Fiber Requirement in Multifiber WDM Networks with Alternate-Path Routing [1]

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Abstract

An important problem in multifiber WDM networks is to decide how many fibers per link are required to guarantee high network performance. The fiber requirement may depend on many factors, e.g., the network topology, traffic patterns, the number of wavelengths per fiber, and the routing algorithm employed in the network. We study the fiber requirement under dynamic traffic in different topologies with alternate path routing (APR) in this paper. A new analytical model is developed to evaluate the blocking performance of such networks. Our analytical and simulation results show that the number of required fibers per link to provide high network performance is slightly higher in the APR than the fixed-path routing (FPR). However, a small number of fibers per link are still sufficient to guarantee high network performance in both the regular mesh-torus networks and the irregular NSFnet with APR. Since multiple fibers have the same effect as the limited wavelength conversion, our analytical model is also applicable in the networks with limited wavelength conversion.

I. Introduction

With the development of the Internet and World Wide Web, the network bandwidth requirements have increased dramatically in recent years. Wavelength division multiplexing (WDM)-based all-optical networks are emerging to utilize the enormous bandwidth in optical fibers to fulfil the bandwidth requirement and deploy new network services. In WDM networks, a connection request encounters high performance degradation because of the wavelength continuity constraint. Wavelength converters have been proposed to overcome the wavelength continuity constraint in WDM networks. However, the technology of all-optical wavelength conversion is not mature yet. The cost of wavelength converters is likely to remain high in the near future. Using multiple fibers on each link in WDM networks is an alternate solution to conquer the wavelength continuity constraint. In multifiber WDM networks, each link consists of multiple fibers, and each fiber carries information on multiple wavelengths. A wavelength that cannot continue on the next hop on the same fiber can be switched to another fiber using an optical cross-connect (OXC) if the same wavelength is free on one of the other fibers. Thus, multiple fibers in WDM networks have the same effect as the limited wavelength conversion. Let F be the number of fibers per link and W be the number of wavelengths per fiber. A network with F fibers per link and W wavelengths per fiber is functionally equivalent to an FW-wavelength network with a partial wavelength conversion of degree F.

Since the cost of a multifiber network is likely to be higher than a single-fiber network (more amplifiers and multiplexer/demultiplexer), the design goal of a multifiber network is to achieve high network performance with the minimum number of fibers per link. Thus an important problem in multifiber networks is to decide how many fibers per link are required to guarantee high network performance that is similar to a network with full-range wavelength converters at every node. This fiber requirement may depend on many factors, e.g., the network topology, traffic patterns, the number of wavelengths per fiber, and the routing algorithm employed in the network. A similar problem has been studied in [15] under the assumption of static traffic. Wavelength assignment algorithms in multifiber networks have been studied in [7, 10]. The performances of multifiber networks with and without wavelength converters are studied in [8]. We study the fiber requirement under dynamic traffic in different topologies with different routing algorithms in this paper. Our study method is different from the ones used in [7, 8, 10], which assume that both F and W are fixed for a network. We assume that the traffic load of a network is fixed, and the number of channels to support this traffic, $C = FW$, is a constant and known beforehand. We vary F from 1 (no wavelength conversion), ... , to FW (full-range wavelength conversion), and $W = C/F$ accordingly. Our study method shows clearly the effect of multiple fibers on the network performance. A network operator could easily pick up a F and W combination with the consideration of both network performance and cost.

There have been considerable interests to analyze the blocking performance of multifiber WDM networks. The independent wavelength load model [3] is extended to multifiber networks in [8]. The results of this model are not numerically accurate for Poisson traffic because of the assumption that the load on one wavelength is independent of those on the other wavelengths on a link. The link load independence model proposed in [2] is extended to multifiber networks in [7]. However, this independent model is not accurate [7]. It overestimates the blocking performance for $F = 1$ and underestimates it for $F > 1$ in a mesh-torus network. The blocking performance models for first-fit wavelength assignment in [9, 11] are also proposed to be applicable in multifiber networks. However, both of these models assume that link loads are independent, which may not be valid for sparse network topologies. A Multifiber Link-Load Correlation (MLLC) model proposed in [12] is more accurate than the models in [7, 8]. The analytical and simulation results in [12] show that a limited number of fibers per link is sufficient to guarantee high network performance in the ring, the mesh-torus, and the NSF T1 backbone networks. However, the results are obtained using a fixed-path routing (FPR) algorithm, i.e., a request is blocked if no wavelength is free on the preselected path between a source-destination (s-d) pair.

