RELATIVE RELUCTANCES IN THICK ALUMINUM SAMPLES

William F. Schmidt
Professor
Department of Mechanical Engineering
University of Arkansas
Fayetteville, AR 72701

Otto H. Zinke
Professor Emeritus of Physics
University of Arkansas
Fayetteville, AR 72701

INTRODUCTION

The effects of sample thicknesses, frequencies, and values of lift-off on real and imaginary reluctances for a modified ac magnetic bridge were found for aluminum. Aluminum test samples with thicknesses varying between 0.406 and 6.35 millimeters were scanned at values of lift-off ranging from 0.1 to 0.38 millimeters and at frequencies between 200 and 5000 Hz.

Real reluctance for magnetic circuits is a concept in common usage. The concept of imaginary reluctance has been introduced for ac magnetic circuits by Zinke and Schmidt [1] in order to create a formalism for ac magnetic circuits very similar to the formalism for impedances in ac electrical circuits. The theory shows that the roles of real and imaginary reluctances are reversed from the roles of real and imaginary impedances in that real and imaginary impedance terms are respectively associated with energy dissipation and storage while the reverse is true with reluctances.

In this work, the term "relative" reluctance is applied to measurements made with modified ac magnetic bridges [2]. "Relative" reluctances are defined by Zinke and Schmidt [3] and are associated with the changes in the real and imaginary reluctances required to null the bridge, first with no sample and then with a sample present. Relative reluctances are not directly related to the real and imaginary reluctances of the sample. However, the specific reluctances of the samples can be extracted from relative reluctances when account is taken of the fact that lift-off produces both a real reluctance in series and a real reluctance in parallel with the real and imaginary reluctance of the sample.

Relative reluctances are used because they can be easily calculated and compared for various samples, and comparison is the basis of most procedures used in NDE. From an examination of relative reluctances presented here, the sensitivity of the modified bridge to hidden corrosion loss can be estimated where the limit of sensitivity results from unintended variations of lift-off (which are, in turn, estimated). In fact, over a wide range of frequencies, the relative imaginary reluctances and total reluctances follow an exponential

Review of Progress in Quantitative Nondestructive Evaluation, Vol. 14
Edited by D.O. Thompson and D.E. Chimenti, Plenum Press, New York, 1995

curve which is a function of a skin depth. The skin depth which is calculated is very close to the value usually accepted for aluminum. For a more restrictive range of frequencies, the real reluctance seems to follow a similar curve.

THE EXPERIMENT

A set of samples of various thicknesses (t) were carefully cut from 2.54-centimeter diameter bar of 2024-T3 aluminum. The thickness of the samples were 0.406, 0.762, 1.08, 1.499, 2.286, 3.175, 4.699, and 6.35 millimeters. The cuts were made slowly with coolant to minimize the possibility of introducing residual stress. After cutting they were carefully ground to final thickness using a 7-inch diameter silicon carbide wheel rotating at 3600 rpm. The samples were then scanned using a modified ac magnetic bridge. This particular bridge has been described in Ref. 3 and elsewhere [4]. The insert in the bridge used in the current work was constructed of copper with a thickness of 1.07 millimeters.

Each test sample was placed so that the plane of the sample was parallel to the plane of the face of the modified bridge. Pieces of plastic of varying thicknesses were placed between the sample and the bridge face to establish lift-off (LO). Lift-off values were 0.102, 0.191, 0.254, 0.318, and 0.381 millimeters. For each lift-off value data were taken at frequencies (f) of 200, 350, 500, 750, 1000, 2000 and 5000 Hz. The ac current to the input of the bridge was adjusted so that the bridge was driven at 12 amp-turns from a 50-ohm outlet of a Hewlett-Packard 651B Signal Generator. The output was fast-Fourier analyzed through the use of a Hewlett-Packard 3582A Signal Analyzer.

