USE OF THE VELOCITY OF HIGHER ORDER LAMB MODES IN THE MEASUREMENT OF TEXTURE

Y. Li and R. B. Thompson

Department of Engineering Science and Mechanics
Ames Laboratory, Iowa State University
Ames, Iowa 50011

INTRODUCTION

The use of ultrasonic velocity measurements to determine the texture (preferred grain orientation) of metal plates has been the subject of considerable recent interest. The foundation for these procedures lies in the mathematical description of texture, in which the crystallite orientation distribution function (CODF) is expanded as a series of spherical harmonics. In the notation of Roe [1,2], the expansion of the ODF takes the form

$$w(\xi,\psi,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=-\ell}^{\ell} W_{\ell mn} Z_{\ell mn}(\xi)\, e^{-im\psi} e^{-im\phi} \qquad (1)$$

where θ,ϕ and ψ are Euler angles describing the orientation of a particular crystallite with respect to the sample axes, $\xi=\cos\theta$, the $Z_{\ell mn}$ are the Generalized Legendre functions, and $W_{\ell mn}$ are the orientation distribution coefficients (ODC's). A similar relation has been developed by Bunge [3], using the expansion coefficients $C_{\ell}^{\mu\nu}$. Knowledge of either set of ODC's fully specifies the CODF, and hence the texture.

Ultrasonic measurements of texture are based on the fact that preferred grain orientations produce an anisotropy in the ultrasonic wave speed. Theoretical models have been been developed relating the ODC's to the anisotropic elastic constants, C_{IJ}, and ultimately to wave speeds. Because of the fourth rank nature of the elastic constants, only the ODC's of order $\ell \leq 4$ influence these wave speeds. For the case of cubic crystallites, the only nonvanishing, independent coefficents are W_{400}, W_{420} and W_{440}.

One of the most promising schemes for measurement of the ODC's has been based on measurements of the velocities of guided modes propagating in the plane of the plate, as shown schematically in Figure 1. In a promising configuration, use is made of the angular variation of the velocities of the SH_0 and S_0 modes [4]. Figure 2 presents the dispersion curves and deformation profiles of these modes for an isotropic plate. It should be noted that the long wavelength limit of the S_0 mode velocity is required, which rigorously entails correction for dispersion. However these corrections are small as long as the wavelength is large with respect to the plate thickness. By measuring the velocities at 0°, 45° and 90°