Routing and wavelength assignment algorithms (RWAA) play a key role in WDM networks [9]. Alternate path routing (APR), in which a request blocked on one path is overflowed to an alternate path, can significantly improve the network performance [14] in single-fiber networks. We study the effects of multiple fibers in WDM networks with the APR in this paper. The question we attempt to answer is how many fibers per link are required to guarantee high performance in a WDM network with the APR. We use and extend the MLLC model in [12] to analyze the performance of WDM networks with the APR. To our knowledge, this is the first analytical model that can by used to predict the performance of multifiber networks with the APR. Our model is a generalized model that can be used in both regular and irregular networks. Since multiple fibers have the same effect as the limited wavelength conversion, our analytical model is also applicable in the networks with limited wavelength conversion.

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This paper is organized as follows. The fundamental ideas of the MLLC model are introduced in Section II. The MLLC model is extended to analyze the performance of the APR in Section III. An iterative approach is proposed to solve the Erlang-map equation introduced by the APR in multifiber networks. The accuracy of the analytical model is assessed in Section IV by comparing the analytical results to the simulation results. The numerical results show that the performance of the APR is much better than that of the FPR in multifiber networks. The number of required fibers per link is slightly higher in the APR than the FPR to provide high network performance. However, a small number of fibers per link are still sufficient to guarantee high network performance in both the regular mesh-torus networks and the irregular NSFnet. We make our concluding remarks in Section V.

II. REVIEW OF THE MULTIFIBER LINK-LOAD CORRELATION MODEL

Much research has been done in obtaining the call blocking performance of WDM networks [2, 3, 4, 5, 6, 16]. A Markov chain model with the consideration of link-load correlation in [6] is accurate and has a moderate complexity. As pointed out in [4], the Markov chain model is an approximate model, because the arrival rates vary with the state of the Markov chain. Based on the Markov chain model, a multifiber link-load correlation (MLLC) model is proposed in [12]. Compared to the link independence model for multifiber WDM networks in [7], the MLLC model is an accurate and general model that is applicable to not only regular networks but also irregular networks. We summarize the basic ideas of the MLLC model in this section. For lack of space, we omit explaining of the details of the model and ask the reader to refer to [12] when necessary. The MLLC model is extended to analyze the performance of the alternate path routing algorithm in the next section.

Assumptions and definitions In the MLLC model, we assume a Poisson input traffic with an arrival rate λ at each node and an exponentially distributed call holding time with mean $1/\mu$. A single path is preselected for each source-destination (s-d) pair, and a wavelength assigned to a connection is randomly selected from the set of free wavelengths on that path with the same probability. The load on link i of a path given the loads on link $1, 2, \ldots, i-1$, depends only on the load on link $i-1$. Let F be the number of fibers per link and W be the number of wavelengths on each fiber. We assume that F and W are the same on all links and fibers, respectively. We also assume that an incoming request can be switched to any output port using OXC as long as the output port has the same wavelength free regardless of which fiber it is on. If the wavelength is not free on all of the F fibers, the request is blocked on this wavelength. The blocked calls will never return to the network. No wavelength converter is available at any node.

We define a Light Channel (LC) as a wavelength on a fiber on a link. A lightpath (LP) is a connection between a s-d pair using the same wavelength on all the links of a path. Note that a lightpath consists of several LCs on successive links. The LCs on a path may or may not be on the same fiber. Let a wavelength trunk (WT) λ_i be a collection of the LCs/LPs using wavelength λ_i on all the fibers. We define a WT "free" on a link if the wavelength is free on at least one of the fibers on the link. A WT is "busy" on the link otherwise. A WT is "free" on a path if that WT is free on all of the links constituting the path. A WT is "busy" on the path otherwise.

