high school percentile rank in the experi-
mental and control groups would have been exceedingly difficult, if
not impossible. The second point to be mentioned is that in the use
of covariance, eventualities occurring such as the unequal frequencies
in the subclasses, must be met. However, appropriate statistical
techniques have been developed and are available to be applied to such
educational studies.
V. EVALUATING THE PREDICTIVE TEST AND THE REGRESSION FUNCTION

With as complete data as possible maintained throughout the course on the students enrolled in Mathematics 14 and throughout Mathematics 15 where enrollment continued, considerable material was available to be used in evaluating work done initially in the study. One problem was that of improving the predictive test to make it more effective in future use. An item analysis of the test provided helpful information towards that end.

A. Analysis of the Predictive Test

In order to make use of the weighted examination score, $Y$, the criterion of achievement in Mathematics 14, in determining which of the predictive test items might be improved, a tabulation was made for each item, listing the corresponding weighted examination score of each paper under the appropriate heading, "right" or "wrong", depending on whether or not the particular paper had the item in question checked right or wrong. Thus, a necessary element, the dichotomous variable, tendency for correct response, was at hand for computation of the statistic, biserial $r$. Under the assumption of the tendency for correct response being normally distributed and a linear relationship existing between $Y$ and the dichotomous characteristic, the biserial $r$ computed for each test item determined the relationship between the response tendency on an item and achievement in Mathematics 14. Where the relationship was not at least significantly different from an assumed population parameter value of zero,
it would appear the item would bear revision in order to increase its contribution in the test toward predicting high or low achievement.

The formula used for the biserial r computations was

\[ r_{\text{bis}} = \frac{M_1 - M}{\sigma_Y} (Z) \]  

(5.01)

where

\[ r_{\text{bis}} = \text{biserial } r \]
\[ M_1 = \text{mean in first dichotomous category} \]
\[ M = \text{general mean of } Y \]
\[ \sigma_Y = \text{standard deviation of } Y \]
\[ p = \text{proportion of cases in first dichotomous category} \]
\[ Z = \text{height of ordinate in normal curve dividing distribution into two parts of } p \text{ and } (1 - p) \]

The scores of 139 students were used for these computations. The results are listed in Table 12. The formula\(^1\) used for testing the significance of the biserial r's was

\[ t = \sqrt{\frac{r^2 (N - 2)}{\frac{E^2}{z^2} - r^2}} \]  

(5.02)

with \((N - 2)\) degrees of freedom.

---

\(^1\)Wert, James E. and Others. The discriminant function and multiple biserial R. Unpublished mimeographed manuscript. The Educational Research Laboratory, 315 Curtiss Hall, Iowa State College, Ames, Iowa. 1949.
Table 12
Item Analysis Data
for Predictive Test

<table>
<thead>
<tr>
<th>Item</th>
<th>$M_1$</th>
<th>$P$</th>
<th>$r_{bis}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>220.62</td>
<td>.56115</td>
<td>.317**</td>
</tr>
<tr>
<td>2</td>
<td>233.94</td>
<td>.34532</td>
<td>.337**</td>
</tr>
<tr>
<td>3</td>
<td>212.07</td>
<td>.54676</td>
<td>.186</td>
</tr>
<tr>
<td>4</td>
<td>256.92</td>
<td>.18705</td>
<td>.415**</td>
</tr>
<tr>
<td>5</td>
<td>218.16</td>
<td>.52518</td>
<td>.259*</td>
</tr>
<tr>
<td>6</td>
<td>216.47</td>
<td>.61151</td>
<td>.286**</td>
</tr>
<tr>
<td>7</td>
<td>226.87</td>
<td>.53957</td>
<td>.390**</td>
</tr>
<tr>
<td>8</td>
<td>216.79</td>
<td>.20863</td>
<td>.132</td>
</tr>
<tr>
<td>9</td>
<td>219.89</td>
<td>.41007</td>
<td>.226*</td>
</tr>
<tr>
<td>10</td>
<td>245.83</td>
<td>.08633</td>
<td>.265</td>
</tr>
<tr>
<td>Part II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>197.18</td>
<td>.63309</td>
<td>-.036</td>
</tr>
<tr>
<td>2</td>
<td>205.53</td>
<td>.53237</td>
<td>.090</td>
</tr>
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<td>3</td>
<td>202.73</td>
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<td>4</td>
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<td>.66187</td>
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</tr>
<tr>
<td>5</td>
<td>231.73</td>
<td>.63309</td>
<td>.567**</td>
</tr>
<tr>
<td>6</td>
<td>228.28</td>
<td>.48201</td>
<td>.366**</td>
</tr>
<tr>
<td>7</td>
<td>242.03</td>
<td>.28058</td>
<td>.380**</td>
</tr>
<tr>
<td>8</td>
<td>206.53</td>
<td>.76978</td>
<td>.191</td>
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<tr>
<td>9</td>
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<td>.27338</td>
<td>.551**</td>
</tr>
<tr>
<td>10</td>
<td>254.94</td>
<td>.24460</td>
<td>.452**</td>
</tr>
</tbody>
</table>