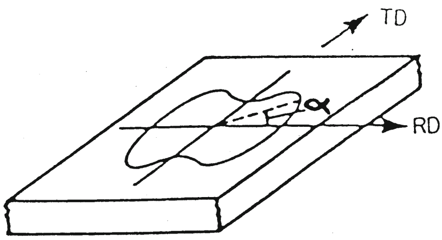

Fig. 1 Angular variation of ultrasonic velocity

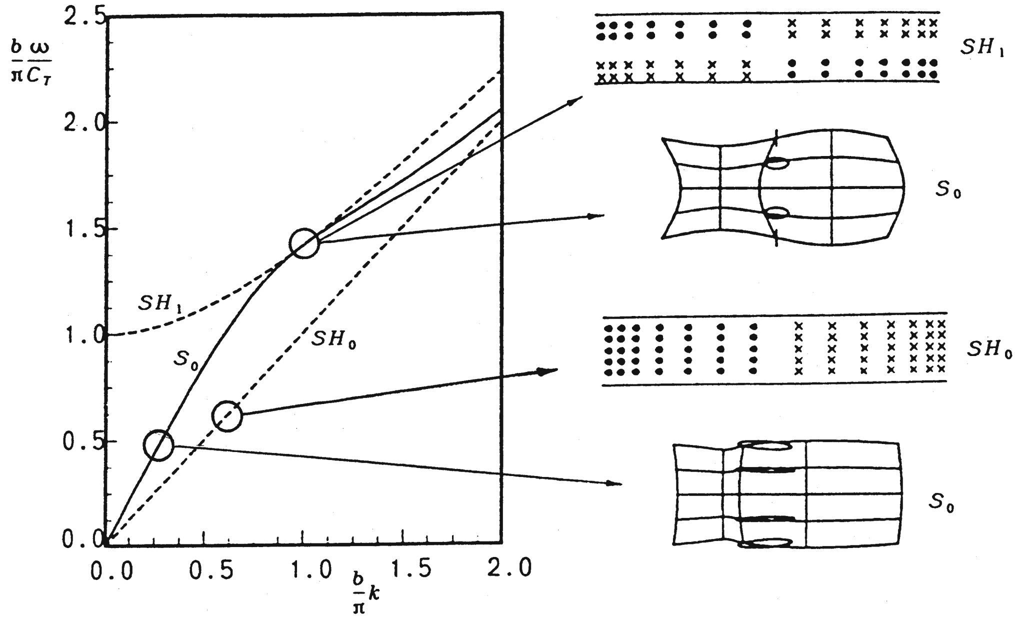

Fig. 2 Isotropic dispersion curves and displacement profiles of SH_1, SH_o and S_o modes.

with respect to the rolling direction, it has been shown that all three ODC's can be deduced from S_o mode data, while W_{400} and W_{440} can be deduced from SH_o mode data [4,5]. For either mode, relative measurements of the angular dependences of velocities can be used to predict W_{420} and W_{440}. However absolute velocity measurements are required to predict W_{400}. This fundamental consequence of the fact that the basis function having the coefficient W_{400} varies only with the polar angle θ, as can be seen from Eq. (1). When the polar axis is chosen normal to the plate, this contribution is independent of rotations of crystallites in the plane of the plate. Consequently, no information regarding W_{400} can be obtained from angular variations of a particular mode velocity in the plane of the plate. The formulae for predicting W_{400} follow [4].

$$W_{400} = \frac{35\sqrt{2}\,\rho}{16\pi^2 C^o}\left[V^2_{SH_o}(45°) + V^2_{SH_o}(0°) - 2(T/\rho)\right] \quad (2a)$$

$$W_{400} = \frac{35\sqrt{2}\,\rho}{32\pi^2[3+8(P/L)+8(P/L)^2]C^o}\left[V^2_{S_o}(0°)+V^2_{S_o}(90°)+2V^2_{S_o}(45°)-4(L-P^2/L)/\rho\right] \quad (2b)$$

