A Tale of Two Supreme Court Rulings on Indian Affirmative Action*

Orhan Aygün† and Bertan Turhan‡

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Abstract

India has implemented a complex affirmative action via a reservation system since the 1950s. In the historic case Indra Shawney vs. Union of India (1992), the judgment formulated three principles for reservation policy to respect. In another historic case—Ashoka Kumar Thakur vs. Union of India (2008) (AKT)—the Supreme Court of India (SCI) mandated a 27 percent reservation to the Other Backward Classes (OBC). It recommended two directives without defining a procedural framework. The first one suggests implementing the OBC reservation as a soft reserve. The second one advises a maximum of 10 points difference between the cutoff scores of OBC and the open category to maintain the standard of excellence. We show that the two directives in AKT conflict with India's fundamental mandates from Indra Shawney (1992) in the sense that no choice rule can implement them concurrently. The incompatibility is resolved by either dropping the AKT directives on de-reservation or weakening the directive on eligibility. We propose path-independent assignment rules in each case.

Keywords: Market design, reserve systems, de-reservation, soft reserves, India.

JEL Codes: C78, D47, D63.

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†orhan.aygun@bogazici.edu.tr; University of Minnesota, Applied Economics Department, 231 Ruttan Hall, 1994 Buford Ave, St. Paul, MN 55108, USA; and Boğaziçi University, Department of Economics, Natuk Birkan Building, Bebek, Istanbul 34342, Turkey.

‡bertan@iastate.edu; Iowa State University, Department of Economics, Heady Hall, 518 Farm House Lane, Ames, IA 50011, USA.

1 Introduction

India has enforced the most intricate affirmative action program in the world to allocate government jobs and public university seats since the 1950s via a highly regulated reservation system that consists of *vertical* and *horizontal* reservations. Both types of reservations were formulated and mandated by the Supreme Court of India (SCI) in a historic judgment in *Indra Shawney vs. Union of India* (1992), henceforth Indra Shawney (1992).[1] After the verdict in Indra Shawney (1992), there have been significant refinements in implementing affirmative action policies through consecutive SCI decisions.

According to the reservation policy, 15, 7.5, 27, and 10 percent of available positions are set aside for *Scheduled Castes* (SC), *Scheduled Tribes* (ST), *Other Backward Classes* (OBC), and *Economically Weaker Sections*[2](EWS), respectively. These are called the *vertical reservations* in Indra Shawney (1992).[3] Individuals not encompassed by these vertical categories are classified as members of the **General Category** (GC). Failure to declare membership in SC, ST, OBC, or EWS relegates candidates to the GC by default. Importantly, the disclosure of vertical category affiliation is discretionary.

The residual 40.5 percent of available positions fall under the **open category**. All applicants, including those from the vertically reserved categories, are eligible for the open category positions.

The assignment of college seats and government job positions in India is fundamentally executed in a **merit-based** system. Applicants in the same vertical reserve category are assigned positions based on merit scores. Mandated in Indra Sawhney (1992), the **inter-se merit** principle requires that given two applicants who belong to the same category, if the one with a lower merit score is assigned a position, then the one with a higher merit score must also be assigned a position. Moreover, when applicants from vertical reserve categories obtain open category positions, they are not counted against reservations of their respective reserve categories. This requirement is called the **over and above** principle. It is crucial to note that open category positions must be allocated to the highest-scoring applicants so that applicants from vertical reserve categories—who cannot obtain a position in the absence of the reservation policy—may obtain a position. Moreover, in each vertical category, all positions must be allocated as long as there are eligible applicants. The last one is a standard **non-wastefulness** property subject to **eligibility.**

The following simple choice procedure—the *over-and-above choice rule*—has been used in India for decades.[4]

[1]The case is available at https://indiankanoon.org/doc/1363234/.

[2]In 2019, the Indian legislative body instituted a 10% vertical reservation for a subset of the General Category, specifically the Economically Weaker Section, defined by an annual income threshold below Rs. 8 lakhs.

