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PLASMA THEORY APPLIED TO
BALL LIGHTNING

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TABLE OF CONTENTS

I. INTRODUCTION

As indicated by the title, this investigation is concerned with the phenomenon variously called ball lightning, electrical meteor, fireball, and kugelblitz. An explanation for this phenomenon will be of primary importance here; but before entering the heart of the matter, let us develop some background material.

A. General Information

First we shall attempt a composite description of ball lightning, discuss something of its observational history, and tentatively classify types of ball lightning in accordance with observed characteristics of behavior. A translation of a general description by Brand (3) can be found in (6, p. 10).

1. Description

Ball lightning is usually reported as a glowing ball with the following details.

a. Size The mean reported diameter is 20 cm (8, p. 1947 and 23, p. 141). The reported sizes vary from about 3 cm to 1.5 meters in diameter (10, p. 3).

b. Shape Almost invariably the shape is given as that of a ball; although in some cases cited in Brand (3), the kugelblitz was pear shaped; and in case 51 (3), the shape was described as that of an egg. Recently observations were recorded of a toroidal form (14, p. 8).

c. Color The color is usually referred to as reddish (23, p. 141) surrounded by a blue contrast region. It has also been called white, yellow, brown, green, and blue.

d. Brightness The ball is often described as bright and shining. As will be discussed later, one estimate was that a 5 inch to 6 inch diameter sphere was as bright as ten 100 watt light bulbs.

e. Heat Some observers comment on the lack of heat, others make no statement either way, and still others point out noticeable heat effects. In about half of the cases quoted by Brand (3), the heat effects were noticeable.

f. Sound Occasionally this phenomenon is described as noiseless (or no comment is made about the sound), although usually an observer refers to a roaring, hissing, humming, or fluttering noise.

g. Smell An odor of burning sulphur, burning rags, or ozone often is present.

h. Time duration The ball usually lasts 3 to 5 seconds, and has been reported to exist for periods of from 1 second (8, p. 1947) to over 15 minutes (18, p. 31).

i. Stability All of the fireball's characteristics appear to be almost constant during existence. Even upon passing through small openings and cracks, the ball returns to its former characteristics after coming out the other side. However, just after formation some balls emit radial sparks.

j. Termination Usually the ball terminates in an explosion; occasionally it just fades away.

k. Spatial motion Often the ball moves slowly and aimlessly, even in the presence of large metal surfaces and mild breezes. Occasionally the velocity is reported as high as 100 meters per second with definite direction. The speed reported most often is about 1 meter per second.

l. Internal motion Many individuals have reported a rolling or tumbling appearance. This suggests the possibility of internal motion or turbulence (14, p. 8).

2. History

Three good bibliographies covering ball lightning from 1665 to 1960 are Brand (3) for 1665 to 1923, Aniol (1) for 1923 to 1954, and "Ball Lightning Bibliography" (22) for 1950 to 1960. In addition, a good summary article on the status of ball lightning research is provided by Hill (8), and there is also a recent book on the subject by Ritchie (14).

Ball lightning was first described by the poet Lucretius in the year 60 B. C. Since then it has occurred frequently enough to attract attention, yet rarely enough so that few people have seen it. Even knowledge of its existence is not widespread. At one time in the middle ages, it was thought to be the devil himself using visible form in order to scare true Christians.

More recently, with the advent of chemistry a mixture of burning gases was proposed as the underlying mechanism. In the last few years standing-waves, plasma, nuclear processes, and molecular dissociation as well as other mechanisms have been suggested. Until the present time, no mechanism has been proposed which has withstood close analysis from such viewpoints as energy content (14, p. 11), time duration, and stability. A long series of experiments by Nauer (22, p. 9) have failed to substantiate any of the above electrical theories (he did not test the standing-wave theory). Even Kapitza's standing-wave theory has some serious problems as indicated by Tonks (21) and Silberg (18,p. 31).

It should also be pointed out that as late as 1950, Schonland (16, pp. 50-52) suggested that ball lightning was an optical illusion. Obviously, not only better theory is required for explanation of the phenomenon, but also better laboratory experiments to verify the theory.

3. Types of fireballs

Fireballs can be classified in a number of different ways. Some of the more usual are by point of origin, mode of termination, and energy effects.

a. Point of origin   Fireballs originate in two general regions--the upper atmosphere, and near or at the surface of Earth.

Those which originate in the upper atmosphere may or may not move towards Earth. The majority of those observed move towards the ground, but this lack of observation of other directions of motion may be due to the atmospheric obstructions to vertical vision which exist during storms. The speed of the electrical meteors has been estimated as high as 100 meters per second.

Those which originate near or at the surface of Earth usually travel at about 1 meter per second. Quite often they will hover or move aimlessly.

b. Mode of termination In the majority of cases the fireball will terminate with an explosion. The explosion is sometimes quite violent, yielding a shock wave as well as a flare of light and heat.

In the minority of cases, the fireball will just fade away as if all of its energy were expended at an almost constant rate in a stable condition. In fact, these modes of termination might be classified as unstable and stable.

c. Energy effects Perhaps this is the most interesting form of classification. Brand (3) classified fireballs into cold and hot categories, and into harmless and dangerous categories. His cold and harmless categories would fall under weak energy effects, and his hot and dangerous under strong energy effects.

Under weak energy effects come the fireballs which give off very little heat and do little, if any, harm to people or property.

Under strong energy effects are the fireballs which radiate noticeable heat, ignite dry material on contact, burn, injure or kill humans or animals, or terminate in powerful explosions.

Occasionally there are reports of fireballs which exhibit weak characteristics during existence and strong characteristics (electrical, thermal, or mechanical) on termination. This suggests a sudden discharge of remaining energy through a change in mode of operation.

B. States of Matter

In order to better evaluate any mechanism proposed as the method of operation of ball lightning, we should attempt to determine the state of matter within the ball. Therefore, a discussion of various states of matter is in order.

