Distributional, allocational, and welfare effects of factor taxes in Applied General Equilibrium Models

by

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Signatures have been redacted for privacy

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I. INTRODUCTION

One of the central themes in economics for about two centuries has been to compute a general equilibrium price.

The general equilibrium model was first formulated by Walras [29], whose law says that if one market at a Walrasian price vector shows excess demand (supply), then at least one of the other markets must exhibit excess supply (demand): in other words, the summation of the value of the excess demands equals zero. Along with this concept, general equilibrium for a competitive economy can be defined as a price vector, the elements of which are the prices of outputs and inputs, such that all markets clear at once. The general equilibrium theory, by utilizing Walras' law and the Pareto optimality, takes account of the interaction of the economic agents through markets [4].

Wald [28] was the first to prove the existence of the Walrasian equilibrium. Under the assumption that an individual's demand for a commodity would rise if the price of any other commodity were to increase, he demonstrated the existence and uniqueness of a vector of competitive equilibrium prices.

J. von Neumann [27] developed a mathematical tool playing an essential role in equilibrium theory under the definitive form as a Brouwer's fixed point theorem, the value of whose tool was powerful enough to show the existence of Nash [14] or


Koopmans [10], meanwhile, showed that an optimum is equivalent to an equilibrium relative to a price vector: if nondistortionary transfers exist, a competitive equilibrium is Pareto efficient and essentially every Pareto efficient allocation can be supported as a competitive equilibrium.

Since the formulation of the Walrasian general equilibrium structure in the form of an abstract economy by Arrow, Debreu, and others such as the above cited, many economists, in the last few years, have developed the so-called "Applied General Equilibrium Analysis" (AGEA) hoping to transform a general Walrasian model into realistic models of actual economies. Using this model makes it possible to evaluate policy options by specifying production and demand parameters reflecting the data of real economies. When calculating empirical estimates of the effects of economic distortions from the perfectly competitive model, the applied general equilibrium models offer a great deal of added generality and flexibility over previous methods, the typical model of which is the two sector-two factor of production general equilibrium model developed by Meade [12] and Johnson [7]. Under the category of this basic model, several studies,
for example, done by Harberger [6] and Johnson and Mieszkowski [8], have been presented which compare the economy with and without the relevant distortion of taxes, tariffs, or other policies. Although Harberger provided a theoretical work for analyzing the effects of the corporate income tax and drew some inferences about the probable incidence of the tax, this sort of approach has faced the following criticisms [21, 22]: 1) it is so based on the local analysis that it is not well-suited for the analysis of the higher rate tax (e.g., 50%) distortion. 2) aggregating the sectors or factors into two is too severe. 3) it is not realistic to assume the fixed factor endowments, and 4) analyzing one distortion at a time can be misleading.

A computer algorithm for calculating general equilibrium prices originally developed by Scarf [17, 18] has been a very important stimulus to overcome the above shortcomings. Now that the general equilibrium model, when cast in a mathematical form, become a complex system of simultaneous equations and inequalities for the determination of all prices and output levels in the economy, the existence of a solution can be guaranteed only by making use of what are known as "fixed point" theorems, which describe conditions under which a continuous mapping leaves at least one point unchanged [17; p. 210]. Scarf's algorithm [16, 17, 18] has provided a general method for the explicit numerical solution of the
neoclassical model.

Shoven and Whalley's works [20, 24], based on Scarf's algorithm, demonstrated a proof of the existence of a competitive equilibrium for an economy with producer and consumer commodity taxes, and developed a computational procedure for the determination of the equilibrium with the taxes being considered. In their model, the government is assumed to be a tax collecting and revenue dispersing agency.

Although Scarf, Shoven, Whalley, and others calculated general equilibrium prices with and without taxes by using the Scarf-type algorithm and by turning the abstract economy into a more realistic one, their method so far did not have full rationality for usefulness in actual economies because they conducted the demonstration in terms of primitive sets as seen at the next section.

This complaint, however, became much less valid after Merrill refined the Scarf's algorithm. The Merrill's fixed point algorithm solves the simultaneous highly-nonlinear equations, finding a general equilibrium price vector.

Shoven and Whalley [23], using the Merrill's algorithm with the Arrow and Hahn's model [2], employed the two sector-two factor-two consumer model to compute general equilibrium prices. The equilibrium conditions in this model are basically equivalent to the four equation model's [25]: market demand equals market supply for all inputs and outputs and profits are zero in each industry.
These conditions produce six equations reduced to four with four unknowns. The equilibrium price vector of these simultaneous equations are solved by the Merrill's algorithm if the parameters of production and consumption functions were specified and the exogenous variables known. It is out of question to compute a general equilibrium price vector with one tax or more taxes if we add one more condition that the transfer payments to consumers equal the tax revenue collected by the government, which is considered as a consumer.

Policy appraisal determining if a particular policy change is welfare-improving usually relies upon a comparison between a no-tax equilibrium and a tax equilibrium. Numerical measures of the gain or loss widely employed are Hicksian compensating (CV) and equivalent (EV) variations associated with the equilibrium comparison. Welfare costs of taxes for the economy as a whole are measured by aggregating the CVs or EVs across individuals [9].

In the review of previous works, I introduce the Scarf's algorithm and Shoven and Whalley's four equation model. These two works will well-represent the developments of the AGEA and show, with other works, that the AGEA provides the valid evaluation of policy options in a real world.

I modify the AGEA model by Shoven and Whalley [25] by allowing labor supply to be endogenous and by adding one more class of consumers. I consider three kinds of factor taxes:
1) tax on a factor in one industry only, 2) tax on a factor in both industries, and 3) tax on both factors in both industries.

By using the up-dated method of the AGEA, I calculate the equilibrium price with and without taxes in the Walrasian general economy, then evaluate the welfare loss or gain among three consumers. Whether any policy change can generate a Pareto superior equilibrium is discussed.
II. REVIEW OF PREVIOUS WORKS

Scarf's algorithm [16, 17, 18], along with Hansen's [5], has provided a general method for the explicit numerical solution of the neoclassical model. For the working of the algorithm, market demand functions are assumed to satisfy Walras' law: \( \sum P_i (X_i - W_i) = 0 \), where \( P_i \) is the price of the \( i \)-th commodity, \( X_i \) is the market demand for the commodity \( i \), and \( W_i \) is the total initial endowments of the commodity \( i \). \( X_i \) is a sum of individual's demand functions (\( \sum x_{ij} \)), each of which is derived from maximizing the individual's utility function subject to the budget constraint, while \( W_i = \sum w_{ij} \) where \( w_{ij} \) is the initial endowment of the \( i \)-th commodity to individual \( j \).

