A Tri-level Model of Centralized Transmission and Decentralized Generation Expansion Planning for an Electricity Market: Part I

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Abstract—We develop a tri-level model of transmission and generation expansion planning in a deregulated power market environment. Due to long planning/construction lead times and concerns for network reliability, transmission expansion is considered in the top level as a centralized decision. In the second level, multiple decentralized GENCOs make their own capacity expansion decisions while anticipating a wholesale electricity market equilibrium in the third level. The collection of bi-level games in the lower two levels forms an equilibrium problem with equilibrium constraints (EPEC) that can be approached by either the diagonalization method (DM) or a complementarity problem (CP) reformulation. We propose a hybrid iterative solution algorithm that combines a CP reformulation of the tri-level problem and DM solutions of the EPEC sub-problem.

Index Terms—Generation Expansion Planning, Transmission Expansion Planning, Equilibrium Problem with Equilibrium Constraints, Mathematical Program with Equilibrium Constraints, Complementarity Problem, Nash Equilibrium.

NOMENCLATURE

A. Sets

N Electricity nodes indexed by i, j, k

L Transmission lines indexed by ij

N_{gen} Set of electricity nodes where a GENCO is located indexed by i, j, k

B. Primal Decision Variable Vectors

z (First level) Binary decision variables for transmission lines, with elements for existing lines set fixed to 1

V^{new} (Second level) Generation capacities after expansion, MW

q (Third level) Demand satisfied at electricity nodes, MW

θ (Third level) Voltage angles at electricity nodes

f (Third level) Electricity flows on transmission lines, MW

y (Third level) Generation amounts at electricity nodes, MW

η (Third level) Price (scalar) at the reference electricity node, \$/MWh

C. Parameter Vectors

a Intercepts of electricity demand prices as linear functions of quantities, \$/MWh

b Slopes of electricity demand prices as linear functions of quantities, \$/MWh/MWh

c Linear coefficient of the generation cost function, \$/MWh

e Quadratic coefficient of the generation cost function, \$/MWh/MWh

c^{exp} Investment costs for generation expansion discounted on an hourly basis, \$/MW

c^{trexp} Investment costs for transmission line expansion discounted on an hourly basis, \$/MW

θ^{max} Maximum values for voltage angles

θ^{min} Minimum values for voltage angles

V Generation capacities at electricity nodes, MW

U Fuel availability, MW

K Capacities of transmission lines, MW

B Negative susceptances of transmission lines, Ω^{-1}

I. INTRODUCTION

INCREASINGLY across the U.S. and worldwide, the wholesale electricity market is composed by separate generation companies (GENCOs), transmission owners (TRANSCOs), distribution companies (DISCOs) and load serving entities (LSEs) [1]. The Independent System Operator (ISO) is charged with monitoring the grid, ensuring reliability and settling the electricity market for a region. The ISOs and regional reliability councils, who conduct transmission planning studies and reliability assessment, must consider how GENCOs' strategic expansion decisions may react to the transmission planning decisions, and how the wholesale markets will perform in response to both the transmission and generation expansions.

To provide reliable and economic electricity supply, planners must not only consider generation expansion to ensure there will sufficient energy to meet future loads, but also take into account the entire wholesale electricity supply system including transmission and market clearing by the ISO. Resource investment decisions have great impact on market outcomes. Transmission congestion due to insufficient transmission capacity can cause spikes in the locational marginal prices (LMPs) or even load curtailment in extreme cases. LSEs, who are the buyers in the wholesale market, play important roles in distributing the electricity to retail

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customers. In restructured markets, expansion decisions may be justified by potential profit increases rather than cost reductions. The profit return received by an investor is determined by an electricity market price settlement. The ISO matches the electricity supply bids and demand offers and settles the LMPs to maximize total market surplus of both buyers and sellers. Typically this is done on an hourly basis in a day-ahead market and every 5 minutes in the real-time market. Moreover, investments in generation capacity will be effective only if the transmission capacity is adequate to transport the newly available power to where the demand is located.

We formulate a market-based transmission and generation expansion problem and propose a method to solve it. In our model, each GENCO anticipates prices settled by an ISO market clearing problem when making its own investment and operational decisions. At the same time, the GENCOs' decisions are also made in response to transmission planning decisions because sufficient transmission capacity is essential for GENCOs to reap additional profits from delivering energy from expanded capacity to the load locations. GENCOs will hesitate to expand if a high level of grid congestion is likely to result in future generation curtailment. On the other hand, too much transmission capacity does not favor generation expansion either, because low electricity prices provide little incentive for investment. Therefore, the transmission expansion planning decision must be considered in a market based generation expansion planning problem. Although in a deregulated market, transmission lines are owned by individual TRANSCOs, transmission expansion planning remains centralized to guarantee reliability of the transmission grid. Therefore, our model includes a centralized transmission planning decision by the ISO, who is mainly in charge of the reliability of the regional market. The ISO conducts a resource adequacy study, anticipates the expansion and dispatch decisions by multiple GENCOs, and decides where to expand the grid. Our market-based model captures both dependence of the GENCOs' expansion decisions on prior transmission plans and their anticipation of wholesale electricity market settlement after expansion.

We formulate the transmission and generation expansion planning problem as a mixed integer tri-level program, where the discrete centralized transmission planning decisions occur in the first level, multi-GENCOs' generation expansion decisions constitute the second level, and an electricity market equilibrium problem forms the third level. Modeling transmission planning in the top level is consistent with a principle that transmission planning should proactively influence generation investment [2]. The lower level interactions are based on our previous model [3], including strategic behavior by the GENCOs. Because the tri-level structure with a sub-problem of bi-level games poses solution difficulties, algorithms will first be proposed to solve the collection of bi-level games. This collection can be reformulated as an equilibrium problem with equilibrium constraints (EPEC), to which two of the currently available methodologies discussed in [4] can be applied. We propose a hybrid iterative algorithm to solve the entire tri-level programming problem by exploiting the advantages of both EPEC solution methods. In part II of this paper, case studies of 6, 30, and 118 bus test systems are presented to illustrate how the algorithm works to optimize the transmission expansion plan in anticipation of generation expansion decisions and market equilibria.

The contributions of this paper are fourfold: 1) We propose a novel formulation of centralized transmission and decentralized generation expansion planning as an integrated tri-level optimization problem with a sub-problem of bi-level games. 2) The solution challenges posed by the problem's multi-level and bi-level games structure are addressed by first reformulating the non-convex sub-problem as an EPEC and solving it by the diagonalization method (DM) as multiple mathematical programs with equilibrium constraints (MPECs). Since the concavity of each maximization MPEC is not guaranteed, we also propose a way to verify the solution as a local (approximate) Nash equilibrium (NE) point. 3) We apply a complementarity problem (CP) reformulation to the entire tri-level programming problem to search for promising transmission expansion plans. 4) We develop a novel hybrid iterative algorithm that can successfully solve the entire tri-level expansion planning model. This approach could be used by a regional transmission planner to identify a good combination of proposed transmission projects to implement.