$\sigma_y = 96.18$

$M = 199.23$

* Significant at 5% level

** Significant at 1% level

137 degrees of freedom
Table 12
(Continued)

<table>
<thead>
<tr>
<th>Item</th>
<th>$M_1$</th>
<th>P</th>
<th>$F_{bis}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>208.13</td>
<td>.88489</td>
<td>.421**</td>
</tr>
<tr>
<td>2</td>
<td>212.89</td>
<td>.84173</td>
<td>.492**</td>
</tr>
<tr>
<td>3</td>
<td>253.36</td>
<td>.31655</td>
<td>.500**</td>
</tr>
<tr>
<td>4</td>
<td>276.43</td>
<td>.10072</td>
<td>.464**</td>
</tr>
<tr>
<td>5</td>
<td>280.05</td>
<td>.15108</td>
<td>.546**</td>
</tr>
<tr>
<td>6</td>
<td>309.00</td>
<td>.05755</td>
<td>.561**</td>
</tr>
<tr>
<td>7</td>
<td>264.33</td>
<td>.10791</td>
<td>.390**</td>
</tr>
<tr>
<td>8</td>
<td>318.30</td>
<td>.07194</td>
<td>.658**</td>
</tr>
<tr>
<td>9</td>
<td>269.25</td>
<td>.05755</td>
<td>.358**</td>
</tr>
<tr>
<td>10</td>
<td>353.00</td>
<td>.05036</td>
<td>.753**</td>
</tr>
<tr>
<td>11</td>
<td>273.74</td>
<td>.33094</td>
<td>.709**</td>
</tr>
<tr>
<td>12</td>
<td>310.10</td>
<td>.14388</td>
<td>.734**</td>
</tr>
<tr>
<td>13</td>
<td>337.83</td>
<td>.04517</td>
<td>.687**</td>
</tr>
<tr>
<td>Part IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>200.19</td>
<td>.92806</td>
<td>.068</td>
</tr>
<tr>
<td>2</td>
<td>211.11</td>
<td>.41007</td>
<td>.131</td>
</tr>
<tr>
<td>3</td>
<td>215.65</td>
<td>.61151</td>
<td>.272**</td>
</tr>
<tr>
<td>4</td>
<td>200.70</td>
<td>.46043</td>
<td>.018</td>
</tr>
<tr>
<td>5</td>
<td>238.19</td>
<td>.51799</td>
<td>.527**</td>
</tr>
<tr>
<td>6</td>
<td>211.47</td>
<td>.24460</td>
<td>.099</td>
</tr>
</tbody>
</table>

$\bar{y} = 96.18$

$M = 199.23$

* Significant at 5% level

** Significant at 1% level

137 degrees of freedom
Some question of the justification of using the biserial r in cases where the value of p is very low or very high, say p less than or equal to .05 or p greater than or equal to .95, might be raised. The latter case did not occur in Table 12. In the case of item 10 and item 13 of Part III, while the p value was only slightly higher than .05 and slightly less than .05, respectively, the "right" category contained the three highest scores in both cases, reinforcing confidence in the relatively high biserial r's. Again it would seem that those items, at least, for which the biserial r's were not significantly different from zero at the 5% level, should have very critical inspection with a view towards revision.