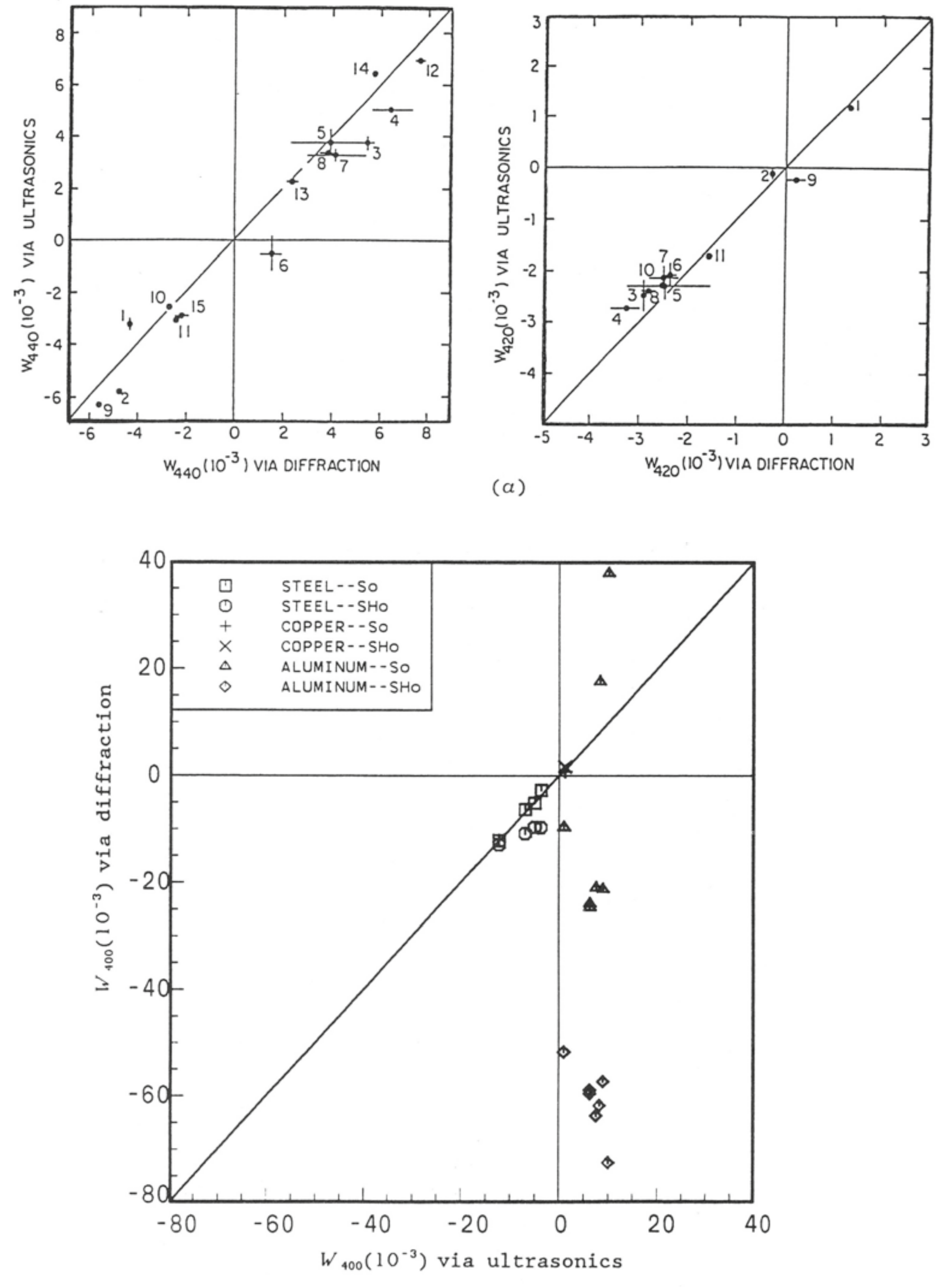

Fig. 3. Predictions of ODC's via ultrasonics. (a) Good correlations for W_{420} and W_{440}. (b) Erratic predictions for W_{400}.

Here L, P and T are moduli of an isotropic polycrystallite, C^o is a measure of the elastic anisotropy, ρ is the density, and the phase velocities are shown as a function of angle with respect to the rolling direction. Comparison of ODC's obtained by ultrasonics to those obtained by x-ray and neutron diffraction have shown good agreement for the cases of W_{420} and W_{440}, as illustrated in Fig. 3a [5,6]. Note that scatter of W values is on the order of 10^{-3}. However, similar comparisons for W_{400} have shown a much more erratic behavior (Fig. 3b). It has been found that for both steel and copper, the agreement between ultrasonic measurements are excellent for the S_o mode with the SH_o mode values being consistently lower. However, for the aluminum samples, there are serious differences between the ultrasonic and diffraction predictions of W_{400}. The fact that the range of the abscissa and ordinate is an order of magnitude greater than that in Fig. 3a makes this disagreement even more severe. Although not fully understood, this greater apparent difficulty in predicting W_{400} in aluminum may arise from the small value of the elastic anisotropy, C^o, which makes the predictions particularly sensitive to errors. Sources of these errors may include the greater difficulty of absolute (as compared to relative) velocity measurements, possible errors in estimates of the isotropic polycrystal moduli (including alloying and second phase effects), and the need for dispersion correction for the S_o mode which becomes more severe as plate thickness increases [6].

