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Effects of surface thermal forcing on stratified flow past an isolated obstacle

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Iowa State University, 1992
Effects of surface thermal forcing on stratified flow
past an isolated obstacle

by

Jon Reisner

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GENERAL INTRODUCTION

Scorer (1968) has pointed out that most experimental studies tend to overemphasize the effects of upstream stratification instead of the equally or perhaps more important effects of the localized heating and cooling of the surface of a three-dimensional (3-D) obstacle. Indeed, as will be shown in this study, surface heating can lead to dramatic changes in the flow such as allowing for the disappearance of the upwind stagnation (defined as surface flow that is either zero or in opposite direction to the incoming flow). To understand how surface heating of mesoscale obstacles, such as the Hawaiian Archipelago, affects the flow, it is important to realize that heating will not only influence the surface flow but, perhaps more importantly, can also modify elevated flow structures such as internal gravity waves, columnar disturbances, and lee vortices.

Previous numerical and experimental studies (Smolarkiewicz and Rotunno 1990, hereinafter SR2; Hunt and Synder 1980) have shown that for low Froude number flow, \( Fr \equiv U/Nh \), where \( U \) is the upstream wind speed, \( N \) is the Brunt-Väisälä frequency, and \( h \) is the height of the obstacle) a strong deceleration of the flow occurs over the upwind side of the obstacle. Typically (for \( Fr \leq 0.5 \)) associated with flow deceleration are two singular points, the upstream attachment point (or equivalently the height of the dividing streamline) and the upstream separation point (see Fig. 1). If rotational effects are neglected,
on the center plane above the attachment point flow goes over the obstacle; whereas below the attachment point, the flow reverses direction and follows back downward until it encounters the upwind separation point. The flow then separates and goes outward around the obstacle. Hereinafter the region of flow between the attachment and separation points will be defined as the upwind stagnation zone. The upwind separation point is associated with a region of fluid upwelling, referred to as the upstream ripple by SR2.

The primary cause of the upwind stagnation and the related phenomena has been attributed to the high-pressure perturbation associated with the standing mountain wave (Smith 1980, SR2). Because the influence of the high pressure perturbations associated with the mountain wave is typically small downstream of the upwind stagnation zone, when surface heating occurs anabatic flow should develop quickly in that area. As the heating cycle continues, hydrostatic reduction of pressure should allow for the anabatic flow to develop farther upstream. For certain upstream conditions this anabatic flow is expected to reach the sea-breeze circulation developing at the coast (see Pielke 1984 and Atkinson 1981 for reviews of sea-breeze and anabatic circulations).

In this study, both the dimensional analysis and particular solution to the (3-D) linearized equations of motion, with surface thermal forcing included, will be used to estimate whether upwind stagnation for a certain set of upstream conditions will disappear with heating or, equivalently, whether there will be a critical transition from the blocked to unblocked flow regime.
Fig. 1. Characteristics of low Fr flow past an isolated obstacle (after Hunt and Synder 1980).

$F_r = 0.4$
For $\beta > 1$, where $\beta$ is the aspect ratio of the obstacle (=across-stream length/along-stream length), wave breaking aloft and columnar modes become important parts of the solutions. According to Pierrehumbert and Wyman (1985) the columnar disturbances enhance offshore flow over the upwind side. How wave activity induces columnar mode production is, however, uncertain. According to Pierrehumbert and Wyman (1985, p. 1002):

The precise role of wave breaking in generating upstream influence remains obscure. Although it seems clear that the onset of upstream influence in the nonrotating case is associated with wave breaking, the physical mechanism responsible for excitation of columnar disturbances has not been identified. It also remains to be seen whether excitation mechanisms independent of wave breaking become operative (p. 1002).

Even though the numerical simulations of SR2 were unable to quantify the relative importance of each effect, their simulations strongly suggest that for obstacles with $\beta \leq 8$ the major cause of flow deceleration is associated with the standing mountain wave.

Unlike the flow over the upwind side, the flow over the lee—even to the first approximation—cannot be described by linear theory. For upstream
Fr between 0.0 and 0.5, Smolarkiewicz and Rotunno (1989, hereinafter SR1) showed that lee vortices exist over the downwind side of the obstacle with the scale and intensity of the leeward vortices linked to the degree to which isentropic surfaces are depressed. SR1 hypothesized that vertical-vorticity of the lee-eddies derives from the tilting of vortex lines that adhere to depressed isentropic surfaces. The degree of the depression in the lee can be estimated by linear theory, but the actual magnitude of the vorticity required to produce flow reversal eluded linear predictions. Furthermore, the numerical study of Crook et al. (1990) provides evidence that the regions of intense vertical vorticity that are often observed in the lee of the obstacles at low Froude numbers may be associated with the highly nonlinear processes of isentropic overturning and wave breaking.

Since a high-pressure perturbation does not initially exist over the lee, both sea-breeze and anabatic circulations should develop quickly with heating. And because a longer Lagrangian time scale of heating (a time over which fluid particles are exposed to heat sources) is typically found in the lee, the thermal circulations that develop should be of greater intensity than those found over the upwind side of the obstacle. The leeward sea-breeze/anabatic circulation should reduce the vertical tilt of the isentropic surfaces in that region. This suggests that the resulting thermal circulations will counteract overturning of the isentropes. For certain flow conditions the thermal circulations over the lee
will either intensify preexisting vortices or generate sufficient vertical vorticity to produce lee vortices. However, as pointed out by Sun and Chern (1992) in the case of preexisting lee vortices, it is difficult to quantify the role that tilting and vortex stretching play in the evolution of the lee vortices. Since thermally induced circulation tends to reduce the vertical tilt of the isentropes behind an obstacle, with the onset of heating the intensity of the reversed flow should theoretically decrease. But with the progression of the heating cycle the magnitude of the reversed flow may increase again as more horizontal vorticity (generated by surface heating) is tilted to the vertical by gravity wave motions.

The issue of the lee eddies has also been addressed within frameworks of simplified physical models. Schär and Smith (1993) demonstrated (by means of numerical experiments) that lee vortex formation in a shallow water system may be traced to vertical vorticity generation across a hydraulic jump. Dempsey and Rotunno (1988) described the development of the lee eddies in terms of a mixed-layer model. They attributed horizontal variations of buoyancy on a constant z-plane to a particular form of the Reynolds stresses implied by the formulation of mixed-layer models. They have shown that vertical vorticity of the eddies is acquired from the curl of the Reynolds stresses. Although both idealizations support lee vortices, the assumptions underlying the shallow water and mixed-layer models are invalid (and poorly supported by observations) for continuously stratified flows past mesoscale obstacles. Hence,
the relevance of these results to natural atmospheric flows is unclear.

Even though it may be difficult to determine the actual physical mechanisms responsible for producing any given vortex, once vortices form they can significantly influence local-scale weather patterns. Naturally, lee vortices induce large vertical wind shear in the lee of the obstacle. They also help to provide a wake region where diabatic effects may allow for greater vertical accelerations than found over the windward side. Thus, with sufficient moisture, the vertical wind shear in combination with a vertical motion field produced by surface heating may allow for the development of long lasting thunderstorm complexes. The latent heat released by these thunderstorm complexes can allow for further intensification of the lee vortices. The southwest vortex found in China (see Kuo et al. 1986, Kuo et al. 1988) and the mesolow found off the southeast coast of Taiwan (Smith 1982) are examples of such phenomena. Upon moving off the Tibetan Plateau, the southwest vortex can bring torrential rainfall to China that frequently results in large losses of both life and property. Severe weather has also been associated with the so-called Denver cyclone (Crook et al. 1990). In fact, a Denver cyclone occurred on the day in which a hailstorm ravaged Denver causing $350,000,000–$650,000,000 in property damage. It is also possible that upon shedding off an obstacle a lee vortex may evolve into a tropical cyclone. When considering how heating will influence low Fr flow past mesoscale obstacles, the effects of the Earth's rotation
and moisture need to be considered. Unlike 2-D flow for which the length scale of flow deceleration is of such extent that the Coriolis force can produce significant velocity perturbations (Pierrehumbert and Wyman 1985), the horizontal extent of flow deceleration over the upwind side of a 3-D obstacle is usually limited; hence the magnitude of the flow perturbations produced by the Coriolis force is expected to be small. But lee vortex shedding, now realized in simulations with the Coriolis force active, should allow for significant differences to occur in the lee between simulations with the Coriolis force active versus those without. As shown by Schär and Smith (1993) vortex shedding can be explained in terms of a localized barotropic instability problem. Any asymmetric perturbation, such as produced by the Coriolis force, of sufficient spatial and temporal extent will excite the instability and produce vortex shedding. Since symmetric heating is applied in the idealized simulations, it is expected that the velocity perturbations produced by heating will tend to locally counteract this instability process. Though the idealized simulations will not include the effects of latent heat release on the flow, the virtual effects of moisture will be included. Low-level moisture (by making the flow more potential) should result in a reduced areal extent of the upwind stagnation zone and weaker lee vortices.
Explanation of Dissertation Format

This dissertation contains one paper to be submitted to a professional journal. After the introduction, the results obtained from dimensional analysis and linear theory will be presented. Once the description of the numerical model and the experimental design is complete, results illustrating different flow responses to heating will be shown. A comparison of the idealized simulations against both observations and flow past Hawaii will be made. Next, several appendixes will present material not discussed in the paper. At the end of the general conclusions, suggestions for future research will be offered. References cited in the dissertation follow the general conclusions.
PAPER:
LOW FROUDE NUMBER FLOW PAST THREE-DIMENSIONAL OBSTACLES:
EFFECTS OF SURFACE THERMAL FORCING
LOW FROUDE NUMBER FLOW PAST THREE-DIMENSIONAL OBSTACLES:
EFFECTS OF SURFACE THERMAL FORCING

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(to be submitted to J. Atmos. Sci.)
ABSTRACT