Next, the dichotomy was formed between the group of students receiving the lowest 40 weighted examination scores and the remaining group, corresponding in proportion to those who received marks of F in Mathematics 14 or who dropped the course and to those who received marks of D or above, respectively. A biserial r was then computed between $X_1^*$, the predictive test total score, and this tendency to success. Under the same assumptions and procedures of the foregoing discussion, the following value was computed with scores for 137 students: $r_{bis} = .733$. The biserial r value indicated a rather high degree of relationship between the prediction test score and the tendency to success in Mathematics 14.

B. Appraising the Regression Function

Two questions were apparent in an attempt to appraise further the
usefulness of the regression equation developed in Chapter III. First, since the regression equation was developed using the predictive-test partial score, \( X_1 \), the possibility arose of the use of the predictive-test total score, \( X'_1 \), giving more effective results. Second, did the best combination of variables predict Mathematics 14 achievement any more effectively for students who proved to be in the low achievement group than for all others in the study; and, likewise, was prediction any more effective within the group of students who proved to be in the high achievement group compared with all others?

In developing an approach to the foregoing questions, the initial step was to compute a regression equation involving the original 118 students and with the same variables except for a substitution of \( X'_1 \) for \( X_1 \). Under the same procedure as described in Chapter III, the resulting equation was

\[
y_e = 10.61391297x'_1 - 0.75810250x_2 - 0.5969104x_4
\]  

(5.03)

A multiple regression coefficient was then computed. This result, \( R = 0.737 \), was considerably higher than the \( R \) for the best combination of variables in Chapter III.

The second step involved the changing of equation (5.03) to raw score form and then computing a \( Y_e \) value for each of the 118 students. Likewise, using the coefficients listed in Table 2 for the combination of variables \( X_1, X_2, \) and \( X_4 \), a \( Y_e \) value was computed for each of the same 118 students. A series of \( D \) values was then computed by taking
the absolute value of the difference between the estimate \( Y_e \) and the weighted examination score \( Y \) for each student, as \( D = |Y_e - Y| \). Similarly, a series of \( D' \) values was computed using \( Y'_e \) as \( D' = |Y'_e - Y| \) for each student.

Under the assumption that these \( D \) and \( D' \) values would provide comparable measures of the effectiveness of prediction of Mathematics 14 achievement, \( t \)-tests on the differences between mean \( D \) and \( D' \) values of both a low achievement group and a high achievement group were performed. Since these data were obviously paired and a high correlation would be expected between the \( D' \)s and \( D'' \)s, a formula of the usual type for paired data was used:

\[
t = \frac{\bar{D} - \bar{D}'}{\sqrt{\frac{\sum d^2 + \sum d'^2 - 2 \sum dd'}{N(N-1)}}}
\]

for \( (N-1) \) degrees of freedom, and where

\[
\bar{D} = \text{mean of } D \text{ values}
\]

\[
\bar{D}' = \text{mean of } D' \text{ values}
\]

\[
d = (D - \bar{D})
\]

\[
d' = (D' - \bar{D}')
\]

\[
N = \text{number of cases in the group}
\]

For the group of students of low achievement in Mathematics 14, approximately the lower 30\% of \( Y \) scores (40 students) were selected, corresponding to the proportion of students who received a mark of \( F \).
in the course. The results of the computations using formula (5.04) follow:

\[
\overline{D} = 68.75 \\
\overline{D'} = 60.60 \\
t = 2.739, \text{ significant at the } 1\% \text{ level with } 39 \text{ degrees of freedom}
\]

This evidence supported the higher multiple R resulting from the regression combination including \(X_1'\) instead of \(X_1\). The mean of the \(D'\) values being significantly smaller than the mean of the \(D\) values was evidence for concluding that the regression equation involving \(X_1'\) was the most effective in prediction of potentially deficient students.