1. Discussion of various states

The ancient Greeks classified the roots of all matter as being earth, water, air, and fire. This was surprisingly accurate when one considers that today the states of matter are believed to be solid, liquid, gas, plasma, "nugas" of nucleons and electrons, and higher energy states (12, p. 1).

a. Solid   This state is characterized by almost constant form and volume.

b. Liquid   This state is characterized by variable form, but almost constant volume.

c. Gas This state has variable form and volume, but is still capable of chemical activity and lies below the energy level of a few ev.

d. Plasma This state is characterized by free electrons and positive ions. The energy range of this state is the widest one, logarithmically, known to man at this time; it stretches from a few ev to 2 Mev.

e. "Nugas" In the range of 2 Mev to 200 Mev, the nucleii have been broken down into nucleons.

f. Higher energy states Above 200 Mev other energy states exist, but it is felt that they are of only academic interest here due to their energy content.

Of the above, plasma is the only state whose energy range encompasses the whole region of energy effects ascribed to ball lightning without taking into account some additional mechanism such as nuclear reactions, electrostatic charge separation, or invisible electric currents. For that reason, the properties of plasma will be briefly reviewed.

Since plasma consists primarily of free electrons and positive ions, Maxwell's equations apply and the plasma is subject to electric and magnetic, as well as thermoelectric and pressure gradient effects. Plasma cannot be contained by matter in a lower energy state. A plasma is capable of generating its own electric and magnetic fields. In addition fluid and momentum equations apply, such as viscosity relationships and Boltzmann's transport equation.

C. State of Matter of Fireball

As discussed above, we are now in a position to determine the state of matter of the fireball provided we assume no continuing external energy source.

1. Energy estimate

Goodlet (6, pp. 32, 55) in 1937 estimated the energy content of a fireball the size of a large orange to be between 0.4×10^7 and 1×10^7 joules. If the diameter were assumed to be 15 cm, the energy density would lie between 2,000 and 6,000 joules/cm^3. This would suggest either an almost completely ionized plasma with few electrons left in the innermost atomic orbits or a singly-ionized plasma with high charged particle energy. A lightning stroke should be more than adequate to produce these conditions.

2. Suggested fireball mechanisms

Since the state of matter of the fireball is also associated with the mechanism involved, it would be advantageous to examine rapidly some of the mechanisms proposed.

a. Chemical combustion (2, 22, p. 9) This approach appears invalid on the grounds of both inadequate energy content and time duration.

b. Electrical luminescence (20) An invisible current of electricity through a conductive gas channel is thought to produce ball lightning at the point where the stroke and counter stroke meet. This solution would not hold up for an occurrence in a steel building, as is cited in Chapter II of this investigation.

c. Eddy current (22, p. 6) Kolobkov has suggested that two oppositely directed lightning strokes could produce eddy currents which would produce the observed phenomenon. This will not explain the single lightning stroke occurrences.

d. Thermonuclear reactions (5) Ball lightning is explained as a form of C^{14} resulting from the action of lightning on atmospheric nitrogen. This would provide too much energy and would also have a serious containment problem. In addition, it could not explain a terminating explosion.

e. Electrons and positive ions bound by atomic forces (22, p. 8) This would provide too restricted an energy region and time duration.

f. Cooled plasma (10) In this theory, the plasma is produced and then cooled. Glowing balls have been produced in the laboratory which are about 1 cm in diameter and last for very short periods of time. Time duration is the problem here as with most plasma explanations. However, it is interesting that the experimental arrangement here suggests the formation of plasmoids (12, p. 157). Since the formation of a plasmoid requires an external magnetic field stronger than that of Earth's, one feels that it is not a valid solution. If the formation of the plasma does not involve an external magnetic field, then the cooled plasma theory would essentially be the same as the theory proposed by Hill (8) and covered under "i" below.

g. Plasma with a magnetic shell (7) Grosu feels that a plasma, which creates its own magnetic field and which is contained in the magnetic shell thus formed, is the most reasonable solution. Such a plasma as he describes, however, would still recombine at too rapid a rate. Also, the betatron type effect he proposes would need an externally produced magnetic field for stability, and would receive its energy from the fluctuating magnetic field.

h. Toroidal plasma (11, p. 51) Ladikov has proposed a mechanism in which the plasma is toroidal in shape, supporting a ring current of up to 200,000 amperes. Unlike Grosu's theory, Ladikov's does not require an external magnetic field, and it does contain all of its required energy at the start. Time duration and lack of ball-like appearance are the problems here.

i. Electromagnetic standing-wave (14, 24) Kapitza's theory also requires a plasma, but it assumes a continual source of energy in the form of a electromagnetic standing-wave. Tonks (21) and Silberg (18) have pointed out some theoretical problems associated with this.

j. Ionized molecules and space charge (8) Hill has avoided most of the problems of rapid plasma recombination rates by assuming ionized molecules. For the energy content necessary, he has assumed space charge separation. However, he has not explained how the space charge separation is maintained for any reasonable period of time. If rapid motion is involved, the relatively low mobilities of ions will prove a serious obstacle to the time duration required. Without rapid motion, the energy content would require such large space charge concentrations that serious doubt exists to the maintenance of the space charge for any significant time interval.

3. Probable state

Based on the above, the most probable state of matter of the fireball is that of plasma. This is because of the energy content involved, and because the explanations which satisfy most of the known conditions involve plasmas. For these reasons, we shall assume for the rest of this investigation that the fireball is in the plasma state.

4. Need for stability condition

The two greatest objections to plasma theories are that the fireball does not expand in size as a non-contained plasma would, and that plasma recombination rates are too rapid. So, if a plasma theory is to be used, it must not only provide for a spherical shape but also for almost constant volume (as did

Grosu's and Ladikov's), and for a low recombination rate.

D. Objective of This Investigation

The objective of this investigation is to postulate an initiating condition which leads to a mechanism satisfying all known facts relevant to ball lightning, to use this mechanism to determine some characteristics of ball lightning, and to examine some of the associated energy losses.

In line with this a specific ball lightning observation will be discussed, an initiating condition will be postulated, applicable basic equations will be derived and solved for initial fireball shape, the fireball structure and characteristics will be examined and the results analyzed to determine the predominant energy-loss mechanism, the energy dependence of the fireball will be examined and the time duration estimated, conclusions will be reached, and recommendations will be made for further study.