The procedure was very simple: The frequency of the signal generator was set at one of the values listed above. The input current was adjusted to 100.5 ± 0.2 milliamps (rms). The output phases and amplitudes of the bridge were manipulated through resistances and capacitances coupled to the arms of the bridge through coils known as "null coils." With no sample in an arm of the bridge, the resistances and capacitances on the null coils of the bridge were adjusted until the bridge output was less than 5 microvolts for the lower frequencies and 10 microvolts for the higher frequencies used. The bridge was then said to be nulled. The values of resistance R and capacitance C required to produce the null were then recorded. These values were labelled R_x, R_y, C_x, or C_y depending on whether the resistance and/or capacitance were on the x or the y arm of the bridge. The nulling procedure produces only one resistance value and one capacitance value. The bridge has four arms. Any two arms on the same side of the output leg (and connected to the same input leg) can be arbitrarily designated the x and y arms. A piece of plastic representing the particular lift-off value was placed on the face of the bridge which contained the modified gap for the x arm. There was no sample in the y arm. Initially, the 0.406-millimeter sample was placed on the plastic and centered over the x gap-face through the use of a jig. The bridge was now nulled with the sample in place. The values of R and C required for nulling were then recorded as one of the two possibilities R_{sx} or R_{sy} and one of the two possibilities C_{sx} or C_{sy} where the subscript s indicates sample and x or y designates the respective bridge arm. The 0.406-millimeter sample was then removed and replaced with the next thickest sample and the measurements repeated until the R and C values for all samples at that frequency were recorded. The empty bridge was renulled after all samples had been measured to determine if the values of R_x or R_y or C_x or C_y had changed. These values rarely changed by more than 1 percent on the capacitance and 0.1 percent on the resistance. Nevertheless, beginning and final values were averaged to be used in the formulae which follow. The frequency was then changed and the process repeated.

The relative real reluctance $\Re$ and the relative imaginary reluctance I were calculated by the following formulae (see [4])

$$\Re = N^2\omega^2 \left\{(C_{sy} - C_y) - (C_{sx} - C_x)\right\} \tag{1}$$

and

$$\mathrm{I} = N^2\omega \left\{\left(\frac{1}{R_{sy}} - \frac{1}{R_y}\right) - \left(\frac{1}{R_{sx}} - \frac{1}{R_x}\right)\right\} . \tag{2}$$

These calculations were carried out for each sample thickness at each frequency and lift-off.

The total relative reluctance can be calculated as follows:

$$\Re_t = (\Re^2 + \mathrm{I}^2)^{1/2} . \tag{3}$$

Reproducibility was tested with two samples at 500, 1000, and 2000 Hz with a lift-off of 0.191 millimeters. The samples were 1.08 and 4.699 millimeters thick. The samples were removed from the bridge and replaced alternately. They were positioned with a jig in the same manner as the samples in the rest of these tests. The error caused by repositioning was typically between 0.5 and 4 percent within the limits of the tests. However, ambient temperature excursions were apparent in those tests where the results were above 2 percent. The variations in the readings producing null with a single sample in place, was about one-fourth of the positioning error. Positioning error was about the same throughout the frequency range and independent of sample thickness. At 500 Hz, the positioning error in the relative total reluctance was between 0.003 and 0.005 mega amps/weber. At 1000 Hz, these limits were 0.009 and 0.01 mega amps/weber.

RESULTS

Imaginary Reluctance

The relative imaginary reluctances calculated from eq. (2) for the frequency range examined are shown as symbols in figure 1. (Hereafter the term "relative" will be dropped). The displayed results are for a lift-off of 0.25 millimeters and are very typical for all other

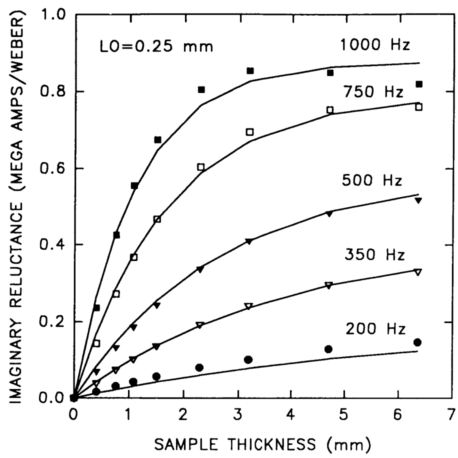

Figure 1. Relative imaginary reluctance versus sample thickness for varying frequencies. Continuous curves are eq. 4.