[3]There are also *horizontal reservations* that are implemented in each vertical category for other groups, such as women and people with disabilities.Sönmez and Yenmez (2022) study the interactions between the vertical and horizontal reservations and relate the Indian judiciary to matching theory.

[4]See Sönmez and Yenmez (2022) and Aygün and Turhan (2024b) for details.

> *First, applicants are selected for open category positions one at a time following the merit score ranking up to the capacity. Then, applicants are selected for SC, ST, OBC, and EWS categories following merit score ranking in each respective category up to their capacities.*

In Indra Shawney (1992), the SCI's language indicates that reservations for the vertically reserved categories (SC and ST at the time) are considered as **hard** reserves, meaning unfilled positions earmarked for these groups are non-transferable to other categories. In governmental employment and admissions to publicly funded educational institutions, reservations for SC and ST have been implemented as hard reserves. While the reservations for OBC have also been treated as hard reserves in the context of governmental job allocations, they were suggested as **soft** reserves in admissions to public universities in the landmark SCI decision *Ashoka Kumar Thakur vs. Union of India* (2008) (henceforth, AKT).[5] More importantly, OBC reservations have been **implemented** as soft reserves in college admissions settings.[6]

The judgment in AKT upheld the validity of 27 percent reservation for OBCs. However, it did not proffer explicit procedural details for reverting unfilled OBC positions to the open category. The absence of judicial guidance has caused considerable ambiguity concerning the eligible beneficiaries of the de-reserved OBC positions—whether they should be exclusively limited to GC candidates or be open to all, including those in reserved categories. The lack of clear rules has led to various inconsistent practices across India (Aygün and Turhan (2022)).

The directive in AKT—to which we refer to the **AKT directive on de-reservation**—states

> *Only non-creamy layer OBCs can avail of reservations in college admissions, and once they graduate from college, they should no longer be eligible for post-graduate reservation. 27% is the upper limit for OBC reservations. The Government need not always provide the maximum limit. Reasonable cutoff marks should be set so that standards of excellence are not greatly affected. The unfilled seats should revert to the general category.*

It is crucial to note here there is widespread confusion in India regarding the position types and applicant types. A cursory search on the internet reveals that the term "general seats" is used to describe open category. The same confusion presents itself in SCI verdicts, as well. In the above quote, by "general category" in the last sentence, the SCI means the "open category." It is how the de-reservation policy is implemented for admissions to technical colleges.[7]

[5]The judgment is available at https://indiankanoon.org/doc/1219385/.

[6]See the recent design for admissions to technical colleges in India in Baswana et al. (2019). Se also Aygün and Turhan (2022).

[7]In Section 3.1 of Aygün and Turhan (2022), the authors show that there are significant unintended consequences if the leftover OBC positions are **only** provided to general category students. They show with a simple example that this will give some SC and ST applicants an incentive not to report their vertical category membership. An easy fix is to allocate surplus OBC positions as open-category following the merit scores. In admissions to technical universities, unfilled OBC positions are allocated as open category slots.

In the same judgment, the SCI recommended a minimum cutoff score for the OBC category to maintain the standard of excellence. The court explicitly stated that there should be a *reasonable difference* in the cutoff scores of the OBC and open category.

> *It is reasonable to balance reservation with other societal interests. To maintain standards of excellence, cutoff marks for OBCs should be set not more than 10 marks out of 100 below that of the general category.*

Effectively, the directive on cutoff scores modifies the eligibility criteria of OBC candidates for reserved OBC positions. Belonging to the OBC category is insufficient to obtain a seat in the OBC category. The candidate must also have enough score.

> *To this end, the Government shall set up a committee to look into the question of setting the OBC cutoff no more than 10 marks below that of the general category. Under such a scheme, whenever the non-creamy layer OBCs fail to fill the 27% reservation, the remaining seats would revert to general category students.*

The directive on cutoff scores may necessitate leaving some OBC positions unfilled to maintain the standard of excellence. The directive on de-reservation requires reverting these vacancies to open category. When reverted open category positions are filled, i.e., more than q_s^o positions are allocated as open category, open category cutoff score becomes lower. In turn, some of the OBC candidates who were ineligible due to their low scores may be eligible with the updated open category cutoff score. In a sense, the two AKT directives contradict with each other.