II. A SPECIFIC OCCURRENCE

At about 1300 Central Standard Time on 13 October 1960, there occurred an especially interesting demonstration of ball lightning. Since this case has not been reported elsewhere, and since some information in it applies directly to proposed theories, it will be covered here.

A. Case History

Fortunately there was a witness to this case of ball lightning who was willing to supply a statement as to the details involved. It is interesting that this witness was a maid who had not heard of ball lightning until after the event under discussion.

1. Statement

The following is a copy of a statement made by the individual who witnessed this particular fireball phenomenon.*

> The following is information pertinent to a ball lightning observation:
>
> a. Time of observation--about 1300 C on 13 Oct. 1960.
> b. Location--Rm 105, 1st floor, south wing of BOQ #432 on Offutt AFB, Nebraska.
> c. Weather--cloudy and windy.
> d. Structure of building--steel walls and roof, asphalt tile floor laid over concrete.
> e. Location of observer--standing just west of center of room 105. Room dimensions 11 ft N-S by 15 1/2 ft E-W. Remained in same spot for duration of observation.

---

*Mrs. Ann Belleisle Perrelet, Omaha, Nebraska, Fireball observation. Private communication. 1961.

f. Fireball information--
   1. Size...about 5 to 6" in diameter.
   2. Shape ...a fuzzy ball with ragged edges.
   3. Color...all orange except towards edge. Light orange center blended into surrounding light blue with white fringe.
   4. Brightness...as bright as ten 100 watt light bulbs.
   5. Sound...a loud roaring and hissing sound during existence, and a loud exploding noise at termination.
   6. Smell...a sulphur odor and burnt match odor.
   7. Time duration...about 30 seconds.
   8. Stability...all above features remained apparently constant in value during observation.

g. Sequence of events--
   1. Crackling noise followed by hissing and roaring.
   2. Fireball first seen moving slowly from vicinity of wall electric power outlet and phone jack outlet. (just west of center of north wall, about 12" from floor.)
   3. Fireball moved slowly and erratically, floating up into the room to a height of about 4 ft.
   4. Fireball floated to about 1 ft off the floor at east wall of room, climbed up east wall about 1 ft north of door edge (steel door hinged on south edge). Hinges located about 9" from south wall of room.
   5. Fireball at height of about 5 ft moved across wall to door edge.
   6. Fireball jumped to middle of open doorway.
   7. Fireball moved into hallway while settling toward floor.
   8. Fireball exploded near floor.
   9. Floor lightly seared in about a 6" blur under where the fireball burst.

h. Associated information--
   1. The building power supply was 120 volts, a.c.
   2. All the hall lights were on before the observation and were out after the observation. Repairmen found a short in the

light just outside of the room in
which I was standing. The south wing
circuit breaker was also burned out.

3. I was scared stiff, and was nervous and jumpy for days.

The above is true to the best of my knowledge.

Observer
26 June 1961

2. <u>Pertinent information</u>

Since this event occurred on a U. S. Air Force Base, a great deal of help was given by Base organizations in obtaining additional information bearing on the event.

a. <u>Work order information</u> The work order section of Civil Engineering verified that at 1340 C on 13 October 1960, workmen were dispatched to perform emergency repairs to the electrical system of the south wing of BOQ #432. This fixes the fireball time at shortly prior to 1340 C, in agreement with the observer.

b. <u>Weather information</u> The Base Operations Weather personnel keep records of all weather affecting the Base. From 1020 C to 2300 C, on 13 October 1960, there was a thunderstorm weather advisory for within a 25 nautical mile radius of the Base. However, the only thunderstorm to pass over the Base did so from 1503 C to 1525 C, about 2 hours after the fireball observation. There were no records of lightning strikes on the Base between 1000 C and 1500 C. Between 1200 C and 1400 C, the surface winds were southwest at

15 knots gusting to 30 knots, and the ceiling varied from 1000 ft to 4000 ft with light rain showers.

c. Electric power information The Base electrical engineering section said there was no power line surge on 13 October 1960, so it is not probable that lightning struck the power line at that time. In addition, lightning arresters would have protected Building #432 in almost any event of lightning striking a power line. The building itself does not have lightning rods, but it is in an area of very low lightning strike frequency. Also, it was pointed out that Building #432 has had a number of shorts in its lighting system in good weather as well as bad.

B. Points of Interest

As mentioned earlier, there are a few items of special interest with regard to this case.

1. No lightning bolt observed

This is possibly a case in which a lightning bolt did not occur in connection with the ball lightning. If so, it suggests that ball lightning can be produced in more than one way, or at least by some method not requiring a visible lightning stroke. Other cases of ball lightning appearing without an accompanying lightning bolt have been recorded (5, p. 11).

2. No power line surge

This eliminates the possibility that lightning could have struck a power line off Base and entered BOQ #432 through the power distribution system.

3. Steel building

Actually, with regard to the building's walls, only the external walls were steel. The internal walls were approximately 4 ft wide by 7 ft high pressed wood panels in steel frames with all doors, door frames, window frames, and corner posts steel. However, in view of the amount of steel in the building, as well as the fact that all outside walls were steel and the floor of the second story was steel, it is very difficult to see how Kapitza's standing-wave theory could be valid. The motion of the fireball would have required too large a frequency change to be explained by a standing wave in a resonant cavity.

4. Circuit breaker burned out

It is significant that other light shorts in the building have not been associated with fireballs. The only difference that is apparent between those shorts and the one reported above is the burning out of what appeared to be a defective circuit breaker. This raises the possibility that this case of ball lightning might have been produced by the 120 volt a.c. source being shorted out in a particular sequence.

5. Timing

There are only four possibilities of interest here.

a) The shorts were unrelated to the ball lightning.

b) The ball lightning produced the shorts.

c) An initiating event produced both the ball lightning and the shorts.

d) The shorts produced the ball lightning.

That the shorts and the ball lightning were unrelated seems very questionable due to their proximity in time. Since the maid heard crackling before the hissing and roaring, and since the fireball did not go near the lights as far as seen, it is also doubtful that the fireball caused the shorts. This leaves the last two possibilities.