In Section II, a thorough literature review is given. The model is presented in Section III. Sections IV and V, respectively, illustrate the algorithms to solve an NE game of bi-level games, and a tri-level programming problem with a bi-level games sub-problem. Section VI concludes the paper.

II. Literature Review

Many recent studies of restructured electricity markets formulate a single decision maker's expansion decision with an ISO market clearing problem as a lower level sub-problem. A review of traditional and market based transmission expansion planning methodologies was summarized in [5]. Transmission expansion with a market equilibrium subproblem was modeled in [6]. Similar models of generation expansion include [7], [8], [9] [10]. Bi-level programming (BLP) models are widely applied to model individual GENCOs' capacity expansion decisions and/or bidding strategies while anticipating the market settlement results [11] [12] [13] [14] [15].

Game theory is widely applied to model and investigate the outcomes when multiple strategic players make their expansion decisions simultaneously. A single level Cournot game of multiple GENCOs making both capacity expansion and operational decision was studied, and an equilibrium solution was iteratively solved by diagonalization [16]. Three models of solving a single level Cournot capacity game under different economic schemes were presented in [17]. Given the parameter assumptions on demand and two types of candidate units, the existence and uniqueness of the Cournot equilibrium solutions were also discussed and proved. A two-tier, multi-period, multi-GENCO equilibrium capacity expansion model was proposed in [18]. A capacity expansion problem of strategic multi-GENCO bi-level games was presented and a co-evolutionary algorithm was applied to search for the NE solution in [19]. Competitive decisions by multiple GENCOs to expand in anticipation of market outcomes can be modeled

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TABLE I
COMPARISON WITH DIFFERENT MODELS PROPOSED IN PREVIOUS RELEVANT LITERATURE REVIEW

	[20]	[21]	[22]	[23] [24]	[25] [26]	Our Model
Transmission Expansion	Centralized; Existing/new line expansion; Maximize net surplus	Decentralized; New line expansion; Maximize net profit	Centralized; Existing/new line expansion; Multi-criteria	Centralized; Existing line augmentation; Minimize operation and investment cost	Centralized; Existing/new line expansion; Minimizing operation and investment cost	Centralized; New line expansion; Maximize net surplus
Generation Expansion	Decentralized; Continuous	Decentralized; Binary	Decentralized; Continuous	Decentralized; Binary	Decentralized; Continuous	Decentralized; Continuous
Multi-Period Expansion	No	Yes	Yes	No	No	No
ISO's Market Problem	Maximize surplus	Minimize system cost, minimize loss of energy probability	Maximize surplus	Minimize operating cost	Minimize operation cost	Maximize surplus
GENCO's Operational Problem	Strategic (Cournot)	Competitve	Strategic (pair of price and quantity)	Strategic (pair of price and quantity)	Competitive	Strategic (Cournot)
Operational Uncertainty	Yes	Yes	No	No	Yes	No
Solution Method	Optimization of Bi-level Games	Simulation of an Iterative Procedure	Search-based and Agent-based Method	Genetic Algorithm	Linearization and MILP Reformulation	Iterative algorithm with Optimization of Bi-level Games

as an equilibrium problem with equilibrium constraints (EPEC). Two of the currently available algorithm to solve an EPEC problem, diagonalization method (DM) and complementary problem (CP) reformulation, are discussed in [4]. Both linearization technique and strong duality theory are adopted in [27] [28] [29] to reformulate an EPEC problem into a set of mixed integer linear constraints and solve it to its optimality. A combination of constructing a linearization of EPEC problem and validating the solution optimality by DM is proposed to solve a multi-GENCO's bilevel capacity expansion problem in [30]. Comparison of open loop and closed loop capacity equilibrium in an electricity market is thoroughly discussed in [31]. This work extended the findings of [32] and found that an EPEC with capacity planning in the upper level and any competition types from perfect to Cournot in the lower level market problem with single load period yield same results as an open loop Cournot equilibrium, where multi-GENCO determine their capacity and quantity to sell at the same time.

When both transmission and generation planning decisions account for interactions among market players, the planning model takes on a more complicated, multi-level structure. Sauma and Oren [20] studied a multi-GENCO equilibrium expansion planning model with anticipation of an ISO market clearing problem, and evaluated the transmission expansion's effect on the social welfare of the system by considering different transmission expansion plans. For various candidate transmission expansion decisions, the bi-level games were solved by an iterative DM algorithm. Roh et al. [21] developed an iterative process to solve a generation and transmission planning problem by simulating the interactions among GENCOs, TRANSCOs and ISO with consideration of uncertainty, profit from the market clearing decision, and transmission reliability. Motamedi et al. [22] proposed a transmission expansion framework to take into account the expansion reaction from decentralized GENCOs and also integrated an operational optimization in restructured electricity market. The problem was formulated as a four level model and it was approached by agent-based system and search-based techniques. Hesamzadeh et al. [23] [24] studied a new framework of transmission augmentation planning problem with strategic generation expansion and operational decision and solved a tri-level program by a genetic algorithm.

Pozo et al. [25] studied a three-level generation and transmission model, and converted it into single level mixed integer linear programming problem. Table I compares our tri-level model with the multi-level generation and transmission expansion models investigated in the previous papers. A level is labeled as "centralized" if decisions are is made by a single entity and "decentralized" if decisions are made separately by individual decision makers. Our formulation is similar to the one in [20] but we include the transmission plan as a decision variable in the optimization problem rather than a parameter. Our tri-level model has a similar structure to that investigated in [25] [26]. However, we consider price-responsive demand functions and strategic interactions among the generators at the operational level. The objective of the system operator, to maximize the total net surplus, cannot be reduced to minimizing cost. The problem structure is also similar to [23] [24] but we consider expansion as new transmission lines rather than augmentation of the existing circuits, price-responsive demand functions, Cournot competition among GENCOs in the operational level, and surplus maximization as the objective function for the system operator.

III. MODEL AND PROBLEM FORMULATION

A. Assumptions

1) For simplicity, we formulate a static model with a single hour of operation and no uncertainty. Thus, the third (operational) level represents a typical hour in a single future scenario for market conditions. At the expense of increased computational time, the model could be extended to incorporate multiple periods and probabilistic scenarios for parameters such as. fuel price and load, or to model infrastructure contingencies.
2) For the transmission expansion, we only consider building new lines and do not consider expanding the capacity of the existing lines. However, the model can be easily extended to include line expansion without changing the structure of the problem.

