SOME INVERSION PROBLEMS IN NONDESTRUCTIVE EVALUATION

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INTRODUCTION

Nondestructive evaluation (NDE) is to materials and structures what CAT scanning is to the human body---an attempt to look inside without opening the body. It is in nature an inversion problem. While such problems present a formidable mathematical challenge, sophisticated new models and software are beginning to yield useful results. In this paper we discuss the solution of several inversion problems using the eddy-current volume-integral code, VIC-3D[1]. These problems, the reconstruction of flaws in steam-generator tubing, the determination of metallic plate thicknesses, and the reconstruction of conductivity profiles versus depth in metallic materials, are typical of the application of eddy-current NDE to process control and in-service inspection.

THICKNESS MEASUREMENTS

One of the simpler inverse problems in nondestructive testing is the determination of plate thickness from measurements obtained with eddy-current probes. We illustrate the procedure by determining the thickness of a brass plate from impedance measurements with an air-core probe. The measurements are those of Burke [1], and cover the range 100 to 10000 Hz. The probe has 408 turns, is 8.8 mm in height, and has inner and outer radii of 9.34 and 18.4 mm.

Often the probe lift-off is poorly known, in some cases due to a non-conducting layer of unknown thickness on the surface. Since the probe response can be very sensitive to this parameter, we demonstrate a scheme that determines this parameter as well as the plate thickness. In principle, the thickness, t, and lift-off, l, can be determined from a single impedance measurement, but more accuracy can be achieved using measurements at multiple frequencies, to obtain an overdetermined system of equations. We minimize

$$\begin{aligned}\Delta R(f) &= R_{\text{meas}}(f) - R_{\text{calc}}(f,t,l) \\ \Delta X(f) &= X_{\text{meas}}(f) - X_{\text{calc}}(f,t,l)\end{aligned} \tag{1}$$

for all frequencies, f. Because the problem is nonlinear, we use a Gauss-Newton iteration scheme, which requires only that we can compute

[1] VIC-3D is a registered trademark of Sabbagh Associates, Inc.

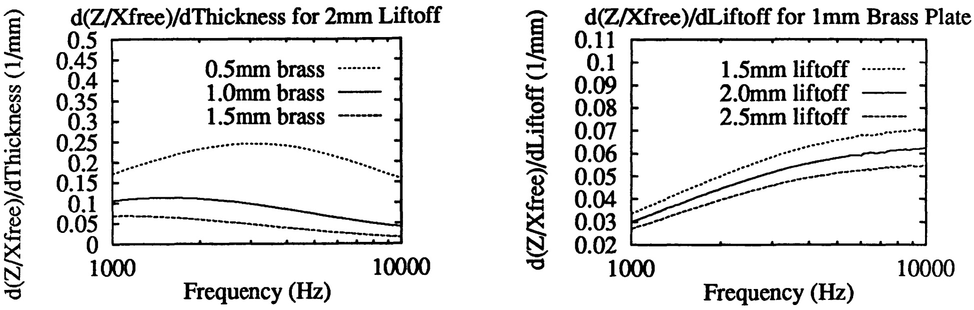

Figure 1: **Derivatives of normalized impedance with respect to thickness and lift-off.**

$R_{\text{calc}}(f,t,l)$, $X_{\text{calc}}(f,t,l)$, and their derivatives with respect to t and l. These are computed using VIC-3D.[2] The computed derivatives of impedance with respect to thickness and lift-off are shown in Figure 1. The range 1 to 2 kHz appears optimum for determining the thickness. If the primary interest had been in determining the lift-off, higher frequencies would be preferable [2].

The analysis was performed a number of times, each using a different number of frequencies, in the range 1 to 2 kHz. Seven frequencies appear sufficient to eliminate the effects of statistical errors in the measurements. The deduced thickness and lift-off are 0.8801 mm and 2.064 mm respectively. The measured values are 0.89 and 2.03 mm. The 1% difference in the thickness values is likely due to systematic errors in the measured or computed impedances. The probe lift-off is not determined quite as well as the thickness. This could be anticipated from Figure 1, which shows the impedance to be less sensitive to the lift-off.

CONDUCTIVITY PROFILE MEASUREMENTS

It is often necessary to inspect materials for hardness, residual stress, fiber-volume content, etc. These properties may be the results of a manufacturing process, or the results of exposure to a harsh environment. Whatever the cause, when these physical attributes can be correlated with electrical conductivity or permeability, then eddy-current inspection can be used to monitor them. We do not address the question of correlating conductivity to the material properties, leaving that to materials scientists. What we will do is show how modeling can 1) tell what is required of the eddy-current instrument in order to acheive satisfactory performance, and 2) help us understand the measurements we obtain from our instruments.

Suppose a flat, layered structure has layers with the conductivities shown (solid) in Figure 2. We can reconstruct this profile using impedance measurements from a simple air-core probe. To model this problem, we use VIC-3D to compute the impedance seen by an air-core probe resting on our layered structure. Our probe has 400 turns, is 1.73 mm in height, and has inner and outer radii of 2.54 and 4.27 mm. To determine the best frequencies for our probe, we look at the derivative of the normalized impedance with respect to the conductivities of the four regions of our structure. For all regions, this derivative peaks between 3 kHz and 3 MHz, as shown in Figure 2.

[2] A complete discussion of the analytical and computational aspects of this problem is given in [2].