6. Conclusion

Although no weather phenomena were recorded in connection with the fireball, we cannot be certain of their absence. We can only conclude that perhaps the weather phenomena went unseen, or perhaps the short caused the fireball. Therefore, this may be an instance in which a man-made fireball was inadvertently produced.

C. Analysis of Fireball Mechanisms

In view of the case just discussed, we can reach some tentative decisions regarding the validity of various proposed fireball mechanisms in this particular instance. The total light energy expended by the ball was about 30 Kilojoules. This provides a light energy density of about 40 joules/cm^3. Assuming a total energy to light energy ratio of 5 to 1, yields 200 joules/cm^3. It is probable that the energy density was something higher than this. Dissociation and single ionization of all atoms in 1 cubic cm of air takes about 150 joules.

1. Chemical combustion

Chemical mechanisms have too low an energy density.

2. Electrical luminescence

It does not seem amenable to the case above.

3. Eddy current

It cannot explain the occurrence inside the steel building.

4. Thermonuclear reactions

Thermonuclear mechanisms have too high an energy level.

5. Plasma with a magnetic shell

This is reasonable with the steel building, but one would expect the plasma to be trapped in the region of shortest magnetic flux path.

6. Electromagnetic standing-wave

It does not seem amenable to the case above.

7. Ionized molecules and space charge

This source is not stable enough to explain the 30 second time duration, unless the molecules are in rapid motion. In that case, there should not be enough energy provided.

8. Self-contained plasma

This is only feasible if some stability condition is imposed. All considered, this is the most reasonable state of matter for this phenomenon. This is in keeping with the assumption of Chapter I that the fireball was composed of plasma.

III. DERIVATION OF BASIC EQUATIONS

As indicated in the Introduction, this chapter is devoted to derivation of basic plasma equations applicable to shape. The postulated initiating condition for ball-type plasma shape is also included, as well as a preliminary analysis of results to be anticipated for the final solution.

A. Reasons for Delaying Energy Loss Considerations

There are good reasons for both including energy losses at this time and for leaving them out until a later point in the discussion. However, of the two reasons listed below for not including energy losses at this time, the first one alone is of such importance that it overrides all objections.

1. Predominant energy-loss mechanism unknown

The plasma motion resulting from the initial condition is not yet known. It is therefore rather difficult to know whether to apply random, gyrotropic, or some other form of motion to the plasma. Also, it is not known whether to apply conductivity or mobility type concepts. Since it is first necessary to know the plasma shape and motion before attempting to determine the predominant energy-loss mechanism, there is no way to consider energy losses at this time.

2. Verification of shape

Unless the initiating condition leads to a ball-type plasma, there is no reason for pursuing this line of inquiry.

In view of that fact, it is essential at the earliest possible point to verify the plasma shape to be anticipated. Since primarily only the duration of the ball-type plasma (and not the shape) is energy-loss dependent, energy-loss considerations can be delayed until later.

B. Assumptions and Initial Condition

The assumptions involved in this approach were intended to approximate reality as closely as possible. This concept also led to the selection of as simple an initiating event, and thus initial condition, as could be consistent with the desired results. Of the eleven items listed below, the first nine are assumptions; the tenth and eleventh are the two parts of the postulated initial condition.

1. <u>All motion non-relativistic</u>

This should at least apply to low energy fireballs, and will avoid what might be unnecessary mathematical complexities.

2. <u>Equations and data in MKSC units</u>

One set of units is necessary for consistency. The few exceptions will be noted.

3. <u>No energy loss or gain involved</u>

This was discussed above.

4. <u>Maxwell's equations hold</u>

No comment is needed.

5. Conservation of charge holds

In this form, the assumption is meant to guarantee that the number of free electrons minus the number of positive charges is a constant. Divergence, recombination, and charge production will be considered.

6. Boltzmann's transport equation is valid

This equation has yielded the most reliable results in past situations dealing with particle transport.

7. The plasma is electrostatically neutral

Since many observations of this phenomenon have indicated that usually the fireball motion is not greatly affected by metal surfaces in close proximity, it appears that no overall electrostatic charge could exist (except for just after formation, when discharges are sometimes observed). Otherwise, the induced surface charge in the metal would have attracted the fireball. We shall assume overall neutrality from the start.

8. A two-fluid plasma exists

This is intended for mathematical simplicity. It avoids the many equations which would be required to represent each type of fluid involved. One fluid will consist of electrons, the other of one type of positive ion.

9. All atoms are nitrogen

This assumption also is intended for simplicity. Since air is composed primarily of nitrogen, little error should result.

10. Initial condition, part 1

Let us assume that initially a rod of electrical current exists in the air with no sources of potential. At first glance, this may appear to be a strange postulate, but it is reasonable that lightning stroke momentum would continue current flow for a short period of time after neutralization of initiating potential. Also, lightning strokes from other sources could neutralize either one or both sources of potential of the original lightning stroke while it was just starting. In some cases, the lightning stroke might not have even travelled far enough to be recognized as such before its sources of potential were neutralized. For our purposes, let us postulate both sources of potential are neutralized.

11. Initial condition, part 2

Normally the current rod is initiated in very turbulent air. Since the center of a violent swirl of air has much lower pressure then ambient, and since low pressure permits easier ionization, let us assume the current rod to be composed of charged particles whose motion is rotation about the rod's axis, as well as linear flow along the axis.

C. Mathematical Expression of Assumptions

Based on the above, equations can be written expressing the physical condition of the plasma. These equations will be expressed in vector notation.