IMPROVED TECHNIQUE

Ideally, one would like to infer W_{400} from relative measurements. However, as noted above, measurements of the variation of velocities in the plane of the plate can not be used to determine W_{400}, which is the coefficient of a basis function which only varies with polar angle. One must then seek a different experimental configuration in which a wave parameter is varied in a cross-section of the plate. This can be accomplished by taking advantage of the properties of higher order Lamb modes. Since these can be viewed as the superposition of partial waves reflecting between the plate surfaces [7,8], and since the angle of these partial waves with respect to the normal depends upon the point of operation on the dispersion curves, measurement of various features of the dispersion can be used to study the angular dependence of wave velocities in the cross-section of the plate. One such scheme based on the dispersion of SH modes has been demonstrated by Armstrong et al. [9,10]. However, that technique generally depends on a precise knowledge of the plate thickness. Although such knowledge is easily obtained in the laboratory, it may not be as accessible in production environments. Hence, alternate procedures are needed.

One technique which appears particularly attractive makes use of the properties of guided Lamé modes, operating at the point at which the isotropic S_o mode and SH_1 mode dispersion curves are tangential, as shown in Fig. 2. Both modes consist of shear waves propagating at 45° with respect to the plate normal. However, the partial waves in the SH_1 mode are polarized parallel to the plane of the plate, while those in the S_o mode are polarized in the sagittal plane. Because of this 90° polarization rotation, one would expect their relative velocities to depend on W_{400}. Furthermore, since the partial waves propagate through the same path at the same angle, one would expect measurement of their relative velocities to be insensitive to small uncertainties in the plate thickness. Thus we expect the presence of W_{400} to split the tangency of the SH_1 and S_o modes at the Lamé point, producing either a mode crossing or a mode separation depending on sign.