In this paper, we formulate the AKT directives as formal axioms and analyze their (in)compatibility with India's fundamental mandates from Indra Shawney (1992). We first show that no choice rule implements the both AKT directives in conjunction with the three mandates from Indra Shawney (1992). In Theorem 1, we show that no choice rule can implement the two AKT directives in conjunction with these three fundamental mandates. We highlight as a policy recommendation that imposing a cap on the difference between categories' cutoff scores may lead to unintended consequences.

We introduce a choice rule that satisfy the *AKT directive on cutoff scores* and the three fundamental mandates of Indra Shawney (1992). In Theorem 2, we show that this choice rule is path-independent.

Finally, we consider choice rules that satisfy the three mandates from the Indra Shawney (1992) and the AKT directive on de-reservations. In this case, the extra eligibility criteria on OBC candidates to achieve a certain minimum score is dropped.

2 The Model

2.1 Preliminaries

There is an institution s with q_s positions and a finite set of individuals $I = \{i_1, ..., i_n\}$. There is a set of reserved categories $\mathcal{R} = \{SC, ST, OBC\}$. The vector $\left(q_s^{SC}, q_s^{ST}, q_s^{OBC}\right)$ denotes the number of positions reserved for SC, ST, and OBC at institution s. To distinguish reserved categories individuals may belong to and position types in an institution, we denote the set of position types by $\mathcal{C} = \{o, SC, ST, OBC\}$. The number of open category slots at institution s is $q_s^o = q_s - q_s^{SC} - q_s^{ST} - q_s^{OBC}$. We write $\mathbf{q_s} = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}\right)$ to denote the initial distribution of positions over categories at institution s.

The function $\mathbf{t} : I \to \mathcal{R} \cup \{\emptyset\}$ denotes individuals' category membership. $\mathbf{t}(i) = \emptyset$ means that individual i does not belong to any reserved category and, hence, is a member of the **general category** (GC). For individual $i \in I$, $\mathbf{t}(i) \in \mathcal{R}$ denotes the reserved category individual i belongs to. In India, reporting membership to reserved categories is optional. Members of the reserved categories who do not report their membership are considered members of GC and are eligible only for open-category positions. Members of reserve category $r \in \mathcal{R}$ are eligible for both open category and category r positions. We denote a profile of reserved category membership by $\mathbf{T} = (\mathbf{t}(i))_{i \in I}$.

The function $\theta : I \to \mathbb{R}_+$ denotes individuals' merit scores. That is, $\theta(i) \in \mathbb{R}_+$ is the merit score of individual $i \in I$. Individuals' merit scores induce a strict ranking, denoted $\succ_s$, a linear order over $I \cup \{\emptyset\}$.[8] We write $i \succ_s j$ to mean that i has a higher priority than j at s. $i \succ_s \emptyset$ means that i is acceptable for institution s. Similarly, we write $\emptyset \succ_s i$ to say that applicant i is unacceptable for institution s.[9]

Each reserved category $r \in \mathcal{C} \setminus \{o\}$ is *exclusive* in the sense that applicants who do not belong to category r are deemed unacceptable for positions in category r. Among the applicants who belong to category r, the ranking $\succ_s$ is preserved.

Given a set of applicants $A \subseteq I$, let $rank_A(i)$ be the rank of applicant i in set A with respect to merit ranking $\succ_s$. That is, $rank_A(i) = k$ if and only if $\mid \{j \in A \mid j \succ_s i\} \mid = k - 1$. Let $A^r = \{i \in A \mid i \succ_s^r \emptyset\}$ be the set of category $r \in \mathcal{R}$ eligible individuals in set A.

Institutions' selection criteria is embedded into their choice rules. A choice rule is a function $C : 2^I \to 2^I$ such that $C(A) \subseteq A$ and $|C(A)| \leq q_s$, for all $A \subseteq I$. An important property of choice rules is **path-independence** (Plott (1973)). When a choice rule is path-independent, regardless of the order in which individuals are considered, the chosen set remains the same. This property is valuable in various scenarios.[10] For instance, in college admissions, path independence

[8] In India, when two or more applicants have the same score, ties are broken with some given objective criteria.