1. Maxwell's equations hold

The magnetic field does not diverge.

$$\nabla \cdot \vec{B} = 0 \qquad (1)$$

Where

$$\vec{B} = \mu\vec{H} \qquad (2)$$

$\nabla \equiv$ grad

$\vec{B} \equiv$ magnetic induction

$\vec{H} \equiv$ magnetic intensity

$\mu = 4\pi \times 10^{-7}$ newtons/amp^2

So

$$\nabla \cdot \mu\vec{H} = 0 \qquad (3)$$

The electric displacement diverges in the form of Gauss's law.

$$\epsilon \nabla \cdot \vec{E}_1 = \rho_i - \rho_e \qquad (4)$$

Where

$$\vec{D}_1 = \epsilon\vec{E}_1 \qquad (5)$$

$\vec{D}_1 \equiv$ electric displacement due to charge distribution

$\vec{E}_1 \equiv$ electric intensity due to charge distribution

$\epsilon = \frac{10^9}{36\pi}$ coulombs^2/newton-meter^2

$\rho_e \equiv$ electron charge density

$\rho_i \equiv$ positive ion charge density

Subscript e is for electrons

Subscript i is for positive ions

The curl of the magnetic intensity is equal to the current density and the displacement current.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{6}$$

Where

$$\vec{J} = \vec{J}_i + \vec{J}_e \tag{7}$$

$$\vec{J}_i = \rho_i \vec{v}_i \tag{8}$$

$$\vec{J}_e = -\rho_e \vec{v}_e \tag{9}$$

$\vec{J} \equiv$ current density

$$\vec{D} = \vec{D}_1 + \vec{D}_2 \tag{10}$$

$\vec{D}_2 \equiv$ electric displacement due to magnetic induction

$\vec{v}_e \equiv$ average velocity of electrons per unit volume

$\vec{v}_i \equiv$ average velocity of positive ions per unit volume

The curl of the electric intensity is equal to the negative of the time rate of change of the magnetic induction.

$$\nabla \times \vec{E}_2 = -\mu \frac{\partial \vec{H}}{\partial t} \tag{11}$$

Where

$\vec{E}_2 \equiv$ electric field strength due to time rate of change of magnetic induction

2. <u>Conservation of charge holds</u>

The electronic charge leaking into or out of a unit volume in unit time, minus the recombination rate and plus the ionization rate, must equal the rate of change of electronic charge in the volume.

$$- \nabla \cdot \rho_e \vec{v}_e + S_e - R_e = \frac{\partial \rho_e}{\partial t} \tag{12}$$

Where

$S_e \equiv$ production rate of electrons in unit volume

$R_e \equiv$ recombination rate of electrons in unit volume

$$R_e = \alpha \frac{\rho_e \rho_i}{e} \tag{13}$$

$$\frac{\partial \rho_e}{\partial t} = - \nabla \cdot \rho_e \vec{v}_e - \alpha \frac{\rho_e \rho_i}{e} + S_e \tag{14}$$

$\alpha \equiv$ electron-positive ion recombination coefficient

For simplification, at present we will assume

$$S_e = 0.$$

So

$$\frac{\partial \rho_e}{\partial t} = - \nabla \cdot \rho_e \vec{v}_e - \alpha \frac{\rho_e \rho_i}{e} \ . \tag{15}$$

Similarly for positive ions

$$\frac{\partial \rho_i}{\partial t} = - \nabla \cdot \rho_i \vec{v}_i - \alpha \frac{\rho_e \rho_i}{e} \ . \tag{16}$$

The charge in a unit volume is $\rho_i - \rho_e$.

3. <u>The plasma is electrostatically neutral</u>

The integral of the electrostatic charge over the plasma volume is zero.

$$\int_V (-\rho_e)\, dV + \int_V \rho_i \, dV = 0 \tag{17}$$

or

$$\int_V (\rho_i - \rho_e)\, dV = 0 \quad \text{for two fluids.} \tag{18}$$

4. Boltzmann's transport equation is valid

This relationship can be represented as two equations in the two fluid case (12, p. 73; 19, p. 18).

Some consideration might be given to the Newtonian equation

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \vec{v}\frac{dm}{dt} + m\frac{d\vec{v}}{dt}. \tag{19}$$

The total derivatives yield:

$$\frac{dm}{dt} = \frac{\partial m}{\partial t} + \frac{\partial m}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial m}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial m}{\partial z}\frac{\partial z}{\partial t} \tag{20}$$

$$= \frac{\partial m}{\partial t} + \vec{v} \cdot \nabla m \tag{21}$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla\vec{v} \tag{22}$$

and

$$\vec{F} = \frac{\partial m}{\partial t}\vec{v} + (\vec{v} \cdot \nabla m)\vec{v} + m\frac{\partial \vec{v}}{\partial t} + m\vec{v} \cdot \nabla\vec{v}. \tag{23}$$

Now, if m varies slowly with time,

$$\frac{\partial m}{\partial t} \approx 0$$

and if m varies slowly with distance in the direction of $\vec{v}$,

$$(\vec{v} \cdot \nabla m)\vec{v} \approx 0. \tag{24}$$

So

$$\vec{F} = m\frac{\partial \vec{v}}{\partial t} + m\,\vec{v} \cdot \nabla\vec{v} \tag{25}$$

where

$$\vec{F} = +\rho_i[\vec{E} + \vec{v} \times \vec{B}] - \nabla p + \text{resistive terms} \tag{26}$$

for a unit volume positive ion gas, and

$\nabla p \equiv$ pressure gradient.

It is interesting to note that this is just the Boltzmann equation after approximations (19, p. 18).

For electrons

$$\left(\frac{m}{e}\right)\rho_e\left(\frac{\partial \vec{v}_e}{\partial t} + \vec{v}_e \cdot \nabla\vec{v}_e\right) = -(\vec{E} + \vec{v}_e \times \vec{B})\rho_e - \nabla\left(\frac{\rho_e}{e}\,\overline{\vec{v}_e\vec{v}_e}\right). \qquad (27)$$

Now, normally terms for the Hall, thermoelectric, and energy loss effects should be included. The Hall effect is included in $\vec{E}_1$, the thermoelectric effict is included in ∇p, and the energy-loss terms will be disregarded until later. In this case, the ∇p term will be neglected. So, the electron equation becomes

$$\frac{\partial \vec{v}_e}{\partial t} = -\vec{v}_e \cdot \nabla\vec{v}_e - \frac{e}{m}\vec{E} - \frac{e}{m}\vec{v}_e \times \vec{B}. \qquad (28)$$