In the MLLC model, we start by analyzing a two-hop path with F fibers on each link and W wavelengths on each fiber. Then the blocking probability on a l-hop path can be computed recursively by viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path. On the two-hop path as shown in Figure 1, we are interested in computing $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$, which is defined as

- $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ = Pr{the probability that $\widehat{N}_{f_2}$ WTs are free on a two-hop path | $\widehat{X}_{f_1}$ WTs are free on the first hop of the path, y_{f_2} LCs are free on the second hop, and z_{c_2} LCs are busy on both of the links occupied by continuing calls from the first to the second hop[2]}.

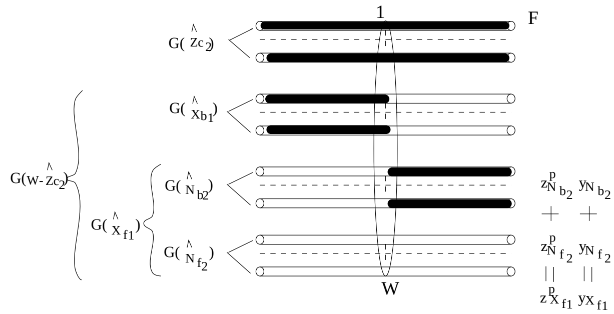

Figure 1: The wavelength trunks on a two-hop path.

The derivation of $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ is shown in the Appendix. Given the steady-state distribution $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ on a two-hop path, we can compute the blocking probability on the l-hop path by viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path. Let $P^{(l)}(\widehat{N}_{f_l}, y_{f_l})$ be the probability that $\widehat{N}_{f_l}$ wavelengths are free on an l-hop path and y_{f_l} LCs are free on hop l . $P^{(l)}(\widehat{N}_{f_l}, y_{f_l})$ can be derived as

$$P^{(l)}(\widehat{N}_{f_l}, y_{f_l}) = \sum_{x_{f_{l-1}}=0}^{FW} \sum_{z_{c_l}=0}^{\min(FW - x_{f_{l-1}}, FW - y_{f_l})} \sum_{\widehat{N}_{f_{l-1}}=0}^{\lfloor \frac{x_{f_{l-1}}}{F} \rfloor} R_{WTLC}(\widehat{N}_{f_l}|\widehat{N}_{f_{l-1}}, z_{c_l}, y_{f_l}) U(z_{c_l}|y_{f_l}, x_{f_{l-1}}) \times S(y_{f_l}|x_{f_{l-1}}) P^{(l-1)}(\widehat{N}_{f_{l-1}}, x_{f_{l-1}}) \,. \quad (1)$$

where $S(y_{f_2}|x_{f_1})$ and $U(z_{c_2}|x_{f_1}, y_{f_2})$ are defined as [6]

- $S(y_{f_2}|x_{f_1})$ = Pr{y_{f_2} LCs are free on the second link of a two-hop path | x_{f_1} LCs are free on the first link of the path }
- $U(z_{c_2}|x_{f_1}, y_{f_2})$ = Pr{z_{c_2} LCs are occupied by continuing calls from the first link to the second link | x_{f_1} LCs are free on the first link, and y_{f_2} LCs are free on the second link }.

Let c_l, c_c, c_e be the number of calls leaving from link 1, continuing from link 1 to link 2, and entering at link 2 on a two-hop path, respectively. Both $U(z_{c_2}|x_{f_1}, y_{f_2})$ and $S(y_{f_2}|x_{f_1})$ are functions of the steady-state probability of

[2]We put a hat on the variables for the number of WTs, to differentiate them from the variables for the number of LCs on a link, throughout this paper.

state (c_l, c_c, c_e) that is given by

$$\pi(c_l, c_c, c_e) = \frac{\frac{(\frac{\lambda_l}{\mu})^{c_l}}{c_l!}\frac{(\frac{\lambda_c}{\mu})^{c_c}}{c_c!}\frac{(\frac{\lambda_e}{\mu})^{c_e}}{c_e!}}{\sum_{j=0}^{F}\sum_{i=0}^{F-j}\sum_{k=0}^{F-j}\frac{(\frac{\lambda_l}{\mu})^i}{i!}\frac{(\frac{\lambda_c}{\mu})^j}{j!}\frac{(\frac{\lambda_e}{\mu})^k}{k!}},$$
$$0 \le c_l + c_c \le F, \quad 0 \le c_c + c_e \le F, \qquad (2)$$

where $\lambda_l, \lambda_c, \lambda_e$ are the rates of calls that leaving the first link, continuing from the first link to the second link, and entering at the second link, respectively. $1/\mu$ is the expected value of the exponentially distributed call holding time.