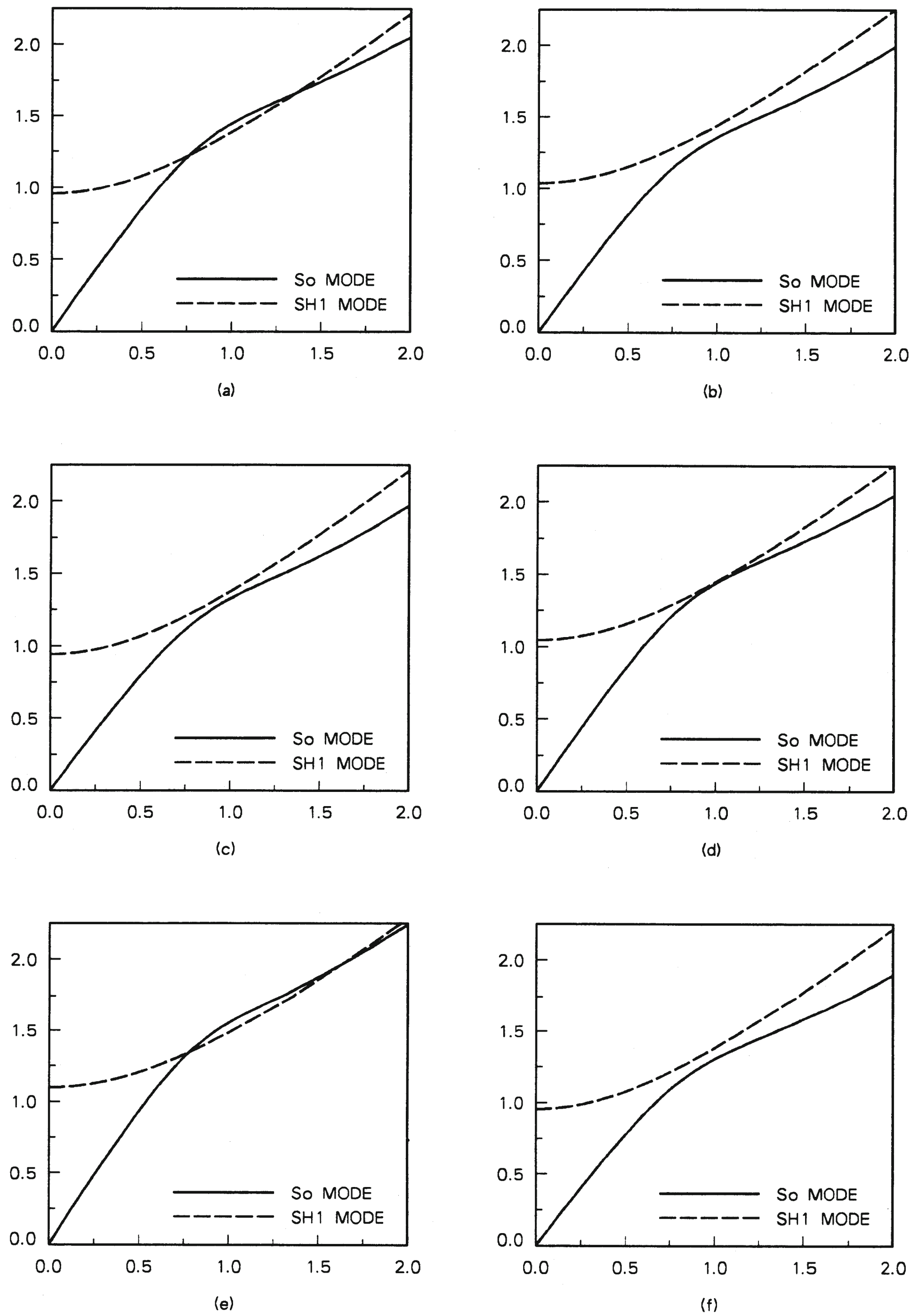

Fig. 4. Dispersion curves for wave propagation in rolling direction. (a) $W_{400}=-0.005$, $W_{420}=W_{440}=0.001$, (b) $W_{400}=0.005$, $W_{420}=W_{440}=0.001$, (c) $W_{400}=W_{440}=0.001$, $W_{420}=-0.005$, (d) $W_{400}=W_{440}=0.001$, $W_{420}=0.005$, (e) $W_{400}=W_{420}=0.001$, $W_{440}=-0.005$, (f) $W_{400}=W_{420}=0.001$, $W_{440}=0.005$. (All abscissae are in $\frac{b}{\pi}$ k, and all ordinates are in $\frac{b}{\pi}\frac{\omega}{C_T}$, where C_T is the through-thickness shear wave velocity and b is the plate thickness).