Similarly, for positive ions

$$\frac{\partial \vec{v}_i}{\partial t} = -\vec{v}_i \cdot \nabla\vec{v}_i + \frac{Ze}{M}\vec{E} + \frac{Ze}{M}\vec{v}_i \times \vec{B}. \qquad (29)$$

D. List of Pertinent Equations

We are now in a position to list the pertinent equations found above. In order, they were:

$$\nabla \cdot \mu\vec{H} = 0 \qquad (3)$$

$$\nabla \cdot \epsilon\vec{E}_1 = \rho_i - \rho_e \qquad (4)$$

$$\nabla \times \vec{H} = \rho_i\vec{v}_i - \rho_e\vec{v}_e + \epsilon\frac{\partial \vec{E}}{\partial t} \qquad (6)$$

$$\nabla \times \vec{E}_2 = -\mu\frac{\partial \vec{H}}{\partial t} \qquad (11)$$

$$\frac{\partial \rho_e}{\partial t} = - \nabla \cdot \rho_e \vec{v}_e - \frac{\alpha}{e} \rho_e \rho_i \tag{15}$$

$$\frac{\partial \rho_i}{\partial t} = - \nabla \cdot \rho_i \vec{v}_i - \frac{\alpha}{e} \rho_e \rho_i \tag{16}$$

$$\int_V (\rho_i - \rho_e) dV = 0 \tag{18}$$

$$\frac{\partial \vec{v}_e}{\partial t} = - \vec{v}_e \cdot \nabla \vec{v}_e - \frac{e}{m} \vec{E} - \frac{e}{m} \vec{v}_e \times \vec{B} \tag{28}$$

$$\frac{\partial \vec{v}_i}{\partial t} = - \vec{v}_i \cdot \nabla \vec{v}_i + \frac{Ze}{M} \vec{E} + \frac{Ze}{M} \vec{v}_i \times \vec{B}. \tag{29}$$

E. Applicable Constants

Numerical values can be ascribed the above constants. The constants involved are ϵ, μ, e/m, e/M, α, and e.

$\epsilon = (36\pi \times 10^9)^{-1}$ coulomb^2/newton-meter^2

$\mu = 4\pi \times 10^{-7}$ newtons/ampere^2

$e/m = 1.759 \times 10^{11}$ coulombs/kg

$e/M = 6.87 \times 10^6$ coulombs/kg

$\alpha = 10^{-13}$ to 10^{-16} meters^3/sec, (13, p. 68). α is not known accurately. It will be assumed that 10^{-13} meters^3/second is the correct value for all electron velocities up to the velocity which is equivalent to twice the ionization potential of the positive ions. For all higher velocities, assume $\alpha = 0$.

$e = 1.602 \times 10^{-19}$ coulombs

F. Analysis of Equations

From the equations listed in Section D, it is apparent that an exact solution would be quite difficult to achieve for any but the most simple cases. If we elect to use a cylindrical geometry to conform with the initial condition of a straight current rod, we can proceed to analyze in stepwise fashion what could be expected as a solution.

1. Reduction to initial condition

In the beginning, a neutrally-charged rod of electrical current exists. There is no electric field in existence. The current rod has produced a strong magnetic field which contains a large portion of the plasma's energy. Electrons travel in the positive z and positive θ directions in the current rod while positive ions travel in the negative z and positive θ directions. The plasma is isotropic in density and energy, and contains no neutral atoms. Therefore, the only forces on the plasma at the start are magnetic constriction and boundary forces (such as density gradient and thermoelectric effect). The plasma will tend to become more dense in the center of the rod.

2. Charge separation

Let us examine Equation 28 more closely, remembering that initially $\vec{E} = 0$ and $\frac{\partial \vec{v}_e}{\partial t} = 0$, except at the boundary.

$$\frac{m}{e}\left(\cancelto{0}{\frac{\partial \vec{v}_e}{\partial t}} + \vec{v}_e \cdot \nabla \vec{v}_e\right) = \cancelto{0}{\vec{E}} - \vec{v}_e \times \vec{B}$$

Since $\vec{v}_e$ already has a given amplitude associated with it, since $-\vec{v}_e \times \mu\vec{H}$ serves to constrict the rod to a smaller radius, by momentum, the electrons at one end of the rod will tend to move out of the rod. The positive ions similarly will tend to move out of the other end of the rod. This will produce an electric dipole opposing the direction of current flow. At this point, we have an electrical current rod of plasma supporting a magnetic field which sustains and contains the rod. The ends of the rod form an electric dipole.

3. Umbrella formation

The region in the vicinity of each electric pole will now be assumed to have reached a steady charge state. In order for the electrons and positive ions to move with about their former velocities, it is necessary for magnetically-induced $\vec{E}_2$ to be at least as great as the $\vec{E}_1$ produced by the electric dipole. So the magnetic field must decay. The magnetic field will also decay as electrons and positive ions recombine. This magnetic field decay causes the remaining charged particles to be accelerated. Then, Equation 11 becomes

$$\nabla \times \vec{E}_2 = -\mu \frac{\partial \vec{H}}{\partial t}$$

and

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \vec{E}_2 .$$

From Equation 28, it is seen at the ends of the rod that although the electrostatic $\vec{E}_1$ and the magnetically-induced $\vec{E}_2$

virtually cancel each other out in the $\vec{z}$ direction (along the rod axis), motion will be induced in the radial direction by the radial component of $\vec{E}_1$.

This means that at each end of the rod, the accumulated charges are splaying out. The effects of the electric dipole and the magnetic field are to cause these charges to assume the distribution of an umbrella at each end of the rod. As seen in Equation 28, each negative charge will attempt to follow an electric flux line from the negative pole to the positive pole, but is arched away from the rod by the magnetic field. Thus, a pair of charge umbrellas are formed at the ends of the rod. Due to the new relative velocities of the charges involved, the negative umbrella is much greater in radius than the positively charge umbrella.

4. Rim juncture

Near the rims of the umbrellas, the magnetic field is weaker and the electric field due to charge separation is not balanced out by the electric field due to magnetic induction. Thus the rims of the umbrellas arch together, roughly forming a ball. This is the shape necessary to warrant further investigation of this approach.