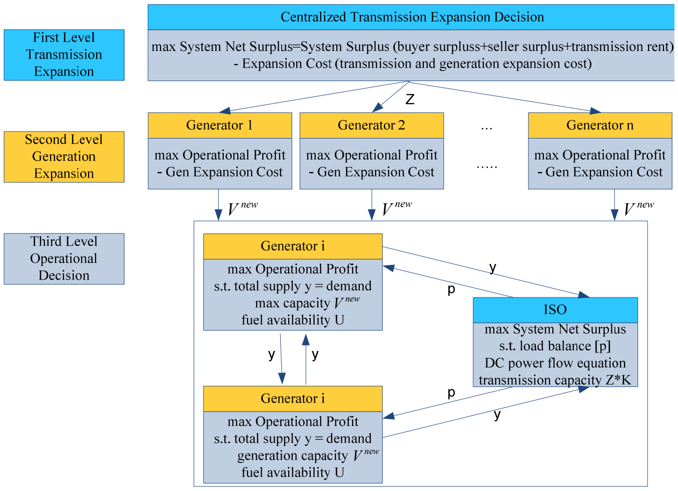

Fig. 1. A Tri-level Integrated Generation and Transmission Expansion Planning Model

3) The transmission and generation expansion costs in the top two levels are both modeled as linear and are discounted to form equivalent hourly costs.
4) Each generator at each bus is owned by a single GENCO. As a Cournot competitor, each GENCO makes his own decision on the generation quantity to sell in the electricity market under a type of bounded rationality *[33]*. A quadratic generation cost function that will not be affected by the capacity expansion is assumed. We formulate one generator per GENCO in the model. However, the model can be easily extended to multiple generators per GENCO.
5) We assume the market equilibrium in the third level is simultaneously determined by Cournot competition among the GENCOs and an ISO market clearing problem. Its equivalent linear complementarity problem (LCP) reformulation generates a unique NE solution due to the concave objective functions and convex feasible regions *[20]*. Although the Cournot model simplifies the actual market structure, its broad market outcomes have been validated against an agent-based simulation of bid and offer matching *[34]*. In contrast, the multiplicity of solutions *[9] [27] [28] [29]* in supply function equilibrium formulations may obscure the effects of upper-level capacity expansion decisions.

B. Formulation

The problem is formulated as a tri-level model as shown in Fig. 1 with the ISO's discrete transmission expansion decisions on the first level, multi-GENCOs' separate generation expansion decisions on the second level, and the multiple market players' operational decisions in the third level. By extending the bi-level model in [3], we decentralize the generation expansion decisions by separate GENCOs where, in the lower level, the GENCOs and ISO simultaneously optimize their own operational benefits. The GENCO decides its generation level, and the ISO allocates energy to the LSEs to maximize the total system surplus. Different from the lower level model in [3], we do not consider a fuel supply problem to account for the fuel availability and fuel transportation capacity. Instead, for a simplified version, we include a fuel capacity constraint in each GENCO's operational decision problem to represent any combination of fuel supply and transportation capacity constraints. Since the generation decisions are influenced by the transmission grid, it is assumed that the ISO makes a centralized transmission expansion decision in anticipation of the expansion decisions made by its followers, the GENCOS.

A full mathematical formulation of the mixed integer tri-level nonlinear programming model is proposed as below, where the variables in the brackets to the right of constraints are their corresponding dual multipliers.

- First Level: From a system point of view, the ISO collects the information about future loads and resources and makes a centralized decision, z, on transmission expansion to maximize system net surplus, equivalent to system total surplus less generation and transmission expansion cost:

$$max_{\ z} \sum_{j\in N}\left(\frac{1}{2}b_j q_j^2 + a_j q_j\right) - \sum_{j\in N_{gen}}\left(c_j y_j + e_j y_j^2\right) - \sum_{ij\in L} c_{ij}^{trexp} K_{ij} z_{ij} - \sum_{j\in N_{gen}} c_j^{gexp}\left(V_j^{new} - V_j\right) \quad (1)$$

- Second Level: Each GENCO k makes its own expansion decision V_k^{new}, given the other GENCOs' decisions and in anticipation of market clearing results. Each GENCO k maximizes its net operating profit from selling the power in the electricity market less the expansion cost:

$$max_{V_k^{new}}\ (p_k - c_k - e_k y_k) y_k - c_k^{gexp}(V_k^{new} - V_k) \quad (2)$$

$$s.t.\quad V_k^{new} - V_k \geq 0\ [\mu n V V_k \geq 0] \quad (3)$$

- Third Level: The ISO chooses q, f, and θ to optimize both the sellers' and buyers' surplus, $\sum_{j\in N}\left(\frac{1}{2}b_j q_j^2 + a_j q_j\right) - \sum_{j\in N} p_j q_j$ and $\sum_{j\in N_{gen}} p_j y_j - \sum_{j\in N_{gen}}\left(c_j y_j + e_j y_j^2\right)$, and transmission rent, $\sum_{ij\in L}(p_j - p_i) f_{ij}$, the total of which reduces to $\sum_{j\in N}\left(\frac{1}{2}b_j q_j^2 + a_j q_j\right) - \sum_{j\in N_{gen}}\left(c_j y_j + e_j y_j^2\right)$. Because $\sum_{j\in N_{gen}}\left(c_j y_j + e_j y_j^2\right)$ remains constant in this optimization problem, it is equivalent to maximize $\sum_{j\in N}\left(\frac{1}{2}b_j q_j^2 + a_j q_j\right)$. Constraint (5) gives the load balance on each electricity nodes. Equations (6) and (7) give the bounds on voltage angles. Equations (8) and (9) are the linearized power flow equations. The thermal transmission limits are enforced by constraints (10) and (11). M_{ij} is a big value so that when z_{ij} is 1, $f_{ij} = B_{ij}(\theta_i - \theta_j)$; otherwise, the constraints (8) and (9) are relaxed. Here, with a direct current optimal power flow approximation, we assume the voltage angle at reference bus, $\theta_{ref} = 0$, and the ranges for all other voltage angles are within ±0.6, so that $M_{ij} = 1.2B_{ij}$ suffices.

$$max_{q,\theta,f} \sum_j \left(\frac{1}{2}b_j q_j^2 + a_j q_j\right) \quad (4)$$

$$s.t.\ q_j + \sum_{ji} f_{ji} - \sum_{ij} f_{ij} = y_j,\ \forall j\in N\ \left[p_j\right] \quad (5)$$

$$\theta_i \leq \theta_j^{max},\ \forall j\in N\ \left[\alpha_j^+ \geq 0\right] \quad (6)$$

$$-\theta_j \leq -\theta_j^{min},\ \forall j\in N\ \left[\alpha_j^- \geq 0\right] \quad (7)$$

$$f_{ij} - B_{ij}\left(\theta_i - \theta_j\right) \leq \left(1 - z_{ij}\right)M_{ij},\ \forall ij\in L\ \left[\gamma_{ij}^+ \geq 0\right] \quad (8)$$

$$-f_{ij} + B_{ij}\left(\theta_i - \theta_j\right) \leq \left(1 - z_{ij}\right)M_{ij},\ \forall ij\in L\ \left[\gamma_{ij}^- \geq 0\right] \quad (9)$$

$$f_{ij} \leq z_{ij} K_{ij},\ \forall ij\in L\ \ [\lambda_{ij}^+ \geq 0] \quad (10)$$

$$-f_{ij} \leq z_{ij} K_{ij},\ \forall ij\in L\ \ [\lambda_{ij}^- \geq 0] \quad (11)$$

$$q_j \geq 0,\ \forall j\in N\ \ [\delta_j \geq 0] \quad (12)$$

Simultaneously, each GENCO i maximizes its operational profit with anticipation of their decisions' effect on the