Let $Q_\alpha^p(i)$ be the probability that the path connecting s-d pair α has i free wavelength trunks. Let $l(p)$ be the length of path p. $Q_\alpha^p(i)$ is given by

$$Q_\alpha^p(i) = \sum_{y_f=0}^{FW} P^{l(p)}(i, y_f) \ . \qquad (3)$$

Implementation and complexity analysis The above equations shows an approach on how to compute the steady-state probability of a path that has i free wavelength trunks. The performance analyses of different routing algorithms in the next section are based on these equations. Comparing to the link-load correlation model for single fiber networks [6], the MLLC model has the same computational complexity except for the computation of the free WT distribution on a two-hop path, $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$. However, $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ does not depend on any network topology and traffic arrival rate. The only parameters needed to compute $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ is the number of fibers per link, F, and the number of wavelengths per fiber, W. Thus $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ can be computed independently. The results can be used repeatedly in different topologies and traffic patterns, as long as they have the same number of fibers per link and wavelengths per fiber.

Different routing algorithms generate different network traffics that determine the network performance. We use the MLLC model to analyze the performances of different routing algorithms in the next section.

III. Analysis of different routing algorithms

We have presented the fundamental ideas of the MLLC model in the last section. The MLLC model provides a method to analyze the status of each path. The stable-state distribution of the number of free wavelength trunks on a path can be computed using Eq. (3). The input parameters of the MLLC model is the network traffic specified by the rate of leaving calls from link i (λ_i^l), the rate of entering calls at link j (λ_j^e), and the rate of continuing calls from link i to link j (λ_{ij}^c). Here link i, j are the links of a network. The carried network traffic is determined by routing and wavelength assignment algorithms in WDM networks. We analyze the performance of two typical routing algorithms, shortest path routing (SPR) and alternate path routing (APR), in this section. We are interested in finding the effects of multiple fibers on the performance of these routing algorithms, i.e., how many fibers per link are required to ensure high network performance. We assume that the paths for each source-destination (s-d) pair are predetermined. The random wavelength assignment algorithm is used in all of the routing algorithms.

A. *Shortest path routing*

In the shortest path routing, if free wavelengths are found on the shortest path, one of the free wavelengths is randomly selected to set up the call. Otherwise, the request is blocked. We assume that the network blocking probability is small such that the effect of the blocking probability on the carried load can be neglected. The results in the next section show that this assumption does not significantly affect the accuracy of the model. Let R_i be the set of routes that pass through link i, and $R_{i,j}^c$ be the set of routes that continue from link i to link j. Let α be a s-d pair and p_α be a path connecting α. λ_i^e, λ_j^l, and λ_{ij}^c are given by

$$\lambda_{i,j}^c = \sum_{P_\alpha \in R_{i,j}^c} \lambda_\alpha \ , \qquad (4)$$

$$\lambda_i^l = \sum_{P_\alpha \in R_i} \lambda_\alpha - \lambda_{i,j}^c \ , \qquad (5)$$

$$\lambda_j^e = \sum_{P_\alpha \in R_j} \lambda_\alpha - \lambda_{i,j}^c \ , \qquad (6)$$

These arrival rates are used to compute the steady-state probability of state (c_l, c_c, c_e) in (2). Let $l(\alpha)$ be the length of a path between a s-d pair. Let $|R|$ be the number of s-d pairs in the network, and P_B be the network-wide average blocking probability. P_B is given by

$$P_B = \sum_\alpha Q_\alpha^{p_\alpha}(0)/|R|. \qquad (7)$$

B. *Alternate path routing*

The shortest path routing algorithm is simple, but suffers poor blocking performance. Alternate path routing is one approach to improve the blocking performance in WDM networks. A set of paths are precomputed statically for each s-d pair and searched sequentially in the alternate path routing. If no free wavelength is found on the first path, the call request is attempted on the second path, and so on. The request is blocked only if all of the candidate paths have no free wavelengths. Note that the number of paths for each s-d pair is restricted to two for easy discussion in this model. The model can be easily extended to consider more than two paths. However, we assume that the overflowed traffic is still Poisson traffic. This assumption may not be valid if more alternate paths are used. Our analytical and simulation results also show that using more than two paths does not significantly improve the network performance.

