"VISION" APPROACH OF CALIBRATION METHODS FOR RADIOGRAPHIC SYSTEMS

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INTRODUCTION

The geometric calibration of radiographic systems cannot be avoided in image reconstruction problems in stereoradiography, X-ray or SPECT tomography. This operation performed usually before the object examination, provides the relative positions of the different parts of the system (object, source, detector). The source-detector geometry is then described by intrinsic parameters, and the scanning motion by extrinsic ones.

We propose in this paper a geometric calibration approach for cone beam geometry systems based on calibration methods used in computer vision [1] (usually applied to robotics). We first recall the principles of classical geometric calibration methods. Second, we describe the "vision" approach and explain how this calibration can be performed using only linear algebra. Third, we show how our approach can be extended to tomographic systems. Finally, we compare the advantages and the drawbacks of this method with the classical one.

CLASSICAL GEOMETRIC CALIBRATION METHODS

First of all, let us recall the principles of classical geometric calibration methods for radiographic and tomographic systems. The geometric parameters are estimated in two steps. First the intrinsic parameters are estimated. These parameters are linked to the acquisition geometry: detector to focal distance, pixel size, projection of the focal point onto the detector. Then, the extrinsic parameters are estimated. In the case of computed tomography the intrinsic parameters are considered as stable during the acquisition and the extrinsic parameters are then given by the rotation axis position.

Estimation of the Intrinsic Parameters

The intrinsic parameters are estimated by analyzing the displacement of the projection of a network of points moved parallel to the detector plane. This method is more or less equivalent to the computation of the projection matrix in the "vision" approach.

Estimation of the Extrinsic Parameters

To estimate the extrinsic parameters of a tomographic system, we place a set of points in an approximately known position in the object set of axes. We then perform an acquisition on this set of points. As the intrinsic parameters are considered as stable during the whole acquisition, we can estimate the extrinsic parameters by minimizing the least square error between the measured positions of the projected points and the computed projection positions using the estimated parameters. We must stress that in the process we obtain directly the parameters we need for reconstruction, but we must minimize a highly non linear expression. So very often a large change on one of the parameters may produce a very small increase of the least square error. Gullberg and al. [2] presented a slightly different process where the extrinsic and the intrinsic parameters are obtained in only one minimization, but they gave almost the same conclusions.

These classical methods are currently used in x-ray CT and in SPECT.

THE "VISION" APPROACH

To overcome the limiting assumptions of classical geometric calibration, we developed the "vision" approach. This method enables us to calibrate geometrically each projection in a reference set of axes. So with this approach we supply algebraic reconstruction or stereoradiography algorithms with the projection matrix for each projection direction. In the case of the tomography the reconstruction set of parameters can be computed from the set of projection matrices.

Like classical geometric calibration methods, the "vision" approach is based on a set of assumptions. First ,we must construct a "calibration object", i.e. a set of 3D points with precisely known relative positions. Then, we must extract accurately 2D image measurements and match 3D points with their respective 2D projections. The acquisitions of this object are performed for all the positions we want to calibrate. Finally, before starting the calibration procedure, sampling steps on the detector plane must have been computed.

Geometric Description of the System: the Set of Vision Parameters

The cone-beam x-ray acquisition geometry can be described by the pinhole camera model. Figure 1 shows the different parameters involved in this description.

The source-detector geometry defines a coordinate system R_F relative to the focal point F and including vectors u and v directed by the pixel grid on the detector plane.
The intrinsic parameters are then the following:

- (p_0,q_0), the coordinates in pixel unit of the projection of the focal point onto the detector plane,
- (α_p,α_q), the scale factors given by:

$$\alpha_p = \frac{FF'}{\Delta_p} \quad , \quad \alpha_q = \frac{FF'}{\Delta_q}$$, Δ_p and Δ_q being the pixel sizes on the detector plane

The calibration object defines another coordinate system R_{obj} at point O.
The extrinsic parameters are then:

- the translation T (t_x,t_y,t_z) from the focal point to the origin O of R_{obj},
- the rotation r defined by its row vectors (r_1,r_2,r_3) which transforms the vectorial set (u,v,w) into (i,j,k). (This rotation can also be expressed according to the three Euler's angles).