The present study investigates basic aspects of the flow of a density-stratified fluid past three-dimensional obstacles for Froude number $\sim O(1)$ and isolated surface thermal forcing representative of diurnally varying mesoscale flows past mountainous islands such as Hawaii. In order to minimize parameter space, we have excluded the effects of friction, rotation, nonuniform ambient flow, and the complexities of realistic surface boundary layer and terrain. Through simple scaling arguments, we deduce that the parameter $\eta^*$

$$\eta^* \equiv \frac{L^2}{\nu R} \left( \frac{g h}{g z} \right)^{-1} \sim O(1) \text{ for mesoscale flows}$$

controls thermally forced flows for a given Froude number, and we provide crude estimates of a flow response for a range of $\eta^*$. The principal question addressed is for what values of $\eta^*$ will a transition occur from the low-Froude-number flow regime, characterized by the stagnation and splitting of the lower upwind flow, to the regime in which flow passes over rather than around the obstacle. We show that the linear theory captures such a tendency consistently with simple scaling arguments. To provide quantitative measures of flow variability with the Froude number and $\eta^*$, we employ an efficient isentropic numerical code and summarize the results of numerous simulations in the form of a regime diagram. The principal result is a simple criterion for the transition of a heated flow from the blocked to unblocked flow regime. We illustrate the relevance of the idealized study to natural flows with an example of applications to a flow past the Hawaiian Archipelago.
I. INTRODUCTION

The influence of mesoscale mountain complexes on local weather and climate is one of the fundamental topics of meteorology. Due to the natural variability with height of potential temperature in the atmosphere, the problem needs to be analyzed in terms of stratified flows past three-dimensional topography. The key parameter characterizing such flows is the Froude number, $Fr = U/Nh$, where $U$ is the far upstream flow speed, $N$ is the Brunt-Väisälä frequency, and $h$ is the height of an obstacle. Considering a typical value of the wind speed in the lower troposphere of $U \sim 10 \text{ m s}^{-1}$, and a typical value of the Brunt-Väisälä frequency of $N \sim 0.01 \text{ s}^{-1}$, one finds $Fr \sim 1/h[\text{km}]$. Thus the flow interaction with mountains of height exceeding approximately 2 km will be in the $Fr \sim \sim 0.5$ flow regime. This flow regime (subsequently referred to as strongly stratified or low-Froude-number flow) is of very special interest. Such flows exhibit upwind influences whose extent and intensity depend on the Froude number and the cross-flow horizontal scale of the mountain. The most pronounced feature of the upwind low-level flow is a remote separation of the flow and a resulting low-level convergence zone. In the separation region, the air parcels are subject to vertical displacements of $Fr \cdot h$. Downwind of the separation region, the strongly stratified flows exhibit low-level flow reversal, which may often be mistaken for katabatic effects or a land breeze. On the
lee-side of the mountain such flows evince a pair of intense vertically oriented vortices of spatial scales comparable to the horizontal extent and height of the mountain.

Although low-Froude-number flows are not yet fully understood, and there is no consensus upon physical processes underlying their realizations, the above-outlined, overall picture has been consistently reproduced in numerous studies. Laboratory experiments of Hunt and Snyder (1980), idealized numerical experiments of Smolarkiewicz and Rotunno (1989, 1990), Crook et al. (1990), and Rotunno and Smolarkiewicz (1992) provide representative examples and discussions of such flows. Theoretical studies of Smith (1980, 1988, 1989) aid interpretation of idealized flows. Salient features of low-Froude-number flows were also identified in studies of flows past the island of Hawaii (Smolarkiewicz et al. 1988, Rasmussen et al. 1989, Ueyoshi and Han 1991, Rasmussen and Smolarkiewicz, 1993) and Taiwan (Sun and Chern 1992), both constituting natural laboratories for strongly stratified mesoscale flows.

Over the last decade, the comprehension of stratified flows past three-dimensional (3-D) obstacles has been considerably advanced. However, as systematic investigations have focused so far on the highly idealized flows, many issues relevant to natural flows (Earth rotation, frictional boundary layers, surface thermal forcing, complexity of the terrain, nonuniformity of the ambient profiles) have been left unresolved, or addressed either in a preliminary man-
ner or in the context of particular applications. One such issue is the response of stratified flows to surface thermal forcing, which is equally or perhaps more important than effects of upstream stratification (Scorer 1968).

Herein we attempt a systematic investigation of idealized, thermally-forced, low-Froude-number flows. We assume uniform ambient wind and stratification, axially-symmetric bell-shaped obstacles with moderate slopes representative of mesoscale mountains, and a simple thermal-forcing function that mimics natural effects over mountainous islands. We neglect the Earth's rotation, surface friction, viscosity, dissipation, and moisture (preliminary results addressing flow sensitivities to some of these factors are discussed in Reisner 1992). With these simplifications we can characterize flows with two parameters: the Froude number and a characteristic scale of thermal forcing (defined later in this paper). The principal tool employed is a hydrostatic, isentropic numerical model whose efficiency allows for numerous simulations at a reasonable computational effort. The results of a series of simulations for a range of the Froude numbers and characteristic scales of heating are summarized in the form of a regime diagram. In order to aid interpretation of numerical results, we provide simple theoretical arguments (in the spirit of the scale analysis) as well as a particular solution to the steady-state, linearized, hydrostatic equations of motion. The relevance of the theoretical considerations and idealized numerical simulations to natural flows is illustrated with an example of an application.
to a flow past the Hawaiian Archipelago. Comparison with field observations collected during the Hawaiian Rainband Project (HaRP) supports our analysis.

This paper is organized as follows. Chapter II offers theoretical estimates (based on dimensional arguments and linear solutions) of the low-Froude-number flow tendencies due to thermal forcing. Chapter III presents the numerical model and the experimental design, whereas Chapter IV discusses the results of a series of idealized simulations. Chapter V provides an example of an application to the flow past the Hawaiian Archipelago and the discussion of relevant observations collected during HaRP.
II. THEORETICAL CONSIDERATIONS

A. Elementary Arguments

Low-Froude-number flows past 3-D obstacles are, for even an ideal fluid, complex and as yet not fully understood. Surface thermal forcing adds further complexity to the problem. Notwithstanding, simple estimates concerning possible flow realizations may be deduced from the elementary analysis of fluid equations. Such an exercise is worthwhile; it aids the interpretation of numerical results and provides guidelines for judging their consistency with a physical intuition. To establish such guidelines, we analyze an ideal, nonrotating, density-stratified Boussinesq fluid of constant buoyancy frequency and uniform ambient flow.

The Boussinesq equations for an ideal fluid may be compactly written as follows:

\[
\frac{d\mathbf{V}}{dt} = -\nabla \phi + kg \frac{\Theta'}{\Theta_o},
\]

(1)

\[
\frac{d\Theta}{dt} = Q,
\]

(2)

\[
\nabla \cdot \mathbf{V} = 0,
\]

(3)

where \(\mathbf{V}\) is the wind vector, \(\phi\) is the pressure departure from a hydrostatic environmental value \(\bar{\phi}\) (both divided by a reference density); \(g\) is the gravitational constant; \(\Theta\) denotes the potential temperature of a fluid parcel; \(\Theta' = \Theta - \Theta_o\) represents its deviation from an ambient, hydrostatic profile \(\Theta_o(z)\); \(\Theta_o = \Theta_o(0)\)
is a reference potential temperature; and \( Q \) represents a heating rate (can be either positive or negative).

Assuming \( Q \neq 0 \) over the region of a characteristic length scale \( L^* \) (\( Q = 0 \) elsewhere) over the obstacle, and integrating (2) along a parcel's trajectory from its datum \((x_o, t_o)\) far upstream to an arbitrary \((x, t)\) above the hill leads to

\[
\Theta(x, t) = \Theta(z - \eta) - \int_{t_o}^{t} Q dt,
\]

where \( \eta \) is a parcel's vertical displacement from its datum. In (4), the relationship \( \Theta(x_o, t_o) = \Theta(z - \eta) \) has been employed to express a parcel's far-upstream temperature through the environmental value; recall that when \( Q \neq 0 \), \( \eta \) does not coincide with the vertical displacement of an isentrope. Expanding the first term on the right hand side (rhs) of (4) into the Taylor series about \( z \), and employing the result in (1), gives the modified momentum equation

\[
\frac{dV}{dt} = -\nabla \phi - k \left[ N^2 \eta + \frac{g}{\Theta_o} \int_{t_o}^{t} Q dt + \mathcal{O} \left( \frac{dN}{dz} \eta^2 \right) \right],
\]

where \( N^2 = \frac{g}{\Theta_o} \frac{\partial \Theta}{\partial z} \) is the Brunt-Väisälä frequency. Note that for typical tropospheric applications \( N \approx \text{const.} \sim 10^{-2} \text{s}^{-1} \), which justifies \( \mathcal{O} \equiv 0 \) in (5).

Normalizing (5) with respect to the linear and velocity scales \( h \) and \( U \), respectively, leads to the dimensionless form of (5)

\[
\frac{dV}{dt} = -\nabla \phi - k \left[ \frac{1}{F \gamma^2} \eta + \frac{gh^2}{\Theta_o U^3} \int_{t_o}^{t} \tilde{Q} dt \right],
\]

where \( x = h^{-1} \tilde{x} \), \( V = U^{-1} \tilde{V} \), \( t = Uh^{-1} \tilde{t} \), \( \phi = U^{-2} \tilde{\phi} \), \( \eta = h^{-1} \tilde{\eta} \), and the tilde refers to dimensional variables.
Insofar as the Boussinesq approximation is concerned, (6) is exact for any $Q$ of compact support. In order to evaluate overall effects of heating, we shall further assume crude estimates of a mean heating rate, $\overline{Q}$, and the dimensionless time during which a fluid particle is exposed to a heat source $\tau \approx L^*/h$.

