On the Use of the Inflation Tax when Non-Distortionary Taxes are Available

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Abstract

Using a pure-exchange overlapping generations model in which money is valued because of a legal restriction, we show the following: a) a benevolent government may make some use of the inflation tax in conjunction with a lump-sum income tax on the young, b) the inflation tax will not be used along with a lump-sum income tax on the old, and c) the welfare-maximizing monetary policy may deviate from the Friedman rule (contract the money supply so as to equate the real return on money and other competing stores of value) depending on how fiscal policy is implemented.

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1 Introduction

Consider money growth rates over fairly long horizons, say, for example, a decade. At this frequency, it is remarkable that the money growth rate is positive in almost every country, i.e., most countries are raising *some* revenue from the inflation tax.[1] It is also true that most countries impose legal restrictions on money holdings. In this paper, we ask whether it is possible that these very legal restrictions make it desirable to inflate the money stock. More precisely, if an inflation tax base has been created via a given reserve requirement, will a benevolent government use the inflation tax as a (partial) source of revenue? Even when a non-distortionary revenue source is available?

There is a large literature on inflationary finance and issues of optimal taxation to which this question is related.[2] In a setting with infinitely lived agents, the Friedman rule, for example, stipulates that a Pareto efficient allocation of resources in an economy can be supported by a policy that sets the money growth rate equal to the subjective time rate of preference of its agents. In the presence of discounting, the Friedman rule thus requires that the money supply should contract. Wallace (1980) studies a similar question in an overlapping-generations economy. With money as the only store of value, Wallace demonstrates that the Pareto efficient allocation can be supported by maintaining a constant money stock. With linear storage as the competing store of value, Wallace shows that the Pareto efficient allocation can be supported by shrinking the money stock so as to equate the returns to storage and money. The Friedman and the Wallace results hold for economies in which either the government does not need to finance any spending or if it does, nondistortionary taxes and transfers are available for that purpose. One thing is clear, then. If nondistortionary taxes were available and governments followed Pareto efficient monetary policies, we should expect to see nonpositive money growth rates. Which still leaves our question open: why do we observe *positive* money growth rates?

The literature has provided at least two answers to our question. First, starting with Phelps (1973), researchers have abandoned the assumption of unavailability of nondistortionary taxes. Both Phelps, and later Helpman and Sadka (1979), derive conditions under which seigniorage is part of an optimal policy package with other *distortionary* taxes.[3,4] The second way is enunciated by Freeman (1987) who studied an overlapping generations model

[1] See Click (1998) for evidence on money growth rates across countries.

[2] See Chari, Christiano, and Kehoe (1996), Mulligan and Sala-i-Martin (1997), and Correia and Teles (1999) for an up-to-date treatment of the issues.

[3] Helpman and Sadka (1979) use an overlapping generations framework. They present two versions of their model, one in which money is the only store of value and is held because there is no second period income, and one in which money co-exists with bonds and is held because it enters the preferences of agents in the tradition of Sidrauski (1967). If lump-sum taxes were available in their world, then it is apparent that in both the versions (especially the latter), the use of seigniorage would not be desirable. The reason we can demonstrate desirability of seigniorage is that we have a fixed unremovable distortion in the form of a binding legal restriction which forces agents to hold some units of a return-dominated asset. In other words, how the rate of return dominance problem of money is solved by Helpman and Sadka, namely putting money in the utility function, matters crucially. We thank an anonymous referee for bringing the Helpman and Sadka paper to our attention, and for suggesting the above line of thinking.

[4] In recent years, Chari, Christiano, and Kehoe (1996) and Correia and Teles (1999) have challenged the Phelps intuition, and succeeded in resurrecting the optimality of the Friedman rule in the presence of distortionary taxes.

with return-dominated money in which a legal restriction forces people to hold money, and seigniorage is the only revenue source for the government. Freeman shows that the stationary utility-maximizing monetary policy is to set the reserve requirement at the minimum feasible level, and inflate the money stock away at an infinite rate. In essence, Freeman's policy mimics a nondistortionary tax since it entirely confiscates the agent's forced holding of real money balances. Freeman's analysis, however, cannot account for why we observe positive yet finite money growth rates around the world.

In this paper, we fill a niche between the Helpman-Sadka analysis and Freeman's work in that we try to justify the place of seigniorage in a welfare-maximizing fiscal tool kit that include nondistortionary instruments. We extend the Freeman setup by allowing the government access to lump-sum taxation alongside seigniorage. Like Freeman, money is valued because it satisfies a legal restriction; unlike Freeman, the legal restriction is not a choice variable for the government. We extend the Helpman and Sadka setup by making nondistorting taxes available. Unlike them, we solve the return-dominance problem by fixing a reserve requirement. Young agents receive a fixed endowment of the consumption good, the old receive nothing. There are two primary assets, storage and money. The latter is rate of return dominated by the former. Agents hold money solely to satisfy an unremovable legal restriction (reserve requirement).[5] There is a government that has to finance a fixed level of useless purchases every period. To that end, it raises the revenue from either a nondistortionary income tax on the young or seigniorage, or some combination of the two. The government's budget is balanced period by period. Private agents take the policies of the government as given, and compute their own decision rules regarding how much to consume in each period. The government, in turn, takes these "policy reaction functions" as given, and chooses the mix of the inflation tax and the non-distortionary tax, so as to maximize the welfare of current and future generations in a stationary setting. We ask, will such a government ever choose the inflation tax as a revenue-raising tool? [6]