5. Closed cycle

Now that the rims have joined, there is a closed conduction path through the center of the rod, out and around the umbrellas, back to the starting point. This is a plasma vortex, in which the umbrellas are supported by the magnetic field. The collapsing magnetic field provides the magnetic induction necessary for charge separation and for current flow. The charge separation provides the electric field necessary to produce the umbrellas. This is now a closed cycle in which the original electric dipole is replaced by generalized charge separation. The configuration will persist until the magnetic field can no longer provide the induction necessary to supply the energy losses. Although this vortex is similar to the one proposed by Hill as a stable plasma configuration, it is not Hill's vortex. This vortex does not have an externally superimposed magnetic field.

6. Internal structure

Due to the relative difference in velocities which is induced by the collapsing magnetic field, electrons will be constricted far more toward the axis of the current rod by the magnetic field than will be the positive ions. The θ motion of the electrons and positive ions will create a contained magnetic field which will tend to separate the electrons and positive ions; the electrostatic forces will tend to hold the electrons and positive ions intermixed. An electron's position is stable with regard to the rod axis when the magnetic force in towards the axis exactly equals the diffusion, centrifugal, and electrostatic forces out. A positive ion's position is stable when the electrostatic force in is equal to the diffusion, centrifugal, and magnetic forces out. Thus there is a tendency for this system to approach the shape of Ladikov's (11) magneto-vortex ring. A more thorough examination will be necessary to determine the extent of the similarities and differences. At this point, it is possible that nearly complete charge separation will occur in this model (see Figure 1). Columnar charge separation has been observed in linear pinch experiments (25, p. 72), but it has resulted from other causes.

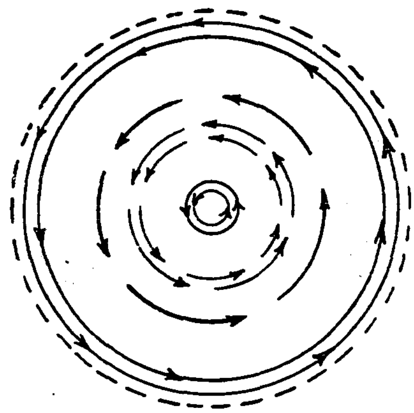
7. External appearance

As discussed above, the plasma might assume a vortex shape of a ball with a rod through it which might also be viewed as a toroid with a zero radius hole in the center. However, the only visible effects of the ball will be seen through photon production. One suspects that most ionization and recombination (and thus photon production) will occur in the shell of the ball. Therefore, the external appearance should be that of a glowing ball, with little rod glow.

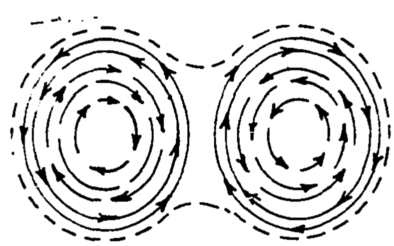
No noticeable magnetic effects should exist at this stage.

Very weak electric effects should exist. Fireballs with high energy should have a much stronger electric separation than fireballs with low energy. So, high energy fireballs should tend to be attracted to metal surfaces (by induced charge effects) much more than low energy fireballs.

Figure 1. Kugelblitz cross sections--flow pattern anticipated from analysis of equations



TOP VIEW



SIDE VIEW

ELECTRON FLOW ——→

POSITIVE ION FLOW →-→

OUTER SURFACE - - - - -

The ionization of oxygen, nitrogen, and other components of air should produce a distinctive odor similar to ozone, burning sulphur, or burning rags.

The ohmic heating as well as the recombination heating of the air should produce noticeable sound on a broad range of frequencies.

The above description sounds strangely like the phenomenon known as ball lightning. However, further steps are necessary to verify this tentative solution and to insure that the time duration and energies involved in the above theory are consistent with the observed phenomenon.

G. Method of Solution

If we were to ignore energy-loss considerations and to go ahead with obtaining an exact mathematical solution at this point, we would find the above equations do not allow separation of variables in any reasonable coordinate system. Due to symmetry, the most obvious choice for solution is the cylindrical coordinate system ($\vec{r}$, $\vec{\theta}$, $\vec{z}$). This reduces us to solving the equations by analysis (as above), by numerical methods, or by a combination of the two. The last method will be used here.

1. Computer

The most straightforward way of obtaining a solution for the initial shape is to program the problem into a computer.

This also has its difficulties, but has the great advantage of yielding accurate, detailed results. Therefore, this will be the method used to verify the initial shape.

2. Analysis

After the initial shape has been verified, various values will be solved for analytically. These include distribution of charge, field configuration, and particle velocities.

3. Need for energy consideration

However, energy losses must be considered. It is possible that the predominant energy-loss mechanism involved with this theory has such a high rate of decay that the fireball would not even form, much less last for any reasonable period of time. Thus energy will be considered after the fireball shape, internal structure, and characteristics have been determined.

IV. SOLUTION OF EQUATIONS

A computer must be used for solution of the basic equations for initial plasma shape. This requires expressing the initial conditions in more explicit form, rewriting the equations in a form which can be better handled, writing the necessary computer program, and interpreting the solutions obtained. These will lead to an intermediate condition shape. Then the kugelblitz intermediate shape will be used to determine intermediate internal structure, and characteristics. These will lead to a stable condition shape. The kugelblitz stable shape will be used to determine stable internal structure and characteristics.

A. Mathematical Expression of Initial Condition

Part 1 of the initial condition postulated in Chapter III was that a rod of electrical current instantaneously existed in the air with no sources of potential. This postulate must be made more explicit for computer use. Therefore, the few facts known about fireballs must form a basis for various combinations of ionization levels and volumes, and for electrical current magnitudes and particle velocities. Then, computer solutions will be obtained for the most reasonable of these possible combinations. If one of these solutions provides an accurate shape for the external appearance of the fireball, that combination of initial condition and computer solution will be used to determine the predominent energy-loss mechanism. We shall neglect Part 2 of the initial condition until later.