[9] Applicants whose scores are under a certain lower bound are deemed unacceptable by the programs.

[10] See Yokote et al. (2023) for a detailed analysis of path independent choice rules.

ensures that the selection of students is not influenced by the sequence in which applications are considered.

DEFINITION. (Plott (1973)) Choice rule C is *path-independent* if

$$C(A \cup B) = C(C(A) \cup B).$$

for any $A, B \subseteq I$.

Aizerman and Malishevski (1981) showed that a choice rule is path-independent if and only if it satisfies the substitutes condition and consistency.

A choice rule C satisfies the *substitutes condition* if, for every $A, B \subset I$, $i \in A \subseteq B$ and $i \in C(B)$ implies $i \in C(A)$.[11] A choice rule C is *consistent* if $C(B) \subseteq A \subseteq B$ implies $C(A) = C(B)$.[12]

2.2 Assignment Function and Induced Choice Rule

In India, the type of positions applicants are assigned to are made public. A choice rule—which selects a subset from any given set of applicants—is insufficient for the problems we consider because it does not specify the position types applicants are chosen under. In what follows, we define an assignment function that specifies, for each applicant, the type of position they are assigned to.

DEFINITION. An **assignment function** for s is a mapping $\eta_s : 2^I \to 2^{I \times \mathcal{C}}$ such that

1. For any $A \subseteq I$, $\eta_s(A) = \{(i,t) | i \in A \text{ and } t \in \mathcal{C}\}$

2. $\forall A \subseteq I$ and for any $i \in A$, we have at most one $(i,t) \in \eta_s(A)$ such that

$$\begin{cases} t \in \{o\} & \text{if } \mathbf{t}(i) = \emptyset, \\ t \in \{\mathbf{t}(i), o\} & \text{if } \mathbf{t}(i) \in \mathcal{R}, \end{cases}$$

3. $|\eta_s(A)| \leq q_s$, and

4. $| \{(i,t) \in \eta_s(A) : t = r\} | \leq q_s^r$, for each $r \in \mathcal{R}$.

An assignment function η_s induces a choice rule C_s, which specifies a selected subset from any given set of applicants. That is, for any given $A \subseteq I$, $C_s(A) \subseteq A$. Formally, given a set of applicants $A \subseteq I$ and assignment function η_s, the **induced choice rule** C_s is defined as

$$C_s(A) = \{i \in A | \exists (i,t) \in \eta_s(A) \text{ for some } t \in \mathcal{C}\}.$$

[11] See Kelso and Crawford (1982) and Roth (1984).

[12] See Alkan and Gale (2003).

3 Policy Objectives as Formal Axioms

We first formulate the principles that were introduced in Indra Shawney (1992).[13]

DEFINITION. An assignment function η_s satisfies the **over and above principle** if for each $A \subseteq I$ and $i \in A$ with $rank_A(i) \leq q_s^o$, $(i, o) \in \eta_s(A)$.

This principle guarantees that the qualifying threshold for the open category exceeds the reserved categories' cutoff scores. It also ensures that high-scoring applicants from reserved categories who receives open category positions are not counted against their respective categories' quota.

DEFINITION. An assignment function η_s is **within-group fair** if for any pair of individuals $i, j \in A$ such that $\theta(i) > \theta(j)$, $(j, t) \in \eta_s(A)$, $t \in \mathbf{t}(i) \cup \{o\}$ imply $(i, t^{'}) \in \eta_s(A)$ for some $t^{'} \in \mathbf{t}(i) \cup \{o\}$.

Within-group fairness requires that the merit scores of applicants are respected in every position category.

DEFINITION. An assignment function η_s satisfies the **quota-filling subject to eligibility** if individual i with $\mathbf{t}(i) = r$ is unassigned, then the number of individuals who are matched to a category r position must be equal to q_s^r, for all $r \in \mathcal{R}$.