For the group of students of high achievement in Mathematics 14, the upper 10%, approximately, of \(Y\) scores (12 students) were selected, corresponding to the proportion of students who received a mark of \(A\) or \(A^+\) in the course. The results of the computations, again using formula (5.04), follow:

\[
\overline{D} = 68.417 \\
\overline{D'} = 60.250 \\
t = .5198, \text{ for } 11 \text{ degrees of freedom, a non-significant value}
\]

At least for these few students with highest achievement, the regression combination including \(X_1'\) instead of \(X_1\) did not predict achievement significantly more effectively.

Using \(D'\) values, a t-test was used to test the difference between
means of the lower 40 students and the remainder of the 118 students. A formula of the usual type for unpaired data was used:

\[ t = \frac{\bar{D}_1' - \bar{D}_2'}{\sqrt{\frac{\Sigma d_1'^2}{N_1} + \frac{\Sigma d_2'^2}{N_2} - 2 \frac{N_1 - N_2}{N_1 N_2}}} \]  

(5.05)

for \((N_1 + N_2 - 2)\) degrees of freedom,

where

- \(\bar{D}_1'\) = mean of \(D'\) values for first group of students
- \(\bar{D}_2'\) = mean of \(D'\) values for second group of students
- \(d_1' = (D_1' - \bar{D}_1')\)
- \(d_2' = (D_2' - \bar{D}_2')\)
- \(N_1\) = number of students in first group
- \(N_2\) = number of students in second group

\(D'\) values were used rather than \(D\) values because of the apparent advantage in prediction already demonstrated in using \(X_1'\) rather than \(X_1\). The results of the computations were as follows, considering the low achievement group as the first group:

- \(\bar{D}_1' = 60.600\)
- \(\bar{D}_2' = 54.321\)
- \(t = 2.784\), a highly significant value for 116 degrees of freedom

With the mean \(D'\) value of the 78 students being smaller than the mean of the 40 students, evidence indicated the best regression combination
was not predicting achievement within the low group as effectively as within the remaining students.

Again using \( D' \) values, a similar t-test using formula (5.05) was invoked to test the difference between means of the upper 12 students and the remaining 106 students. Results of computations were as follows, considering the upper 12 students the first group:

\[
\bar{D}_1' = 60.250 \\
\bar{D}_2' = 56.019 \\
t = 0.364, \text{ a non-significant value for 116 degrees of freedom}
\]

While the mean of the second group, the 106 students, was somewhat smaller than that of the first group, there was little evidence to support a conclusion that the best regression combination, involving \( X_1' \), predicted achievement less effectively for the high achievement group than all others in the study. However, there was no evidence of more effective prediction within the high achievement group, one of the groups for which use of the regression equation had been anticipated.

C. The Ultimate Concern, Achievement in Mathematics of Finance

The foregoing remarks have all had to do with achievement in Mathematics 14. The second semester course Mathematics 15 was the ultimate concern of both students and faculty. The consequent question was whether or not there was statistical evidence available to
show any degree of correlation between achievement in the first course and achievement in the second semester's work.

One approach to the question raised that was possible from data at hand was additional use of biserial r. A dichotomy was set up on the variable tendency to succeed in Mathematics 15: first, those students who did not enroll in the course, those who dropped from the course, and those who received the 23 lowest scores of a weighted examination score in Mathematics 15, similar in detail to that for Mathematics 14; second, those students who received the 29 highest scores in the Mathematics 15 weighted examination score, corresponding somewhat to successful achievement in the course. The 53 students with weighted examination scores available were only a portion of those enrolled in the second semester course, and the 23 lowest scores probably correspond to marks of F and D. The first dichotomy included a total of 106 students. Assuming a linear relationship between Y, Mathematics 14 criterion of achievement, and the tendency to succeed in Mathematics 15, and also assuming this tendency to be normally distributed, biserial r was computed for the two variables using formula (5.01): \( r_{\text{bis}} = .815 \). This evidence indicated a definite relationship between Mathematics 14 achievement and the tendency to succeed in Mathematics 15.