Production is described by an arbitrary activity analysis matrix labelled "\( A_{n \times m} \)" (\( m > n \)). Each column of the \( A \) matrix refers to a feasible activity, the first \( n \) columns (negative \( n \times n \) identity matrix) indicating free disposibility. An \( a_{ij} \) (an element of the \( A \) matrix) is positive for an output and negative for an input.

A competitive equilibrium is defined by a price vector, \( P^* \), and a vector of activity levels, \( y^* \), such that

1. demand equals supply in each market
   \[ X_i (P^*) = W_i + \sum a_{ij} y_j^*, \quad i = 1, \ldots, n \]
2. profit is no better than the break even
   \[ \sum P_i a_{ij} \leq 0 \] for all \( j \), with equality if \( y_j^* > 0 \)

To find a \( P^* \) and associated \( y^* \) which approximately meet
the above two conditions, the $B_{n^*k}$ matrix $(n<k\leq m)$ is created from the $A$ matrix by the specific rules so as to satisfy the theorem: there exists a primitive set, $pj_1, \ldots, pj_n$, so that the columns, $jl, \ldots, jn$, form a feasible basis for $By=W$. Such primitive set, which is an $n*n$ price matrix, approximates a competitive equilibrium price. The usefulness of the algorithm is due to the fact that it allows a computer to rapidly find a primitive set which approximates a competitive equilibrium rapidly.

Shoven and Whalley \cite{20, 22, 24} and Shoven \cite{21} added a tax system to the Scarf's algorithm to compute an equilibrium price with producer and/or consumer commodity taxes. All government revenue is distributed to consumers as transfer payments, which affect a consumer's budget constraint.

The market demand functions, which depend on the $n$ commodity prices $(P_1, \ldots, P_n)$ and the tax revenue $(R)$, satisfy the Walras' law $(\sum P_iX_i(P,R) = \sum P_iW_i + R)$.

One change to the Scarf's $A$ matrix is to add one more row. Thus, the last row of each column of the $A$ matrix becomes the tax revenue $(r_j)$ from the corresponding column, where $r_j = \sum P_it_{ij}a_{ij}$ (t$_{ij}$ is the ad valorem tax rates). An equilibrium price vector, $P^*=(P_1, \ldots, P_n, R)$, and nonnegative activity levels, $y^*$, satisfy the two equilibrium conditions: 1) supply is equal to demand for each commodity, and 2) after-tax profit is maximized at the price vector, $P^*$. At the above two conditions with the Walras' law, it was shown that the revenue
distributed among consumers was same as that generated on the production and/or consumption side.

Shoven and Whalley not only showed the proof of the existence of an equilibrium price with the producer and/or consumer taxes by modifying the Scarf's algorithm, but also calculated the efficiency loss and the incidence effects of a differential taxation by comparing the equilibria in both the presence and the absence of the surtax.

Harberger [6], meanwhile, introduced in the '60s a general equilibrium model of taxation to examine the interindustry distortion from the corporate income tax by the way of the differential calculus. He classified industries into the corporate (heavily taxed) and non-corporate (lightly taxed) sectors according to the U.S. taxation of capital income during 1953-59. Each sector produced a single output in perfect competition using homogeneous and perfectly mobile labor and capital, available fixed aggregate supplies. He assumed there was one consumer, thereby representing consumer preferences as identical and homothetic.

In equilibrium, his solution was obtained by total differentiation, so that the model was valid only for small changes in the tax rate. The results on the burden of the corporate income tax depended in part on the substitutability of labor and capital in production. He concluded that capitalists bear the full burden of the corporate income tax.
when a 50% tax on capital income in one sector was levied.

Shoven's recomputation of Harberger's two sector-two factor model employing the above algorithm, did not differ much from the corrected Harberger's results [21]. However, the algorithmic method took advantages of disaggregating the model into the 12 sectors. The dead weight loss estimates increased an average of about 40% from that of the two-sector model, giving valuable information for the making of policy decisions.

Piggot and Whalley [15] presented the impact of 1973 changes in the United Kingdom by making use of the above method of simultaneously incorporating several tax distortions. They employed a 33-product and 100-household type model to evaluate structural characteristics of the U.K.'s tax/subsidy system with CES utility and production functions. They concluded that the existing U.K. tax system yielded a distorting loss of 6-9% of NNP per year, that subsidies to local authority housing area were a significant source of welfare loss, and that there existed significant redistributive effects of taxes in the U.K. tax/subsidy system.

Serra-Puche [19], employing the algorithmic method, analyzed tax incidence by replacing indirect turnover taxes with consumption value-added taxes as was instituted in Mexico in 1981. He disaggregated the demand side into 10 consumer groups, the government, and the foreign sector; the production
side into 14 industries, and assumed there were Cobb-Douglas utility and production functions. He concluded that the policy change caused resource allocation to move in favor of the government target sectors, such as agriculture and food stuffs, and the income distribution "improved," reducing differentials between urban and rural households.

As seen from the previous works, some of the advantages of the AGEA are as follows:

1. no localization assumptions are required.
2. it is simple to incorporate many commodities and classes of consumers.
3. the effect of several simultaneous distortions can be analyzed by using the empirical data.
III. OBJECTIVES

The Scarf-type computer algorithm has provided an excellent tool to solve the highly nonlinear equations yielded by general equilibrium analysis. By using it, the AGEA went beyond traditional two-sector general equilibrium models, thereby allowing for much more detail and complexity.

My major objective in this paper is to employ AGEA to compute general equilibrium prices with and without a tax for the hypothetical two-sector, three-consumer model. I modify the Shoven and Whalley's example [23] to allow labor-leisure choices. Also, I compare pre-, and post-tax equilibria to see who gains, or loses from the tax programs being considered. The Hicksian compensating (CV) and equivalent (EV) variations are also computed to facilitate this comparison.
IV. THE MODELS AND RESULTS

A. Assumptions

Taking some assumptions from traditional microeconomic theories in this paper, I employ the Arrow and Hahn's model [2], which is commonly used for the AGEA.

The number of consumers is specified. Each of them has initial endowments of the commodities and a set of preferences, which generate an individual demand function for each commodity. Each demand function depends on all prices in the no-tax system (the tax revenue is added in the tax model), are continuous, nonnegative, homogeneous of degree zero in all prices, and satisfy Walras' law. Since market demands are simply a sum of each consumer's demands, the characteristics of market demands are the same as the above individual demand function's.

On the production side, technology exhibits constant returns to scale (CRS), and producers maximize profits (this implies that producers always satisfy the cost minimization). The factor demand functions are derived from the cost minimization. The properties of the minimizing cost function are that it is non-decreasing, homogeneous of degree zero, and concave in factor prices, and separable into the output and a function of factor prices if the production function exhibits homotheticity. For the CRS case, the cost function should exhibit costs that are linear in the level of output: \[ TC = \]
Q*M(R), where TC is a cost function, Q is output, and M(R) is a function of a price vector of factors. Inputs are perfectly mobile and homogeneous, and outputs are homogeneous.