In the alternate path routing, the traffic load of a link consists of two parts: (1) the loads carried on the first paths that pass through the link; (2) the loads overflowed from the first paths and carried on the second paths that pass through the link. Thus, the link load cannot be directly obtained from the offered traffic without knowledge of the blocking probability on each path. The probability of blocking is in turn dependent on the arrival rate to the links. This leads to a system of coupled non-linear equations called the Erlang map [13]. Let R_i^1 be a set of the first paths that use link i and R_{ij}^1 be a set of the first paths that continue from link i to link j. Let R_i^2 be a set of the second paths that use link i and R_{ij}^2 be a set of the second paths that continue from link i to link j. Let $P_{B_\alpha}^1$ be the blocking probability on the first path of a s-d pair α, and $P_{B_\alpha}^2$ be the blocking probability on the second path of α. Denote p_α as a path connecting the s-d pair α. Let ϵ be a small positive number that is used as convergence criterion. The Erlang map equation can be solved recursively using the following procedure:

1. Set $P_B = P_B{}^1_\alpha = P_B{}^2_\alpha = 0$;
2. Compute the traffic loads entering at link i (λ_i), leaving from link j (λ_j), and the load continuing from link i to link j (λ_{ij}):

$$\lambda_{ij} = \sum_{p_\alpha \in R^1_{ij}} \lambda_\alpha (1 - P_B{}^1_\alpha) + \sum_{p_\alpha \in R^2_{ij}} \lambda_\alpha P_B{}^1_\alpha (1 - P_B{}^2_\alpha);$$

$$\lambda_i = \sum_{p_\alpha \in R^1_i} \lambda_\alpha (1 - P_B{}^1_\alpha) + \sum_{p_\alpha \in R^2_i} \lambda_\alpha P_B{}^1_\alpha (1 - P_B{}^2_\alpha) - \lambda_{ij};$$

$$\lambda_j = \sum_{p_\alpha \in R^2_j} \lambda_\alpha (1 - P_B{}^1_\alpha) + \sum_{p_\alpha \in R^2_j} \lambda_\alpha P_B{}^1_\alpha (1 - P_B{}^2_\alpha) - \lambda_{ij};$$

3. Compute the new values of the blocking probability on the first and second path of α, $\widehat{P_B}{}^1_\alpha$ and $\widehat{P_B}{}^2_\alpha$, respectively, using Eq. 7;
4. Compute the new value of the average blocking probability

$$\widehat{P_B} = \frac{\sum_\alpha \widehat{P_B}{}^1_\alpha \widehat{P_B}{}^2_\alpha}{|R|}$$

5. If ($|P_B - \widehat{P_B}| < \epsilon$), exit; otherwise let $P_B{}^1_\alpha = \widehat{P_B}{}^1_\alpha$, $P_B{}^2_\alpha = \widehat{P_B}{}^2_\alpha$, $P_B = \widehat{P_B}$, go to Step 2.

IV. Numerical results and analysis

In this section, we assess the accuracy of our analysis model by comparing it with the simulation results. The MLLC model is applied to a 5×5 mesh-torus network, and an irregular NSF T1 backbone network (NSFnet, a figure of this network can be found in [16].) with the shortest path routing and alternate path routing. We are interested in finding the effect of multifibers on these networks with different routing algorithms. The question we attempted to answer is how many fibers are required to provide similar performance as that in a full-wavelength-convertible network.

In the networks we studied, the link capacity is fixed at 24 light channels, i.e., $FW = 24$ on each link. We vary the number of fibers on each link, F, from $1, 2, 3, 4, 6, 8, 12$ to 24, and the number of wavelengths on each fiber by $W = 24/F$ accordingly. We assume Poisson traffic arrives at each node, and the destination for an arrival request is uniformly distributed among other nodes[3]. We adjust the traffic load such that the blocking probabilities are around 10^{-3}. Each data point in the simulations was obtained using 10^6 call arrivals. In the approximate analysis of the APR, multiple iterations are required and the convergence criteria is set to be 10^{-5} for the blocking probabilities.