Further evidence of such a relationship, though overlapping somewhat the foregoing, was found by computing a correlation coefficient between the weighted examination scores for the two courses, for those 52 students on whom data were available. This gave a value of \( r = .6122 \).
From the evidence described, it appeared there was some justification for assuming the existence of at least an ordinary degree of relationship between achievement in Mathematics 14 and Mathematics 15. Any effort to aid the low ability students in the basic work of the first course appears to have some justification with a view toward eventually successful achievement in the second course. This entails a continuing appraisal of the predictive scheme, perhaps of the sort described in this chapter, in order to secure maximum effectiveness in its ability to classify the potentially deficient student.
VI. SUMMARY

The two semesters' work in mathematics required of Commerce College students at Drake University is the source of great difficulty for many students. The first semester course, Mathematics 14, covers elementary work through simple algebra, preparatory to the second course, Mathematics 15. The latter course deals with fairly complex problems of finance such as bonds, sinking funds, and amortization, all from viewpoints of both the investor and creditor. Many commerce students have little background or aptitude for the work, and mortality in the courses has been exceedingly high. Too, the work of the first course is of such an elementary nature that students of superior background and ability are seldom challenged.

A comprehensive study of the foregoing problem involved many research techniques and a variety of statistical treatments. A discussion of the approach and method used was the purpose of this study.

Of first concern was the development of a predictive scheme for classifying potentially deficient and superior students in Mathematics 14. A predictive test was developed as part of the scheme, consisting of simple arithmetical examples and stated problems, simple algebraic equations, and functional relationships. Preliminary administration of a similar test provided valuable information for determining items to include in the predictive test. Scores were developed for each student in two forms: $X_{1}$, a total score; and $X_{1}$, a partial score, which did not include results of that section of the test involving
A regression equation was then developed using \( X_1 \) as one of the independent variables. There were 118 students enrolled in Mathematics 14 in the fall of 1949 for whom needed data were available for this part of the study. Other independent variables, selected from readily available records, were the ACE Q-score \( X_2 \), the ACE L-score \( X_3 \), and the high school graduating class percentile rank \( X_4 \). Dependent variable \( Y \) was a weighted examination score developed from departmental examinations in Mathematics 14, the criterion-of-achievement in the course.

Each of the foregoing independent variables was deleted in turn from the respective regression combination, beginning with \( X_4 \) out of the combination of the four variables \( X_1, X_2, X_3, X_4 \). The loss due to such deletion was tested and found to be non-significant only in the case of \( X_3 \). The resulting best combination of variables gave the equation, deviation form:

\[
y_e = 10.24862289x_1 + 2.04695186x_2 + .84474443x_4
\]

where \( y_e \) is the estimate of \( y \). The multiple correlation coefficient's value was .661.

Development of such a regression equation at the beginning of the study was impossible because of the lack of a criterion of achievement variable. Consequently, in order to move ahead with aid to students in Mathematics 14 and to provide a basis for evaluating such aid, an experiment was set up involving the predictive test alone. This
experiment involved only predicted low-achievement students, for a critical review of the predictive-test results gave little basis for classifying any student as potentially superior.

Since the predictive-test partial score did not include work in algebra and many of the students in Mathematics 14 lacked background in algebra, the partial score was used as the criterion of potential deficiency. The potentially deficient students were already enrolled in the seven sections of the course, when the predictive test was administered. Random selection of the predicted low students was made from the seven sections, 33 students assigned to the experimental group and 33 students to the control group. The experimental group received approximately two hours per week of regular review work under student tutors. The control group received the usual treatment afforded Mathematics 14 students, which included access to the five instructors involved for assistance. Despite efforts to maintain enrollment, by the end of the semester the number of students for whom complete data were available dropped to 16 experimental students and 17 control students.