In competitive equilibrium, a price vector and level of production in each industry satisfy the following conditions:

1. market demand equals supply in inputs.
2. market demand equals supply in outputs.
3. zero profit conditions hold in industries in case of CRS.

B. No-Tax Equilibrium and Results

By following Arrow and Hahn's model and employing traditional microeconomic theories, I consider a model with two final goods (manufacturing and nonmanufacturing), two factors of production (capital and labor), and three classes of consumers (the rich, the middle, and the poor). Consumers have initial endowments of capital, some units of which are owned by the rich, the middle, and the poor. Labor is supplied by the labor-leisure choice among consumers. Production of each good is represented by a CRS constant elasticity of substitution (CES) production function and each consumer class has a demand function derived from maximizing a CES utility function subject to his or her budget constraint. Since the CES function is quasi-concave, the demand function for the input and output has the characteristics cited in the
previous section.

The production functions are given by

\[ (1) \quad Q_i = a_i [b_i L_i (c_i - 1)/c_i + (1 - b_i) K_i (c_i - 1)/c_i] \quad c_i/(c_i - 1), \]

where

i: 1 and 2 (1: manufacturing, 2: nonmanufacturing),
Q_i: output of the i-th industry,
a_i: scale or units parameters,
b_i: distribution parameters,
K_i: capital inputs employed by the industry i,
L_i: labor inputs employed by the industry i, and
c_i: elasticity of factor substitution.

By the first-order conditions of the cost minimization, we have

\[ (2) \quad \frac{K_i}{L_i} = \left( \frac{1 - b_i \; P_W \; c_i}{b_i \; P_k} \right) \]

where

P_W: price of labor, and
P_k: price of capital.

(3) \[ k_i = \frac{K_i}{L_i} \quad \text{will be defined as the capital intensity.} \]

From (1) and (2), the factor demand functions become

\[ (4) \quad L_i = \frac{1}{a_i} Q_i [b_i + (1 - b_i) \left( \frac{b_i \; P_k \; (1 - c_i)}{1 - b_i \; P_W} \right)] \quad c_i/(1 - c_i) \]
Meanwhile, the total cost functions of each sector become

\[
T_{Ci} = P_{W}L_{i}[1 + \frac{K_{i}P_{k}}{L_{i}P_{W}}] 
\]

Plugging (2) and (4) into (5) gives

\[
T_{Ci} = \frac{1}{a_{i}} \frac{Q_{i}b_{i}c_{i}}{(1 - c_{i})} \frac{P_{W}(1 - c_{i})}{(1 - c_{i})} + \left(\frac{1 - b_{i}}{b_{i}}\right) P_{k}(1 - c_{i}) \frac{1}{1 - c_{i}} 
\]

The above cost functions (7) show their separability representing the characteristics of the CRS CES production function. By the definition of the derivative of \( T_{Ci} \) with respect to \( Q_{i} \) as the marginal cost (MC), which is equal to the price of its output in a perfectly competitive market,

\[
M_{C} = \frac{1}{a_{i}} \frac{b_{i}c_{i}}{(1 - c_{i})} \frac{P_{W}(1 - c_{i})}{(1 - c_{i})} + \left(\frac{1 - b_{i}}{b_{i}}\right) P_{k}(1 - c_{i}) \frac{1}{1 - c_{i}} 
\]

Since MC of an industry showing CRS is always equal to the price of its output for positive production, the profit is zero. Thus, the equation (8) is equivalent to the zero profit condition.

The CES utility functions are given by

\[
U_{j} = \frac{a_{1}l_{j}}{s_{j}} x_{1j}(s_{j}-1)/s_{j} + \frac{a_{2}l_{j}}{s_{j}} x_{2j}(s_{j}-1)/s_{j} \\
+ \frac{a_{3}l_{j}}{s_{j}} R_{j}(s_{j}-1)/s_{j} s_{j}/(s_{j}-1)
\]
where

\( j: 1 \) (the rich), 2 (the middle), and 3 (the poor),
\( a_{1j} + a_{2j} + a_{3j} = 1, \)

\( U_j: \) utility of the i-th consumer

\( X_{1j}, X_{2j}: \) quantity of the good 1 and 2 demanded by the j-th consumer,

\( R_j: \) leisure hour of the consumer j,

\( a_{1j}, a_{2j}, a_{3j}: \) share parameters, and

\( s_j: \) elasticity of substitution.

The budget constraint of the consumer j is

(10) \( E_j = P_k W_{kj} + P_{w} W_{lj} \Rightarrow P_1 X_{1j} + P_2 X_{2j}, \)

where

\( E_j: \) earned income of the consumer j,

\( W_{kj}: \) capital endowment to the consumer j,

\( W_{lj}: \) labor hour supplied by the consumer j, and

\( P_1, P_2: \) consumer prices of the manufacturing and nonmanufacturing output.

Since \( R_j + W_{lj} = 24, \) (10) becomes

(11) \( I_j = P_k W_{kj} + 24P_{w} \Rightarrow P_1 X_{1j} + P_2 X_{2j} + P_{w} R_j \)

In (11) \( I_j \) is interpreted as the full income of the consumer j.

By the first-order conditions of the utility maximization subject to the budget constraint (11),

(12) \( X_{2j} = \frac{a_{2j} P_1 s_j}{a_{1j} P_2} X_{1j}, \) and
Substituting (12) and (13) into (11) gives the following demand functions and leisure demand functions:

(14) \[ X_{lj} = \frac{a_{lj} \frac{I_j}{p_1 s_j}}{p_1 s_j} X_{lj}, \]

(15) \[ X_{2j} = \frac{a_{2j} \frac{I_j}{p_2 s_j}}{p_2 s_j} X_{2j}, \]

(16) \[ R_j = \frac{a_{3j} \frac{I_j}{p_w s_j}}{p_w s_j} X_{3j} \]

where

\[ D_j = a_{lj} p_1 (1-s_j) + a_{2j} p_2 (1-s_j) + a_{3j} p_w (1-s_j) \]

and

\[ I_j = p_k W_{kj} + 24 p_w. \]

In this model, there are fifteen parameters whose values need to be specified: six production function parameters affecting the supply of two products \((a_i, b_i, \text{and } c_i \text{ for } i=1, 2)\) and nine utility function parameters which determine the demand for each of two products by each of three consumers \((a_{lj}, a_{2j}, \text{and } s_j \text{ for } j=1, 2, 3)\). There are three exogenous variables whose values must also be specified: the endowments of capital \((W_{k1}, W_{k2}, \text{and } W_{k3})\).