Wavelength converters are useful to reduce the blocking probability in the mesh-torus networks. We first study a 5×5 mesh-torus network. The call blocking probability against the number of fibers per link is plotted in Figure 2 and 3 for the shortest path routing and alternate path routing, respectively.

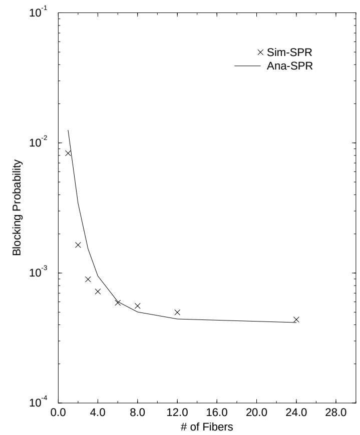

Figure 2: Blocking probability versus number of fibers in a 5×5 mesh-torus network using the shortest path routing. The traffic load is 12 Erlangs per node. The number of LCs per link is fixed at 24.

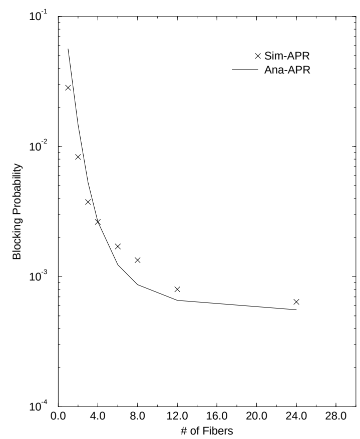

Figure 3: Blocking probability versus number of fibers in a 5×5 mesh-torus network using the alternate path routing. The traffic load is 17 Erlangs per node. The number of LCs per link is fixed at 24.

The traffic load is 12 Erlangs per node for the shortest path routing, and 17 Erlangs per node for the alternate path routing. The analytical results closely match the simulation results in both the shortest path routing and the alternate path routing, which indicates that the model is adequate in analytically predicting the performance of SPR and APR in mesh-torus networks.

We observed from the figures that the network performance of using a full-range wavelength converter (F=24, W=1) at every node is much better than using no wavelength conversion (F=1, W=24) in the shortest path routing (more than one order of magnitude). Using full wavelength conversion improves the blocking performance even further, around two order of magnitudes, if the alternate path routing is used. However, the network blocking probability decreases sharply with the increasing number of fibers per link F, when F is small. The performance improvement becomes less significant after 4 fibers per link are used in the shortest path routing and 6 fibers per link in the alternate path routing. Thus, routing algorithms affect the benefits of using multiple fibers per link in WDM networks. The alternate routing requires more fibers per link than the shortest path routing to achieve high network performance. However, only very limited number of fibers per

[3]The MLLC model could also be used for non-uniformly distributed traffic using Eqs. (4), (5) and (6). The uniform distribution assumption is made only for simplicity. Note that link loads in NSFnet are non-uniformly distributed.

link are sufficient for both the shortest path routing and the alternate path routing.

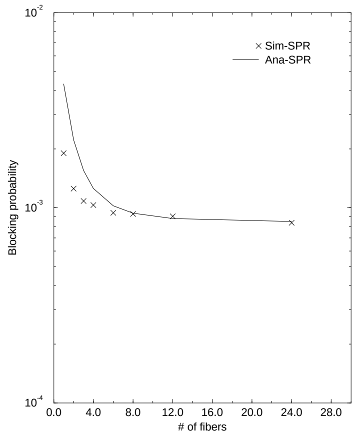

Figure 4: Blocking probability versus number of fibers in NSFnet using the shortest path routing. The traffic load is 12 Erlangs per node. The number of LCs per link is fixed at 24.