Our main result is easily summarized. A benevolent government may wish to raise some positive fraction of its revenue from the inflation tax even when a nondistortionary tax is available. The intuition for this result is as follows. Suppose the government increases the portion of its spending financed by nondistortionary taxes, effectively decreasing the fraction of spending financed by seigniorage. On the "good" side of the seigniorage Laffer curve, money growth rates can now be lowered. With slower money growth, the real return to saving increases. Even when saving is totally unresponsive to its return, ceteris paribus, the income effect of an increase in the interest rate raises second period consumption. On the other hand, the increase in tax payments when young results in less first-period disposable income, thereby reducing the quantity of saving, and hence possibly, second period consumption. On balance, lifetime utility of current and future generations may increase or decrease in response

[5] We rely on some outside (unmodelled) factor to motivate the existence of the reserve requirement. For instance, during the Great Depression, the Congressional Record indicates that deposit insurance was foremost in policymakers minds' as they debated the application of this tool. Throughout this paper, we treat the reserve requirement as exogenously given and focus exclusively on the public finance considerations that arise when reserve requirements are present.

[6] We are searching among policies that maximize stationary lifetime utility. In a second-best world, Pareto efficient policies may or may not maximize welfare. We will also be ignoring the welfare of the initial old in our welfare assessments.

to an increase in nondistortionary taxes. We derive conditions in which lifetime welfare is greater with some use of the inflation tax.

Notice that monetary policy can be used to undo the distortion to saving caused by the legal restriction. The Friedman rule accordingly advises the central bank to contract the money stock so that money and the linear storage technology offer the same return. We show that this advice is dominated in a stationary welfare sense by a policy that combines a lump-sum tax on the young and some seigniorage. Why? If the government followed the Friedman rule, taxes on the young would have to finance the government spending in addition to making up for lost seigniorage. This "overtaxing" of the young may be detrimental to welfare. Our exercise is therefore a classic application of the theorem of the second best. Indeed, as Woodford (1990) states, "...in the presence of additional distortions, no available policy may achieve a 'first-best' allocation, and among the allocations that are attainable, the best one need not be any of the ones that happen to reduce the nominal interest rate to zero. This idea is familiar from the 'theory of the second best' in public finance" (p.1086).

There is a caveat though. Thus far, we have compared the inflation tax with an exogenously specified alternative revenue source, a lump-sum tax on the young. If instead, we compare the inflation tax against a lump-sum tax on the *old*, we find that the welfare-maximizing monetary policy is to hold the money stock fixed. Moreover, we show that implementing the Friedman rule (shrinking the money stock) via a lump-sum tax on the old, may produce *higher* welfare than using the combination of lump-sum taxes on the young and seigniorage. We go onto show that implementing the Friedman rule via a lump-sum tax on the old may produce higher welfare than if the same were implemented via a lump-sum tax on the young.

In sum, one of the things we have shown is that in an overlapping-generations model with reserve requirements, welfare-maximizing monetary policy may deviate significantly from the Friedman (equal-real-return) rule, depending on *how* fiscal policy is implemented. The novelty of this result lies partly in the fact that such a result could not be obtained in a model with infinitely-lived representative agents. In those models, the Friedman rule can be implemented in only one way: by changing the transfers to the agents at each date. There the entire issue is whether the variations in money growth required by the Friedman rule are brought about via lump-sum transfers or not. The question of timing of these transfers is moot. In contrast, in an overlapping generations model, as is well-known from Wallace (1980) and Sargent (1987), there are many competing ways of implementing the Friedman rule. Additionally, the timing of policy typically matters. One of our contributions then is to show that *how* the Friedman rule is implemented matters in determining whether welfare-maximizing monetary policy deviates from it or not, an insight that cannot be generated using monetary models with infinitely lived agents.

The rest of the paper is organized as follows. Section 2 describes the economic environment for the case where the inflation tax is compared to an alternative revenue source: a nondistortionary income tax on the young. It contains a statement (and detailed discussion) of our main result: a benevolent government would raise some positive fraction of its revenue from the inflation tax even when a nondistortionary tax is available. Section 3 considers the model in which the inflation tax is compared to a nondistortionary income tax on the old. Here we show that the taxing the old is preferred by current and future generations to raising seigniorage. We conclude in Section 4.

2 The model with a lump-sum tax on the young

We consider an economy consisting of an infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely-lived government. Let t be the time index, with $t = 0, 1, 2, ...$At each date $t \geq 1$, a new generation comprised of N identical agents appears. Each such agent is endowed with y units of a perishable consumption good only when young. They all have identical preferences over their young-age and old-age consumption summarized by a time-separable utility function,

$$U(c_1, c_2) = u(c_1) + v(c_2), \tag{1}$$

where c_1 (c_2) denotes the consumption when young (old). We assume that u and v are twice continuously differentiable, and strictly concave; formally, $u', v' \geq 0$, $u'', v'' < 0$.

There are two primary assets in this economy: storage and money. Each unit of the consumption good placed into storage at date t yields $x > 1$ units of the consumption good at date $t+1$. Let p_t denote the time t price level. Because fiat money does not pay any explicit interest, its gross real return between t and $t+1$ is $\frac{p_t}{p_{t+1}}$.

Following Freeman (1987) and others, we assume that all storage activity is intermediated. Specifically, there is a composite asset, called "deposits", that are sold by banks. Banks operate in a perfectly competitive environment, taking the price of deposits and the gross real return on storage goods as given. There is no cost to creating these deposits. Let the gross real return on deposits between t and $t+1$ be represented by r_t.