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reference price, η, and determines its quantity to sell [33]. Equation (14) implies balance of total demand and generation, (15) indicates that the generation level must not exceed its capacity. Inequality (16) imposes a constraint on fuel availability, which also implies an upper bound on the generation level y and expanded capacity V_i^{new}, that restricts the feasible region and makes the problem computationally easier to solve.

$$max_{y_i,\eta}\ (\eta + \phi_i - c_i - e_i y_i)\, y_i \tag{13}$$

$$s.t.\, y_i + \eta \sum_j \frac{1}{b_j} = \sum_j \frac{\phi_j - a_j}{b_j} - \sum_{j \neq i} y_j\ \ [\beta_i] \tag{14}$$

$$y_i \leq V_i^{new}\ \ [\mu_i \geq 0] \tag{15}$$

$$y_i \leq U_i\ \ [\rho_i \geq 0] \tag{16}$$

$$y_i \geq 0\ \ [\zeta_i \geq 0] \tag{17}$$

The reference node LMP and price premia at non-reference nodes that appear in (13) and (14) are respectively defined in equations (18) and (19), which link the dual variables, p_j, in the ISO's problem and the reference price, η, in the GENCO's problem.

$$\eta = p_{ref} \tag{18}$$

$$\phi_j = p_j - \eta,\ \forall j \in N \tag{19}$$

Working from the bottom to the top, the sub-problem in the third level is an equilibrium problem formed by combining equations (13) – (17) for each GENCO with (4) – (12), (18), and (19), given the decision variables in the upper two levels, z_{ij} and V_k^{new}, as fixed parameters. The bi-level sub-problem in the lower two levels consists of the objective function (2) subject to constraint (3) and the decision variables (q, etc.), with corresponding dual variables, being optimal in the lower level equilibrium problem (4) – (19), given the decision variables in the first level, z_{ij}, as fixed parameters. Finally, the entire tri-level program includes the objective function (1) subject to the decision variables V^{new} being optimal in the bi-level sub-problem (2) – (19).

IV. Algorithm to Solve the EPEC Sub-Problem

To approach the optimization of the tri-level expansion problem with electricity market, we first study its equilibrium sub-problem, given in equations (2)–(19), without considering the ISO's centralized transmission expansion decisions in the first level. This game involves generation expansion decisions from multiple GENCOs, where each of their separate optimization problems is a bi-level problem with multiple followers including all GENCOs and the ISO. Each GENCO's bi-level problem, in equations (2)-(19), can be reformulated as an MPEC by replacing the lower level optimization problems in equations (4)-(17) with their equivalent first-order optimality conditions given in equations (20)-(35) [4]. Each perpendicular constraint; e.g., $\theta^{max} - \theta_j \perp \alpha_j^+$, can be further converted to an equivalent nonlinear reformulation; e.g., $(\theta^{max} - \theta_j)\alpha_j^+ = 0$. Therefore, for each equilibrium constraint, there are three corresponding dual variables: two for the equations or inequalities and one for the nonlinear reformulation of the perpendicular relationship. For example, in equation (24) $\mu\theta_{kj}^+ \geq 0$ and $\mu\alpha_{kj}^+ \geq 0$ are dual variables for equations $\theta^{max} - \theta_j \geq 0$ and $\alpha_j^+ \geq 0$, respectively while $\mu\theta\alpha_{kj}^+$ is the dual variable for the equation $(\theta^{max} - \theta_j)\alpha_j^+ = 0$. In each GENCO k's problem, the other GENCOs' capacity expansion decisions are considered as fixed parameters. The subscript of dual variables starting with k indicates the specific sets of dual variables for GENCO k. Also the dual variables for (18) and (19) are, respectively, $\mu\eta_k$ and $\mu\phi_{kj}$.

$$b_j q_j + a_j - p_j + \delta_j = 0,\ \forall j \in N\ \ [\mu d q_{kj}] \tag{20}$$

$$-\alpha_j^+ + \alpha_j^- - \sum_{i,} \gamma_{ij}^+ B_{ij} + \sum_i \gamma_{ji}^+ B_{ji} + \sum_i \gamma_{ij}^- B_{ij} - \sum_i \gamma_{ji}^- B_{ji} = 0 \quad \forall j \in N\ \ [\mu d\theta_{kj}] \tag{21}$$

$$p_j - p_i - \gamma_{ij}^+ + \gamma_{ij}^- - \lambda_{ij}^+ + \lambda_{ij}^- = 0,\ \forall ij \in L\ \ [\mu d f_{kij}] \tag{22}$$

$$q_j + \sum_{ji} f_{ji} - \sum_{ij} f_{ij} = y_j,\ \forall j \in N\ \ [\mu q y_{kj}] \tag{23}$$

$$\theta^{max} - \theta_j \geq 0 \perp \alpha_j^+ \geq 0, \forall j \in N \quad [\mu\theta_{kj}^+ \geq 0, \mu\alpha_{kj}^+ \geq 0, \mu\theta\alpha_{kj}^+] \tag{24}$$

$$\theta_j - \theta^{min} \geq 0 \perp \alpha_j^- \geq 0, \forall j \in N \quad [\mu\theta_{kj}^- \geq 0, \mu\alpha_{kj}^- \geq 0, \mu\theta\alpha_{kj}^-] \tag{25}$$

$$-f_{ij} + B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})B_{ij}1.2 \geq 0 \perp \gamma_{ij}^+ \geq 0, \forall ij \in L \quad [\mu f_{kij}^+ \geq 0, \mu\gamma_{kij}^+ \geq 0, \mu f\gamma_{kij}^+] \tag{26}$$

$$f_{ij} - B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})B_{ij}1.2 \geq 0 \perp \gamma_{ij}^- \geq 0,\ \forall ij \in L \quad [\mu f_{kij}^- \geq 0, \mu\gamma_{kij}^- \geq 0, \mu f\gamma_{kij}^-] \tag{27}$$

$$z_{ij}K_{ij} - f_{ij} \geq 0 \perp \lambda_{ij}^+ \geq 0, \forall ij \in L \quad [\mu K_{kij}^+ \geq 0, \mu\lambda_{kij}^+ \geq 0, \mu K\lambda_{kij}^+] \tag{28}$$

$$z_{ij}K_{ij} + f_{ij} \geq 0 \perp \lambda_{ij}^- \geq 0, \forall ij \in L \quad [\mu K_{kij}^+ \geq 0, \mu\lambda_{kij}^+ \geq 0, \mu K\lambda_{kij}^+] \tag{29}$$

$$q_j \geq 0 \perp \delta_j \geq 0, \forall j \in N\ \ [\mu q_{kj} \geq 0, \mu\delta_{kj} \geq 0, \mu q\delta_{kj}] \tag{30}$$