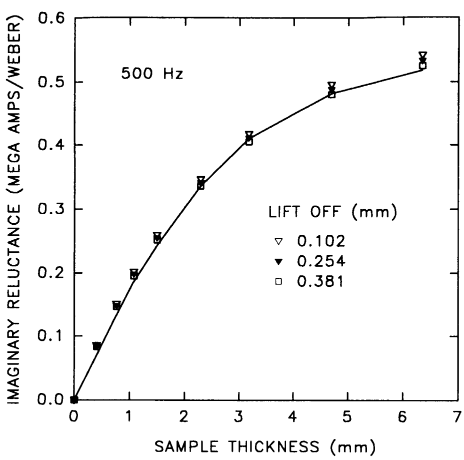

Figure 2. Relative imaginary reluctance versus sample thickness for varying lift-off. The continuous curve is eq. 4.

values of lift-off. The effect of lift-off on the imaginary reluctance is very small and is shown for the 500 Hz data in figure 2. The variations seen in the imaginary reluctance are similar for all other frequencies in the range from 200 Hz to 2000 Hz. Notice that a factor of almost 4 in the lift-off has very little effect on the imaginary reluctance. This is of importance when using the bridge on real systems which may have rough or uneven surfaces.

The continuous curves in figures 1 and 2 were obtained through the following procedure: Each of the curves represented by constant frequency symbols in figure 1 was matched through a least-squares technique to the following formula:

$$I = A\,(1 - exp\,(-\frac{t}{\zeta})) \qquad (4)$$

where the amplitudes A and the absorption coefficients ζ were parameters selected to fit the curves. Pairs of values of A and ζ each corresponding to a separate frequency were produced. The values of ζ did not measurably depend on lift-off. Between frequencies of 350 and 1000 Hz, ζ could be fitted linearly to the inverse of the square root of frequency using least squares through the following formula:

$$\zeta = \frac{\delta}{f^{0.5}} + \beta \ . \qquad (5)$$

The quantity δ then bears approximately the same relation to ζ as skin depth does to the electromagnetic absorption coefficient except for the intercept β. The fit of ζ can be seen in figure 3. The 200-Hz data could be fit to eq. 4 but the resulting ζ did not fall on the straight line seen in figure 3. Consequently, the 200-Hz data were deleted from this analysis. The least-squares technique yielded values for δ and β which were respectively 114±9 and -2.47.

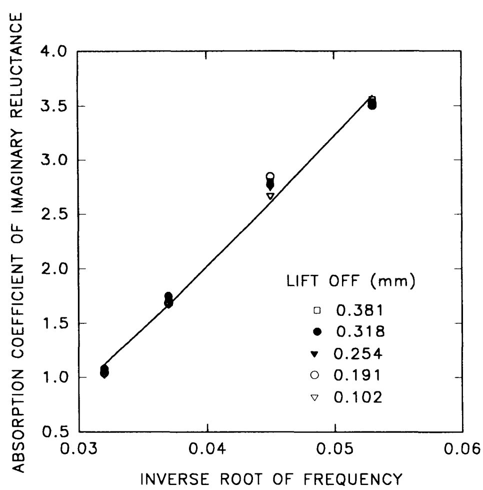

Figure 3. The absorption coefficient for imaginary reluctance versus the frequency. The continuous curve is eq. 5.

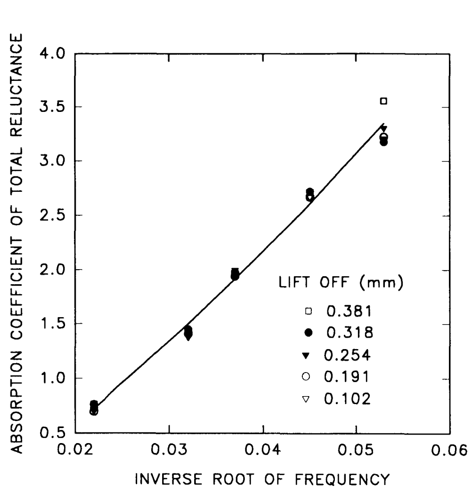

Figure 4. The absorption coefficient for the total reluctance versus the frequency. The continuous curve is eq. 7.

There seems to be no obvious physical interpretation for β. However,the reluctance here is the relative imaginary reluctance and not the imaginary reluctance of the sample.

The value of the skin depth δ seems reasonable. For pure aluminum, δ should have a value of 85. For an alloy such as is examined here, it is not surprising that the skin depth would be somewhat greater, i.e., that the depth of penetration of the electromagnetic field should be deeper within the sample.

The amplitude A was approximated by

$$A = [1-0.113\ (LO-0.102)]\ (-0.182\ + 0.00203\,f - 0.956\ \cdot 10^{6} f^{2})\,. \qquad (6)$$

From figure 2 and eq. 6, it can be seen that the equation has a very weak dependence on lift-off (LO).