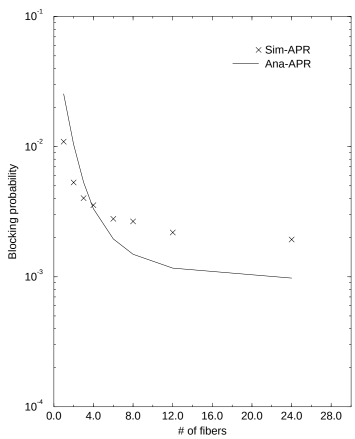

Figure 5: Blocking probability versus number of fibers in NSFnet using the alternate path routing. The traffic load is 17 Erlangs per node. The number of LCs per link is fixed at 24.

The mesh-torus network has been intensively studied in the literature [1, 8]. However, few analytical models are applicable to irregular multifiber networks. We apply the analytical model to the NSFnet and show the results in Figure 4 and 5. The traffic load per node is 12 Erlangs for the shortest path routing and 17 Erlangs for the alternate path routing. It is seen that the analytical results follow the trend of the simulation results for both of the routing algorithms. In the NSFnet, full wavelength conversion (F=24, W=1) at every node does not improve much the performance compared to no wavelength conversion (F=1, W=24). It is also interesting to note that only 4 fibers per link in the shortest path routing, and 6 fibers per link in the alternate path routing, are sufficient to provide similar performance to using full wavelength converters in the NSFnet.

V. Conclusions

We study the effect of multiple fibers in all-optical WDM networks with alternate path routing. We use and extend a multifiber link-load correlation model [12] to evaluate the blocking performance of such networks. We have shown that the model is accurate for a variety of network topologies by comparing the analytical results with the simulation results. We observed that the number of fibers required providing high performance using alternate path routing is slightly higher than in the shortest path routing. However, very limited number of fibers is sufficient to guarantee high performance in both shortest path routing and alternate path routing. Since multiple fibers have the same effect as the limited wavelength conversion in WDM networks, our analytical model is also applicable in limited-wavelength-convertible networks.

An important conclusion of our study is that a multifiber network has similar blocking performance to a full-wavelength-convertible network if we select the wavelength-fiber-pairs adequately. Most of the current optical networks are built on multiple fibers. Multifiber WDM networks without wavelength conversion is not only a feasible, but also a desirable choice under current technologies.

VI. Appendix

In this appendix, we derive the expression of $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ defined in section II. The difficulty in computing $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ results from the continuing calls from the first hop to the second hop. To simplify the computation, we divide the W WTs on the two-hop path into different groups as shown in Figure 1. Each wavelength trunk consists of F fibers. A filled slot in the figure indicates that that wavelength trunk is busy, that is, it is fully occupied on the link. An unfilled slot indicates that the wavelength trunk is free, that is, the wavelength trunk may be partially occupied or free on every fiber. The conditional distribution of continuing calls is computed in each group.

Notations We define the following steady-state probabilities that are used in obtaining $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$.

- $P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ = Pr{ $\widehat{Z}_{c_2}$ busy WTs are occupied completely by continuing calls | $\widehat{X}_{f_1}$ WTs are free on the first hop, y_{f_2} LCs are free on the second hop, and z_{c_2} busy LCs continue from the first to the second hop}.
- $P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})$ = Pr{$z^p_{X_{b_1}}$ continuing calls are in the subgroup $G(\widehat{X}_{b_1})$ | $z^p_{c_2}$ calls are randomly distributed in groups $G(\widehat{X}_{b_1})$ and $G(\widehat{X}_{f_1})$, and no busy WT is occupied completely by $z^p_{c_2}$ continuing calls [4] }.
- $P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})$ = Pr{$y_{X_{b_1}}$ LCs are free on the second hop in group $G(\widehat{X}_{b_1})$ | $z^p_{X_{b_1}}$ LCs are in group $G(\widehat{X}_{b_1})$, $z^p_{X_{f_1}}$ LCs are in group $G(\widehat{X}_{f_1})$, and y_{b_2} calls are randomly distributed in groups $G(\widehat{X}_{b_1})$ and $G(\widehat{X}_{f_1})$ }.
- $P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})$ = Pr{$\widehat{N}_{f_2}$ WTs are free on the two-hop path | all of the WTs are free on the first hop, $y_{X_{f_1}}$ LCs are free on the second hop, and $z^p_{X_{f_1}}$ calls continue from the first hop to the second hop }.