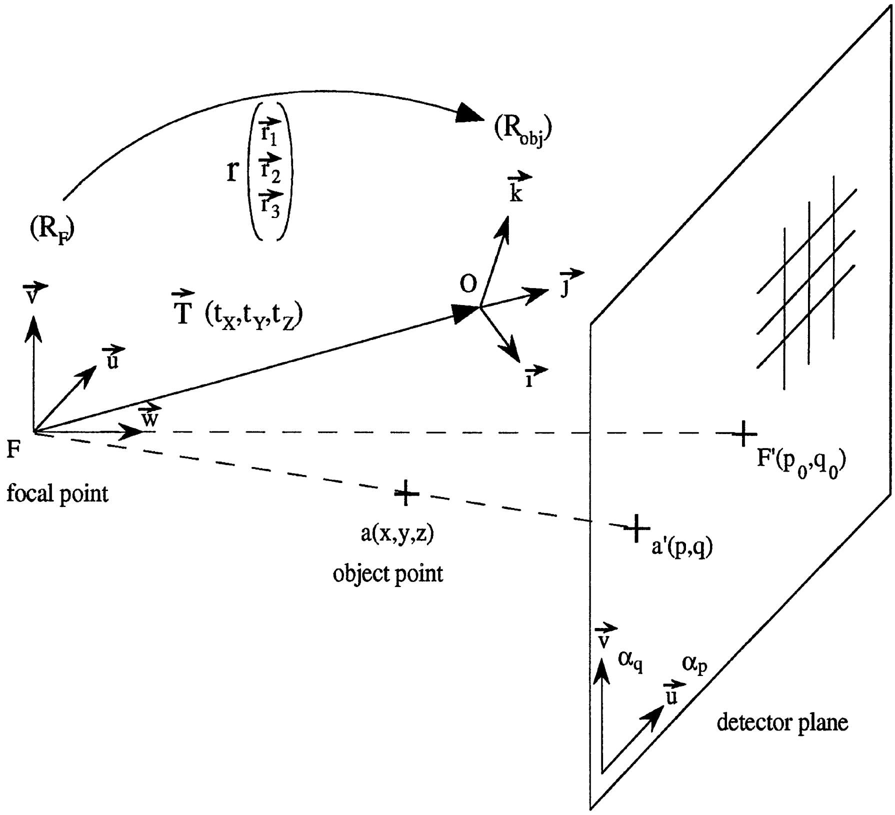

Fig. 1   Description of the acquisition geometry

For each position of this object, the calibration procedure is divided into the four following steps.

<u>Computation of the Projection Matrix and the Intrinsic Parameters</u>

Although the projection is a non linear transformation, we can note the projection of a point a(x,y,z) with a linear matrix formula expressed in homogeneous coordinates by equation (1) (where s is the scale factor of the transformation):

$$s.\begin{pmatrix} p \\ q \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_p.\vec{r_1} + p_0.\vec{r_3} & \alpha_p.t_X + p_0.t_Z \\ \alpha_q.\vec{r_2} + q_0.\vec{r_3} & \alpha_q.t_Y + q_0.t_Z \\ \vec{r_3} & t_Z \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{1}$$

This transformation matrix, denoted (M), relates each point of the calibration object to their projected image. Thus, for each of them we can write

$$s.\begin{pmatrix} p \\ q \\ 1 \end{pmatrix}_j = (M)\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_j = \begin{pmatrix} \vec{m_1} & m_{14} \\ \vec{m_2} & m_{24} \\ \vec{m_3} & m_{34} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_j \tag{2}$$

If we have at least 6 object points, we can compute the matrix (M) by a least mean square estimation. We then derive intrinsic parameters taking into account that row vectors of the rotation matrix are perpendicular to each other. Therefore:

$$p_0 = \vec{m_1} \cdot \vec{m_3} \qquad \alpha_p = \| \vec{m_1} \wedge \vec{m_3} \|$$
$$q_0 = \vec{m_2} \cdot \vec{m_3} \qquad \alpha_q = \| \vec{m_2} \wedge \vec{m_3} \| \tag{3}$$

Once this first step has been carried out, computation of extrinsic parameters must be done. In order to perform only linear estimation in our calibration process and thus to find more stable solutions, the estimation of the rotation must be separated from the estimation of the translation [3].

Separation of Rotation and Translation Computations

The projection formulas (2) give for all object points $\vec{a_j} = (x,y,z)_j^T$; $j=1,\ldots,n_{points}$ the following system:

$$\left\{ \begin{array}{l} \quad\quad\quad \ldots \\ (p_j \vec{m_3} - \vec{m_1}).\vec{a_j} + (p_j\, m_{34} - m_{14}) = 0 \\ (q_j \vec{m_3} - \vec{m_2}).\vec{a_j} + (q_j\, m_{34} - m_{24}) = 0 \\ \quad\quad\quad \ldots \end{array} \right. \tag{4}$$

By replacing the elements of matrix M by their expressions, this can also be written:

$$\left\{ \begin{array}{l} \quad\quad\quad \ldots \\ \alpha_p\, (\vec{r_1}.\vec{a_j} + t_X) + (p_0 - p_j)(\vec{r_3}.\vec{a_j} + t_Z) = 0 \\ \alpha_q\, (\vec{r_2}.\vec{a_j} + t_Y) + (q_0 - q_j)(\vec{r_3}.\vec{a_j} + t_Z) = 0 \\ \quad\quad\quad \ldots \end{array} \right. \tag{5}$$

We notice in system (5) that for each point of the focal line (see figure 1) the second terms of all the equations equal zero. These points are projected on the detector plane at (p_0,q_0). Therefore, a particular solution a_0 of system (4) is obvious, and using it we can eliminate the translation parameters in system (5):

$$\left\{ \begin{array}{l} \quad\quad\quad \ldots \\ \alpha_p \vec{r_1}(\vec{a_j} - \vec{a_0}) + (p_0 - p_j)\vec{r_3}.(\vec{a_j} - \vec{a_0}) = 0 \\ \alpha_q \vec{r_2}(\vec{a_j} - \vec{a_0}) + (q_0 - q_j)\vec{r_3}.(\vec{a_j} - \vec{a_0}) = 0 \\ \quad\quad\quad \ldots \end{array} \right. \tag{6}$$