$$\eta + \phi_j - c_j - 2e_j y_j - \beta_j - \mu_j - \rho_j + \zeta_j = 0, \forall j \in N_{gen} \quad [\mu d y_{kj}] \tag{31}$$

$$y_j + \beta_j \sum_i \frac{1}{b_i} = 0,\ \forall j \in N_{gen}\ \ [\mu d\eta_{kj}] \tag{32}$$

$$V_j^{new} - y_j \geq 0 \perp \mu_j \geq 0, \forall j \in N \quad [\mu n V_{kj} \geq 0, \mu\mu_{kj} \geq 0, \mu n V\mu_{kj}] \tag{33}$$

$$U_j - y_j \geq 0 \perp \rho_j \geq 0, \forall j \in N \quad [\mu U_{kj} \geq 0, \mu\rho_{kj} \geq 0, \mu U\rho_{kj} \geq 0] \tag{34}$$

$$y_j \geq 0 \perp \zeta_j \geq 0,\ \forall j \in N\ [\mu y_{kj} \geq 0, \mu\zeta_{kj} \geq 0, \mu y\zeta_{kj}] \tag{35}$$

A. Diagonalization Method (DM)

One way to find an equilibrium solution, if one exists, is to iteratively solve each GENCO's problem by fixing the other GENCOs' expansion decisions to their current optimal solutions, which is called DM in [4]. In other words, the optimal solution determined by each GENCO should be identical to the value that the other GENCOs assume as a model parameter of their own optimization models. However, the existence of a pure Nash equilibrium (NE) strategy is not guaranteed for the EPEC sub-problem, and the GENCOs' expansion decisions, V_k^{new} , can oscillate, usually among two or more different values within a small range, generally by 1-3% and at most 5% from our computational experience. Therefore, we define a maximum number of iteration cycles and an approximate NE solution as the average of the subsequential limiting solutions, which will be further illustrated in Part II of this paper. The model of producers in a Cournot game under a type of bounded rationality drastically limits the number of possible Nash equilibria compared to a supply function equilibrium formulation as in [23]. In our numerical studies described in Part II of this paper, multiple Nash equilibria are not observed.

The DM algorithm is illustrated in Table II. First, we solve the MPEC for GENCO 1 by initializing the values of the other GENCOs' expansion decisions $V_{2,..|N_{gen}|}^{newequ}$, equal to their existing capacity, $V_{2,..|N_{gen}|}$ as model parameters and GENCO 1's existing capacity V_1 as starting point for V_1^{new}. Once the optimal solution V_1^{new*} for GENCO 1 is obtained, it is considered as a model input V_1^{newequ} to the next MPEC problem of GENCO 2 while $V_{3,..,|N_{gen}|}^{newequ}$ remain the same for the new problem. The iteration continues with starting point for V_k^{new} as updated V_k^{newequ} until the MPEC problem of the final GENCO is solved. Because the decision V_k^{new} can only be changed when the MPEC of GENCO k is solved, V_k^{new} is updated once during a round of n iterations (one for each GENCO). If, after a round of iterations, the changes in value of each GENCO's decision are all within the predefined threshold ε, we conclude that an equilibrium has been identified and stop the iteration process. Otherwise, we continue the iteration until the predetermined limit is reached. The MPEC problem can be solved in GAMS by using the solver NLPEC [35], which first reformulates our MPEC into a nonlinear program (NLP), and then calls the NLP solver CONOPT [36] to solve the problem.

TABLE II
SOLVING THE EPEC SUB-PROBLEM BY DM ALGORITHM

DM Algorithm

```
Input parameters V_{2,..,|N_gen|}^{newequ};
Let ConvergenceFlag = 0, Cycle=0;
While (ConvergenceFlag = 0 and Cycle ≤MaxCycle)
    Let ConvergenceFlag = 1;
    For GENCO  k = 1 to |N_gen|
        Cycle = Cycle+1;
        Solve GENCO k's MPEC problem, Equations (2), (3), (18), (19),
(20)-(35), with optimal solution V_k^{new*};
        If |V_k^{new*} − V_k^{newequ}| ≥ ε
            ConvergenceFlag = 0;
        End If;
        Let V_k^{newequ} = V_k^{new*};
    End For
End While
Output V_{1,..,|N_gen|}^{new} = V_{1,..,|N_gen|}^{newequ};
```

B. Complementarity Problem (CP) Reformulation

Another way to find a NE solution of the equilibrium bi-level sub-problem is through CP reformulation [4]. Combining the MPEC problem for each GENCO results in an equilibrium problem with equilibrium constraints (EPEC). The CP reformulation combines the KKT conditions of each MPEC to reformulate the EPEC into a mixed complementarity problem (MCP). Given the lack of convexity of each MPEC, optimality to the CP reformulation is only a necessary condition for a solution to be an equilibrium of the original bi-level games, but not a sufficient condition. Specifically, the solution found by CP reformulation is a stationary point of the original EPEC problem. The derivation of the CP reformulation can be found in Appendix A.

V. A Hybrid Iterative Algorithm to Solve the Tri-level Programming Problem

Upon CP reformulation of equations (2) – (19), the tri-level problem (1) – (19), can be converted into a single level optimization problem with a set of nonlinear, linear and complementarity constraints (38) – (95) as shown in Appendix A. The reformulated problem, consisting of the equations (1) and (38) – (95), is a generalized MPEC with mixed integer, linear, nonlinear and equilibrium constraints, and can be solved by the NLPEC solver in GAMS. The NLPEC solver provides several different reformulation methodologies to approach MPEC problems [35]. The type of MPEC reformulation we found the most successful and reliable is to penalize violation of the equilibrium constraints in the objective function by first converting each equilibrium constraint, $0 \le x \perp y \ge 0$, to its equivalent constraint set: $x \ge 0, y \ge 0, xy = 0$, and then including a term, $1/\mu^{pen}xy$, in the objective function. As the reciprocal penalty parameter μ^{pen} iteratively decreases to zero, the penalty applied to $xy \neq 0$ increases until solutions eventually approximate $xy = 0$ [35].