The continuous curves of figures. 1, 2 and 3 were calculated through the use of eqs. 4 and 5 with the values of ζ, β, and A determined as outlined above. However, these results do not apply below 300 Hz or above 1000 Hz. The 200-Hz relative imaginary reluctances display an extremum which eq. 4 would not predict, and the 5000-Hz relative imaginary reluctances display an excursion in the negative direction.

Total Reluctance

The total relative reluctance, which corresponds to the amplitude of the impedance in electrical circuits, is defined by eq. 3. The same procedures were followed with the total

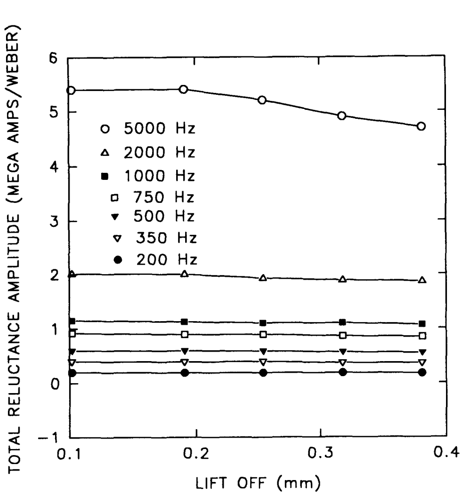

Figure 5. Relative total reluctance versus sample thickness for varying frequencies. The continuous curves are eq. 4.

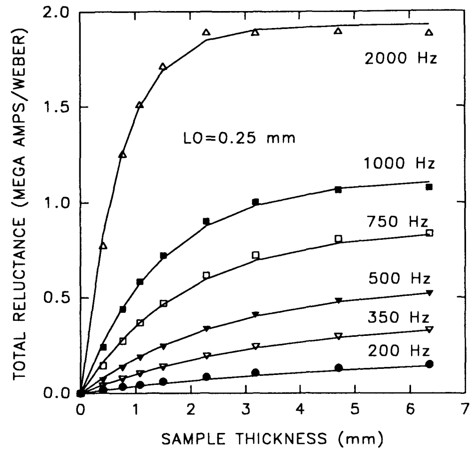

Figure 6. The amplitude of total reluctance versus lift-off for various frequencies. The continuous curves are given by eq. 8.

reluctance as were followed with the imaginary reluctance. The absorption coefficient was found to fit the values of ζ as follows:

$$\zeta = \frac{84}{f^{0.5}} - 1.167 \ . \tag{7}$$

The mean-square fit is shown in figure 4. Again, the reason for the intercept -1.167 is not obvious. The amplitude followed the equation

$$A = (1 - 2.182 x10^{-4} f(LO - 0.254)) \\ (-0.07994 + 0.001397 f - 0.1964 x10^{-6} f^2) \ . \tag{8}$$

Again, this equation is not unique, but it did yield good agreement between prediction and results for eq. 4.

The smooth curves appearing in figure 5 are calculated for the total reluctance through eq. 4 using eqs. 7 and 8. The agreement between the calculated total reluctance and the calculations of these values from the data as exhibited in figure 5 seems better than the agreement between the calculated imaginary reluctance and the data results as seen in figure 1. Certainly, the agreement was over a wider range of frequencies for total reluctance (200-2000 Hz) than for imaginary reluctance (350-1000) Hz.

The total reluctances calculated from measurements are relatively independent of lift-off. The only measureable lift-off variation is in the amplitude A of eq. 8. The independence of total reluctance from lift-off is typical of all frequencies but 5000 Hz as can be seen from the variation of the amplitude with frequency as shown in figure 6. The variations of the amplitudes, A, with lift-off becomes a factor only at the highest frequency, 5000 Hz, where a downward trend is seen with increasing lift-off.

ANALYSIS

As an example of an application of the above formalism to NDE, the total reluctance will be used to calculate the ability of the bridge to detect hidden corrosion as compared to the interfering effect of fluctuating lift-off. The assumption will be made that hidden corrosion amounts to loss of conductive material. The results will be compared to measurements made by Zinke and Schmidt [5] on a 1.194-millimeter thick aluminum plate simulating the skin of an aircraft. Corrosion was simulated by milling slots in the hidden side of the plates. Measurements were made at 1920 Hz. The above analysis allows an answer to the question as to whether 1920 was the best frequency to use. Although Zinke and Schmidt used two plates in juxtaposition with the milled portions in one in various positions with respect to the bridge to simulate corrosion in seams on the inside of the outside plate, on the outside of the inside plate, or on the inside of the inside plate, the example here will confine itself to the simple example, hidden corrosion in a single plate.