Multifiber link-load correlation model Let w be the number of considered WTs on a link, $w \leq W$. w is used as a subscript in the expressions of this paper to indicate that the

[4] Since the busy WTs that are occupied completely by continuing calls are in the $G(\widehat{Z}_{c_2})$ group, the number of continuing calls that only partially occupy a WT in other groups are indicated by z^p.

computation of the expressions is on w WTs. The probabilities $P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})_w$, $P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w$, $P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w$, and $P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w$ can be derived using Figure 1 as follows:

$$P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})_w = \begin{cases} 0 & \widehat{Z}_{c_2} + \widehat{X}_{f_1} > W, \\ 0 & y_{f_2} + z_{c_2} > WF, \\ \dfrac{\binom{W}{\widehat{Z}_{c_2}} f(z^p_{c_2}, w, F)}{\binom{wF}{z_{c_2}}} & otherwise, \end{cases} \tag{8}$$

where $z^p_{c_2} = z_{c_2} - \widehat{Z}_{c_2}F$, and $f(z, w, F)$ is the number of ways of distributing z LCs to w WTs such that every WT is free. Note that each WT consists of F fibers. Suppose $j = \lfloor \frac{z}{F} \rfloor$. $f(z, w, F)$ is given by [7]

$$f(z, w, F) = \begin{cases} 0 & z > (w-1)F \\ \binom{wF}{z} & j = 0 \\ \binom{wF}{z} - \sum_{i=1}^{j} \binom{w}{i} f(z - iF, w - i, F) & otherwise. \end{cases} \tag{9}$$

$$P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w = \frac{f(z^p_{X_{b_1}}, \widehat{X}_{b_1}, F) f(z^p_{X_{f_1}}, z^p_{c_2} - \widehat{X}_{b_1}, F)}{f(z^p_{c_2}, w, F)}. \tag{10}$$

$$P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w = \frac{\binom{\widehat{X}_{b_1}F - z^p_{X_{b_1}}}{y_{X_{b_1}}} \binom{\widehat{X}_{f_1}F - z^p_{X_{f_1}}}{y_{b_2} - y_{X_{b_1}}}}{\binom{wF - z^p_{X_{b_1}} - z^p_{X_{f_1}}}{y_{b_2}}}. \tag{11}$$

$$P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w = \sum_{z^p_{N_{b_2}}=0}^{z^p_{X_{f_1}}} \frac{\binom{\widehat{X}_{f_1}}{\widehat{N}_{f_2}} f(z^p_{N_{b_2}}, \widehat{N}_{b_2}, F) g(\widehat{N}_{f_2}, z^p_{X_{f_1}} - z^p_{N_{b_2}}, y_{X_{f_1}} - y_{N_{b_2}})}{f(z^p_{X_{f_1}}, \widehat{X}_{f_1}, F) \binom{\widehat{X}_{f_1}F - z^p_{X_{f_1}}}{y_{X_{f_1}}}}. \tag{12}$$

where $g(\widehat{N}, z, y)$ is the number of ways to distribute z continuing calls and y entering calls to $\widehat{N}$ WTs such that every WT is free. $g(\widehat{N}, z, y)$ is given by

$$g(\widehat{N}, z, y) = \binom{\widehat{N}F}{z} \binom{\widehat{N}F - z}{y} - \sum_{i=1}^{\lfloor \frac{z+y}{F} \rfloor} \sum_{j=0}^{min(iF,z)} \times \binom{\widehat{N}}{i} \binom{iF}{j} g(\widehat{N} - i, z - j, y - (iF - j)).$$

A closed-form expression of $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ is obtained as

$$R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2}) = \sum_{\widehat{Z}_{c_2}=0}^{\lfloor \frac{z_{c_2}}{F} \rfloor} \sum_{z^p_{X_{b_1}}=0}^{z_{c_2} - \widehat{Z}_{c_2}F} \sum_{y_{X_{b_1}}=0}^{(W - \widehat{Z}_{c_2})F - z_{c_2} - y_{f_2}} P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2}) \times P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) \times P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}}). \tag{13}$$

VII. References

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