The quota-filing subject to eligibility (QFE) requires that an applicant cannot be unassigned if there are vacancies in the categories of positions for which she is eligible.

We now formulate the directive from the AKT judgment. The AKT directive on cutoff scores modifies the **eligibility criteria for the OBC category**. We reformulate the quota-filling subject to eligibility property under the AKT directive.

DEFINITION. An assignment function η_s satisfies the **QFE under the AKT directive** if for any set of individuals $A \in I$ and

1. individual i with $\mathbf{t}(i) = r \in \mathcal{R} \setminus \{OBC\}$ is unassigned, then the number of individuals who are matched to a category r position must be equal to q_s^r,

2. individual i with $\mathbf{t}(i) = OBC$ is unassigned and $\theta(i) \geq min\{\theta(j)|(j, o) \in \eta_s(A)\} - \kappa$, then the number of individuals who are matched to an OBC position must be equal to q_s^{OBC},

3. individual i with $\mathbf{t}(i) = OBC$ and $\theta(i) < min\{\theta(j)|(j, o) \in \eta_s(A)\} - \kappa$, then $(i, OBC) \notin \eta_s(A)$.[14]

[13] See also Sönmez and Yenmez (2022) and Aygün and Turhan (2024b).

[14] In the AKT judgment, $\kappa = 10$.

Since the QFE under the AKT directive (possibly significantly) reduces the demand from the OBC members by making many ineligible, it may create vacancies. Realizing this effect, the judgment recommends a policy to provide unfilled OBC positions to others. We formulate this recommendation, the AKT directive on de-reservation, as a formal axiom.

DEFINITION. An assignment function η_s satisfies the **AKT directive on de-reservation** if for any set of individuals $A \subseteq I$,

$$|\{(i,t) \in \eta_s(A) : t \in \{o, OBC\}\}| < q_s^o + q_s^{OBC} \implies |\eta_s(A)| = |A|.$$

Theorem 1. *No assignment function satisfies the over and above principle, within-category fairness, QFE under the AKT directive, and AKT directive on de-reservation.*

Proof. Consider an institution with four positions. One position is reserved for SC, one for ST, and one for OBC. The remaining position is open category. Consider the set of individuals $A = \{i_1, i_2, i_3, i_4, i_5\}$ with the following scores and category membership:

$Individual$	$Category$	$Score$
i_1	GC	100
i_2	GC	90
i_3	OBC	89
i_4	SC	81
i_5	ST	80

The over and above principle, within category-fairness, and QFE under the AKT directive imply that

$$(i_1, o), (i_4, SC), (i_5, ST) \in \eta_s(A).$$

Since there are four positions, given the category memberships of individuals i_2 and i_3, one of the following must hold.

1. $\eta_s(A) = \{(i_1, o), (i_3, OBC), (i_4, SC), (i_5, ST)\}$.
2. $\eta_s(A) = \{(i_1, o), (i_2, o), (i_4, SC), (i_5, ST)\}$.
3. $\eta_s(A) = \{(i_1, o), (i_3, o), (i_4, SC), (i_5, ST)\}$.
4. $\eta_s(A) = \{(i_1, o), (i_4, SC), (i_5, ST)\}$.

The assignment function in (1) violates the QFE under the AKT directive because i_3 is assigned to an OBC position even though her score is 11 points lower than the cutoff score of the open category.

The assignment function in (2) violates the QFE under the AKT directive because the cutoff score of the open category is 90 (i.e., the score of i_2) while i_3 was not chosen even though she is eligible for the OBC position.

The assignment function in (3) violates the within-group fairness because i_2 has a higher score than i_3, assigned an open-category position.

Finally, the assignment function in (4) violates the AKT directive on de-reservation because, from the open-category and OBC combined, only i_1 is chosen. However, $|\eta_s(A)| = 3$ and $|A| = 5$.

Thus, no assignment function can satisfy these four axioms concurrently.

□

4 Designing Assignment Functions

Building on the impossibility result in Theorem 1, in this section, we introduce assignment functions by (i) dropping the AKT directive on de-reservation and (ii) weakening the QFE under the AKT directive to the Indra Shawney (1992)'s version of QFE property.