Banks are subject to a standard currency reserve requirement in that they are required to hold money balances worth at least γ goods for each unit deposited with them.[7] With $x > \frac{p_t}{p_{t+1}}$, the reserve requirement will bind in equilibrium. Let m denote nominal money balances per young person. Then, $m_t = \gamma p_t d_t$ holds. The gross real return to deposits is a weighted average of the returns to storage and money, the weights being pinned down by the reserve requirement. Formally,

$$r_t = (1-\gamma)\, x + \gamma \frac{p_t}{p_{t+1}}. \tag{2}$$

The government has a fixed "purposeless" spending of g units (per young person) each period. The revenue needed to fund this expenditure comes from the revenues raised by the two wings of the government, the treasury and the central bank. The former collects lump-sum taxes from the young. The latter controls the (aggregate) nominal money stock, M, contributing to the government's revenue needs by creating money. Let ϕ denote the fraction of the government's spending that lump-sum taxes will cover (henceforth, referred to as the tax-responsibility parameter).

Let τ be the quantity of goods that each young person pays in the form of a lump-sum tax. The representative agent born at date $t \geq 1$ chooses non-negative combinations of c_1 and c_2 such that (1) is maximized subject to the following per-period budget constraints:

$$y - \tau = c_1 + d_t,$$

[7] Our formulation of the reserve requirement is standard and follows Freeman (1987) and Woodford (1990; appendix A.4). The government faces a fixed, time-invariant γ.

and

$$r_t d_t = c_2.$$

The first order conditions for an interior optimum are given by

$$c_2^* = r_t c_1^*. \tag{3}$$

The optimal quantity of deposits, d^*, is defined as

$$d_t^* \equiv d(r_t, y) = \arg\max_{d_t} \; [u(y - \tau - d_t) + v(r_t d_t)]. \tag{4}$$

The government budget constraint is represented (in per-young person terms) as

$$g = \tau + \frac{m_t - m_{t-1}}{p_t}. \tag{5}$$

Let $\tau = \phi g$ and $\left(\frac{m_t - m_{t-1}}{p_t}\right) = (1-\phi)\, g$. We do *not* restrict the value of ϕ to the $[0,1]$ interval. $\phi > 1$ is equivalent to a case in which the money stock shrinks; then, lump-sum taxes would have to cover both the spending *and* the loss in seigniorage revenue. The special case, $\phi = 1$, is one in which taxes fully back the level of government spending. In contrast, when $\phi = 0$, the government's spending is funded entirely through money creation. The government chooses ϕ to maximize a representative agent's lifetime welfare in a stationary setting.

Throughout the analysis, we assume that nominal money growth is dictated by the rule, $M_t = \theta M_{t-1}$, where θ is the gross rate of money growth. It is then apparent that money growth plays a role in government financing whenever $\phi < 1$.[8] In equilibrium, the government budget constraint (5) may be rewritten as:

$$g = \tau_t + \frac{m_t}{p_t}\left(1 - \frac{1}{\theta}\right). \tag{6}$$

Here, θ is endogenous in the sense that changes in ϕ will prompt the central bank to adjust θ in order to satisfy (6) for all $t \geq 1$.

In steady states, the money market clearing condition implies that $\frac{p_t}{p_{t-1}} = \frac{1}{\theta}$. Thus, using (2), it follows that $r = (1-\gamma)\, x + \frac{\gamma}{\theta}$. In addition, the central bank's steady state seigniorage revenue responsibility is defined by $\left(\frac{m_t - m_{t-1}}{p_t}\right) = \gamma d^* \left(1 - \frac{1}{\theta}\right) = (1-\phi)\, g$ from where we can compute the money growth rate as

$$\frac{1}{\theta} = 1 - \frac{(1-\phi)\, g}{\gamma d^*}$$

and the return to deposits as,

[8] The analysis below is conducted with ϕ as the variable of interest. Equivalently, one could choose θ, taking τ as the residual that ensures the government budget constraint is satisfied. Our task ahead is to show that a benevolent government would choose a $\phi \in (0,1)$ or equivalently, a $\theta > 1$.

$$r = (1-\gamma)\,x + \gamma\left[1 - \frac{(1-\phi)\,g}{\gamma d^*}\right]. \tag{7}$$

The steady state level of welfare for all future generations is obtained by substituting the equilibrium decision rules into the agent's utility function. Formally,

$$W(\phi) = u\left[y - d^*(r,y) - \phi g\right]\} + v\left[r \cdot d^*(r,y)\right] \tag{8}$$

From (8), the reader can see the different channels through which changes in the tax responsibility parameter affect lifetime welfare. In addition to the direct impact, there are two channels reflecting the additional effects that changes in ϕ have on welfare.

The direct effect is captured by the last term inside $u(.)$. Here, an increase in the tax-responsibility parameter, for example, results in a decline in the agent's first-period disposable income. If things stopped here, welfare would be decreasing in the tax responsibility parameter. However, the welfare analysis is complicated by the fact that both the equilibrium level of deposits and the equilibrium gross real return to deposits are affected by changes in the tax-responsibility parameter.

To illustrate the real-return effect, suppose, for now, that deposits are invariant to changes in ϕ. Equation (7) indicates that an increase in tax-finance responsibility results in a higher gross real return to deposits, holding the level of deposits constant. With non-distortionary taxes bearing a larger share of the financing, money creation supports a smaller portion. With constant deposits, the economy is on the "good" side of the seigniorage Laffer curve; hence, the government budget constraint is satisfied at a lower money growth rate. With a decline in the money growth rate, the gross real return to deposits increases. It follows then second-period consumption (or equivalently, old-age utility) would increase.