With these approximations, (6) becomes

$$\frac{dV}{dt} = -\nabla \phi - k \frac{1}{F r^2} (\eta - \eta^*),$$

(7)

where

$$\eta^* \equiv \frac{L^* \overline{Q}}{U h} \left( \frac{\partial \theta}{\partial z} \right)^{-1}.$$  \hspace{1cm} (8)

(7) and (8) are useful for estimating overall effects of heating on stratified flows. In the limit of $\eta^* = 0$, (8) reproduces the familiar result that for a given boundary condition the realizations of stratified flows past 3-D obstacles are controlled solely by $Fr$. In particular, as $Fr \to \infty$ the second term on the rhs of (7) vanishes, and in the absence of frictional boundary layer (free-slip assumption) the solution is given by a 3-D potential flow with simply-connected isentropes and a flow passing over the obstacle. In contrast, $Fr \to 0$ implies $\eta = 0$ and $k \nabla \times \mathbf{V} = 0$, i.e., the solution is given by a 2-D, horizontal potential flow with multiple connected isentropes and a flow going around the obstacle.

Between these two limits, the ideal fluid flows evolve from these described by the small-perturbation linear theory (Smith 1980) toward those approximated by the asymptotic solutions of Drazin (1961) (cf. Smolarkiewicz and Rotunno 1989, Smolarkiewicz and Rotunno 1990).
Although $\eta^* \neq 0$ substantially complicates possible flow realizations, some simple conclusions can be deduced. For instance, if $\eta^* \geq 1$ the stratified flow should go over an obstacle regardless of the value of the Froude number. If $|\eta^*| \ll 1$, the effects of heating are expected to be negligible. Perhaps the most fascinating case is where $\eta^* \sim Fr \lesssim 1$. Then, the character of the solution may depend upon a subtle balance between the magnitudes of $\eta^*$ and $Fr$ (recall that for thermally unforced low-Froude-number flows $\eta \sim Fr$). Apparently, a critical transition from a blocked (upwind stagnation and separation) to unblocked (no upwind stagnation) flow regime may occur.

The relationship (7) merely provides a guideline for identification of different regimes of thermally forced stratified flows past mesoscale obstacles. Recall that while deriving (7), the uniform Lagrangian time scale $\tau = L^*/h$ was adopted. In general, however, $\tau$ varies between the trajectories arriving at different locations $(x, t)$ over the hill. To a first approximation, $\tau$ is about twice as large for the leeward locations as for the windward locations. Also, one may expect that a returning wake flow in the lee will further contribute toward such a differentiation. Consequently, the overall effects of thermal forcing are expected to be substantially more pronounced in the lee than on the upwind side of the obstacle.

B. Linear Theory

Dimensional analysis in the preceding section offers crude estimates of
possible effects due to the thermal-forcing in stratified flows past an isolated obstacle. Further estimates may be obtained by considering linearized equations of motion in a steady-state limit. Although the small-perturbation linear theory is too crude a tool to provide quantitatively meaningful predictions, it sheds some light on flow tendencies dependent on the Froude number and characteristics of thermal forcing. In order to evaluate a closed-form, particular solution to the linearized problem, we shall follow closely the development in Smith (1980).

The linearization of the hydrostatic Boussinesq system [(1) to (3), with \( w = 0 \) on the rhs of (1)] results in

\[
\begin{align*}
U u'_{x} &= -\phi'_{x} , \\
U v'_{x} &= -\phi'_{y} , \\
\phi'_{z} &= b' , \\
u'_{x} + v'_{y} + w'_{z} &= 0 , \\
Ub'_{x} + w'N^{2} &= \frac{Qg}{\Theta_{o}} ,
\end{align*}
\]  

(9a) (9b) (9c) (9d) (9e)

where \( U \) is a uniform undisturbed flow (assumed in the \( x \)-direction), \( b' \) denotes the buoyancy, and primes refer to perturbed quantities; other symbols have their usual meaning.

Eliminating appropriate variables in (9) and employing the steady-state kinematic relationship

\[
w' = U\eta_{x}
\]

(10)
leads to a single equation for the vertical displacement of fluid parcels,

\[ \eta_{zzzz} + \frac{N^2}{U^2} \nabla_H^2 \eta_z = \frac{g}{\Theta_o U^3} \nabla_H^2 Q \]  \hspace{1cm} (11)

[cf. (3) in Smith 1980]. Assuming a vertically attenuated heating function \( Q \equiv Q(x, y) \exp(-\alpha z) \), and representing \( \eta \) and \( Q \) by the Fourier integrals

\[ \eta(x, y, z) = \int \int \int_{-\infty}^{\infty} \hat{\eta}(k, l, z) e^{i(kx + ly)} dk dl \]  \hspace{1cm} (12)

\[ Q(x, y, z) = \int \int \int_{-\infty}^{\infty} \hat{Q}(k, l) e^{-\alpha z} e^{i(kx + ly)} dk dl \]  \hspace{1cm} (13)

leads to

\[ \hat{\eta}_{zz} + m^2 \hat{\eta} = iQ'e^{-\alpha z}, \]  \hspace{1cm} (14)

where

\[ Q' \equiv -\frac{\hat{Q}g(k^2 + l^2)}{\Theta_o U^3 k^3} , \]  \hspace{1cm} (15)

and

\[ m^2 = \frac{N^2 (k^2 + l^2)}{U^2 k^2} , \]  \hspace{1cm} (16)

or alternatively

\[ m^2 = \frac{N^2}{U^2 \cos^2 \psi} , \]  \hspace{1cm} (17)

with \( \psi \) denoting the angle of the horizontal wavenumber vector. The general solution to (14) is

\[ \hat{\eta}(k, l, z) = A e^{imz} + B e^{-imz} + \frac{iQ'e^{-\alpha z}}{\alpha^2 + m^2} . \]  \hspace{1cm} (18)
The arbitrary constants A and B in (18) are determined by requiring \( \hat{\eta}(k, l, 0) = \hat{h}(k, l) \) at the lower boundary, and the radiation boundary condition at \( z \to \infty \) (the latter implies \( B = 0 \); cf. Smith, 1980). Applying these boundary conditions and substituting (18) into (12), the result may be written as

\[
\eta(x, y, z) = \int \int_{0}^{\infty} \hat{h} e^{imz} e^{i(kz+ly)} dk dl
- \frac{ig}{\Theta U} \int \int_{0}^{\infty} \hat{Q} e^{i(kz+ly)} (e^{-\alpha z} - e^{imz}) \frac{dk dl}{k(U^2 \alpha^2 \cos^2 \psi + 2)}.
\]  

(19)

The first term on the rhs of (19) (hereinafter \( \eta_S \)) can be readily identified with the rhs of (11) in Smith (1980), whereas the second term (hereinafter \( \eta_R \)) closely matches that appearing in (7) of Lin (1986). Assuming axially-symmetric bell-shaped obstacles, \( z_h(x, y) = h[1 + (r/a)^2]^{-3/2} \) with \( r \equiv (x^2 + y^2)^{1/2} \), Smith (1980) provided closed-form asymptotic solutions to (19) with \( \hat{Q} = 0 \), at the surface, far from the obstacle, and for large altitudes above the mountain. Due to additional complications, no attempts were made to provide closed-form solutions for the thermally forced case in Lin (1986).

The primary motivation for the current exercise is not to evaluate characteristics of natural flows (this is deferred to the following chapters), but to determine whether a simple linear model predicts tendencies for disappearance of the upwind stagnation [captured adequately (Smolarkiewicz and Rotunno 1990) by the linear predictions for thermally unforced flows (Smith 1980)]. In consequence, (19) may be further manipulated to provide a closed-form expression for the velocity perturbation at the center-plane of the surface flow.
Integrating (9a) over $x$ and assuming $p' = u' = 0|_{z=-\infty}$ leads to

$$Uu' = \frac{-p'}{\rho_o},$$

(20)

whereupon the integration of (9c) over $x$ results in

$$-\frac{p'}{\rho_o} = \int_z^\infty g\theta' \frac{\partial \Theta}{\partial z} dz,$$

(21)

where $p' = 0|_{z=\infty}$ has been assumed. Integrating, then, (9e) over $x$ and implementing (10) leads to

$$\theta' + \eta \frac{d \Theta}{dz} = \int_{-\infty}^x \frac{Q}{U} dx,$$

(22)

with $\theta' = \eta = 0|_{z=-\infty}$ postulated. Combining (19) to (22) gives

$$u' = \frac{g}{\Theta_o U^2} \int_z^\infty \int_{-\infty}^x Q dx dz - \frac{N^2}{U} \int_z^\infty (\eta_S + \eta_R) dz + C,$$

(23)

where $C$ is an integration constant implied by the boundary conditions adopted in (20). Assuming that the mountain and heating distributions are bell-shaped, leads finally (Reisner, 1992) to the closed-form expression for the velocity perturbation on the center plane at $z = 0$:

$$\frac{u'(x,0,0)}{U} = \frac{1}{Fr} \frac{x}{a} \left[ 1 + \left(\frac{x}{a}\right)^2 \right]^{3/2} + \frac{\eta^*}{Fh^2} \left[ 1 + \left(\frac{x}{b}\right)^2 \right]^{1/2}$$

$$+ \frac{\eta^*}{(1 + Fh^2)^{1/2}} \left[ \left(\frac{x}{b}\right) + Fh \right] \left[ 1 - \left(\frac{x}{b}\right)^2 \left(\frac{1 + Fh^2}{1 + (x/b)^2}\right)^{1/2} \right] \left(\frac{\partial \Theta}{\partial z}\right)^{-1},$$

(24)

where $a$ and $b$ are the horizontal scales of the mountain and heating, respectively; $Fh \equiv \frac{U \alpha}{N}$, and $\eta^* \equiv \frac{Q_o b \alpha}{U} \left(\frac{\partial \Theta}{\partial z}\right)^{-1}$, where $Q_o \equiv F_o \alpha/(\rho_o C_p)$ is the
normalized surface heating rate with $F_o$ representing the magnitude of surface
heat flux ($\rho_o$ denotes the reference density, and $C_p$ is the specific heat at con-
stant pressure). The parameters $F_h$ and $\eta^*$ are analogous, respectively, to the
Froude number defined with respect to the vertical scale of heating $\alpha^{-1}$, and
$\eta^*$ in (8) but defined with respect to the vertical and horizontal length-scales of
heating.