Despite the small number of students with data available for analysis and the likelihood of indecisive results, statistical analysis was developed. Since unequal frequencies existed in the subclasses of the experimental design, covariance analysis as developed by Tsao\(^1\)

---

was used. Under the assumption of expected equal frequencies in the subclasses of the population, which assumption was tested, this procedure gave an approximate solution to the problem. The weighted examination score \( Y \) was used again as the criterion of achievement in Mathematics 14. The mean of the experimental-group weighted-examination scores was 141.87 compared favorably to a mean value of 130.11 for the control group. With control on variables \( X_1, X_2, \) and \( X_4 \) as was indicated by the previous work in regression, however, this difference proved to be of no significance. Despite the lack of statistical significance in the difference between the groups receiving and not receiving tutoring aid, any conclusion of ineffectiveness of such aid must be withheld pending an experiment with larger groups than were available in this study.

A continuing appraisal of the predictive scheme would be desirable in an effort to secure maximum ability of the scheme to differentiate potentially deficient students or potentially superior students. Following are some of the techniques used and conclusions reached in attempting such evaluation:

1. Biserial r's computed for each item of the predictive test using Mathematics 14 achievement criterion and response tendency, "right" or "wrong," gave an indication of the contribution of each item.

2. A rather high degree of relationship existed between the predictive-test total score and the tendency to succeed in Mathematics 14 as indicated by a biserial r value of .733.
(3) Not only did a regression equation involving the predictive-test total score, as opposed to the one involving the partial score, predict more accurately for low achievement students, but also more accurately for all students as shown by the higher $R$ value of .737.

(4) The regression equation involving the predictive-test total score did not predict significantly more effectively for high achievement students than did the one involving the partial score.

(5) Prediction within the group of students of low achievement was less effective than within the remaining group.

(6) Prediction within the high achievement group of students was less effective, but not significantly, than within the remaining group.

(7) At least an ordinary degree of relationship existed between Mathematics 14 achievement and achievement in Mathematics 15, tending to give some justification to efforts to aid students in the basic work in the hope of eventual success in Mathematics 15.

From the foregoing account, the two ensuing inferences seemed warranted with respect to introductory courses in Mathematics of finance. Potentially deficient students may be predicted with a degree of accuracy justifying the time, effort and expense of such prediction in student personnel routine. Student tutorial aid probably is effective but lack of a significant difference, with the small group of available students, suggests the possibility of exploring other teaching procedures, such as changing from a three to a four-hour course, for potentially deficient students.
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VIII. APPENDIX
Estimation  PART I  8 minutes

Write the letter of the closest answer to your estimation in the answer blank.

1. $720.9 \div 18.327$
   Ans: (a) 40  (b) 33  (c) 45  (d) .04  (e) 500
   1. __

2. $18.726 \times 423.3$
   Ans: (a) 802.3  (b) 9363.7  (c) 8023  (d) 7065  (e) 79,360
   2. __

3. $720.9 + 163.4 - 411.36 + 18.2 - 187.36$
   Ans: (a) 470  (b) 300  (c) 250  (d) 190  (e) 400
   3. __

4. $\frac{147.3}{63.1} \div 67.2$
   Ans: (a) .65  (b) .52  (c) .04  (d) 3.1  (e) .40
   4. __

5. A refrigerator costs $259.95 less 15% discount for cash.
   - What is the cash price?
   Ans: (a) $230  (b) $210  (c) $220  (d) $250  (e) $240
   5. __

6. Four-sevenths is what percent?
   Ans: (a) 62  (b) 55  (c) 47  (d) 74  (e) 68
   6. __

7. If 7# nails cost 18 cents, what would 100# nails cost?
   Ans: (a) $2.80  (b) $2.00  (c) $2.50  (d) $1.80  (e) $3.00
   7. __

8. $2637.3 \times .0012$
   Ans: (a) 65.2  (b) 7.53  (c) .94  (d) .26  (e) 83.3
   8. __

9. $4 \frac{1}{2} \times 6 \frac{2}{3}$
   Ans: (a) 28  (b) 24 $\frac{2}{15}$  (c) 24 $\frac{3}{8}$  (d) 26 $\frac{1}{2}$  (e) 25
   9. __

10. Express $3 \frac{1}{4}$ as %.
    Ans: (a) 3.25  (b) 325  (c) .0325  (d) .325  (e) 32500
    10. __
Write your answer in the indicated blank.