Of course, the equilibrium level of deposits will vary with ϕ and this effect further muddles our efforts to assign a direction of change to lifetime welfare. Formally, the total derivative of lifetime utility with respect to the tax-responsibility parameter is[9]

$$W'(\phi) = -u'(c_1^*)[d_\phi^* + d_r^* r_\phi - g] + v'(c_2^*)[r_\phi d^* + r d_r^* r_\phi + r d_\phi^*].$$

Using (3), we can further reduce this expression to

$$W'(\phi) = v'(c_2^*)[d^* \cdot r_\phi - g] \tag{9}$$

Define ϕ^* as a solution to $W'(\phi^*) = 0$. Our central question may then be posed as: is $\phi^* \in (0,1)$? Can it be that a benevolent government would choose to use some seigniorage even when a non-distortionary revenue source is available? Among other things, the answer will depend on the size of g. Moreover, since higher lump-sum taxes imply less reliance on seigniorage, the money growth rate should fall with an increase in the tax-responsibility parameter; as such, the return to deposits should go up (i.e., it seems likely that $r_\phi > 0$ holds). One approach that will yield more definitive answers would be to adopt specific functional forms.

[9] In what follows, we introduce the notation $X_y \equiv \frac{\partial X}{\partial y}$.

2.1 An example with logarithmic utility

Let preferences be represented by

$$U(c_1, c_2) = \ln(c_1) + \ln(c_2). \tag{10}$$

For this specification, the decision rule for deposits is

$$d^* = \frac{y - \tau}{2}.$$

With $\tau = \phi g$, this reduces to

$$d^* = \frac{y - \phi g}{2}. \tag{11}$$

For an interior solution for deposits, feasibility requires $y > g$. Throughout our analysis, we will assume the feasibility condition is satisfied.

Also,

$$r = (1-\gamma)x + \gamma\left[1 - \frac{(1-\phi)g}{\gamma d^*}\right] = (1-\gamma)x + \gamma\left[1 - \frac{(1-\phi)g}{\gamma\left(\frac{y-\phi g}{2}\right)}\right].$$

Substitute (11) into the agent's budget constraints when young, yielding

$$c_1^* = y - d^* - \tau = \frac{y - \phi g}{2}.$$

To derive the equilibrium decision rule for old-age consumption, first substitute for r to obtain

$$c_2^* = rd^* = \left\{(1-\gamma)x + \gamma\left[1 - \frac{(1-\phi)g}{\gamma d^*}\right]\right\} d = (1-\gamma)xd^* + \gamma d^* - (1-\phi)g$$

then substitute for d^*, yielding

$$c_2^* = (1-\gamma)x\left(\frac{y-\phi g}{2}\right) + \gamma\left(\frac{y-\phi g}{2}\right) - (1-\phi)g.$$

Thus, steady state welfare is given by

$$W(\phi) = \ln\left[\frac{y-\phi g}{2}\right] + \ln\left[(1-\gamma)x\left(\frac{y-\phi g}{2}\right) + \gamma\left(\frac{y-\phi g}{2}\right) - (1-\phi)g\right]. \tag{12}$$

The following proposition computes the exact expression for the welfare-maximizing ϕ.

Proposition 1 *Suppose*

$$\gamma > \hat{\gamma} = 1 + \frac{g}{y(1-x)}.$$

Then, a)

$$\phi^* = \frac{g - y\left[x + \gamma(1-x) - 1\right]}{g[2 - x + \gamma(x-1)]}, \tag{13}$$

and b)

$$\phi^* \in (0, 1).$$

Note that $\hat{\gamma}$ can be negative in which case the condition $\gamma > \hat{\gamma}$ would have no bite. If $\gamma > \hat{\gamma}$ holds (i.e., there is a minimum size reserve requirement in place[10]), then Proposition 1 states that a benevolent government would always choose to make *some* use of the inflation tax even when non-distortionary taxes on the young were available.

2.1.1 Remarks

Recall that the government chooses ϕ so as to maximize $W(\phi)$. The first order conditions for an *interior* solution (c_1^*, c_2^*) set

$$\frac{c_2^*}{c_1^*} = 2 - \gamma - (1-\gamma)x.$$

From (3), it follows then that

$$r = 2 - \gamma - (1-\gamma)x.$$

Notice that, as long as $\gamma \in (0,1)$ holds, $r < 1 < x$ holds at an optimum. Thus, for cases depicted in Proposition 1, $c_1^* > c_2^*$ holds. In contrast, with $\gamma = 0$, $c_2^* = xc_1^*$ $(x > 1)$ would hold, or that $c_1^* < c_2^*$ would obtain. Not surprisingly, the reserve requirement distorts the agent's pattern of lifetime consumption. Each unit of the good deposited with the bank earns only r units. Had it not been for the reserve requirement, each unit invested would have earned x. Therefore, ceteris paribus, for every unit saved when young, the agent is losing $(x - r)$ units of income when *old* because of the binding reserve requirement. It is in this sense then that the old agents are effectively getting taxed, if the inflation tax gets used. Why might this be a move in the "right" direction?