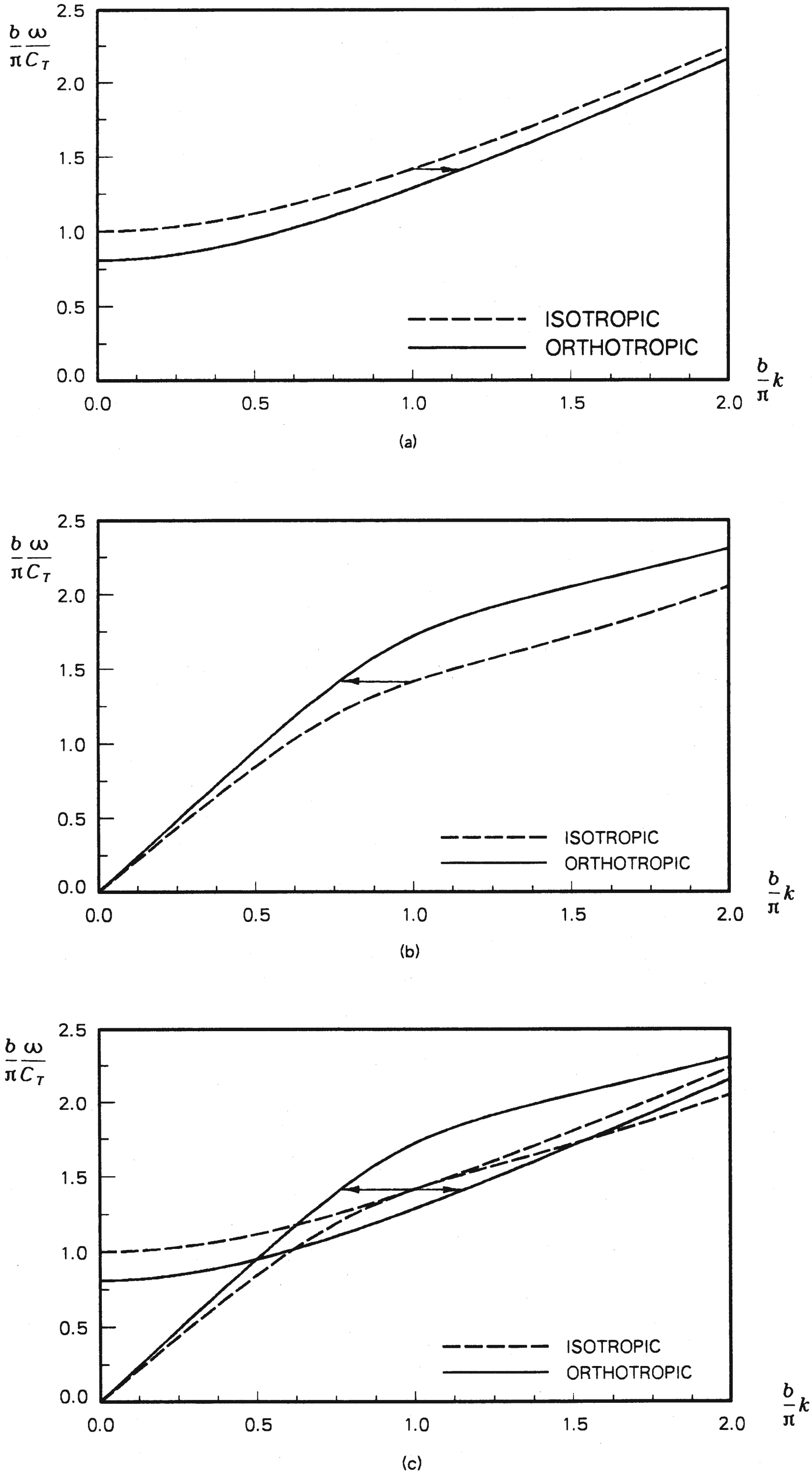

Fig. 5. Inversion algorithm strategy. (a) Shift in k for SH_1 mode, (b) shift in k for S_o mode, (c) total shift in k for SH_1 and S_o modes.

NUMERICAL EVALUATION

In order to evaluate this expectation, an exact theory has been used to calculate the dispersion curves of Lamb waves in an anisotropic plate [11]. Fig. 4 presents dispersion curves for waves propagating along the rolling direction of a plate. In each case, one of the ODC's (W_{400}, W_{420} or W_{440}) has been varied between ±0.005 with the other two held constant at 0.001, and the Hill averaging scheme was employed. As expected, introduction of texture causes the tangency to be broken, with either mode splitting or crossing occurring depending on the sign of W. The coefficients W_{400} and W_{440} are seen to have an influence of comparable magnitude, somewhat greater than that of W_{420}.

INVERSION

Having established the sensitivity of this special feature of the dispersion curves, it is necessary to seek an inversion algorithm that will allow the three ODC's to be separately determined from experimental data. Fig. 5 presents the strategy. Let (ω_o, k_o) define the point of Lamé mode propagation in the isotropic medium. When anisotropy is introduced, the dispersion curves will be shifted. Define

$$\Delta k \triangleq \Delta k_{SH_1} - \Delta k_{S_o} \tag{3}$$

to be the relative shift in wave vector at the frequency ω_o. From perturbation theory, one can compute $\Delta K(\theta)$, where θ is the angle of propagation with respect to the rolling direction. Explicitly examining the results at $\theta = 0°$, 45° and 90°, one obtains a set of linear equations in the ODC's which can be solved with the result,