For $\eta^*=0$ (24) informs us that a tendency for upwind stagnation is a con-
sequence of the high-pressure perturbation associated with the standing moun-
tain wave; whereas for $\eta^* \neq 0$, this tendency also depends upon the magnitude
of a pressure perturbation due to thermal forcing offsetting the high-pressure
perturbation associated with the standing mountain wave. The first term on
the rhs of (24) represents the wave response to the mountain forcing [cf. Smith
1980, (39)], and the second and third terms describe, respectively, the convec-
tive and wave response to the imposed heating/cooling function. Figure 1 il-
lustrates z-variability and relative contributions of the three terms to the total
solution (parameters selected are $Fr = 0.33$, $\eta^* = 3.6$, $Fh = 1.7$, and $b = 2a$;
their relevance to meteorological applications will be explained later in this sec-
tion). Consistent with simple arguments of the preceding section, the resulting
solution exhibits substantially larger perturbations in the lee than over the up-
wind side of the obstacle.

The relationship (24) might be exploited [in the spirit of Smith's analysis
for the upwind stagnation; see his (57) and the accompanying discussion] to establish linear predictions for the existence and position of incipient stagnation points on the upwind and downwind sides of the hill as a function of $Fr$, $Fh$, $\eta^*$, and the aspect ratio of the characteristic length scales $a$ and $b$. Below, we discuss specific predictions addressing the variability of the lower upwind flow for the selected parameters representative of mesoscale low-Froude-number flows.

Figure 2a shows the upwind surface-flow perturbations as predicted by (24) for $Fr=0.11$, 0.33, and 0.66 ($U = 10$ m s$^{-1}$, $h = 3$ km). The light curves are for the thermally unforced case, whereas dark curves represent the heated problem with $b = 2a = 50$ km, $F_o = 800$ W m$^{-2}$, and $\alpha = 1/600$ m$^{-1}$. Other relevant constants ($g$, $\rho_o$, $C_p$, and $\Theta_o$) assume typical atmospheric values. This corresponds to $Fh \approx 0.5$, 1.7, 3.4, and $\eta^* \approx 0.4$, 3.6, 14.4 for $Fr = 0.11$, 0.33, 0.66, respectively. Note that the corresponding values of $\eta^*$ defined with respect to the mountain height $h$, instead of the heating scale $\alpha^{-1}$, would be five times smaller. The apparent trend observed in the linear solutions is that heating counteracts the mountain-induced deceleration of the upwind flow. For $Fr = 0.11$ and 0.33 heating reduces (by about a factor of three) the negative velocity perturbation associated with the standing mountain wave, whereas for $Fr = 0.66$ it actually overcomes the mountain forcing resulting in accelerated surface flow. At $Fr = 0.33$ the heating imposed sufficed
for a transition from blocked to unblocked flow, consistent with the simple predictions of the preceding section.

In the remaining part of this paper, we shall discuss nonlinear solutions for low-Froude-number flows representative of atmospheric applications. Figure 2b precedes detailed discussion of the model results and shows numerical solutions corresponding to those in Fig. 2a. Although the design of numerical experiments assumes spatially uniform heating over $L^* = 3a$ evolving according to the diurnal cycle (the solutions shown are at peak heating), the overall trend of the nonlinear predictions is similar to that captured by the linear theory. This documents that, although the linear theory is technically invalid for these Froude numbers, it provides qualitatively meaningful predictions for the flow tendencies. Apparent differences, however, may be noted. For $Fr = 0.11$, the linear solutions substantially deviate from the numerical results as they underpredict (by approximately a factor of two) the magnitude of the heating-induced velocity perturbations. This is not surprising, as in this flow regime the total velocity perturbations are large (compared to $U$) for both thermally forced and unforced problem. For all three values of $Fr$, the linear theory tends to overestimate the magnitude of the heating-induced perturbations in the proximity of the obstacle's origin. A possible explanation for this is that the linear theory does not account for the returning flow of the lee eddies which (as will be shown later) intensifies with heating and extends further
upwind.

The dimensional analysis and the linear theory discussed in this chapter merely formalize intuitive predictions for the thermally forced low-Froude-number flows. In order to better comprehend the temporal evolution of such flows, as well as their morphology and variation with $Fr$ and $\eta^*$, a series of numerical simulations has been performed. The numerical tools applied will be described in the next chapter, whereas in subsequent chapters the experience gained from the theoretical considerations will aid our interpretation of both the idealized experiments and simulations of the flow past the Hawaiian Archipelago.
III. MODEL DESCRIPTION AND DESIGN OF THE SIMULATIONS

A. Model Description

The primary concerns of this study are with tropospheric low-Froude-number flows past mesoscale mountains of aspect ratios \( h/L \sim \mathcal{O}(0.1) \). For uniform ambient wind \( U \), constant Brunt-Väisälä frequency \( N \), and \( \eta^* = 0 \), such flows are well captured by the hydrostatic approximation (Smolarkiewicz and Rotunno 1989; page 1161). Following the discussion in chapter II.A, this should hold for \( \eta^* \sim \mathcal{O}(1) \) which is representative of atmospheric applications (e.g., \( \eta^* \approx 0.5 \) is typical for Hawaii). The main tool employed in this study is a computationally efficient hydrostatic, isentropic model. For the sake of completeness, a number of numerical experiments have also been performed using an anelastic nonhydrostatic code of Clark (1977) and Clark and Farley (1984) employed in previous studies of low-Froude-number flows (Smolarkiewicz and Rotunno 1989, Smolarkiewicz and Rotunno 1990, Crook et al. 1990, Rotunno and Smolarkiewicz 1992). For a class of the applications addressed, both models produce similar results (cf. Reisner 1991 for a discussion of \( \eta^* = 0 \) flows); however, they depart where \( \eta^* \sim \mathcal{O}(10) \). In this paper, the results discussed will be from the hydrostatic simulations. Since the principles of the model employed have been documented in the literature, we shall describe only its basic formulation and those aspects that are directly related to the present work.

At the \( \theta \) isentrope, the prognostic equations for mass continuity and mo-
momentum take a simple form

\[
\frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \theta} \right) + \nabla \cdot (p \frac{\partial \mathbf{v}}{\partial \theta}) + \frac{\partial}{\partial \theta} \left( \theta \frac{\partial p}{\partial \theta} \right) = 0 \tag{25}
\]

\[
\frac{\partial q_x}{\partial t} + \nabla \cdot (\mathbf{q}_x) + \frac{\partial \theta q_z}{\partial \theta} = -\frac{\partial p}{\partial \theta} \frac{\partial M}{\partial x} - f q_y, \tag{26a}
\]

\[
\frac{\partial q_y}{\partial t} + \nabla \cdot (\mathbf{q}_y) + \frac{\partial \theta q_v}{\partial \theta} = -\frac{\partial p}{\partial \theta} \frac{\partial M}{\partial y} + f q_x, \tag{26b}
\]

where \( p \) is the pressure; \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \); \( \mathbf{v} \) is a horizontal velocity vector; and \( \theta \) represents parameterized diabatic forcings. \( q \equiv \mathbf{v} \frac{\partial p}{\partial \theta} \) is a vector of the horizontal momentum; and \( M = g \chi + \theta \Pi \) is the Montgomery potential with \( \chi \) denoting the height of the isentropic surface, and \( \Pi \) the Exner function \( \Pi = C_p(p/p_0)^{R_d/C_p} \) \( (R_d \) is the gas constant for dry air, and \( p_0 \) is a reference pressure). \( f \) is the Coriolis parameter. The conservation laws (25) and (26) are coupled through the diagnostic relationship of the hydrostatic balance of the fluid

\[
\frac{\partial M}{\partial \theta} = \Pi. \tag{27}
\]

The upper boundary condition incorporates the free-surface assumption, \( (p = \text{constant})_{\theta=\infty} \), whereas for the lower boundary a material surface is assumed.

The second-order-accurate finite-difference approximation to the system (25)-(27) closely follows those discussed in Smolarkiewicz (1991a) and Smolarkiewicz and Margolin (1992). In the horizontal, all variables are defined at the same grid-point positions; in the \( \theta \)-direction, the pressure is staggered with respect to all other variables. The forward-in-time discretization of (25) and (26) employs the nonlinear, sign-preserving, conservative MPDATA schemes
documented in a series of publications (Smolarkiewicz 1983, 1984, 1991a; Smolarkiewicz and Clark 1986). The sign-preservation property of the algorithm is essential: it prevents the development of spurious negative pressure thicknesses of isentropic layers (which could lead to "convective" instability) and bounds the total "energy" of the scheme, necessary for the nonlinear stability of the system (cf. Smolarkiewicz, 1991b). In order to prevent spurious accelerations due to pressure forces from zero-thickness layers (see Bleck, 1984, Bleck and Smith, 1990, for discussions) the horizontal derivative of the Montgomery potential is approximated with the second-order-accurate pressure-thickness-weighted average of the one-sided derivatives. In order to simulate an infinite extent of the fluid, a gravity-wave absorber (Klemp and Lilly, 1978) is employed in the upper portion of the model; and the Davies (1983) relaxation scheme is incorporated at the lateral boundaries of the computational domain. Except for the absorbing lateral and upper boundary regions, the solver employs no numerical filters or other means of explicit artificial viscosity. Diabatic-forcing terms are approximated, using a one-dimensional version of MPDATA in the sense of a time-splitting approach.