4.1 Dropping the AKT Directive on De-reservation

We first propose an assignment function that satisfies

- the over-and-above principle,
- within-group fairness, and
- QFE under the AKT directive.

To maintain the standard of excellence, the OBC candidates scoring more than ten marks lower than the lowest-scoring applicant who obtains an open category position are deemed ineligible.

Assignment Function η_s^{AKT}

Given an initial distribution of positions $q_s = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}\right)$, a set of applicants $A \subseteq \mathcal{I}$, and a profile reserve category membership $\mathbf{T} = (\mathbf{t}(i))_{i \in A}$ for the members of A, the assignment function $\eta_s^{\text{AKT}}(A)$ make its selection as follows:

Step 1. Consider open category positions. Individuals are chosen one at a time following merit scores up to the capacity q_s^o. Let $C_s^o\,(A, q_s^o)$ be the set of chosen applicants in the open category. For each $i \in C_s^o(A, q_s^o)$, add (i, o) into $\eta_s^{AKT}(A)$.

Step 2.1 Among the remaining applicants $A' = A \setminus C_s^o(A, q_s^o)$, for each reserve category $t \in \{SC, ST\}$, consider applicants in reserved category t. Select applicants one at a time following merit scores up to the capacity q_s^t. Let $C_s^t\left(A', q_s^t\right)$ be the set of chosen applicants for reserve category $t = SC, ST$. For each $i \in C_s^t(A', q_s^t)$, add (i, t) into $\eta_s^{AKT}(A)$.

Step 2.2 Let $i \in C_s^o(A, q_s^o)$ be the lowest score applicant chosen in Step 1 with the score $\theta(i)$. Remove all OBC applicants with a score less than $\theta(i) - \kappa$. Consider the remaining OBC applicants and choose one at a time following merit scores up to the capacity q_s^{OBC}. Let $C_s^{OBC}\left(A', q_s^{OBC}\right)$ be the set of chosen OBC applicants. For each $i \in C_s^{OBC}(A, q_s^{OBC})$, add (i, OBC) into $\eta_s^{AKT}(A)$.

The induced choice rule from the assignment function η_s^{AKT}, denoted $C_s^{\text{AKT}}(A, q_s)$, is defined as follows:

$$C_s^{\text{AKT}}(A, q_s) = \{(i \in A : \exists (i, t) \in \eta_s^{AKT}(A) \text{ for some } t \in \mathcal{C}\}.$$

Theorem 2. *The choice rule C_s^{AKT} is path-independent.*

4.2 Weakening the QFE under AKT

We now consider choice rules that satisfy

- the over-and-above principle,
- within-group fairness, and
- quota-filling subject to eligibility (the Indra Shawney version), and
- the AKT directive on de-reservations.

The backward transfer assignment function introduced in Aygün and Turhan (2022) satisfies all four axioms. Furthermore, Aygün and Turhan (2024a) introduced a family of assignment functions that satisfy these four axioms. A critical assignment function in this family, the *forward transfer assignment function*, fills leftover OBC positions as open category positions after filling all reserved category positions. This assignment function is shown to maximize merit among the assignment functions in this family.

We describe the forward and backward transfer assignment functions in the Appendix for completeness.

5 Conclusion

In this paper, we show that the fundamental mandates and directives in two historic cases in India, Indra Shawney (1992) and Ashoka Kumar Thakur (AKT) (2008), the SCI's recommendations are conflicting. We provide positive results by either dropping one of the recommendations in AKT or weakening another directive in the same judgment.

References

AIZERMAN, M. AND A. MALISHEVSKI (1981): "General theory of best variants choice: Some aspects," *IEEE Transactions on Automatic Control*, 26, 1030–1040.

ALKAN, A. AND D. GALE (2003): "Stable schedule matching under revealed preference," *Journal of Economic Theory*, 112, 289–306.

AYGÜN, O. AND B. TURHAN (2022): "How to de-reserve reserves: Admissions to technical colleges in India," *Management Science*, 69, 6147–6164.