Because of possible occurrence of non-existing pure NE solution, the MPEC problem might not even be feasible, instead of solving the entire MPEC problem by itself, we relax it and adopt it to identify a promising transmission plan. We then evaluate the generation expansion and market outcomes. Two of the currently available approaches to solve bi-level games as an EPEC are DM and CP reformulation. Based on our computational studies reported in Part II, given a certain transmission expansion planning decision, the performance of DM is quite stable in successfully identifying the (approximate) Nash equilibrium (NE) of the bi-level games; while the CP reformulation, since it is not an equivalent reformulation of the original bi-level game, can only find a stationary point and provide a bound for the original problem. However, the benefit of CP reformulation is to be able to solve the entire tri-level problem as a single level problem that includes the transmission expansion decisions z_{ij}. With all these considerations in mind, we propose a hybrid iterative algorithm that takes advantage of both methods. It first solves a master problem, transformed from the CP reformulation, to propose a transmission expansion decision $z^{master-n}$ in the n-th iteration. Given that transmission expansion plan, it employs DM to find an (approximate) NE point of the game of bi-level games. The iterative solution procedure is illustrated in Figure 2, and a detailed explanation of the hybrid algorithm is in Appendix 4.B of [37]. The steps of the algorithm are as follows:

- Step 1: Set n to 0. Initialize the best found system net surplus, F^{best}, to 0, and let the objective value lower bound constraint (36) of MINLP master problem be greater than F^{best}. Go to Step 2.

$$F(z,\Omega) \ge F^{best},$$
$$\text{where } \Omega = \{V_j^{new}, q, y\},\ F(z,\Omega) = \sum_j \left(\tfrac{1}{2} b_j q_j^2 + a_j q_j\right) - \sum_{j\in N_{gen}}\left(c_j y_j + e_j y_j^2\right) - \sum_{ij\in L} c_{ij}^{trexp} K_{ij} z_{ij} - \sum_{j\in N_{gen}} c_j^{gexp}\left(V_j^{new} - V_j\right) \quad (36)$$

- Step 2: Solve a MINLP master problem A-n. If the MINLP master problem A-n can be successfully solved to an optimal solution, we fix the transmission expansion decision $z^{master-n}$ and continue to solve the EPEC sub-problem B with DM. Go to Step 3. Otherwise, the algorithm is

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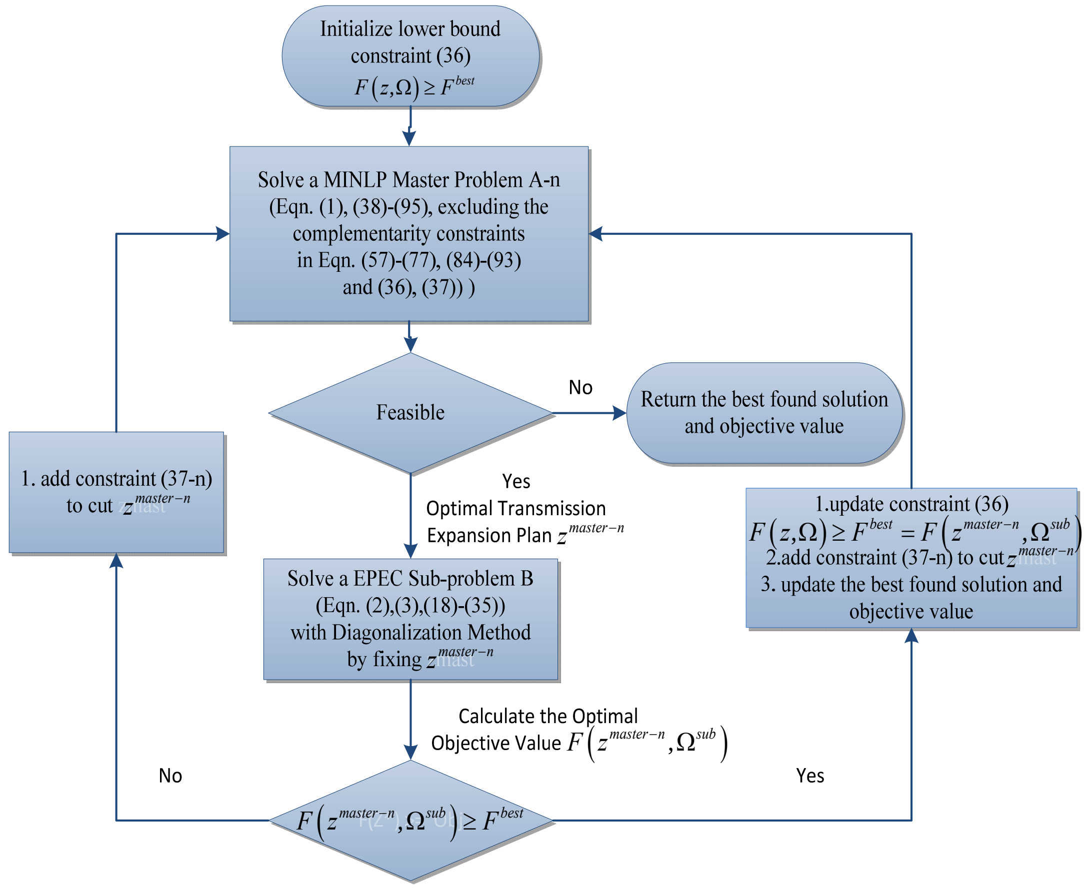

Fig. 2. A Hybrid Iterative Algorithm to Solve a Tri-level Problem with an EPEC Sub-problem

terminated and best solution found so far is returned as the final solution.

- Step 3: With optimal solution $\Omega^{sub} = \{V_j^{newsub}, q^{sub}, y^{sub}\}$ found by DM, the system net surplus $F(z^{master-n}, \Omega^{sub})$ is calculated. If $F(z^{master-n}, \Omega^{sub}) \ge F^{best}$, go to Step 4. Otherwise, go to Step 5.
- Step 4: Update constraint (36) with $F^{best} = F(z^{master-n}, \Omega^{sub})$, add a constraint (37-n) to cut $z^{master-n}$ point and update the best found solution and its objective value, $F(z^{master-n}, \Omega^{sub})$. Let $n = n + 1$, and go to Step 2.

$$\sum_{ij, z_{ij}^{master-n}=1} (1 - z_{ij}) + \sum_{ij, z_{ij}^{master-n}=0} (z_{ij}) \ge 1 \quad (37\text{-n})$$

- Step 5: Add a constraint (37-n) to cut $z^{master-n}$. Let $n = n + 1$, and go to Step 2.

The hybrid algorithm is not guaranteed to find a global optimal solution of the tri-level model. However, the numerical results for two of the case studies presented in Part II of this paper are validated as the global optimal solutions.

VI. Conclusions

In this paper, we consider an integrated transmission and generation expansion planning problem in a restructured electricity market environment. We propose a novel tri-level programming model, where a centralized transmission expansion planning decision in the top level is made in anticipation of the multi-GENCOs' responses in terms of their generation expansion decisions in the middle level, while each GENCO also makes its capacity expansion decision by anticipating the electricity market equilibrium results achieved by all the GENCOs making their generation decisions, and an ISO's market clearing problem in the bottom level.

The tri-level programming model includes an EPEC problem, which can be approached by either the diagonalization method (DM) or a complementarity problem (CP) reformulation. To solve the tri-level optimization problem, a hybrid iterative algorithm is proposed by taking advantage of the strengths of both DM algorithm and CP reformulation. The benefit of applying CP reformulation is its capability to transform the tri-level model into a single level MINLP problem, to which we can apply the DICOPT solver [38], and identify a promising transmission planning decision in each iteration. Because of the model's nonconvexity, there is no guarantee that a global optimum will be identified by any solver designed for convex MINLPs. Among these, DICOPT has been found to be relatively fast on a variety of problems [39] and, in our computational experience described in Part II, produced good results. A thorough study of which MINLP solver performs best in this context is a subject for further research. On the other hand, given a preselected transmission expansion decision, the DM algorithm works more reliably and efficiently to find the corresponding (approximate) Nash Equilibrium (NE) point for the generation expansion bi-level games.