1. Add: 6287 + 382 - 1476 + 18

2. Multiply: 5 yds. 1 ft. 10 in. by 3

3. Divide: 7 hrs. 10 min. 8 sec. by 3

4. 28% of 28 is what?

5. 54 is what % of 75?

6. \( \frac{11}{15} \div \frac{4}{5} = ? \)

7. \( 2 \frac{3}{5} + 1 \frac{2}{3} - \frac{3}{4} = ? \)

8. \( 180.3 \times 1.06 = ? \)

9. 312 is 32% of what number?

10. \( 4874.1 \div 21.22 = ? \)
PART III
12 minutes

Write your answer in the indicated blank.

1. Solve for \( x \): \( 3x = 7 \)

2. Solve for \( x \): \( \frac{x}{3} = 5 \)

3. Solve for \( x \): \( \frac{2x}{3} \) \(- \frac{1}{2} = \frac{3}{4} \)

4. Solve for \( x \): \( \frac{7}{2x} \) \(- \frac{1}{3x} = 2 \)

5. Solve for \( x \) and \( y \): \( 2x - 3y = 5 \)

\(... x - 4y = 5 ...

\( x = \)(

\( y = \)(

6. If \( \log \) 325.47 = 2.51251, what does \( \log \) .0032547 equal?

7. If \( \log \) 2.261 = .35430 and \( \log \) 2.262 = .35449, what does \( \log \) of 2.2617 equal?

8. \( 4^{-1} = \) ?

9. \( \frac{3}{2} = \) ?

10. If \( \log \) 2 = .3010 and \( \log \) 3 = .4771, what is \( \log \) \( 6^2 \)?

11. Given: \( I = PRT \). Solve for \( R \).

12. Given: \( S = \frac{N}{2} (A + L) \). Solve for \( L \).

13. \( \frac{5x}{2} + \frac{3x}{6} + 2 - \frac{7x}{4} = ? \)
**PART IV**

5 minutes

Write the letter of the correct answer in the answer blank.

1. One day a boy caught $F$ fish, then gave $A$ of them away. The next day he caught $G$ fish more and sold half of them. How many fish did he have left?

   (a) $F + B - \frac{A}{2}$  
   (b) $F - A + \frac{G}{2}$  
   (c) $F + A + \frac{G}{2}$  
   (d) $F + \frac{A}{2} - G$  
   (e) None of these

2. One roll of wire will reach $T$ feet. How many rolls of such wire are needed to reach $F$ feet?

   (a) $\frac{F}{T}$  
   (b) $\frac{T}{F}$  
   (c) $T \times F$  
   (d) $T + F$  
   (e) None of these

3. John has $\$2$ more than Jim. If John has $\$X$, how much does Jim have?

   (a) $2 - x$  
   (b) $2 + x$  
   (c) $2x$  
   (d) $x - 2$  
   (e) None of these

4. If 5 books are bought for $\$x$ per book and 10 books are bought for $\$y$ per book, what is the average cost of each book?

   (a) $\frac{x + y}{15}$  
   (b) $\frac{x + y}{2}$  
   (c) $\frac{5x + 10y}{2}$  
   (d) $\frac{5x + 10y}{15}$  
   (e) None of these

5. Given: $I = PRT$. If $P$ and $T$ are unchanged and $R$ is doubled, what happens to $I$?

   (a) Remains unchanged  
   (b) Is halved  
   (c) Is doubled  
   (d) None of these

6. Given: $I = PRT$. If $I$ and $R$ are unchanged, and $P$ is halved, what happens to $T$?

   (a) Remains unchanged  
   (b) Is halved  
   (c) Is doubled  
   (d) None of these