——— (2023): "Priority design for engineering colleges in India," *Indian Economic Review*, 1–11.

——— (2024a): "How to Implement Soft Reserves," *Iowa State University Working Paper*.

——— (2024b): "The Over-and-Above Implementation of Reserve Policy in India," *Iowa State University Working Paper*.

BASWANA, S., P. P. CHAKRABARTI, S. CHANDRAN, Y. KANORIA, AND U. PATANGE (2019): "Centralized admissions for engineering colleges in India," *INFORMS Journal on Applied Analytics*, 45, 338–354.

KELSO, A. S. AND V. P. CRAWFORD (1982): "Job matching, coalition formation, and gross substitutes," *Econometrica: Journal of the Econometric Society*, 1483–1504.

PLOTT, C. R. (1973): "Path independence, rationality, and social choice," *Econometrica: Journal of the Econometric Society*, 1075–1091.

ROTH, A. E. (1984): "Stability and polarization of interests in job matching," *Econometrica: Journal of the Econometric Society*, 47–57.

SÖNMEZ, T. AND M. B. YENMEZ (2022): "Affirmative action in India via vertical, horizontal, and overlapping reservations," *Econometrica*, 90, 1143–1176.

YOKOTE, K., I. E. HAFALIR, F. KOJIMA, AND M. B. YENMEZ (2023): "Representation Theorems for Path-Independent Choice Rules," *arXiv preprint arXiv:2303.00892*.

6 Appendix

6.1 Proof of Theorem 2

We first show that C_s^{AKT} satisfies the substitutes condition. Consider $i, j \in I$ and $A \subset I \setminus \{i, j\}$ such that $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We need to show that $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Since $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$, then $\nexists t \in \mathcal{C}$ such that $(i,t) \in \eta_s^{AKT}(A \cup \{i\})$. Let $\mathbf{t}(i) = GC$. Then, it must be the case that $rank_{A\cup\{i\}}(i) > q_s^o$. When individual j joins we still have $rank_{A\cup\{i,j\}}(i) > q_s^o$. Thus, we have $i \notin C_s^{AKT}(A \cup \{i,j\}, q_s)$.

Now let $\mathbf{t}(i) \in \{SC, ST\}$. W.l.o.g., suppose $\mathbf{t}(i) = SC$. $i \notin C_s^{AKT}(A\cup\{i\}, q_s)$ implies that (1) $rank_{A\cup\{i\}}(i) > q_s^o$ so that i could not obtain an open-category position and (2) $rank_{A'_{SC}\cup\{i\}}(i) > q_s^{SC}$, where $A'_{SC} \cup \{i\}$ is the set of remaining unassigned SC applicants after the open-category positions were filled. When j is added to the set $A \cup \{i\}$, both (1) and (2) still hold. Thus, $i \notin C_s^{AKT}(A \cup \{i,j\}, q_s)$.

Now, consider $\mathbf{t}(i) = OBC$. $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that $rank_{A\cup\{i\}}(i) > q_s^o$ and either (1) $rank_{A'_{OBC}\cup\{i\}}(i) > q_s^{OBC}$, where $A'_{OBC} \cup \{i\}$ is the set of remaining unassigned OBC applicants after the open-category positions were filled, or (2) $\theta(i) < min\{\theta(j)|(j,o) \in \eta_s(A)\} - 10$. Including j to $A \cup \{i\}$ does not impact either condition. Therefore, $i \notin C_s^{AKT}(A \cup \{i,j\}, q_s)$.

Next, we show that C_s^{AKT} satisfies consistency. Suppose $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We must have $rank_{A\cup\{i\}}(i) > q_s^o$. By definition of C_s^{AKT}, the same set of individuals are assigned to open-category positions when i is removed from the set of alternatives.

If $\mathbf{t}(i) \in \{SC, ST\}$, then the positions in category $\mathbf{t}(i)$ must be filled with individuals with higher scores. Thus, removal of i from the set of alternatives does not impact who are chosen in categories SC and ST.