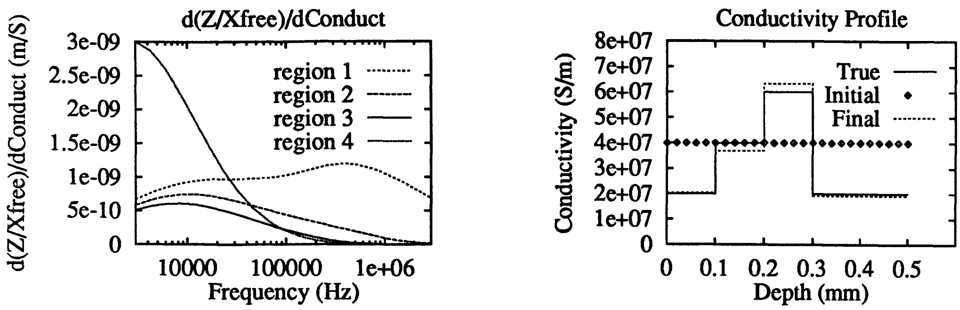

Figure 2: A model problem with three layers over a half-space. Derivative of normalized impedance with respect to conductivity, versus frequency (left), and conductivity profile. Region 1 lies between 0 and 0.1 mm; region 4 lies below 0.3 mm.

We use this frequency range.

We use the same nonlinear least-squares algorithm (Gauss-Newton) that was used in the thickness measurements, and is described in [2], to recover the conductivities from the impedances. The starting point for our iterative algorithm is a uniform conductivity of 4×10^7 S/m. Three to six frequencies, in a fairly narrow band, are sufficient to exactly reconstruct the conductivities, if our computed ''noiseless'' impedances are used. To investigate the effect of 'noise' in the data, we perturb the computed impedances, which are our input data, by the fractional amounts shown in Figure 3. This is not representative of any particular noise model, but simply illustrates the effects that perturbations have on the reconstructions. Additional frequencies are now required to produce a satisfactory inversion. The conductivites obtained using 21 frequencies are shown (dashed) in Figure 2. The frequencies chosen are simply those that form a geometric progression between 3 KHz, and 3 MHz. Ten iterations were required.

We can better understand the quality of the reconstruction we can expect from our measurements by referring to the derivatives shown in Figure 2. Clearly, at low frequencies, the impedance is much more sensitive to the

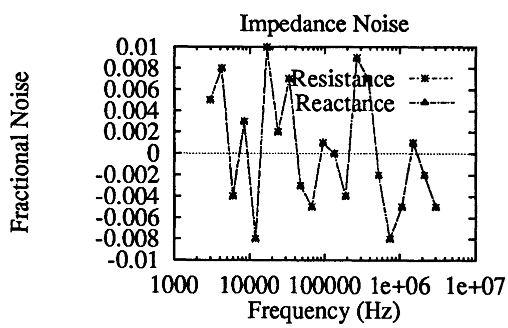

Figure 3: Perturbations in the computed impedances at twenty-one frequencies. This is the input data that represents the effects of 'noise.'

conductivity of the half-space, while at high frequencies it is much more sensitive to that of the top layer. Hence, our recovered conductivity profile is most accurate in these regions.

Our modeling allows us to test our inversion algorithm, determine the proper frequencies to measure, establish noise requirements for our measurements, and understand the quality of reconstruction we can expect.

We believe that this is an important point to stress. Eddy-currents have long been applied to the measurement of thickness and conductivity, but the techniques have generally been quite empirical, and used discrete standards. The results of blindly using the nearest available instrument have often been disappointing. Recent work has begun to offer a theoretical underpinning to experimental measurements [3-4], and our work continues this line of inquiry. Modeling must be performed before equipment is purchased, or expensive experiments undertaken.

FLAW RECONSTRUCTIONS

Eddy-current nondestructive methods are routinely used in the inspection of tubing, particularly by the nuclear power industry. The methods generally used today require data from a series of calibration flaws typical of the type to be measured and covering the range of dimensions expected. We describe here the results of a flaw inversion algorithm superior to these methods in two respects: 1) it describes the shape as well as the length and depth of the flaw; and 2) it requires calibration data from only a single flaw of known type, shape, and dimensions.

The algorithm [5] begins by constructing rigorous integral equations from electromagnetic theory. These equations are then discretized on a grid covering the flaw region, using the method of moments. The resulting nonlinear equations are then linearized by using a Born approximation, in which the field at the flaw is approximated by the unperturbed field produced by the probe. The difference between the EMF's produced by the flaw and the measured values is minimized using a least squares algorithm.

The technique is designed for detecting axially orientied flaws in tubing walls, using a conventional differential bobbin coil. It has been successfully applied to the reconstuction of narrow rectangular and triangular notches cut into a 304 stainless-steel pipe with a 22.3 mm OD and 1.22 mm wall thickness. The results of the reconstruction are shown in Table 1. Flaws D

Table 1: Results of flaw reconstructions.

	Nominal Values		From Inversion		Relative Error (%)	
Flaw ID	Length (mm)	Depth (mm)	Length (mm)	Depth (mm)	Length	Depth
A	5.16	0.241	5.59	0.254	8.4	5.0
B	2.54	0.483	2.41	0.483	5.0	0.0
C	5.16	0.483	5.33	0.483	3.4	0.0
D	5.08	0.610	3.91	0.559	23	9.0
E	5.16	0.737	5.33	0.762	3.4	3.4
F	5.08	0.737	5.08	0.533	0.0	28

and F are triangular; the others are rectangular.

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