B. Design of the Numerical Simulations

Conclusions of this study are based on numerous simulations with different designs addressing sensitivities of the solutions to various physical and numerical specifications. Below, we outline the setups of the selected simulations whose results will be discussed in more detail in the remaining part of this pa-
The experimental design assumes a bell-shaped mountain shape with an elliptic cross section

\[ z_h(x, y) = h \left[ 1 + \left( \frac{x - x_0}{L} \right)^2 + \left( \frac{y - y_0}{\beta L} \right)^2 \right]^{-3/2}, \]

placed in the center of the domain. The selected parameters of the hill are \( L = 25 \text{ km} \) and \( h = 0.12L \), and \( \beta = 1 \) (experiments with \( \beta > 1 \) are discussed in Reisner 1992). For \( \left( \frac{(x - x_o)}{L^*} \right)^2 + \left( \frac{(y - y_o)}{\beta L^*} \right)^2 \leq 1 \) and \( L^* = 3L \), the diabatic source is parameterized as

\[ \dot{\theta} = \frac{\theta R_d}{p C_p \alpha} F_o \exp(\alpha (z - z_h)) \sin(\omega(t - t_o)), \]

where the amplitude of the surface flux \( F_o \) is \( 800 \text{ W m}^{-2} \) during the day and \(-75 \text{ W m}^{-2} \) at night, with respective values of the attenuation coefficient \( 1/600 \text{ m}^{-1} \) and \( 1/75 \text{ m}^{-1} \) assumed. \( \omega = \pi/12h^{-1} \) is the frequency of the diurnal cycle. \( \dot{\theta} = 0 \) if \( \left( \frac{(x - x_o)}{L^*} \right)^2 + \left( \frac{(y - y_o)}{\beta L^*} \right)^2 > 1 \). With such a selection of parameters, (29) attempts to mimic the surface-boundary-layer thermal forcing characteristic of Hawaii (Garrett 1980, Smolarkiewicz et al. 1988).

The initial condition assumes flat isentropes and an impulsive startup. Until quasi-steady state is achieved (\( \tau = tU/L \approx 9 \), cf. Smolarkiewicz and Rotunno 1989), \( \eta^* = 0 \) in all simulations; afterward, surface heating is activated \([t = t_o \text{ in (29)}]\), and the model simulation continues for 12 h. A special-purpose experiment continues for an additional 24 h.

In the horizontal, the majority of simulations spans a computational domain \( 20L \times 20L \) covered with \( 50 \times 50 \) uniform grid intervals. The long-term
simulation spans double the horizontal domain at the same grid-resolution. In
the vertical, the majority of experiments spans a 4.8$h$ domain covered with
$48\Delta Z_o$ intervals, where $\Delta Z_o$ is the initial vertical separation of the isentropes.
Simulations with $Fr = 0.66$ span double the vertical domain (at the same grid
resolution) in order to accommodate the gravity wave absorber that fills the
upper ($\approx \lambda_z = 2\pi U/N$ deep) portion of the domain. The regions adjacent
to the lateral boundaries of width $\approx 2L$ are designated for absorbing bound­
ary schemes. The temporal increment $\Delta t = 10$ s is limited by the propagation
speed of the external mode.

In order to illustrate the relevance of the idealized study to natural low-
Froude-number flows, simulations of flows past the entire Hawaiian Archipelago
have been performed using the isentropic model discussed above. In these ex­
periments, the horizontal domain covers the region $1500 \times 1000 \text{ km}^2$ resolved
with a uniform mesh of $\Delta X = \Delta Y = 10$ km. In the vertical, the model design
is the same as that adopted for the majority of idealized simulations. The nu­
merical experiment discussed in this paper is for the uniform easterly flow with
$U = 6.8 \text{ m s}^{-1}$ and constant $N = 0.013 \text{ s}^{-1}$ which represent average ambient
conditions during HaRP (Nash 1992).
IV. RESULTS

A. Overview

In order to facilitate further discussion of different flow regimes depending upon $Fr$ and $\eta^*$ [defined in (8) based on an average value of $\dot{\theta} = 2$ K° h$^{-1}$ in (29)], a summary of over 20 different simulations for nonrotating flows with an axially symmetric obstacle is presented in the format of a flow regime diagram in Fig. 3. The asterisks represent the simulations performed (for clarity, only the simulations with $0.05 < \eta^* < 4$ are marked in the figure), and the light solid curves represent lines of constant $U$. The heavy solid line (near the bottom of the figure) separates solutions for which there is a critical transition from blocked to unblocked flow regime (the portion of the figure above the curve) from those where upwind flow exhibits upwind stagnation during the entire 12-h heating cycle (the portion of the figure below the curve). The critical curve satisfies approximately the relationship $2\eta^* + \frac{4}{3}Fr = 1$, which quantifies our earlier assertion that for $\eta^* \geq 1$ the stratified flow may go over an obstacle regardless of the value of $Fr$. In particular, it suggests that the critical value of $\eta^*$ is about a factor of two less than that expected from dimensional arguments. The heavy dashed line in Fig. 3 separates solutions where flows are dominated by the mountain forcing (portion of the figure below the curve) from those where heating dominates the flow response (portion of the figure above the curve), i.e., the solutions strongly diverge from those charac-
teristic of thermally unforced flows with $Fr \sim \mathcal{O}(1)$. The primary criterion for this curve is whether the lower upwind flow accelerates or decelerates as it approaches the hill, and it derives from the linear predictions in Fig. 2a.

Each point of the regime diagram represents a different, time-dependent flow realization, but a detailed discussion of all of these flows exceeds the scope of this paper. In order to highlight the flows' dependence on $Fr$ and $\eta^*$, as well as their temporal evolution, we adopt the following strategy. In the following section we discuss flow variability with $\eta^*$ for the fixed Froude number $Fr = 0.33$ at peak heating. In the subsequent section, we focus on flow with $Fr = 0.33$ and $\eta^* = 0.45$ (representative of the flow past Hawaii) and describe its temporal evolution. In the following chapter we shall refer to other points from the regime diagram while discussing flows past the Hawaiian Archipelago.

B. Flow Response For $Fr = 0.33$ with Variations of $\eta^*$

Figures 4a to 4e display flow realizations for $\eta^*$ varying along a $Fr = 0.33$ direction in Fig. 3. Figure 4a is for $\eta^* \equiv 0$; 4b for $\eta^* = 0.13$ (the point on $U = 15$ m s$^{-1}$ line, right below the heavy solid curve in Fig. 3); 4c for $\eta^* = 0.45$ (the point on $U = 10$ m s$^{-1}$ line, right above the heavy solid curve in Fig. 3); 4d for $\eta^* = 1.1$ (the point on $U = 7.5$ m s$^{-1}$ line in Fig. 3); and Fig. 4e is for $\eta^* = 3.6$ (the point on $U = 5$ m s$^{-1}$ line, right above the heavy dashed curve in Fig. 3). The $\eta^* \equiv 0$ solution (hereinafter called the reference solution) is at $\tau = tU/L \approx 9$, whereas thermally forced solutions are shown at
the peak heating. The reference solution compares well with those discussed in the literature (see Reisner 1991 for a detailed discussion). It shows the upwind stagnation zone (the region encompassed by the heavy-dashed line on the windward side of the obstacle) and the lee eddies (whose returning flow is encompassed by the heavy-dashed line on the leeward side of the hill). The solution at \( \eta^* = 0.13 \) (Fig. 4b), although it retains the general character of the low-Froude-number flow, differs substantially from the reference solution. In accord with predictions deduced from the dimensional arguments and the linear theory, the most pronounced differences are observed in the lee. The vortices intensify and grow in both the horizontal and the vertical. On the upwind side, the stagnation zone shrinks, and the upwind influence of the mountain diminishes. At \( \eta^* = 0.45 \) (Fig. 4c) the upwind stagnation disappears, and there is some evidence of deceleration of the lower upwind flow at \( x \approx -4L \). Lee eddies shrink and approach the obstacle; however, their intensity nearly doubles as measured by the maximal speed of the return flow. The latter is attributed to the increased upslope flow in the lee which constitutes an element of the anabatic/breeze circulation. At \( \eta^* = 1.1 \) (Fig. 4d) the lower upwind flow is nearly constant, whereas well-defined anabatic/breeze circulation develops aloft (note the reversed shear at \( x \approx -2L \) in the upper panel). The lee eddies are dominated by the thermal circulation and further approach the mountain. At \( \eta^* = 3.6 \) the lower upwind flow accelerates, constituting a part of the thermal
circulation centered at the obstacle’s origin.

C. Temporal Evolution of the Flow with $Fr = 0.33$ and $\eta^* = 0.45$

Figures 5a to 5g display long-term evolution of the flow, starting from the reference solution at $t = t_0$ in (29) (Fig. 5a) and progressing up to 1.5 diurnal cycles (Fig. 5g).

With the onset of heating, onshore flow quickly develops downwind of the upwind stagnation zone and over the lee, leading to a disappearance of the upwind stagnation after $\approx 3h$ (Fig. 5b). 3 h later (Fig. 5c), the flow departs considerably from the reference solution (cf. discussion of Fig. 4c in IV.B). At the end of the heating period (Fig. 5d), the upwind stagnation reappears again (incipient stagnation, not shown, occurs 2 h earlier); however, there is still a notion of weak breeze/anabatic circulation over the upwind slopes, which disappears 4 h later (not shown). In contrast, at this time the lee eddies reach their maximal intensity and depth and the closest position to the obstacle’s origin.

6 h later (Fig. 5e), at peak cooling, upwind flow closely resembles that for the reference solution with only a slightly more pronounced stagnation zone. Over the lee slopes, the breeze/anabatic circulation due to daytime heating has already disappeared, and the vertically propagating mountain wave has been restored aloft. However, the vortex doublet does not return to its original state.