Finally, if $\mathbf{t}(i) = OBC$, then $i \notin C_s^{AKT}(A\cup\{i\}, q_s)$ implies that either $\theta(i) < min\{\theta(j)|(j,o) \in \eta_s(A)\} - 10$ or $rank_{A'_{OBC}\cup\{i\}}(i) > q_s^{OBC}$, where $A'_{OBC} \cup \{i\}$ is the set of remaining unassigned OBC applicants after the open-category positions were filled. In both cases, removing i from the set of individuals avsailable does not change the set of chosen individuals for OBC positions.

Thus, we conclude

$$C_s^{AKT}(A \cup \{i\}, q_s) = C_s^{AKT}(A, q_s).$$

Aizerman and Malishevski (1981) showed the equivalence of path-independence and the conjugation of substitutes condition and consistence. This concludes our proof.

6.2 Formal Descriptions of Backward & Forward Transfer Assignment Functions

6.2.1 *Backward Transfer Assignment Function*

The following description follows Aygün and Turhan (2023).

Step 1. Consider open category positions. Individuals are chosen one at a time following merit scores up to the capacity q_s^o. Let $C_s^o(A, q_s^o)$ be the set of chosen applicants in the open category. For each $i \in C_s^o(A, q_s^o)$, add (i, o) into $\eta_s^{BT}(A)$.

Step 2.1. Among the remaining applicants $A^{'} = A \setminus C_s^o\,(A, q_s^o)$, consider the remaining OBC applicants and choose one at a time following merit scores up to the capacity q_s^{OBC}. Let $C_s^{OBC}\left(A^{'}, q_s^{OBC}\right)$ be the set of chosen OBC applicants. For each $i \in C_s^{OBC}(A, q_s^{OBC})$, add (i, OBC) into $\eta_s^{BT}(A)$. If q_s^{OBC} applicants are chosen, continue with step 3. Otherwise, let $\delta = q_s^{OBC} - |C_s^{OBC}(A, q_s^{OBC})|$ and continue with step 2.2.

Step 2.2. Consider the remaining applicants and choose one at a time following merit scores up to the capacity δ. For each chosen applicant i, add (i, o) into $\eta_s^{BT}(A)$.

Step 3. Among the remaining applicants $A^{'} = A \setminus C_s^o\,(A, q_s^o)$, for each reserve category $t \in \{SC, ST\}$, consider applicants in reserved category t. Select applicants one at a time following merit scores up to the capacity q_s^t. Let $C_s^t\left(A^{'}, q_s^t\right)$ be the set of chosen applicants for reserve category $t = SC, ST$. For each $i \in C_s^t(A^{'}, q_s^t)$, add (i, t) into $\eta_s^{BT}(A)$.

6.2.2 *Forward Transfer Assignment Function*

The following description follows Aygün and Turhan (2024a).

Step 1. Consider open category positions. Individuals are chosen one at a time following merit scores up to the capacity q_s^o. Let $C_s^o\,(A, q_s^o)$ be the set of chosen applicants in the open category. For each $i \in C_s^o(A, q_s^o)$, add (i, o) into $\eta_s^{FT}(A)$.

Step 2. Among the remaining applicants $A^{'} = A \setminus C_s^o\,(A, q_s^o)$, for each reserve category $t \in \{SC, ST, OBC\}$, consider applicants in reserved category t. Select applicants one at a time following merit scores up to the capacity q_s^t. Let $C_s^t\left(A^{'}, q_s^t\right)$ be the set of chosen applicants for reserve category $t = SC, ST, OBC$. For each $i \in C_s^t(A^{'}, q_s^t)$, add (i, t) into $\eta_s^{FT}(A)$. If q_s^{OBC} applicants are chosen for reserve category OBC, terminate the procedure. Otherwise, let $\delta = q_s^{OBC} - |C_s^{OBC}(A, q_s^{OBC})|$ and continue with step 3.

Step 3. Consider the remaining applicants and choose one at a time following merit scores up to the capacity δ. For each chosen applicant i, add (i, o) into $\eta_s^{FT}(A)$.