The solution to this model is characterized by 17 variables, the four prices, \(P_1, P_2, P_w, \text{and } P_k\), nine demand quantities, \(X_{11}, X_{21}, R_1, X_{12}, X_{22}, R_2, X_{13}, X_{23}, \text{and } R_3\), two labor inputs, \(L_1 \text{ and } L_2\), and two capital intensities, \(k_1 \text{ and } k_2\).
The equilibrium conditions in this model are factor market clearing, output market clearing, and zero profits:

1. factor market clearing

(17) \[ L_1 + L_2 = 72 - R_1 - R_2 - R_3 \]

(18) \[ k_1 L_1 + k_2 L_2 = k \] (k is the total market endowment), where \( L_1, L_2, k_1, \) and \( k_2 \) are given by (3) and (4),

2. output market clearing

(19) \[ X_{11} + X_{12} + X_{13} = Q_1 \]

(20) \[ X_{21} + X_{22} + X_{23} = Q_2 \], where \( X_{1j} \) and \( X_{2j} \) are given by (14) and (15), and \( Q_i \) are given by (1),

3. zero profit

(21) \[ P_1 = \frac{1}{a_1} \left( \frac{b_1 c_1}{(1-c_1)} \right) \left[ \frac{p_w (1-c_1)}{1/(1-c_1)} + \left( \frac{1-b_1}{b_1} \right) \frac{c_1}{p_k (1-c_1)} \right] \]

(22) \[ P_2 = \frac{1}{a_2} \left( \frac{b_2 c_2}{(1-c_2)} \right) \left[ \frac{p_w (1-c_2)}{1/(1-c_2)} + \left( \frac{1-b_2}{b_2} \right) \frac{c_2}{p_k (1-c_2)} \right], \]

where these equations are from (8).

Once the parameters of these production and demand functions are specified and the capital endowments are known, a solution to satisfy the simultaneous highly-nonlinear equations (17)-(22) above is obtained.

Putting numerical values for all the parameters and the exogenous variables in Table 1 below, I solved the equations using a computer package named TK Solver.
What I have to note in this specification is that the curvature of the isoquant of the sector 1 is more rapid than that of the sector 2 because of the size of the value of the elasticity of factor substitution (c_i) of both sectors, and that every consumer prefers enjoying leisure to consuming other goods, while the elasticity of substitution (s_j) of each consumer differs much from each other.

The equilibrium solution with the labor price as numeraire is shown in Table 2. As seen from Table 2, at the equilibrium prices, total demand for each output exactly matches the amount produced, and producer revenues equal consumer expenditures. Labor supplied by the labor-leisure
choice and capital endowments are fully employed, and consumer factor incomes equal producer factor costs. Because of the

Table 2. Equilibrium Solution for the No-tax General Equilibrium model
(M: Manufacturing, N: Nonmanufacturing)

<table>
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<tr>
<th>Equilibrium Prices</th>
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<td>Pk = .304</td>
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<td></td>
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<tr>
<td>Capital Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>33.631</td>
<td>25.309</td>
<td>49.381</td>
</tr>
<tr>
<td>N</td>
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<tr>
<td>Total</td>
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<th>Total Cost</th>
<th>Per-unit Cost</th>
<th>Capital Intensity</th>
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<td>N</td>
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<td>17.373</td>
<td>23.646</td>
<td>.648</td>
<td>1.187</td>
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<tr>
<td>Total</td>
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<td>48.955</td>
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<td>15.269</td>
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<td>The Middle</td>
<td>12.193</td>
<td>9.647</td>
<td>15.431</td>
<td>9.346</td>
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<td>The Poor</td>
<td>10.102</td>
<td>11.552</td>
<td>15.092</td>
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<tr>
<td>Total</td>
<td>33.631</td>
<td>36.468</td>
<td>48.955</td>
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<td>The Rich</td>
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</tbody>
</table>
assumption of CRS in production, the per-unit cost in each industry equals the selling price, meaning that economic profits are zero. Utility is the numerical value of the utility function (9), which is same as that of indirect utility, which will play an important role when calculating CVs and EVs. Since only relative prices affect behavior in general equilibrium models, I have chosen labor as numeraire.

C. General Equilibrium with Taxes and Results

The interdependence of demands and supplies, and tax revenues should be considered when taxes are being incorporated into this model. For a given tax program (that is, for a specified tax rate imposed on a particular output or factor), tax revenues will be determined once demands, production levels, and factor employments are known. Also, demands depends on tax proceeds since these are redistributed to consumers, shifting individual budget constraints. The solution, thus, will be not only for equilibrium prices, but also for equilibrium tax revenues. In this model with taxes, I assume all government revenues are redistributed to consumers: tax revenues collected by government equal the transfer payments to consumers, shares of which sum to unity.

When ad valorem taxes are levied on factors, production possibility frontiers are changed, as are the equilibrium marginal rate of technical substitution to the extent that taxes are levied of factors according to where they are
Let $t_{k1}$, $t_{l1}$, $t_{k2}$, and $t_{l2}$ denote ad valorem tax rates levied on capital and labor used in sector 1 and sector 2, respectively. Then, without changing the prices of labor and capital involved in demand functions (because consumers receive the net-of-tax prices), those in production functions are slightly modified from the no-tax model: $P_k$ and $P_w$ in each sector become $(1+t_{k1})P_k$ and $(1+t_{l1})P_w$ in sector 1, and $(1+t_{k2})P_k$ and $(1+t_{l2})P_w$ in sector 2. Tax revenues derived from these taxes, denoted as $T_1$, $T_2$, $T_3$, and $T_4$, respectively, will be the multiplication of the tax rate, price of the factor, and the amount of the factor employed. It can be shown arithmetically as follows: $T_1 = t_{k1}P_kK_1$, $T_2 = t_{l1}P_wL_1$, $T_3 = t_{k2}P_kK_2$, and $T_4 = t_{l2}P_wL_2$. Thus, the total tax revenue, $T$, is the sum of $T_1$, $T_2$, $T_3$, and $T_4$.

One also needs to specify how tax revenue is redistributed to consumers; I assume the revenue is redistributed evenly to the consumer groups, so each consumer receives $T/3$. This condition will be included in the budget constraint of each consumer.

Preliminary arrangements for the tax equilibrium model are so far done. The conditions for the equilibrium with taxes are same as those for the no-tax model: 1) demand equals supply in inputs, 2) demand equals supply in outputs, and 3) zero profit conditions hold in both industries. Values of the parameters and exogenous variables are assumed to be
same as given in Table 1. The full arrays of the equations are shown in Appendix (of course, they are same as the no-tax equilibrium equations when the tax rates are all zero).

The tax programs considered are (tax rates are arbitrarily chosen):

1. tax program 1; 50% tax on capital income in sector 1 only.
2. tax program 2; 50% tax on capital income in both sectors.
3. tax program 3; 50% tax on all factor income in both sectors.