It may be helpful to see why seigniorage is a move in the welfare-maximizing direction by considering a non-monetary economy. Suppose a benevolent government wants to raise g by either taxing the old or the young or both in some combination. Here, the agent's problem would be

$$\max \ln c_1 + \ln c_2$$

s.t

$$c_1 = y - \tau_1 - s$$
$$c_2 = Rs - \tau_2$$

where τ_1 and τ_2 are lump-sum taxes, s is savings, and R is the gross real return to savings. Suppose $\tau_1 = \varphi g$ and $\tau_2 = (1-\varphi)g$, i.e., the government is financing g by a combination of lump-sum taxes on the young and the old. First-order conditions reveal

$$\frac{c_2^*}{c_1^*} = R$$

[10]To get a sense of what this minimum has to be, consider some plausible values of g, y, and x taken from long-run averaged US data. If we set $g/y = 0.2$ (the postwar average) and $x = 1.02$, then $\hat{\gamma} < 0$. Clearly as x rises, $\hat{\gamma}$ increases. So one's interpretation of the real return in this model will have a lot to do with whether the condition will be satisfied in real-world economies. For the parameters listed above, suppose in addition that $\gamma = 0.1$. Then, for these values of the parameters, it may be checked that $\phi^* = 0.93$ implying that about 7% of government spending would be raised via seigniorage in this model economy.

from where it follows that

$$s^* = \frac{y - \tau_1}{2} + \frac{\tau_2}{2R}$$

Then, steady state welfare is given by

$$W(\phi) = \ln\left[\left(\frac{y - \varphi g}{2} - \frac{(1-\varphi)g}{2R}\right)\right] + \ln\left[R\left\{\left(\frac{y - \varphi g}{2} + \frac{(1-\varphi)g}{2R}\right)\right\} - (1-\varphi)g\right]$$

It is easy to check that welfare is maximized at $\varphi = 0$ when $R > 1$. With $R > 1$, the young can store one unit of the good and receive R goods in old age thereby more than offsetting the tax when old. Hence, taxing only the old is most preferred.

There is an analogy in our monetary economy. First note that if the money stock is constant, the return to deposits — $r = (1-\gamma)\,x + \gamma$ — (analogous to R above) is greater than one. If lump-sum taxes on the old were possible, then using them would be best, as we have just seen. In their absence, however, the next best thing would be to try and mimic a tax on the old. Seigniorage does that. For every unit saved when young under the seigniorage-use case, the agent loses $(x - r)$ units of income when *old* because of the binding reserve requirement.[11] It is in this sense that it is ultimately the old agents who are effectively getting taxed, if the inflation tax is employed. This makes use of seigniorage a move in the "right" direction. Our results indicate that the mere fact that use of seigniorage indirectly taxes the old is not sufficient justification for its utilization. Indeed, as we have shown in Proposition 1, the reserve requirement has to be large enough to merit the use of seigniorage. If the reserve requirement is too small, the gap between returns to storage and returns to deposits, $(x - r)$, is too small, implying that the old cannot be "taxed" *enough*, and definitely not enough to satisfy the government's financing needs. In other words, the seigniorage tax base must be large enough for it to be desirable for the government to use the inflation tax.

2.2 Connection with the Friedman rule

How does our result match up with those obtained in the optimal inflation tax (i.e., Friedman rule) literature? Would a government ensure a higher stationary utility for its citizens if it simply implemented the Friedman rule? Below, we show that the answer is not an unqualified yes.

Thus far, we have focussed on a case in which money is dominated in rate of return and is held solely to satisfy a legal restriction. The legal restriction creates a wedge between the return to storage and the return to saving. This distorts the incentives of agents to save. Agents would like to get the high return from storage but cannot on their own unless their portfolios also include money balances. The efficient allocation of resources from the point of view of the agents thus clearly requires that the returns to storage and money be equalized.[12] This is achieved if $\frac{1}{\theta} = x > 1$; that is, the Friedman rule. Therefore, the money supply must contract at a rate equal to the inverse of the real return to storage. This is the Pareto efficient monetary policy.

[11] Seigniorage therefore acts as a proportional tax on the incomes of the old agents. In this case then, a move to a proportional (hence, distortionary) tax on the old dominates a non-distortionary tax on the young.

[12] See Woodford (1990; appendix A.4) for a nice discussion.

Consider a case in which the government has to finance a positive spending. Suppose it follows the Friedman rule and raises all its revenue from lump-sum taxes. Would such a policy necessarily maximize stationary utility? Intuitively, it is clear that following the Friedman rule would undo the distortion to saving caused by the legal restriction (though not remove the restriction itself) and reinstate the right incentives to save. In this case, however, the government's financing needs would necessitate that lump-sum taxes on the young pay for the entire government spending *and* the lost seigniorage. In other words, the young would face a tax *higher* than what the government would have chosen for them if it made some use of seigniorage. As such, it is quite possible that following the Friedman rule may not produce the maximum welfare.

To formally explore this, consider a case where the government follows the Friedman rule and sets $\theta = \frac{1}{x}$. In this case, the reserve requirement is there but does not bind any longer since money earns the same as storage. Government budget balance requires

$$\tau = g + m(1 - \frac{1}{\theta}).$$

If money demand is positive, then a contracting money supply ($\theta < 1$) implies $\tau > g$: the government loses seigniorage and so, taxes on the young back spending as well as help retire money.[13]

It is easy to check that deposits under the Friedman rule policy (implemented via a lump-sum tax on the young), denoted d^{Fy}, is given by $d^{Fy} = \left(\frac{y-\tau}{2}\right)$. Since $m = \gamma d^{Fy}$ by virtue of the reserve requirement, it follows that $\tau = g - \gamma d^{Fy}(1-x)$ using which yields

$$d^{Fy} = \frac{y-g}{2-\gamma(1-x)}.$$

Thus, under the Friedman rule policy, stationary welfare is given by

$$W^{Fy} \equiv \ln\left(y - \tau - d^{Fy}\right) + \ln\left(x \cdot d^{Fy}\right)$$

which when simplified yields

$$W^{Fy} = \ln\left[\left(\frac{y-g}{2}\right)\left\{1 + \frac{\gamma(1-x)}{2-\gamma(1-x)}\right\}\right] + \ln\left[\frac{x(y-g)}{2-\gamma(1-x)}\right].$$

Can $W(\phi^*)$ with $\phi^* \in (0,1)$, as defined in (??), be greater than W^{Fy}? Below, we illustrate with an example that the answer can be yes.