6 h later (Fig. 5f), after completion of the diurnal cycle, the flow realization is similar to that in Fig. 5e, except the lee-eddies propagate further downstream
and weaken. During the next 12 h of the second diurnal cycle, solutions closely match those for the first cycle. The nighttime cooling does not delay the disappearance of upwind stagnation, and the windward flow is nearly the same as those in Figs. 5b and c. In the lee, the differences between the two cycles are more pronounced. With heating the lee eddies move back to the approximate position from the previous cycle; however, the wake region far downstream is now established (Fig. 5g vs. 5d). As the initial condition of the reference solution did not include effects due to the preceding diurnal cycle, a precise reversibility of the solution cannot be expected. With the Coriolis force included (Reisner 1992), the irreversibility of the leeward flow is even more pronounced as the vortices shed (in response to the asymmetric perturbation imposed by the rotation; cf. Schär and Smith 1993).
V. APPLICATIONS TO FLOW PAST THE HAWAIIAN ARCHIPELAGO

Due to different heights and length scales of the individual islands in the Hawaiian Archipelago, each island yields different $Fr$ and $\eta^*$. Hence, each island may be subject to a different regime of flow. The estimated values of $Fr$ and $\eta^*$ (based on average ambient conditions during HaRP) are: $Fr = 0.52, 0.70, 1.3, 0.17, 0.17$ and $\eta^* = 0.32, 0.42, 0.79, 0.13, 0.35$ for Kauai, Oahu, Molokai, Maui, and Hawaii, respectively (the respective values of $h$ and $L^*$, in kilometers, were assumed as $h \approx 1.0, .75, .40, 3., 3.1$, and $L^* \approx 20., 20., 20., 25., 70.$). The islands of Nihau, Lanai, and Kahoolawe are excluded from the current discussion as they are in wake regions of the larger islands. With respect to the relationship $2\eta^* + \frac{4}{3}Fr = 1$ approximating the critical stagnation line in the regime diagram (Fig. 3), the islands of Maui and Hawaii fall below the line, while all other islands fall above the line.

Figure 6a illustrates the reference solution with $\eta^* \equiv 0$ after 12 h of integration. Analysis of the model results indicates that flows past Kauai, Maui, and Hawaii exhibit salient features of low-Froude-number flows (the upwind stagnation and lee eddies). The corresponding solution at the peak heating (integrated for 6 h with $\eta^* \equiv 0$) is shown in Fig. 6b. As expected from the regime diagram, the upwind stagnation disappears over Kauai but does not disappear over Maui and Hawaii. In Maui, splitting of the surface flow is still evident at peak heating. In Hawaii, however, the upwind stagnation zone shrinks con-
siderably to a small region upwind of Mauna Kea volcano. These predictions agree reasonably well with observations (Roy Rasmussen, personal communication). In accord with idealized simulations, the vortices in the lee of Kauai, Maui, and Hawaii intensify with heating (as measured by the magnitude of the reversed flow; not shown).

During HaRP, Portable Automated Mesonet (PAM) stations were placed over Hawaii. Figures 7a and b (after Nash 1992) show surface flow (averaged over the length of the field program) at midday and at night, respectively. They document that except for the northern and southern tips of the island, the average surface wind was onshore at midday and offshore at night. Figure 7a indicates that even during the day, flow over the upwind side of the obstacle is decelerated compared to its far upstream value. The directions of the wind arrows in Figs. 7b and 7a are in good agreement with directions of surface flows over the big island in Figs. 6a and 6b.

Nash (1992) has found that the onset time of onshore flow is a function of the distance from the center of the island. Onshore flow in the vicinity of the saddle starts within the first 1 h of heating, whereas it takes about 3 h for the flow to reverse at the coast line. This behaviour is consistent with that observed in idealized simulations for, e.g., $Fr = 0.11$ and $\eta^* = 0.05$. Although there is no transition from blocked to unblocked flow regime at this $Fr$ and $\eta^*$ (see Fig. 3), there is an apparent reversal of the flow near the obstacle's origin (Fig. 2b).
VI. CONCLUDING REMARKS

This paper has addressed flows past mountainous islands such as Hawaii or Taiwan. In previous studies, basic aspects of such flows have been investigated in an idealized framework of nonrotating, inviscid, density stratified flows with uniform wind and constant stratification past bell-shaped obstacles. In this paper, we have expanded this conceptual model by including an idealization of the surface thermal forcing. Within the confines of the idealization, we were able to characterize the flows by the two parameters, Froude number and $\eta^*$, where the latter parameter has a sense of normalized displacement of fluid particles due to diabatic effects. To a first approximation, $\eta^*$ may be determined based on ambient profiles, representative height of the mountain, characteristic horizontal length scale of the thermal forcing, and climatological heating rate (8).

By means of elementary scaling arguments, we have arrived at the hypotheses that for $\eta^* \geq 1$ stratified flows are expected to go over the mountains regardless of the Froude number, that for $|\eta^*| \ll 1$ the effects of thermal forcing are negligible, that for $\eta^* \sim Fr < 1$ a critical transition from the blocked to unblocked flow regime may occur, and that thermal effects should be substantially more pronounced in the lee than over the upwind side of the mountain. We have shown that these semi-intuitive predictions are supported by the linear theory. In order to quantify simple arguments, we conducted a series of
numerical simulations whose results have led us to empirical criteria for estimating whether a critical transition from a blocked to unblocked flow regime will occur (Fig. 3).

We have described the flows' responses for a range of $\eta^*$ at fixed Froude number $Fr = 0.33$ as well as a diurnal evolution of the flow at $Fr = 0.33$ and $\eta^* = 0.45$ that are representative of Hawaii. In order to illustrate the applicability of our results to atmospheric flows, we conducted a simulation with a flow past the Hawaiian Archipelago, where every island is subject to different values of $Fr$ and $\eta^*$, and show that the experience gained from this idealized study may be useful for predicting regimes of natural flows.
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Fig. 1. Relative contributions of the three terms on the rhs of (24) to the total solution (light-solid, short-dashed, long-dashed, and heavy-solid curve, respectively).
Fig. 2. The x component of the velocity perturbation at the center plane \( (y = 0) \) of the flow for \( Fr = 0.11 \) (—1—), \( Fr = 0.33 \) (—3—), and \( Fr = 0.66 \) (—6—); (a) thermally unforced (light lines) and heated (heavy lines) linear solution; (b) thermally unforced (light lines) and heated (heavy lines) numerical solutions.
Fig. 3. Regime diagram of flow response to heating as a function of $Fr$ and $\eta^*$. 
Fig. 4a. Reference solution for flow with $Fr = 0.33$. Top panel shows wind vectors and isentropes on the x-z center-plane; bottom panel shows surface wind vectors and contours of $u' / U$ with positive/negative values ranging from (.3 to 2.5)/(.2 to 2.5) with interval of 0.1. Bold dashed lines correspond to $u' / U = -1$. Heavy solid contours represent dimensionless heights of the obstacle for $z/h = 1/30, 1/3, 2/3$, with the first contour encompassing the domain of thermal forcing.
Fig. 4b. As in Fig. 4a but for $\eta^* = 0.13$ solution at peak heating.
Fig. 4c. As in Fig. 4b but for $\eta^* = 0.45$. 
Fig. 4d. As in Fig. 4b but for $\eta^* = 1.1$. 


Fig. 4e. As in Fig. 4b but for $\eta^* = 3.6$. 
Fig. 5a. Reference solution for flow with $F_T = 0.33$ and $\eta^* = 0.45$. The plotting convention is the same as in Figs. 4a–4e.
Fig. 5b. As in Fig. 5a but after 3 h into the heating cycle.
Fig. 5c. As in Fig. 5a but at the peak heating.
Fig. 5d. As in Fig. 5a but at the end of the heating cycle.
Fig. 5e. As in Fig. 5a but at the peak cooling.
Fig. 5f. As in Fig. 5a but at the beginning of the next heating cycle.
Fig. 5g. As in Fig. 5a but at the end of the next heating cycle.
Fig. 6a. Surface flow past the Hawaiian Archipelago for \( \eta^* = 0 \).
Fig. 6b. As in Fig. 6a but results are shown at peak heating.
Fig. 7. Average (over the length of the field project) surface winds (after Nash 1992) at (a) 14 local standard time (LST) and (b) 02 LST. Wind speeds are half-barb = 0.5 m s\(^{-1}\), full barb = 1.0 m s\(^{-1}\), and a triangle = 5 m s\(^{-1}\).
APPENDIX A: LINEAR THEORY

Evaluation of the first term in equation (24) in Reisner and Smolarkiewicz (1992) at z=0.

By assuming a bell-shaped distribution of heating,

\[ Q = \frac{Q_0e^{-ax}}{(1 + (x/b)^2 + (y/b)^2)^{3/2}} \]  

(1)

the first integral in equation (24) can be readily evaluated to give

\[ \frac{gQ_0}{\theta_0U^2\alpha} \left[ \frac{x}{\sqrt{x^2/b^2 + 1}} + b \right] \]  

(2)

Evaluation of the third term in equation (24) in Reisner and Smolarkiewicz (1992) at z=0.

To proceed with the integration, inverse Fourier transforms of the heating and topography functions are needed. The inverse Fourier transforms are:

\[ \hat{h}(k, l) = \frac{1}{2\pi} ha^2e^{-a\phi} \]  

(3)

\[ \hat{Q}(k, l) = \frac{1}{2\pi} Q_0b^2e^{-b\phi} \]  

(4)

where \( \phi = \sqrt{k^2 + l^2} \) is the magnitude of the horizontal wavenumber vector and \( a \) and \( b \) are the horizontal scales of the mountain and heating respectively.