The new equilibrium solutions corresponding to the above schemes are shown in Tables 3, 4, and 5. I will compare each of the solutions with the no-tax equilibrium in Table 2.

From Table 3, which shows the results of the tax program 1, we see that the 50% tax on capital income in sector 1 causes the net-of-tax return on capital to fall. Capital employment in sector 1 falls, while that in sector 2 rises. Because this tax is avoidable through factor mobility, capital intensity in sector 1 naturally falls while that of sector 2 rises because of the decline in the net rental rate on capital. Less manufacturing, and more nonmanufacturing output, is produced because of the tax. Price of the manufacturing output, therefore, increases, and that of nonmanufacturing one decreases. The incomes of the poor increase while the other two consumers' incomes decrease even
Table 3. Equilibrium Solution for General Equilibrium Model with 50% Tax on Manufacturing Capital (M: Manufacturing, N: Nonmanufacturing)

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Revenue</th>
<th>Capital</th>
<th>Net Capital Cost</th>
<th>Tax Paid</th>
<th>Total Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>40.331</td>
<td>24.005</td>
<td>25.947</td>
<td>5.597</td>
<td>0.000</td>
<td>5.597</td>
</tr>
<tr>
<td>Total</td>
<td>71.896</td>
<td>48.634</td>
<td>70.000</td>
<td>15.099</td>
<td>4.751</td>
<td>19.850</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Labor Cost</th>
<th>Total Cost</th>
<th>Per-Unit Cost</th>
<th>Capital Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>10.376</td>
<td>24.629</td>
<td>.780</td>
<td>4.246</td>
</tr>
<tr>
<td>N</td>
<td>18.408</td>
<td>24.005</td>
<td>.595</td>
<td>1.410</td>
</tr>
<tr>
<td>Total</td>
<td>28.784</td>
<td>48.634</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Demands

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>N</th>
<th>Expenditure</th>
<th>Labor Hours</th>
<th>Labor Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Middle</td>
<td>11.613</td>
<td>10.568</td>
<td>15.350</td>
<td>9.452</td>
<td>9.452</td>
</tr>
<tr>
<td>The Poor</td>
<td>10.231</td>
<td>13.054</td>
<td>15.752</td>
<td>13.089</td>
<td>13.089</td>
</tr>
<tr>
<td>Total</td>
<td>31.569</td>
<td>40.331</td>
<td>48.634</td>
<td>28.784</td>
<td>28.784</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Capital Income</th>
<th>Transfers</th>
<th>Total Income</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Rich</td>
<td>9.706</td>
<td>1.583</td>
<td>17.532</td>
<td>42.079</td>
</tr>
<tr>
<td>The Middle</td>
<td>4.314</td>
<td>1.584</td>
<td>15.350</td>
<td>35.841</td>
</tr>
<tr>
<td>The Poor</td>
<td>1.079</td>
<td>1.584</td>
<td>15.752</td>
<td>33.484</td>
</tr>
<tr>
<td>Total</td>
<td>15.099</td>
<td>4.751</td>
<td>48.634</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Equilibrium Solution for General Equilibrium Model with a 50% Tax on Capital Income in Manufacturing and Nonmanufacturing Industries (M: Manufacturing, N: Nonmanufacturing)

**Equilibrium Prices**

\[
P_1 = 0.755 \quad P_2 = 0.649 \quad P_k = 0.204 \quad P_w = 1.000
\]

**Production**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Revenue</th>
<th>Capital</th>
<th>Net Capital Cost</th>
<th>Tax Paid</th>
<th>Total Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>33.764</td>
<td>25.479</td>
<td>49.389</td>
<td>10.064</td>
<td>5.033</td>
</tr>
<tr>
<td>N</td>
<td>36.517</td>
<td>23.707</td>
<td>20.611</td>
<td>4.200</td>
<td>2.100</td>
</tr>
<tr>
<td>Total</td>
<td>49.186</td>
<td>70.088</td>
<td>112.464</td>
<td>14.264</td>
<td>7.133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor</th>
<th>Total Cost</th>
<th>Per-unit Cost</th>
<th>Capital Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>10.382</td>
<td>25.479</td>
<td>0.755</td>
</tr>
<tr>
<td>N</td>
<td>17.407</td>
<td>23.707</td>
<td>0.649</td>
</tr>
<tr>
<td>Total</td>
<td>27.789</td>
<td>49.186</td>
<td></td>
</tr>
</tbody>
</table>

**Demand**

<table>
<thead>
<tr>
<th>Income</th>
<th>Labor</th>
<th>Labor</th>
<th>Expenditure</th>
<th>Labor Hours</th>
<th>Labor Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Rich</td>
<td>10.543</td>
<td>14.379</td>
<td>17.366</td>
<td>5.819</td>
<td>5.819</td>
</tr>
<tr>
<td>The Middle</td>
<td>12.305</td>
<td>9.753</td>
<td>15.617</td>
<td>9.164</td>
<td>9.164</td>
</tr>
<tr>
<td>The Poor</td>
<td>10.816</td>
<td>12.386</td>
<td>16.203</td>
<td>12.806</td>
<td>12.806</td>
</tr>
<tr>
<td>Total</td>
<td>33.764</td>
<td>36.517</td>
<td>49.186</td>
<td>27.789</td>
<td>27.789</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capital Income</th>
<th>Transfers</th>
<th>Total Income</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Rich</td>
<td>9.170</td>
<td>2.377</td>
<td>17.366</td>
</tr>
<tr>
<td>The Middle</td>
<td>4.075</td>
<td>2.378</td>
<td>15.617</td>
</tr>
<tr>
<td>The Poor</td>
<td>1.019</td>
<td>2.378</td>
<td>16.203</td>
</tr>
<tr>
<td>Total</td>
<td>14.264</td>
<td>7.133</td>
<td>49.186</td>
</tr>
</tbody>
</table>
Table 5. Equilibrium for General Equilibrium Model with 50% Tax on Capital and Labor Income in Manufacturing and Nonmanufacturing Industries (M: Manufacturing, N: Nonmanufacturing)