Example 1 *Let the parameters of the economy be as follows:* $y = 1$, $g = 0.08$, $\gamma = 0.173$, *and* $x = 1.07$. *Then,* $\phi^* = 0.29$ *and* $W(\phi^*) = -1.4935$, *while* $W^{Fy} = -1.4974$. *That is, stationary welfare is higher with partial use of seigniorage and lump-sum taxes on the young than under the Friedman rule implemented via a lump-sum tax on the young. Similar examples are easy to generate.*

[13]There is more than one way to implement the Friedman rule to get a Pareto optimal equilibrium. For example, one could impose a lump-sum tax on the old to retire the currency, and still tax the young to finance g. There is a unique Pareto optimum here only because we have ruled out taxes on the old.

In the presence of distortions, following the Friedman rule may achieve a 'first-best' allocation, but among the allocations that are attainable, the best one need not be the one attained via the Friedman rule. Another way to restate this insight would be: welfare-maximizing monetary policy may deviate significantly from the Friedman rule in the presence of unremovable legal restrictions. The next section deals with a caveat to this result.

3 The model with a lump-sum tax on the old

To recap, our results thus far indicate that it may be welfare-maximizing to use seigniorage *if* the alternative revenue source is a lump-sum tax on the young. This in itself is novel in that it challenges the textbook Friedman rule wisdom that usage of the inflation tax is "suboptimal" when non-distortionary taxes are available. However, if we are to place our result somewhere amidst the larger issue of optimality of seigniorage, we would, at the very least, have to address the question of exogeneity of the alternative form of taxation, and its impact on our result. Is our result immune to the timing of the alternative tax or is it that the demographic structure of the overlapping generations model imposes strictures on the *set* of alternative tax sources that would leave our result unaffected? The latter is a very general question largely outside the scope of the current paper.

In this section we attempt a limited and more modest answer to this last question. To that end, we study the welfare impacts of the inflation tax alongside a lump-sum tax on the old. To foreshadow, we show the following: it is never welfare-maximizing to use the inflation tax if the alternative revenue source is a lump-sum tax on the old. We use a model identical in all aspects, except one, to the model economy we studied above; the exception is that the treasury can impose a lump-sum tax on the old instead of the young. To save on space, we use the same notation as above, and only sketch some of the details below.

We assume that the agent maximizes a program with additively-separable logarithmic preferences, facing a lump-sum tax of τ when old. Hence, max $\ln c_1 + \ln c_2$ s.t $c_1 = y - d$,and $c_2 = rd - \tau$. In equilibrium, deposits are given by $d^* = \frac{y}{2} + \frac{\tau}{2r}$. The returns to deposits are $r = (1-\gamma)x + \gamma\left[1 - \frac{(1-\phi)g}{\gamma d^*}\right]$. The next lemma computes the exact expression for r and studies its comparative static properties.

Lemma 2 *Define* $q \equiv [(1-\gamma)x + \gamma]$ *and* $X \equiv \left\{yq + 2g\left(\frac{\phi}{2} - 1\right)\right\}$. *Then,*
a)

$$r = \frac{\left(\frac{X}{y}\right) \pm \sqrt{\left(\frac{X}{y}\right)^2 + \frac{4\phi g q}{y}}}{2}$$

b)

$$r_\phi > 0$$

As before, the government chooses ϕ by maximizing stationary welfare. Recall that in equilibrium, $c_1^* = y - d^*$, and $c_2^* = rd^* - \phi g$. Then, steady state welfare is given by $\ln c_1^* + \ln c_2^*$ which after simplification yields,

$$W(\phi) \equiv \ln\left[\frac{y}{2} - \frac{\phi g}{2r}\right] + \ln\left[\frac{ry}{2} - \frac{\phi g}{2}\right].$$

Then, straightforward differentiation reveals

$$W'(\phi) = \frac{1}{c_1^*}\left(\frac{-g}{2}\right)\left\{\frac{1}{r} - \frac{\phi}{r^2}r_\phi\right\} + \frac{1}{c_2^*}\left[\frac{y}{2}r_\phi - \frac{g}{2}\right]. \tag{14}$$

Using (3), it is easy to check that (14) reduces to

$$2c_2^* W'(\phi) = \left[\frac{\phi g}{r} + y\right] r_\phi - 2g.$$

Proposition 3 *$W'(\phi) > 0$ holds. In other words, if lump-sum taxes on the old are available, then the welfare-maximizing monetary policy is to hold the money stock constant.*

The intuition behind 3 is straightforward. Imagine that the government needs to obtain one unit of the good to balance its budget. It can raise the revenue from seigniorage or from lump-sum taxes on the old or some combination. In either case, as we have discussed before, it is the old that pay the tax. There is no difference in the wealth effect of the two policies. However, seigniorage carries with it an additional distortionary effect. Accordingly, a benevolent government prefers to use the lump-sum tax. As we have discussed in Section 2.1.1, shifting the taxes to the old instead of the young is welfare improving if the gross return to savings exceeds unity (which it does here when the money stock is held fixed).