For ease of evaluation transform the integral from \( dkdl \) space to \( d\psi d\phi \) space to produce

\[ Q' \int_0^\infty \int_0^{2\pi} \int_0^\infty \frac{e^{-b\phi}e^{i\phi r\cos(\psi-\theta)}(e^{-ax} - e^{imz})d\phi d\psi dz}{\cos \psi(U^2\alpha^2\cos^2\psi + N^2)} \]  

(5)
where \( x = r \cos \theta, y = r \sin \theta, k = \phi \cos \psi, l = \phi \sin \psi, \) \( dkdl = \phi d\psi d\phi, \) and

\[
Q' = \frac{i Q_o b^2 g N^2}{2 \pi \theta_o U^2}
\]

When evaluating equation (5) over \( z \) to obtain \( u' \) stationary phase arguments (see Smith (1980) Appendix IV) allow contributions to the integral from the upper limit to be set to zero. After integrating and evaluating the first integral, equation (5) becomes

\[
Q' \int_0^{2\pi} \int_0^\infty e^{\phi(\imath r \cos(\psi - \theta) - b)} \left( \frac{1}{\alpha} - \frac{i}{m} \right) \frac{d\phi d\psi}{\cos \psi(U^2 \alpha^2 \cos^2 \psi + N^2)}
\]

Next integrate and evaluate equation (6) over \( d\phi \) to obtain

\[
-Q' \int_0^{2\pi} \left( \frac{1}{\alpha} - \frac{i}{m} \right) \frac{d\psi}{\cos \psi(U^2 \alpha^2 \cos^2 \psi + N^2)(\imath r \cos(\psi - \theta) - b)}
\]

The remaining integral can most easily be evaluated on the center planes \( (\theta = 0, \pi/2, \pi, \ldots) \).

After setting \( \theta = 0 \) in equation (7), multiply (7) by

\[
\frac{ix \cos \psi + b}{ix \cos \psi + b}
\]

to obtain

\[
Q' \int_0^{2\pi} \left( \frac{1}{\alpha} - \frac{i}{m} \right) \frac{d\psi(ix \cos \psi + b)}{\cos \psi(U^2 \alpha^2 \cos^2 \psi + N^2)(x^2 \cos^2 \psi + b^2)}
\]

Next, note that the real part of

\[
i \left( \frac{1}{\alpha} - \frac{i}{m} \right) (ix \cos \psi + b)
\]
is

$$\cos \psi \left( \frac{bU}{N} - \frac{x}{\alpha} \right)$$

so that equation (8) becomes

$$Q'' \int_0^{2\pi} \left( \frac{bU}{N} - \frac{x}{\alpha} \right) \frac{d\psi}{(U^2 \alpha^2 \cos^2 \psi + N^2)(x^2 \cos^2 \psi + b^2)}$$

(9)

where

$$Q'' = \frac{Q_o b^2 g N^2}{2\pi \theta_o U^2}$$

the expression

$$\frac{1}{(U^2 \alpha^2 \cos^2 \psi + N^2)(x^2 \cos^2 \psi + b^2)}$$

can be broken into two expressions by using the method of partial fractions

$$\frac{1}{(U^2 \alpha^2 \cos^2 \psi + N^2)(x^2 \cos^2 \psi + b^2)} = \frac{A}{\alpha^2 U^2 \cos^2 \psi + N^2} + \frac{B}{x^2 \cos^2 \psi + b^2}$$

where

$$A = \frac{\alpha^2 U^2}{\alpha^2 U^2 b^2 - x^2 N^2}$$

and

$$B = \frac{x^2}{x^2 N^2 - \alpha^2 U^2 b^2}$$

After breaking apart (3) into two expressions, the resulting integrals are of the form,

$$\int_0^{2\pi} \frac{dx}{a + b \cos^2 x}$$

This integral can be evaluated to obtain

$$\frac{2\pi}{\sqrt{a(a + b)}}$$
Thus, equation (9) finally becomes,

$$\frac{Q_{og} Nb^2}{\theta U^2 \alpha (\alphaUb + xN)} \left[ \frac{\alpha^2 U^2}{\sqrt{N^2 (N^2 + \alpha^2 U^2)}} - \frac{x^2}{\sqrt{b^2 (b^2 + x^2)}} \right]$$  

Equation (10), dividing by $1/U$ and rewriting, is the third term in equation (24) of Reisner and Smolarkiewicz (1992).
APPENDIX B: $Fr \neq 0.33$

This appendix will detail results from simulations whose reference solution is different from that of Fig. 4a in Reisner and Smolarkiewicz (1992). In particular, results (both above and below the heavy curves shown in Fig. 3 of Reisner and Smolarkiewicz, 1992) will be shown for $Fr = 0.11$ and $Fr = 0.66$.

The experimental setup and domain for the simulations is the same as that described in their paper.

A. $Fr = 0.11$

The reference solution for $Fr = 0.11$ (see Fig. B1) shows a large upwind stagnation zone, small isentropic displacements, and shallow lee vortices. Significant perturbations are produced by heating for the simulation with $\eta^* = 0.4$ (below the stagnation curve in Fig. 3), but upwind stagnation does not disappear with heating (see Fig. B2). The intensity of the thermal circulations is not surprising, considering that $\eta^* \sim Fr$. For the simulation with $\eta^* = 0.7$ (see Fig. B3) (above the stagnation curve in Fig. 3) the flow is directed toward the center of the mountain, with the surface flow being controlled by the thermal circulations. Though the flow at the surface is strongly influenced by heating, strong stratification prevents the thermal circulations from extending to great depths above the obstacle.
B. $Fr = 0.66$

Greater isentropic displacements and wave activity, a smaller upwind stagnation zone, and weaker return flow in the lee are the main differences between the reference solutions of $Fr = 0.66$ versus $Fr = 0.11$ (see Fig. B4). For the simulation with $\eta^* = 0.05$ (below the stagnation curve in Fig. 3) upwind stagnation is still evident at peak heating (see Fig. B5). The lee vortices increase in size; however, it is difficult to determine if the increase is related to heating. For $\eta^* = 0.2$ (see Fig. B6) upwind stagnation disappears quickly with heating. Heating allows the mountain wave to be brought close to the surface. The downward motions associated with the mountain wave are partially responsible for the both the weakening and downstream advection of the lee eddies. Notice that in Fig. B6 the flow in the lee is not symmetric with respect to the x-z center plane. For flow regimes in which the isentropes become almost vertical, such as wave breaking regimes, it appears that round-off errors are being excited in the model. Though heating allows for the disappearance of upwind stagnation, the most interesting aspects of the effects of heating noted in the simulation (above the heavy dashed line in Fig. 3) with $\eta^* = 1.8$ occur in the lee. Like the previous simulation, within the first few hours of heating the strong downward motions associated with the mountain wave reach the surface; however, unlike the previous simulation, as the heating cycle progresses the lee eddies intensify and move closer the center of the mountain (see Fig. 
B7). In sharp disagreement with previous simulations, for simulations with 
$\eta^* = 15.0$ (both hydrostatic and nonhydrostatic models were used) significant
differences (not shown) exist between the results produced by the hydrostatic
versus nonhydrostatic model. In this flow regime the nonhydrostatic model
produces weaker vertical velocities but stronger horizontal inflow than that of
the hydrostatic model. Even though the hydrostatic model is unable to accu-
rately predict certain areas of the flow, it should be stressed that the upstream
conditions used in this simulation are rarely if ever observed on Earth.
Fig. B1. Reference solution for flow with $Fr = 0.11$. The plotting convention is the same as in Fig. 4a of Reisner and Smolarkiewicz (1992).
Fig. B2. As in Fig. B1 but for $\eta^* = 0.4$ solution at peak heating.
Fig. B3.  As in Fig. B2 but for $\eta^* = 0.7$. 
Fig. B4. Reference solution for flow with $Fr = 0.66$. The plotting convention is the same as in Fig. 4a of Reisner and Smolarkiewicz (1992).
Fig. B5. As in Fig. B4 but for $\eta^* = 0.05$ solution at peak heating.
Fig. B6. As in Fig. B5 but for $\eta^* = 0.2$. 
Fig. B7. As in Fig. B5 but with results shown at the end of the heating cycle for $\eta^* = 1.8$. 
APPENDIX C: WAVE DRAG

The horizontal resolution used in the simulations enables the isentropic model to resolve flow features such as internal gravity waves; however, because of computer limitations, current global climate models are unable to resolve such features. While the frequency of internal gravity waves is at the outer limits of the Rossby wave spectrum, several studies (McFarlane 1987, Palmer et al. 1986) have noted that neglect of gravity waves can lead to unrealistically strong zonal jet streams. As shown by Miranda and James (1992), 2-D surface wave drag parameterizations, which include both linear and nonlinear effects, greatly overestimate surface wave drag for a 3-D obstacle. In the same manner the value of wave drag offered by 3-D linear theory (Smith 1980) also overestimates the wave drag for a 3-D obstacle for $Fr < 0.5$. Evidently, if surface heating allows for the disappearance of upwind stagnation, the value of the surface wave drag may increase to the point where it recovers the linear prediction. Thus, using the value offered by linear theory as a diurnal average might be sufficient for parameterization purposes.

Figure C1 illustrates that for typical atmospheric flow conditions, such as for $\eta^* = 0.45$ and $Fr = 0.33$, the value of the surface wave, $D_s = \int p \frac{\partial u}{\partial z} dx dy$, is initially below the value offered by 3-D linear theory; however, with heating the value slightly exceeds the linear value. The increase in surface wave drag with heating can be attributed mainly to the decrease in surface pressure
over the lee. The temporal behaviour of the wave drag would suggest that using the value offered by 3-D linear theory as a mean value to parameterize the diurnal fluctuations of wave drag for flows below the heavy dashed line and above the heavy solid line in Fig. 3 of Reisner and Smolarkiewicz (1992) may be appropriate. For simulations above the dashed line, the wave drag during the heating cycle goes into an oscillatory type pattern. For the simulation with $\eta^* = 3.6$ and $Fr = 0.33$ the curve shows some small oscillations; whereas for $\eta^* = 15.$ and $Fr = 0.66$ the wave drag appears to be undergoing rapid oscillations. The oscillations in the wave drag for the larger $\eta^*$ simulations are supported by similar results from the nonhydrostatic model. Because of this behavior, parameterization of gravity wave drag for flow in this regime is not suggested. And finally, for flow regimes beneath the solid curve in Fig. 3, using the steady-state values offered by Miranda and James (1992) might be appropriate.
Fig. C1. Surface wave drag as a function of time is \( Fr = 0.11 \) (dashed line, \(-1-\)), \( Fr = 0.33 \) (light solid line, \(-3-\)), and \( Fr = 0.66 \) (solid line, \(-6-\)) with corresponding \( \eta^* \) (top figure)/(bottom figure) 0.05/0.4, 0.45/3.6, and 1.8/15.0, respectively. Values of drag are normalized by \( \frac{\pi}{4} \rho NUa h^2 \).
A number of numerical simulations have been conducted that have included physical factors such as variations of $\beta$, where $\beta$ is the aspect ratio of the obstacle (= across-stream length/along-stream length), the Coriolis force, and low-level moisture. This appendix will describe how these factors along with surface heating modify the flow.