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>N</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equilibrium Prices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_1$ = .951</td>
<td>$P_2$ = .908</td>
<td>$P_K$ = .231</td>
<td>$P_W$ = 1.000</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity Revenue</td>
<td>$29.329$</td>
<td>$27.351$</td>
<td>$52.709$</td>
</tr>
<tr>
<td>Net Capital</td>
<td>$27.885$</td>
<td>$24.824$</td>
<td>$70.000$</td>
</tr>
<tr>
<td>Cost Paid</td>
<td>$52.866$</td>
<td>$17.134$</td>
<td>$16.190$</td>
</tr>
<tr>
<td>Total Capital Cost</td>
<td>$12.227$</td>
<td>$3.963$</td>
<td>$8.095$</td>
</tr>
<tr>
<td><strong>Net Labor Tax</strong></td>
<td>$6.114$</td>
<td>$1.981$</td>
<td>$24.285$</td>
</tr>
<tr>
<td>Per Labor Unit</td>
<td>$8.341$</td>
<td>$5.944$</td>
<td></td>
</tr>
<tr>
<td><strong>Demands</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditure</td>
<td>$8.643$</td>
<td>$6.363$</td>
<td>$18.950$</td>
</tr>
<tr>
<td>Labor Hours</td>
<td>$7.670$</td>
<td>$3.181$</td>
<td>$9.474$</td>
</tr>
<tr>
<td>Total</td>
<td>$17.330$</td>
<td>$18.880$</td>
<td>$52.709$</td>
</tr>
<tr>
<td>Capital Income</td>
<td>$18.950$</td>
<td>$18.950$</td>
<td>$38.900$</td>
</tr>
<tr>
<td>Transfer</td>
<td>$9.474$</td>
<td>$12.587$</td>
<td>$22.061$</td>
</tr>
<tr>
<td>Total Income</td>
<td>$28.424$</td>
<td>$31.537$</td>
<td>$59.961$</td>
</tr>
<tr>
<td>Utility</td>
<td>$41.501$</td>
<td>$35.698$</td>
<td>$77.199$</td>
</tr>
</tbody>
</table>
though the labor supply of the poor falls, while that of the rich and middle rises, increasing the total labor supply. The utility of the rich falls while that of the middle and poor rises. Details about welfare changes will be discussed in the next section.

The equilibrium solution for the tax program 2 is shown in Table 4. Since the 50% tax is levied on capital income in both sectors, this tax can not be avoided through shifting capital between sectors. Naturally, the net-of-tax price of capital falls, though the gross cost of capital to firms is essentially unchanged (it increases from $0.304 to $0.306, measured in labor units). Overall, the tax - since it is on a factor in fixed supply - affects resource allocation only because it redistributes income and because consumers have different preferences. Because of the tax, capital and labor employment in sector 1 slightly increase while in sector 2 labor employment increases and capital employment decreases. Since gross capital cost rises in both sectors, capital intensity falls, so the tax leads to a less capital intensive method of production in both industries. The relative price of manufacturing to nonmanufacturing output is barely changed, leading to little change in production of outputs. The incomes of the rich fall, while the middle and poor increase their expenditures. The labor supply of the rich increases, and that of the middle and poor decreases. The utility of the rich falls, while that of the middle and poor rises because of
the income redistribution effects of the tax.

Table 5 shows the results of the tax program 3, the 50% tax on labor and capital income in both sectors. The tax on capital, which is unavoidable, affects resource allocation only through its impact on the income distribution; however, the tax on labor income could be avoided by reducing the supply of labor hours. As we see from Table 5, the tax increases the relative price of labor (since labor supply shrinks), causing both sectors to adopt more capital intensive production techniques. Overall, labor employment in both sectors falls, while capital employment rises in sector 1 (falls in sector 2). The tax on labor income, which distorts the supply of labor, causes production of both outputs to decrease. The incomes of the rich fall, and those of the middle and poor rise; the labor supply of each group falls because of the labor tax. The utility of the rich and middle falls, while that of the poor rises.

D. Welfare Measures of the Gain or Loss

Policy appraisal using this sort of technique usually relies upon a comparison between an existing equilibrium and a counterfactual equilibrium computed with modified policies (i.e., with changed tax programs). For numerical welfare measures of the gain or loss, CVs and EVs are widely employed. The CV is defined as the amount of income we can take away
from, or give to, an individual after an economic change, while leaving him as well off as he was before it. The EV is defined as the amount of income we would need to give to or take from an individual, if an economic change did not happen, to make him as well off as if it did.

A reasonable measure to adopt is the money metric utility function (i.e., the expenditure function). The expenditure function, \( E(q; p, I) \), which is defined as \( e(q, v(p, I)) \), where \( v(p, I) \) is the indirect utility, measures how much income the consumer would need at prices \( q \) to be as well off as he or she would be facing prices \( p \) and having income \( I \) [26; p. 264]. By using the expenditure function, CVs and EVs can be shown in mathematical forms as follows:

\[
(22) \quad EV = E(p^0; p', I') - E(p^0; p^0, I^0) = E(p^0; p', I') - I^0
\]

\[
(23) \quad CV = E(p'; p', I') - E(p'; p^0, I^0) = I' - E(p'; p^0, I^0),
\]

where the superscript of "o" and "'" indicates the old prices and income, and the new (at the proposed project) prices and income, respectively. As seen from the above two equations, the EV is the difference between the old income and the income, at old prices, required to reach the new utility. The CV is the difference between the new income and the income, at new prices, required to reach the old utility. The CV and EV have the same sign as the direction of the change in welfare: for a welfare loss, both are negative. Although the CV seems reasonable for some compensation scheme at the new prices, the
EV is better for measuring the "willingness to pay" [26; p. 265]. There are two reasons for this. First, the EV measures the income change at current prices and it is much easier for decision makers to judge the value of a dollar at current prices than at some hypothetical prices. Second, if we are comparing more than one proposed policy change, the CV keeps changing the base prices while the EV keeps the base prices fixed at the old prices. Thus, the EV is more suitable for comparisons among various tax programs.

For measuring the welfare change for the economy as a whole, the welfare costs of the tax being considered are measured by aggregating the CVs or EVs across the individuals. The sum of EVs is a more easily interpreted measure because old incomes and prices are used and are typically the same in the sequence of pairwise comparisons [9]. This is also supported by the second reason of the superiority of the EV to the CV.

In calculating the CVs and EVs in practice, the expenditure function is obtained as below by inverting the indirect utility function, which is derived by substituting (14), (15), and (16) into (9) on the page 16-18:

\[
I_j = V_j (a_1jP_1^{1-s_j} + a_2jP_2^{1-s_j} + a_3jP_w^{1-s_j})^{1/(1-s_j)}
\]

Therefore, the CV and EV in (22) and (23) become

\[
CV_{jh} = I_{jh} - V_j0(a_1jP_1^{1-s_j} + a_2jP_2^{1-s_j} + a_3jP_w^{1-s_j})^{1/(1-s_j)}
\]
EVjh = Vjh(a1jP1θ1 - sj + a2jP2θ1 - sj + a3jPwθ1 - sj)1/(1-sj) - Ijθ,

where

θ: values at the no-tax equilibrium,

j: consumers 1, 2, and 3,

h: values at the tax programs 1, 2, and 3.