3.1 Connection with the Friedman rule

Suppose the government implements the Friedman rule (shrinks the money stock so as to equalize the returns on money and storage) via a lump-sum tax on the old. Will such a policy produce higher stationary welfare than the combination policy of using lump-sum taxes on the young and seigniorage? In this case, it is easy to verify that deposits under this Friedman rule implementation, denoted d^{Fo}, is given by

$$d^{Fo} = \frac{y}{2} + \frac{\tau}{2x}$$

and hence, stationary welfare is given by

$$W^{Fo} \equiv \ln\left[y - d^{Fo}\right] + \ln\left[x \cdot d^{Fo} - \tau\right]$$

where $\tau = g - \gamma d^{Fo}(1 - x)$. The following example shows that stationary welfare can be lower with partial use of seigniorage and lump-sum taxes on the young than under the Friedman rule implemented via lump-sum taxes on the old.

Example 2 *Let the parameters of the economy be as follows: $y = 1$, $g = 0.08$, $\gamma = 0.173$, and $x = 1.07$. Then, $\phi^* = 0.29$ and $W(\phi^* = 0.29) = -1.49342$, while $W^{Fo} = -1.48604$. That is, stationary welfare is lower with partial use of seigniorage and lump-sum taxes on the young than under the Friedman rule implemented via lump-sum taxes on the old. Also notice that $W^{Fo} > W^{Fy}$ implying that implementing the Friedman rule by a lump-sum tax on the old produces higher welfare implementing the same via a lump-sum tax on the young. Similar examples are easy to generate.*

Intuitively, it is clear that following the Friedman rule removes the distortion to saving. Also, a lump-sum tax on the old increases saving. Suppose the government has to raise one unit of the good as revenue. Since the return on saving (x) is greater than one, agents are better off if they are taxed one unit of the good while old and are allowed to get $x > 1$ in return. As we have seen before, the same would not be true if the tax was on the young. To summarize, our results indicate that the question of whether the Friedman rule coincides with the welfare-maximizing monetary policy or not can only be answered once it is known *how* the Friedman rule is implemented.

4 Concluding remarks

In this paper, we examine welfare-maximizing monetary policy in a second-best world with reserve requirements. We derive our results in a simple general equilibrium model with finitely lived agents. In the model, there is a benevolent government that must finance a fixed level of spending through a package of lump-sum taxes and seigniorage. Interestingly, the inflation tax may be part of a welfare-maximizing policy package if the only alternative is taxing young agents. A tax on the young means less disposable income when young, which could potentially lower lifetime utility more than what an agent would obtain from a higher return when old. We also demonstrate that the inflation tax will not be used if the government has access to lump-sum taxes for the old. We go on to show that the welfare-maximizing monetary policy may deviate significantly from the Friedman rule (equalizing real returns), depending on the nature of the fiscal policy.

There are two features of this model economy that are crucial to obtaining our result about the inflation tax being part of a welfare-maximizing tool kit. First, in the economy, there is an unremovable reserve requirement which the government takes as given. If the government were instead given the option to choose the utility-maximizing reserve requirement in the presence of lump-sum taxes, then it seems clear that the optimal reserve requirement would be zero; the government would not use the inflation tax at all. For our purposes, it is essential that there is some reason – other than public finance – why the reserve requirement is present. Governments, for instance, routinely legislate reserve requirements to impose discipline on financial institutions. Second, our result is also predicated on the requirement that the government not have access to taxes on the old. We think that taxes only on the young are not so far fetched either, especially in light of the prevalence of old-age pensions around the world.

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A Proof of Proposition 1

We differentiate (12) with respect to ϕ,set the resulting expression to zero and obtain,

$$\frac{c_2^*}{c_1^*} = 2 - \gamma - (1-\gamma)x$$

Using (3), we can get,

$$r = (1-\gamma)x + \gamma\left[1 - \frac{(1-\phi)g}{\gamma\left(\frac{y-\phi g}{2}\right)}\right] = 2 - \gamma - (1-\gamma)x$$

which after simplification yields

$$2 - \gamma - (1-\gamma)x = (1-\gamma)x + \gamma - \frac{2(1-\phi)g}{y-\phi g}.$$

From here, one can solve for the optimal value, ϕ^* :

$$\phi^* = \frac{g - y[x + \gamma(1-x) - 1]}{g[2 - x + \gamma(x-1)]} \tag{A.1}$$

For a strictly positive solution, we need that

$$g > y[x + \gamma(1-x) - 1]$$

which is true when $\gamma > \hat{\gamma}$. To see if $\phi^* < 1$, suppose instead that $\phi^* \geq 1$. Then, (A.1) implies that

$$g - y[x + \gamma(1-x) - 1] \geq g[2 - x + \gamma(x-1)]$$

which upon simplification implies $x \leq 1$ which is a contradiction. ■

B Proof of Lemma 2

a) Use the expression for optimal deposits to get

$$r = (1-\gamma)x + \gamma - \frac{(1-\phi)\,g}{\frac{y}{2} + \frac{\phi g}{2r}}$$

which after substantial simplification yields

$$\frac{yr}{2} - \frac{\phi g}{2r}\left[(1-\gamma)x + \gamma\right] = \frac{y}{2}\left[(1-\gamma)x + \gamma\right] + g\left(\frac{\phi}{2} - 1\right)$$