For the simulation ($Fr = 0.33$ and $\eta^* = 0.45$) with the Coriolis force active, $f = 2\Omega\sin\phi$ with $\phi = 19$ degrees, and for the simulation ($Fr=0.33$ and $\eta^* = 0.45$ based on the stratification above 1500 m), which included virtual effects of low-level moisture, $N = 0.0016 \text{s}^{-1}$ below 1500 m. The domain for the simulations ($Fr=0.33$ $\eta^* = 0.45$) with $\beta > 1$ was $60L$ in the $y$-direction and $40L$ in the $x$-direction; for the simulation with the Coriolis force active the domain was $40L \times 40L$; and for the simulation with low-level moisture the domain was $20L \times 20L$. The depth of domain for all simulations was $4.8h$. For the simulation with the Coriolis force active the integration period was the same as for the long-term simulation discussed (see III.C) in Reisner and Smolarkiewicz (1992). For other relevant information concerning the simulations, see Reisner and Smolarkiewicz (1992).
A. Variations in $\beta$

For a given $Fr$ the boundary conditions and diurnal variations of $\eta^*$ will partially determine how the flow will respond to heating. This section will illustrate how variations in surface boundary conditions for a fixed $Fr$ and $\eta^*$ influence the flow.

Comparing Fig. D1 (for $\beta$=8, $\eta^*$ = 0.45, and $Fr$ = 0.33) with Fig. 4a reveals that the areal extent of both the upwind stagnation zone and the lee vortices increases with $\beta$. In contrast, the magnitude of the return flow of the lee vortices first increases until $\beta \approx 3$ (not shown, but based on inspection of simulations for $\beta$=2, 3, 4, and 8) and then starts decreasing as $\beta$ increases. The weak dependence on $\beta$ for the magnitude of the return flow of the lee vortices is evident not only in the reference solution but also during the heating cycle as well (see Fig. D2).

Though the areal coverage of the upwind stagnation zone increases with $\beta$, the disappearance of upwind stagnation during the heating cycle occurs at approximately the same time for the entire range of $\beta$ (see Fig. D3 for solution at peak heating for $\beta$ =8). Two possible explanations exist for why the upwind stagnation zone disappears as fast for the larger obstacles as for smaller obstacles. First, as $\beta$ increases the upwind extent of the flow deceleration zone increases. Hence for large $\beta$, an air parcel will remain over a heated region longer than for a smaller $\beta$. Second, the magnitude of surface offshore flow associated
with columnar disturbances (see general introduction for discussion) decreases with heating. This statement is based upon inspection of the temporal variations of horizontally integrated momentum flux, $u'w'$, which decrease considerably with heating above the obstacle.

B. Coriolis and Low Level Moisture

For simulations with $\beta=1$, the upwind flow both at $t = t_0$ and during the heating and cooling cycles was not significantly influenced by the Coriolis force. This implies that for obstacles with length scales ($L \approx 10^2$ km), the position of both the solid and dashed curves in Fig. 3 of Reisner and Smolarkiewicz (1992) may not be substantially modified by the Coriolis force.

Because of vortex shedding, during the day/night the intensity of the onshore/offshore flow at any particular location in the lee is modulated by the strength of the lee vortex in that area (see Fig. D4 and compare with Fig. 5c). The symmetric heating applied in the simulations tends to destroy some of the asymmetries related to the shedding process; however, the vortex that is initially weaker and hence allows for a longer time scale of heating is typically the stronger vortex at peak heating. The interaction between thermal forcing and a given vortex makes accurate prediction of the intensity of the onshore or offshore flow at a given location in the lee difficult.

Beside the Coriolis force, another element of atmospheric flows is moisture. Though low level moisture leads to small changes in vertical displacements of
the isentropes, these displacements are responsible for producing a smaller flow deceleration zone, weaker flow perturbations related to the mountain wave, and smaller and less intense lee vortices (compare Fig. D5 with Fig. 4a). In fact, for the current simulation upwind stagnation is not evident in steady state. With heating the trend of stronger onshore flow over the upwind side continues; however, unlike for the steady-state flows heating produces lee vortices that are slightly stronger and deeper than their dry counterparts.

Of interest is that the effects of low level moisture on the overall stability of the atmosphere is small, and hence only small changes in the $Fr$ occur when modifications to the mean stability of the atmosphere based on moisture content is made. However, as these simulations demonstrate, low-level moisture can have a great impact on flow in the lowest layers of the atmosphere. Another important point to consider is that, given sufficient upstream moisture, orographic clouds can form with heating over the obstacle. Even though the clouds will limit the intensity of heat flux reaching the ground, the latent heat released by condensation combined with onshore flow may compensate for the reduction in diabatic heat fluxes.
Fig. D1. Reference solution for $Fr = 0.33$, $\eta^* = 0.45$, and $\beta = 8$. The plotting convention is the same as in Fig. 4a of Reisner and Smolarkiewicz (1992).
Fig. D2. Characteristics of flow over the lee as a function of time, for \( \eta^* = 0.45 \) and \( Fr = 0.33 \), are for \( \beta = 1 \) (light solid line; \(-1-\)), \( \beta = 3 \) (dashed line; \(-3-\)), and \( \beta = 8 \) (solid line, \(-8-\)). Top panel is maximum return flow \((u/U)\) in the lee as a function of time. Bottom panel is the depth of lee vortices as a function of time \((d/h)\).
Fig. D3. As in Fig. D1 but results shown at the peak heating.
Fig. D4. As in Fig. 5e of Reisner and Smolarkiewicz (1992) but with the Coriolis force active.
Fig. D5. As in Fig. 4a of Reisner and Smolarkiewicz (1992) but with low level moisture present.
GENERAL CONCLUSIONS

How low $Fr$ flow responds to heating as a function of $Fr$ and $r^*$ has been identified in this dissertation. For "standard" atmospheric conditions ($Fr = 0.33$ and $r^* = 0.45$) and for isolated heating, the upwind stagnation zone disappears with heating, while over the lee an intensification of the lee vortices occurs. The thermal circulations over both the windward and lee sides of the obstacle are typically a combined anabatic/sea breeze circulation. The intensity of the thermal circulations (predicted roughly by dimensional analysis and linear theory) was found to depend on a variety of factors, such as time of day, upstream flow conditions, and boundary conditions.

Before applying the results of the simulations to atmospheric flows, the realization that factors not considered in this study, such as latent heating, temporal and spatial variability of upstream conditions, wind shear, and turbulence might significantly influence the flow needs to be addressed. The final few paragraphs of this dissertation give suggestions for future research projects directed toward understanding processes not considered in this study.

Analyse of data obtained during the Taiwan Area Mesoscale Experiment, TAMEX, suggest that several lee vortices were modified by both surface heating and latent heating. And in the numerical study of Intensive Observing Period II, Sun and Chern (1992) found that latent heating can significantly modify the evolution of an individual vortex. Because limited offshore observations in the lee were made during TAMEX, future studies, both numerical
and observational, are needed to document the life cycle of an individual lee vortex. These observational studies should attempt to examine the role that latent heating plays in the development of the vortices. With sufficient latent heating, the circulations associated with the convection will either strengthen the vortex or weaken it. If the cloud circulations strengthen the vortex, then a long-lasting mesoscale convective system might result.

Under certain circumstances, rapid temporal variations of the large-scale pressure gradient force may lead to unpredictable results. As demonstrated with Hurricane Iniki (18 h before landfall it was predicted to be 500 miles west of the Archipelago), when a tropical storm passes to the south of an obstacle the geostrophic forcing may quickly change from an easterly direction to more of a southeasterly direction. Thus, for certain synoptic conditions, particularly when a large-scale upper trough is to the northeast of an island chain in the northern hemisphere, a tropical disturbance will first be deflected by the obstacle and then, once over the lee, be directed back towards the island. Fortunately, this rapid temporal change of the large-scale forcing is uncommon for the Hawaiian Archipelago; however, over Taiwan and other tropical islands in the northern hemisphere it may be more common. Numerical simulations, possibly of Iniki, are needed to understand the complex interactions between a tropical storm with that of low $Fr$ flow past isolated obstacles.

In the real atmosphere wind shear is almost always present. Even with no surface forcing, the degree to which the shear will modify flow structures, such as the upwind stagnation zone, columnar disturbances, and lee vortices has
not been extensively studied. Thus, to quantify how variations of upstream Richardson number influence the flow, numerical and observational studies need to be conducted. A possible site for an observational study would be New Zealand. Because New Zealand is located far enough south in the southern hemisphere, substantial upstream wind shear should be present during its winter season. New Zealand is also broken up into two major islands. Comparison of the simulations of the Hawaiian Archipelago (a weak wind-shear case) against observations and simulations of flow past New Zealand may be useful for understanding how shear modifies flow past multiple isolated obstacles.

The idealized simulations of the current study did not allow for interactions between mean and turbulent flow quantities. At present, parameterization of turbulent quantities, especially over a sloping surface, is rather ill defined and turning a parameterization "nob" slightly may produce differences in mean quantities both at the surface and in gravity wave breaking regions. But, even with grid nesting and efficient memory management, present computer limitations make explicit representation of turbulent quantities over large three-dimensional domains impossible. However, it is currently possible to resolve turbulence over select regions of the domain. The information gained from such a study could be quite useful. For example, understanding how surface heating influences gravity waves breaking in the lee could lead to better parameterization of wave drag in climate models.
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