Table 6 shows the values of the CVs and EVs comparing the no-tax equilibrium price with each of equilibrium prices of three tax programs.

Table 6. Welfare Measure of Impacts of Three Tax Programs

<table>
<thead>
<tr>
<th></th>
<th>Tax Program 1</th>
<th>Tax Program 2</th>
<th>Tax Program 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EV</td>
<td>CV</td>
<td>EV</td>
</tr>
<tr>
<td>The Middle</td>
<td>.021</td>
<td>.021</td>
<td>.334</td>
</tr>
<tr>
<td>The Poor</td>
<td>1.524</td>
<td>1.503</td>
<td>1.843</td>
</tr>
<tr>
<td>Total</td>
<td>.310</td>
<td>.303</td>
<td>0.000</td>
</tr>
</tbody>
</table>

As seen from the Table 6, at the tax program 1, the rich become worse off and the middle and poor better off, leaving the economy "as a whole" worse off. At new prices due to the 50% tax on capital income in sector 1, we must give the rich 1.827 more units of income at new incomes to compensate them for a price change such that utility is unchanged, while the middle and poor are willing to pay 0.021 and 1.503 units of income from the new incomes for the pre-tax satisfaction. These income changes would be necessary to compensate the
consumers for the price change. Meanwhile, at old prices (i.e., at no-tax equilibrium prices), the old income minus 1.855 units is needed to reach the new utility for the rich while 0.021 (and 1.524) more units of income at the old incomes will make the middle (and the poor) reach the new utility in this tax system. These income changes at the new prices would be equivalent to the proposed change.

At the tax program 2, comparing no-tax equilibrium to equilibrium with 50% tax on capital income in both sectors, the economy as a whole does not experience any welfare change because the tax is non-distortionary, even though the rich become worse off and the middle and poor better off. At new prices due to the taxes, we must give the rich 2.18 units of income at current incomes to compensate for a price change such that utility is unchanged, while the middle and poor are willing to pay 0.335 and 1.845 units of income from the current incomes to achieve no-tax utility. Meanwhile, at old prices (i.e., at no-tax equilibrium prices), the old income minus 2.177 units is needed to reach the new utility for the rich while 0.334 (and 1.843) more units of income at the old incomes will make the middle (and the poor) reach the new utility.

At the tax program 3, 50% tax on labor and capital income in both sectors, the economy as a whole becomes worse off even though the poor improve their welfare and the rich and middle
become worse off. At new prices due to the taxes, we must give the rich and middle 2.674 and 0.114 units of income at current incomes and we can take 0.781 units of income away from the old income of the poor to compensate for a price change such that utility is unchanged. Meanwhile, at old prices (i.e., at no-tax equilibrium prices), the old incomes of the rich and middle minus 2.347 and 0.099 units is required to reach the new utility for them, while 0.459 more units of income at the old incomes will make the middle reach the new utility.

What must be emphasized once again is that the EV measures the income change at current prices, so we can judge the value of a dollar at current prices and the EV uses the status-quo prices (i.e., no-tax equilibrium prices) as the base for every comparisons with prices of each of three tax programs. Thus, the sum of EVs is a more easily interpreted measure for the economy as a whole. With this concept of the sum of EVs, the economy as a whole in the tax program 1 and 3 decreases 0.31 and 1.787 units of income from the old one (at no-tax equilibrium) to reach the new utility whatever the levels of each consumer group's utility are, while the tax program 2 keeps the same income at the current prices as the old income. Caution, however, follows in interpreting the welfare change of the economy as a whole: each utility of the three consumers at the changed utility possibility frontiers can not be said Pareto superiority or inferiority to the
utility at the old utility possibility frontiers of the no-tax model.
V. SUMMARY AND CONCLUSIONS

The AGEA converts the Walrasian general equilibrium structure from an abstract representation of an economy into realistic models of actual economies. Techniques developed by it calculate not only equilibrium prices with and without taxes and associated levels of production, but also the tax revenues to evaluate policy options.

I simulate a tax model of the AGEA. I take the two sector, two factor, and three consumer model with the labor-leisure choice. Production of each good takes place according to a CRS CES production function and utility is a form of a CES function. Factor demands are derived from cost minimization, while commodity demands from utility maximization subject to the budget constraint. The model satisfies three equilibrium conditions, which provide solutions of general equilibria with and without tax by a computer package.

Three tax programs are considered. When 50% tax on capital income in the manufacturing industry is levied, the net-of-tax price of capital falls. Since the gross-of-tax price of capital in sector 1 increases compared to the no-tax model, capital intensity of sector 1 falls while that of sector 2 rises. Less manufacturing and more nonmanufacturing output is produced, and the relative price of manufacturing to nonmanufacturing output rises. Incomes of the poor
increase and those of the other two decrease, while the labor supply of the poor falls and that of the rich and middle rises. The rich become worse off and the middle and poor better off because of this tax. For the rich to reach the new utility, the old income minus 1.855 is required while \( 0.021 \) (1.524) more units of income at old incomes makes the middle (poor) reach the new utility.

When 50% tax on capital income in both sectors is levied, both industries can not escape this tax. Capital intensity in both sectors falls very slightly because the gross-of-tax price of capital increases from 0.304 to 0.306, leading to a less capital intensive method of production in both industries. This very marginal change is at large due to the redistribution of income. The relative price of manufacturing to nonmanufacturing output and production levels of both sectors are little changed. The incomes of the rich fall while those of the middle and poor rise, and the labor supply of the rich increases and that of the middle and poor decreases because of the income effect of the transfer. The rich become worse off and the middle and poor better off. At no-tax equilibrium prices, the old incomes of the rich minus 2.177 is required to reach the new utility while 0.334 (1.843) more units of income at the old incomes will make the middle (poor) reach the new utility.

Neither industry can avoid the 50% tax on labor and capital income in both sectors. Since relative price of
capital to labor falls, both sectors use more capital intensive method of production, employing more capital and less labor in sector 1 and less capital and labor in sector 2. The relative price of manufacturing to nonmanufacturing output falls. The labor supply is distorted and reduced because of the labor tax, causing production of both outputs to decrease. The incomes of the rich fall and those of the middle and poor rise with a decrease of labor supply of every consumer. The rich and middle become worse off and the poor better off in this tax system unlike the other two. At no-tax equilibrium prices, the old incomes of the rich and middle minus 2.347 and 0.099 units are required to reach the new utility, while 0.659 more units of income at the old income will make the poor reach the new utility.