$$W_{400} = \frac{7\sqrt{2}\ T}{40\pi^2 C^o k_o}\left[\Delta k(0°) + \Delta k(90°) + 2\Delta k(45°)\right] \tag{4a}$$

$$W_{420} = \frac{7\sqrt{5}\ T}{8\pi^2 C^o k_o}\left[\Delta k(90°) - \Delta k(0°)\right] \tag{4b}$$

$$W_{440} = \frac{\sqrt{35}\ T}{24\pi^2 C^o k_o}\left[\Delta k(0°) + \Delta k(90°) - 2\Delta k(45°)\right] \tag{4c}$$

where T is the shear modulus.

The stability of this inversion scheme has been numerically evaluated by using the exact theory to calculate Δk and then inverting to obtain the ODC's by Eqs. (4). To test the sensitivity to small fluctuations in moduli, Hill's averaging procedure was used to compute the isotropic moduli in the exact calculation of dispersion and the Voigt, Hill and Reuss procedures were each used in the data inversion. The material was assumed to be polycrystalline copper, having $W_{400} = W_{420} = W_{440} = 1 \times 10^{-3}$. Table I presents the results of the inversion.

TABLE I - TEST OF INVERSION

	Voigt	Hill	Reuss
$W_{400}(10^{-3})$	1.12	1.02	0.91
$W_{420}(10^{-3})$	1.33	1.21	1.08
$W_{440}(10^{-3})$	1.13	1.03	0.91

CONCLUSIONS

Previously proposed techniques have shown the ability to determine W_{420} and W_{440} from the angular dependence of the velocities of SH_o and S_o guided elastic modes of plates. Those procedures require absolute velocities for the prediction of W_{400}, and difficulties have been encountered under conditions of weak anisotropy (aluminum) or thick plates ($b/T_{S_o} \gtrsim 1/3$). An improved technique has been proposed based on the texture induced splitting of the tangency of the S_o and SH_1 Lamé modes. Numerical calculations and perturbation theory have been used to verify the procedure. Experimental evaluations are in progress, utilizing EMAT's to excite these special modes [12].

ACKNOWLEDGEMENT

Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under contract No. W-7405-Eng-82. This work was supported by the Director for Energy Research, Office of Basic Energy Sciences.

REFERENCES

1. R.-J. Roe, J. Appl. Phys., 36, 2024 (1965).
2. R.-J. Roe, J. Appl. Phys. 37, 2069 (1966).
3. H. J. Bunge, Texture Analysis in Materials Science (Butterworth, London, 1982).
4. R. B. Thompson, S. S. Lee and J. F. Smith, Ultrasonics 25, 133 (1987).
5. R. B. Thompson, J. F. Smith, and Y. Li, Ultrasonic Measurement of Rolling Texture in Non-ferrous Plates", Presented at 8th International Conference on Texture of Materials, Santa Fe, New Mexico, Sept. 21-25, 1987.
6. R. B. Thompson, J. F. Smith, S. S. Lee and G. C. Johnson, manuscript in preparation.
7. R. D. Mindlin, Waves and Vibrations in Isotropic, Elastic Plates, pp. 199-232, Structural Mechanics, Pergamon, New York, 1960.
8. B. A. Auld, Acoustic Fields and Waves in Solids (Wiley Interscience, New York, 1973).
9. J. F. Smith, G. A. Alers, P. E. Armstrong, and D. T. Eash, J. Nondes. Eva., Vol. 4, No. 3/4, 157 (1984).
10. P. E. Armstrong, D. T. Eash, J. A. O'Rourke, and J. F. Smith, J. Nondes. Eva., Vol. 6, No. 1, 33 (1987).
11. Y. Li and R. B. Thompson, this proceedings.
12. R. B. Thompson and C. F. Vasile, J. Appl. Phys., 34, 128 (1979).