From eq. 4, the change in total reluctance ΔR_t with respect to a thickness loss Δt of the sample can be calculated as

$$\Delta R_t = \frac{A}{\zeta} \, exp \, (\frac{-t}{\zeta}) \, \Delta t \; . \tag{9}$$

Also, from eq. 4, the change in total reluctance δR_t with respect to changes in lift-off can be calculated as

$$\delta R_t = (1 - exp \, (\frac{-t}{\zeta})) \, \Delta A \tag{10}$$

where

$$\Delta A = (1 - 2.1819 \; x \, 10^{-4} f \; \Delta LO) \cdot \\ (-\,0.07994 \; = + \, 0.001397 \, f - 0.1964 \; x \, 10^{-6} f^2) \tag{11}$$

with ΔLO equal to the change in lift-off.

Assume that the detection of a change of 5 percent in the sample thickness is desired. Since the single-plate thickness was approximately 1.2 mm, Δt=0.06 mm. Further, suppose that lift-off can be regulated to 0.025 mm so that ΔLO=0.025 mm. In addition, suppose that the experiment is carried out with an average lift-off of 0.102 mm as it was in the work given in [4]. For a frequency of 1920 Hz, the formulae above together with the information developed in the previous section yield a ΔR_t = 0.032 mega amps/weber while the lift-off variation yields a δR_t = 0.016 mega amps/weber. Therefore, the 5 percent corrosion loss would produce a detection signal roughly twice the signal from fluctuating lift-off. If the experiment had been conducted at a lift-off of 0.25 mm, calculations show that these results would be essentially unchanged.

If Zinke and Schmidt [5] had used a frequency of 750 Hz the calculations produce a ΔR_t=0.015 and a δR_t=0.002, i.e., the corrosion loss would have produced an effect roughly 7 times that of the lift-off variation. These can be compared with something between 0.005 and 0.010 mega amps/weber which might arise with positioning error resulting from bridge placement on the small sample used here. However, it is unclear how positioning error would occur in scans on flat plates. Again, the data presented in [5] show variations which, on the basis of percents, is far less than the positioning error measured with the discrete samples here.

At 1920 Hz, reference 5 figure 5a shows detection of a simulated 7% corrosion loss producing a response considerably above the fluctuations in a scan over an unflawed section of the plate. This indicates that the variations in lift-off in that experiment were considerably less than 0.025 mm. In fact, the order of magnitude of the change in the reluctances along the normal scan, where the only variation expected would be from lift-off, are about a factor of 10 less than those predicted here. So, lift-off variations must have been of the order of 0.0025 millimeters.

CONCLUSIONS

Through analyses of the relative real, imaginary, and total reluctances generated by examining aluminum samples of different thicknesses at different values of lift-off, a formalism has been developed through which the operating conditions of the ac magnetic bridge can be optimized for certain types of NDE problems. The results indicate that the best operating range for a non-ferrous meterial is in the range of 500-1000 hz.

REFERENCES

1. O. H. Zinke and W. F. Schmidt, "Linear ac Magnetic Circuit Theory," *IEEE Transactions on Magnetics*, Vol. 29, pp. 2207-2212, 1993.
2. O. H. Zinke, U.S. Patent No. 4,901,017, 13 February 1990.
3. W. F. Schmidt and O. H. Zinke, "Application of the Quasitatic Model to the Modified Bridge," to be published in *Review of Progress in Quantitative Nondestructive Evaluation*, edited by D. O. Thompson and D. E. Cimenti, Vol. 13, 1994.
4. W. F. Schmidt and O. H. Zinke, and S. Nasrazadani, "Characterization of a Crack in an Aluminum Bar Using an ac Magnetic Bridge," accepted for an STP publication by ASTM to appear in 1994.
5. O. H. Zinke and W. F. Schmidt, "Modified ac Magnetic Bridge Scanning patterns of Samples Simulating Flaws in Aircraft Seams," *Review of Progress in Quantitative Nondestructive Evaluation,* Vol. 12, 2011-2019, Edited by D. O. Thompson and D. E. Cimenti, Plenum Press, New York, 1993.