Although tax programs 1 and 3 make the economy as a whole worse off, and tax program 2 is neutral based on that the sum of EVs is a more easily interpreted measure because old incomes and prices are used, the utility of three consumers can not be said Pareto superiority or inferiority at the changed utility possibility frontier in interpreting the welfare of the economy as a whole. This is because taxes in tax programs 1 and 3 are distortionary, and those in tax program 2 are non-distortionary.
VI. BIBLIOGRAPHY


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VIII. APPENDIX : EQUILIBRIUM CONDITIONS FOR FACTOR TAXES

When a factor tax is imposed, marginal product of a factor equals the gross-of-tax income of its factor. Thus, MRTS between capital and labor in each sector is equal to the ratio of the gross-of-tax income of labor to that of capital:

\[
\frac{\partial Q_i}{\partial L_i} = \frac{\frac{b_i L_i}{1-b_i K_i} - \frac{1}{c_i}}{\frac{P_w(1+t_{li})}{P_k(1+t_{ki})}}
\]

where

- \(Q_i\): production functions given in (1) on p. 15,
- \(t_{li}\): ad valorem tax rate on labor income in the sector \(i\),
- \(t_{ki}\): ad valorem tax rate on capital income in the sector \(i\),
- \(P_w\): net-of-tax price of labor,
- \(P_k\): net-of-tax price of capital, and
definitions of other notations are same as in no-tax equilibrium and results.

This gives us the following:

\[
A1 \quad K_i = \frac{1-b_i (1+t_{li})P_w}{b_i (1+t_{ki})P_k} c_i
\]

\[
A2 \quad k_i = \frac{K_i}{L_i} \quad \text{will be defined as the capital intensity as}
\]
Before.

From the production functions, (A1), and (A2), the factor demand functions become

\[
L_i = \frac{1}{a_i} Q_i \frac{b_i}{1 - b_i} \left( \frac{(1 + t_{ki}) P_k}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)} + \frac{1 - b_i}{1 - b_i} \left( \frac{(1 + t_{li}) P_w}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)}
\]

\[
K_i = \frac{1}{a_i} Q_i \frac{1 - b_i}{b_i} \left( \frac{(1 + t_{li}) P_w}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)} + \frac{1 - b_i}{1 - b_i} \left( \frac{(1 + t_{ki}) P_k}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)}
\]

Meanwhile, the total cost functions of each sector become

\[
T_C_i = (1 + t_{li}) P_w L_i \frac{(1 + t_{ki}) P_k}{1 + t_{li} P_w L_i}
\]

Plugging (A2) and (A3) into (A5) gives

\[
T_C_i = \frac{1}{a_i} Q_i (b_i) \frac{c_i}{1 - c_i} \left( \frac{(1 + t_{li}) P_w}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)} + \frac{1 - b_i}{b_i} \left( \frac{(1 + t_{ki}) P_k}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)}
\]

The marginal cost derived from (A6) equals the price of output of each sector:

\[
\frac{1}{a_i} Q_i (b_i) \frac{c_i}{1 - c_i} \left( \frac{(1 + t_{li}) P_w}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)} + \frac{1 - b_i}{b_i} \left( \frac{(1 + t_{ki}) P_k}{1 - c_i} \right) \frac{1}{c_i/(1 - c_i)}
\]

The CES utility functions are same as before:
\[ U_j = \left[ a_{ij1}/sj x_{1j}(sj-1)/sj + a_{i2j}/sj x_{2j}(sj-1)/sj \right. \]
\[ \left. + a_{i3j}/sj R_j(sj-1)/sj \right]/(sj-1) \]

Since I assume all government revenues are evenly redistributed to consumers, the budget constraint of each consumer contains a share of the total tax revenue, \( T/3 \). Each of tax revenues from the tax rates levied on factor incomes in both sectors is denoted as \( T_i \); that is,

\[ T_1 = t_{k1} P_k K_1, \]
\[ T_2 = t_{l1} P_w L_1, \]
\[ T_3 = t_{k2} P_k K_2, \]
\[ T_4 = t_{l2} P_w L_2. \]

The total tax revenue, \( T \), is a sum of the above \( T_1, T_2, T_3, \) and \( T_4 \). Thus, the budget constraint of each consumer becomes

(A9) \[ I_j' = P_k W k_j + 24 P_w + T/3 \Rightarrow P_1 X_{1j} + P_2 X_{2j} + P_w R_j \]

By the first-order conditions of the utility maximization subject to the budget constraint (A9), we have the following commodity and leisure demand functions:

(A9) \[ X_{1j} = \frac{a_{1j} I_j'}{P_1 S_j D_j}, \]

(A10) \[ X_{2j} = \frac{a_{2j} I_j'}{P_2 S_j D_j}, \] and

(A11) \[ R_j = \frac{a_{3j} I_j'}{P_w S_j D_j}, \]

where
\[ D_j = a_1 p_1 (1-s_j) + a_2 p_2 (1-s_j) + a_3 p_w (1-s_j) \] and
\[ I_j' = P_k W_k j + 24 P_w + T/3. \]

The three equilibrium conditions for the tax model are same as those for the no-tax model:

1. demand equals supply in inputs
   
   \((A12)\) \quad L_1 + L_2 = 72 - R_1 - R_2 - R_3
   
   \((A13)\) \quad k_1 L_1 + k_2 L_2 = k \) (k is the total market endowment of capital)

   where L_1, L_2, k_1, and k_2 are given by \((A2)\) and \((A3)\),

2. demand equals supply in outputs
   
   \((A14)\) \quad X_{11} + X_{12} + X_{13} = Q_1
   
   \((A15)\) \quad X_{21} + X_{22} + X_{23} = Q_2

   where X_{1i} and X_{2i} are given by \((A9)\) and \((A10)\)

3. zero profit conditions hold

   \((A15)\) \quad P_1 = \frac{1}{a_1} \frac{(b_1 c_1)}{(1-c_1)} \frac{1/((1+t_1 l) P_w (1-c_1))}{\frac{1-b_1 c_1}{b_1}}

   \((A17)\) \quad P_2 = \frac{1}{a_2} \frac{(b_2 c_2)}{(1-c_2)} \frac{1/((1+t_2 l) P_w (1-c_2))}{\frac{1-b_2 c_2}{b_2}}

With the same parameter values as the no-tax model, the solution to this tax model is characterized by 22 variables, the four prices, \(P_1\), \(P_2\), \(P_w\), and \(P_k\), nine demand quantities, \(X_{11}, X_{21}, R_1, X_{12}, X_{22}, R_2, X_{13}, X_{23}\), and \(R_3\), two labor and capital inputs, \(L_1, L_2, k_1,\) and \(k_2\), two capital intensities,
k1 and k2, and five tax revenues, T1, T2, T3, T4, and T.

The no-tax model is same as all tax rates being zero in this tax model. At the tax program 1, tk1 = 0.5, and t11 = tk2 = t12 = 0. At tax program 2, tk1 = tk2 = 0.5, and t11 = t12 = 0. At tax program 3, tk1 = tk2 = t11 = t12 = 0.5.