The problem we consider in this paper is a static model that considers only a single hour in a future year, which will always result in the generation levels y being equal to the new

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capacity levels V^{new}. In the future, we can extend the model and the algorithm to take into account multiple periods and uncertainties. Instead of having one equilibrium problem in the third level, the model should include multiple equilibrium problems under different scenarios for, e.g., demand levels and fuel prices. Each generator's MPEC problem would be to maximize average net profit. The resulting buyers' and sellers' surplus amounts would be averaged in the top level objective with appropriate weighting factors.

Part II of the paper will continue with numerical results.

Appendix A. CP Reformulation for Multiple GENCO's EPEC Sub-Problem

The CP reformulation for the EPEC sub-problem, equations (38)-(95), are derived as following, based on *[4]*. L represents the Lagrangian function of the second level problem (2) – (19), with a nonlinear reformulation (20) – (35) replacing the third level problem, given by equations (4) – (19).

A. Partial derivatives of Lagrange function

There are all together 19 sets of equality constraints with dimension $|N_{gen}|+7|N_{gen}||N|+5|N_{gen}||L|+6|N_{gen}||N_{gen}|$. They have the same number of unrestricted variables to match them, respectively μdq_{kj}, $\mu d\theta_{kj}$, μdf_{kij}, μdy_{kj}, $\mu d\eta_{kj}$, $\mu U\rho_{kj}$, μqy_{kj}, $\mu\theta\alpha^+_{kj}$, $\mu\theta\alpha^-_{kj}$, $\mu f\gamma^+_{ij}$, $\mu f\gamma^-_{ij}$, $\mu K\lambda^+_{ij}$, $\mu K\lambda^-_{ij}$, $\mu q\delta_{kj}$, μyy_{kj}, $\mu nV\mu_{kj}$, $\mu y\zeta_{kj}$, $\mu\eta_k$, $\mu\phi_{kj}$.

$$L_{nV_k} = -c_k^{gexp} + \mu nVV_k + \mu nV_{k,k} + \mu nV\mu_{kk}\mu_k = 0, \forall k \in N_{gen}, k \in N_{gen} \quad (38)$$

$$L_{q_j} = \mu dq_{kj}b_j + \mu qy_{kj} + \mu q_{kj} + \mu q\delta_{kj}\delta_j = 0,\ \forall k \in N_{gen}, j \in N \quad (39)$$

$$L_{\theta_j} = -\mu\theta^+_{kj} - \mu\theta\alpha^+_{kj}\alpha^+_j + \mu\theta^-_{kj} + \mu\theta\alpha^-_{kj}\alpha^-_j - \sum_i(\mu f^+_{kij}B_{ij} + \mu f\gamma^+_{kij}\gamma^+_{ij}B_{ij} - \mu f^-_{kij}B_{ij} - \mu f\gamma^-_{kij}\gamma^-_{ij}B_{ij}) + \sum_i(\mu f^+_{kji}B_{ji} + \mu f\gamma^+_{kji}\gamma^+_{ji}B_{ji} - \mu f^-_{kji}B_{ji} - \mu f\gamma^-_{kji}\gamma^-_{ji}B_{ji}) = 0, \forall k \in N_{gen}, j \in N \quad (40)$$

$$L_{f_{ij}} = -\mu qy_{kj} + \mu qy_{ki} - \mu f^+_{kij} - \mu f\gamma^+_{kij}\gamma^+_{ij} + \mu f^-_{kij} + \mu f\gamma^-_{kij}\gamma^-_{ji} - \mu K^+_{kij} - \mu K\lambda^+_{kij}\lambda^+_{ij} + \mu K^-_{kij} + \mu K\lambda^-_{kij}\lambda^-_{ij} = 0, \forall k \in N_{gen}, ij \in L \quad (41)$$

$$L_{y_j} = p_{j=k} - c_{j=k} - 2e_{j=k}y_{j=k} - 2e_j\mu dy_{kj} + \mu d\eta_{kj} + \sum_j \mu yy_j - \mu nV_{kj} - \mu nV\mu_{kj}\mu_j + \mu y_{kj} + \mu y\zeta_{kj}\zeta_j - \mu qy_{kj} = 0, \forall k \in N_{gen}, j \in N_{gen} \quad (42)$$

$$L_\eta = \sum_j \mu dy_{kj} + \mu\eta_k + \sum_j \mu\phi_{kj} = 0, \forall k \in N_{gen} \quad (43)$$

$$L_{\phi_j} = \mu dy_{kj} + \mu\phi_{kj} = 0, \forall k \in N_{gen}, j \in N \quad (44)$$

$$L_{p_j} = y_{j=k} - \mu dq_{kj} + \sum_i \mu df_{kij} - \sum_i \mu df_{kji} - \mu\phi_{kj} = 0, \forall k \in N_{gen}, j \in N \quad (45)$$

$$L_{\alpha^+_j} = -\mu d\theta_{kj} + \mu\alpha^+_{kj} + \mu\theta\alpha^+_{kj}(\theta^{max} - \theta_j) = 0, \forall k \in N_{gen}, j \in N \quad (46)$$

$$L_{\alpha^-_j} = \mu d\theta_{kj} + \mu\alpha^-_{kj} + \mu\theta\alpha^-_{kj}(\theta_j - \theta^{min}) = 0, \forall k \in N_{gen}, j \in N \quad (47)$$

$$L_{\gamma^+_{ij}} = -\mu d\theta_{kj}B_{ij} + \mu d\theta_{ki}B_{ij} - \mu df_{kij} + \mu\gamma^+_{kij} + \mu f\gamma^+_{kij}[-f_{ij} + B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})M_{ij}] = 0, \forall k \in N_{gen}, ij \in L \quad (48)$$

$$L_{\gamma^-_{ij}} = \mu d\theta_{kj}B_{ij} - \mu d\theta_{ki}B_{ij} + \mu df_{kij} + \mu\gamma^-_{kij} + \mu f\gamma^+_{kij}[f_{ij} - B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})M_{ij}] = 0, \forall k \in N_{gen}, ij \in L \quad (49)$$

$$L_{\lambda^+_{ij}} = -\mu df_{kij} + \mu\lambda^+_{kij} + \mu K\lambda^+_{kij}(z_{ij}K_{ij} - f_{ij}) = 0, \forall k \in N_{gen}, ij \in L \quad (50)$$

$$L_{\lambda^-_{ij}} = \mu df_{kij} + \mu\lambda^-_{kij} + \mu K\lambda^-_{kij}(z_{ij}K_{ij} + f_{ij}) = 0, \forall k \in N_{gen}, ij \in L \quad (51)$$