Straightforward rearrangement yields

$$yr - \frac{\phi g}{r}q = X$$

or, the quadratic,

$$r^2 - \left(\frac{X}{y}\right) r - \frac{\phi g q}{y} = 0 \tag{B.1}$$

The result follows.

b) (B.1) implies

$$yr^2 - r\left[yq + g\phi - 2g\right] - \phi g q = 0$$

Then, straightforward differentiation yields

$$\frac{\partial r}{\partial \phi} = \frac{g\left(r+q\right)}{2yr - yq - g\phi + 2g} = \frac{g\left(r+q\right)}{2yr - X}$$

Also (B.1) implies that $yr - \frac{\phi g q}{r} = X$, and so we have

$$\frac{\partial r}{\partial \phi} = \frac{g\left(r+q\right)}{2yr - yr + \frac{\phi g q}{r}} = \frac{g\left(r+q\right)}{yr + \frac{\phi g q}{r}} > 0.$$

■

C Proof of Proposition 3

We have to show that

$$2c_2^* W'(\phi) = \left[\frac{\phi g}{r} + y\right] \frac{\partial r}{\partial \phi} - 2g > 0$$

To see this, note that

$$\left[\frac{\phi g}{r} + y\right] \frac{\partial r}{\partial \phi} = \left(\frac{\phi g}{r} + y\right) \frac{g\left(r+q\right)}{yr + \frac{\phi g q}{r}} = \frac{g\left(r+q\right)\left(\phi g + ry\right)}{yr^2 + \phi g q}$$

Then,

$$2c_2^* W'(\phi) = \left[\frac{\phi g}{r} + y\right] \frac{\partial r}{\partial \phi} - 2g = g\left[\frac{\left(r+q\right)\left(\phi g + ry\right)}{yr^2 + \phi g q} - 2\right]$$

which simplfies to

$$\frac{g}{yr^2 + \phi g q}\left[\left(q - r\right)\left(ry - \phi g\right)\right]$$

Recall $c_1^* = \frac{1}{2r}\left(ry - \phi g\right) > 0$. Also, $q > r$ always holds. Then, the proof is done.■

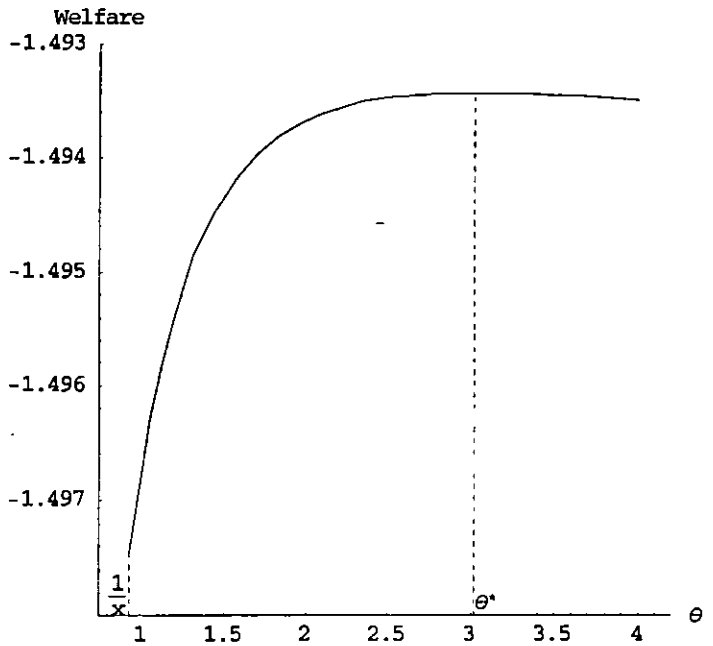

Figure 1: Dependence of welfare on the money growth rate, θ, when only lump-sum taxes on the young are allowed

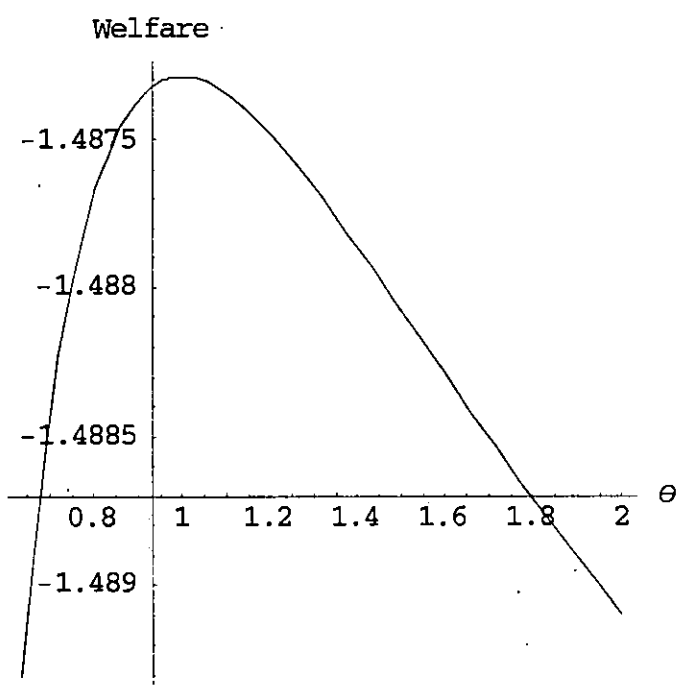

Figure 2: Dependence of welfare on the money growth rate, θ, when only lump-sum taxes on the old are allowed