Computation of Rotation Parameters

Let n_{1j}, n_{2j}, and n_{3j} be the vectors such that

$$n_{1j}\begin{pmatrix} \alpha_p \\ 0 \\ p_0 - p_j \end{pmatrix}, \; n_{2j}\begin{pmatrix} 0 \\ \alpha_q \\ q_0 - q_j \end{pmatrix}, \; n_{1j} \wedge n_{2j} = n_{3j}\begin{pmatrix} \alpha_q\,(p_0 - p_j) \\ \alpha_p\,(q_0 - q_j) \\ \alpha_p \alpha_q \end{pmatrix} \tag{7}$$

In system (6), the rotation parameters have been isolated. These equations mean that for each point indexed j, the vector $(r)(a_j - a_0)$ is perpendicular to the vectors n_{1j} and n_{2j}; therefore its direction is given by the vectorial product n_{3j} of these two vectors. The problem becomes then to find an optimal rotation (r) transforming the vectorial set $(a_j - a_0)$ into n_{3j} (for j equals 1 to n_{points}, the number of object points).

To solve this problem, we use the unit quaternions [4] which provide by a linear estimation the optimal solution as a set of four parameters (angle and unit vectorial axis of rotation). These four parameters can be easily converted into Euler's angles or into the rotation matrix (r).

Computation of Translation Parameters

The last step of the calibration procedure is to estimate the translation from the system (8) where all of the other parameters are known. These three remaining translation parameters are estimated by least mean square.

$$\begin{cases} \dots \\ \alpha_p t_X + (p_0 - p_j) t_Z + ((\alpha_p \vec{r_1} + (p_0 - p_j)\vec{r_3}).\vec{a_j} = 0 \\ \alpha_q t_Y + (q_0 - q_j) t_Z + ((\alpha_q \vec{r_2} + (q_0 - q_j)\vec{r_3}).\vec{a_j} = 0 \\ \dots \end{cases} \tag{8}$$

GENERALIZATION TO TOMOGRAPHIC SYSTEMS

If we intend to generalize this approach to calibrate tomographic systems using classical analytical reconstruction codes, we must find the tomographic set of geometric parameters which is often relative to the rotation axis of the system. Then different positions of the calibration object are needed, these positions are taken by rotating the object by arbitrary angles about the axis we would like to determine the position. We assume in this case that the source-detector geometry is stable (we consider an analytical reconstruction code for which intrinsic parameters do not change during the acquisition), so that the previous procedure must be modified like that:

- intrinsic parameters are computed for each position of the calibration object,
- we take then their mean values,
- extrinsic parameters are finally computed for each position.

Once the different positions of the calibration object have been calibrated, we must convert the "vision" set into the tomographic set. We proceed then as follows:

- a reference position of the object is chosen,
- the set of rotation axes between each position and the reference one is computed from the set of extrinsic parameters,
- the mean position of the rotation axis is computed,
- finally the tomographic parameters are derived from this axis position and the values of the averaged intrinsic parameters.

We must stress here that only two positions of the calibration object are sufficient to carry out this procedure if the system is stable on a complete rotation.

DISCUSSION

The difference between the classical geometric calibration and the "vision" calibration implementations is that for classical methods we do not need to construct a particular 3D calibration object. Neither do we need to match these points with 2D image measurements for different positions of the object in order to calibrate the system. We directly compute the required set of parameters from the trajectory of a test point.

We must also stress that the "vision" method uses linear estimation algorithms. Thus, no initialisation problem comes up at any step of the calibration procedure, and the stability of the results is ensured. Moreover, this method can be adapted to any kind of source trajectories -- it does not require a regular angular sampling or a stable acquisition geometry.

CONCLUSION

We have described a "vision" method for calibrating radiographic systems. We have stressed that if we can define a perfectly known set of 3D points and extract accurately 2D image measurements, this method is easy to carry out and can be adapted to a large variety of geometry for applications in radiography, x-ray CT and SPECT. Moreover with such a method, the calibration does not need to be done by a specialist because the only experimental work to be performed is to place the calibration object in the field of view. The experimental application of this method is under development.

REFERENCES

1. P. Puget and T. Skordas, in *Calibrating a mobile camera*, Image and Vision Computing, Vol.8 (4), pp 341-348, November 1990.
2. G. T. Gullberg, B. M. W. Tsui, C. R. Crawford, J. G. Ballard and J. T. Hagius, in *Estimation of geometrical parameters and collimator evaluation for cone beam tomography*, Medical Physics 17(2), pp. 264-272, March/April 1990.
3. O. Faugeras and M. Hebert, in *Representation, recognition and locating 3D objects*, International Journal of Robotics Research, Vol.5 (3), pp 27-52, 1986.
4. B. K. P. Horn, in *Relative orientation revisited*, Journal of Optical Society of America, Vol.8 (10), pp 1630-1638, October 1991.