$$L_{\delta_j} = \mu dq_{kj} + \mu\delta_{kj} + \mu q\delta_{kj}q_j = 0, \forall k \in N_{gen}, j \in N \quad (52)$$

$$L_{\beta_j} = -\mu dy_{kj} + \mu d\eta_{kj}\sum_j \frac{1}{b_j} = 0, \forall k \in N_{gen}, j \in N_{gen} \quad (53)$$

$$L_{\mu_j} = -\mu dy_{kj} + \mu\mu_{kj} + \mu nV\mu_{kj}(V_j^{new} - y_j) = 0, \forall k \in N_{gen}, j \in N_{gen} \quad (54)$$

$$L_{\rho_j} = -\mu dy_{kj} + \mu\rho_{kj} + \mu U\rho_{kj}(U_j - y_j) = 0, \forall k \in N_{gen}, j \in N_{gen} \quad (55)$$

$$L_{\zeta_j} = \mu dy_{kj} + \mu\zeta_{kj} + \mu y\zeta_{kj}y_j = 0, \forall k \in N_{gen}, j \in N_{gen} \quad (56)$$

B. KKT conditions derived from the optimization problems in the second level:

There are all together 21 sets of inequality constraints with dimension of $|N_{gen}|+6|N_{gen}||N|+8|N_{gen}||L|+6|N_{gen}||N_{gen}|$. They have the same number of positive variables to match them, shown as the dual variables in the constraints.

$$V_k^{new} - V_k \ge 0 \perp \mu nVV_k \ge 0, \forall k \in N_{gen} \quad (57)$$

$$\theta^{max} - \theta_j \ge 0 \perp \mu\theta^+_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (58)$$

$$\theta_j - \theta^{min} \ge 0 \perp \mu\theta^-_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (59)$$

$$-f_{ij} + B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})M \ge 0 \perp \mu f^+_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (60)$$

$$f_{ij} - B_{ij}(\theta_i - \theta_j) + (1 - z_{ij})M \ge 0 \perp \mu f^-_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (61)$$

$$z_{ij}K_{ij} - f_{ij} \ge 0 \perp \mu K^+_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (62)$$

$$z_{ij}K_{ij} + f_{ij} \ge 0 \perp \mu K^-_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (63)$$

$$q_j \ge 0 \perp \mu q_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (64)$$

$$\alpha^+_j \ge 0 \perp \mu\alpha^+_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (65)$$

$$\alpha^-_j \ge 0 \perp \mu\alpha^-_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (66)$$

$$\gamma^+_{ij} \ge 0 \perp \mu\gamma^+_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (67)$$

$$\gamma^-_{ij} \ge 0 \perp \mu\gamma^-_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (68)$$

$$\lambda^+_{ij} \ge 0 \perp \mu\lambda^+_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (69)$$

$$\lambda^-_{ij} \ge 0 \perp \mu\lambda^-_{kij} \ge 0, \forall k \in N_{gen}, ij \in L \quad (70)$$

$$\delta_j \ge 0 \perp \mu\delta_{kj} \ge 0, \forall k \in N_{gen}, j \in N \quad (71)$$

$$V_j^{new} - y_j \ge 0 \perp \mu nV_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (72)$$

$$U_j - y_j \ge 0 \perp \mu U_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (73)$$

$$y_j \ge 0 \perp \mu y_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (74)$$

$$\mu_j \ge 0 \perp \mu\mu_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (75)$$

$$\rho_j \ge 0 \perp \mu\rho_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (76)$$

$$\zeta_j \ge 0 \perp \mu\zeta_{kj} \ge 0, \forall k \in N_{gen}, j \in N_{gen} \quad (77)$$

C. Equivalent KKT conditions derived from the optimization problems in the third level

There are all together 18 sets of constraints, among which there are 8 sets of equality constraints with dimension of $1+2|N_{gen}|+4|N|+|L|$ and 10 sets of inequality constraints with dimension of $3|N_{gen}|+3|N|+4|L|$. The equality constraints have the same amount of unrestricted variables to match them, respectively q_j, θ_j, f_{ij}, y_j, η, p_j, β_j, ϕ_j, while the inequality constraints have the same number of positive variables to match them, shown as the dual variables in the constraints.

$$b_jq_j + a_j - p_j + \delta_j = 0, \forall j \in N \quad (78)$$

$$-\alpha^+_j + \alpha^-_j - \sum_{i,ij\in L}(B_{ij}\gamma^+_{ij} - B_{ij}\gamma^-_{ij}) +$$

$$\sum_{i,ji\in L}\left(B_{ji}\gamma_{ji}^{+}-B_{ji}\gamma_{ji}^{-}\right)=0,\forall j\in N \quad (79)$$
$$p_j-p_i-\gamma_{ij}^{+}+\gamma_{ij}^{-}-\lambda_{ij}^{+}+\lambda_{ij}^{-}=0,\forall ij\in L \quad (80)$$
$$\eta+\phi_j-c_j-2e_jy_j-\beta_j-\mu_j-\rho_j+\zeta_j=0,\forall j\in N_{gen} \quad (81)$$
$$y_j+\beta_j\sum_j\frac{1}{b_j}=0,\forall j\in N_{gen} \quad (82)$$
$$q_j+\sum_i f_{ji}-\sum_i f_{ij}=y_j, j\in N \quad (83)$$
$$\theta^{max}-\theta_j\geq 0\perp\alpha_j^{+}\geq 0,\forall j\in N \quad (84)$$
$$\theta_j-\theta^{min}\geq 0\perp\alpha_j^{-}\geq 0,\forall j\in N \quad (85)$$
$$-f_{ij}+B_{ij}\left(\theta_i-\theta_j\right)+\left(1-z_{ij}\right)M_{ij}\geq 0\perp\gamma_{ij}^{+}\geq 0,\forall ij\in L \quad (86)$$
$$f_{ij}-B_{ij}\left(\theta_i-\theta_j\right)+\left(1-z_{ij}\right)M_{ij}\geq 0\perp\gamma_{ij}^{-}\geq 0,\forall ij\in L \quad (87)$$
$$z_{ij}K_{ij}-f_{ij}\geq 0\perp\lambda_{ij}^{+}\geq 0,\forall ij\in L \quad (88)$$
$$z_{ij}K_{ij}+f_{ij}\geq 0\perp\lambda_{ij}^{-}\geq 0,\forall ij\in L \quad (89)$$
$$q_j\geq 0\perp\delta_j\geq 0,\forall j\in N \quad (90)$$
$$V_j^{new}-y_j\geq 0\perp\mu_j\geq 0, j\in N_{gen} \quad (91)$$
$$U_j-y_j\geq 0\perp\rho_j\geq 0, j\in N_{gen} \quad (92)$$
$$y_j\geq 0\perp\zeta_j\geq 0, j\in N_{gen} \quad (93)$$
$$\eta=p_{ref} \quad (94)$$
$$\phi_j=p_j-\eta,\forall j\in N \quad (95)$$

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