1. Plasma condition

In order to simplify the mathematics, it will be assumed that the energy of the lightning bolt not only creates a plasma rod, but also imparts to this plasma rod both kinetic and inductive energy. The electrons will have a velocity vector parallel to the plasma rod core, while the positive ions will also have a velocity vector parallel to the core but opposite in direction to the electrons. Overall charge neutrality must be taken into account at this point. In this way Equation 18 of Section III-D will be satisfied for the initial condition.

a. Plasma energy The average energy of a lightning bolt is 10^{10} joules (9, p. 3372) and the average length is 6,000 meters. So, the energy per unit length is about 1.67 x 10^6 joules/meter. And since the average fireball diameter is 15 cm, we will observe that the lightning bolt energy per 15 cm length is about 2.5 x 10^5 joules. This is somewhat less than the 4 x 10^6 to 10^7 joules estimated by Goodlet (6, pp. 32, 55) for a specific case, but it is close to the value estimated (1.5 x 10^5 joules) for the particular case history discussed in Chapter II. It might be noted that Goodlet's fireball would be classified under "strong energy effects", while the case cited in this investigation would fall under "weak energy effects". The difference in energy density between maximum energy and average energy lightning bolts could easily account for this apparent discrepancy. For our purposes here, we shall at first use the most accurate estimate of energy and size available (Goodlet's) and assume that the initial energy is about 5×10^6 joules.

b. Ionization energy Let us now find the ionization energy, W_I, required for 28 different conditions. These 28 different conditions will consist of the basic 7 different levels of ionization possible in nitrogen, with 4 different volumes of air. The ionization potentials of nitrogen are 14.48, 29.47, 47.40, 77.0, 97.4, 130.0 (estimated), and 150.0 (estimated) ev (9, p. 2546). The last two values were not given in the source referenced. The number of atoms per cubic meter of air are approximately $\rho = 5.38 \times 10^{25}$ at S.T.P., and $\rho_e = 8.61 \times 10^6$ coulombs/meter^3. Thus, in cylinders of air of lengths $\ell = 14, 12, 10, 8$ cm and diameters $D = 14, 12, 10, 8$ cm where $\ell = D$, there are $N = 1.16 \times 10^{23}$, 7.30×10^{22}, 4.23×10^{22}, and 2.16×10^{22} atoms respectively. Then

$$W_I = W_i \times N$$

where

$$W_i \equiv \text{ionization energy per atom.}$$

The range of energies in Table 1 is obviously adequate for both weak and strong energy effects. However, it would not be

Table 1. Ionization energies of plasma

Z	W_i (joules)	W_I (joules): ℓ = D = 14	12	10	8 cm
1	2.32×10^{-18}	2.69×10^5	1.69×10^5	9.81×10^4	5.01×10^4
2.	7.04	8.16	5.14	2.98×10^5	1.52×10^5
3.	1.46×10^{-17}	1.69×10^6	1.07×10^6	6.18	3.16
4.	2.69	3.12	1.97	1.14×10^6	5.81
5.	4.25	4.93	3.10	1.80	9.18
6.	6.34	7.35	4.63	2.68	1.37×10^6
7.	8.75	1.01×10^7	6.39	3.70	1.78

reasonable to assume that the plasma could support a particular ionization level unless less than 1/3 of the total plasma energy ($W_T = 5 \times 10^6$ joules) were in the form of ionization energy, W_I. Therefore, only those values of W_I above the line through Table 1 will be considered ($W_I \leq 1.67 \times 10^6$ joules). By having at least twice as much energy involved in charged particle motion as is involved in ionization, it is believed that the ionization level will be supported for some noticeable period of time.

2. Electric current

Assuming that all of the energy difference between ionization energy and total energy is in the form of inductive energy, we have

$$W_L = W_T - W_I \tag{30}$$

where

$$W_L = \tfrac{1}{2} LI^2. \tag{31}$$

Now

$$L = 2 \times 10^{-7} \ell [\ln \frac{4\ell}{D} - 0.75 + \frac{D}{2\ell}] \tag{32}$$

henries (9, p. 3257).

a. Plasma rod inductance From the dimensions of the plasma rod, the values for L can be found.

Table 2. Plasma rod inductance

$\ell = D$ (cm)	$[\ln \frac{4\ell}{D} - 0.75 + \frac{D}{2\ell}]$	L (henries)
14	1.14	3.19×10^{-8}
12		2.74
10		2.28
8		1.82

b. Inductive energy If we assume that $W_L = W_T - W_I$ as stated above, then the electrical current can be found by noting that

$$I = [\frac{2W_L}{L}]^{\frac{1}{2}}. \tag{33}$$

Table 3. Plasma rod currents

Z	ℓ=D (cm)	W_I (joules)	$W_L = W_T - W_I$ (joules)	I (amperes)
1	14	2.69×10^5	4.73×10^6	1.72×10^7
	12	1.69	4.83	1.88
	10	9.81×10^4	4.90	2.07
	8	5.01	4.95	2.33

Table 3. (Continued)

Z	ℓ=D (cm)	W_I (joules)	$W_L = W_T - W_I$ (joules)	I (amperes)
2	14	8.16×10^5	4.18×10^6	1.62×10^7
	12	5.14×10^5	4.49×10^6	1.81×10^7
	10	2.98	4.70	2.03
	8	1.52	4.85	2.31
3	12	1.07×10^6	3.93	1.69
	10	6.18×10^5	4.38	1.96
	8	3.16	4.68	2.27
4	10	1.14×10^6	3.86	1.84
	8	5.81×10^5	4.42	2.20
5	8	9.18	4.08	2.12
6	8	1.37×10^6	3.63	2.00

It is interesting to note that all of the above currents lie between 1.62×10^7 and 2.33×10^7 amperes. This is somewhat in excess of 2.5×10^5 amperes, the observed maximum stepped-leader portion of lightning strokes (15, p. 78). However, it is not inconceivable that irregularities could exist within the lightning bolt so that an electron avalanche could be produced over a limited distance, as it is in the Geiger-Muller voltage range of an ionization tube. This would yield the necessary high current.

<u>c. Effective masses</u> Because of the energy storage capability of the electromagnetic field, each electron and positive ion has an effective mass somewhat larger than its rest mass. Let us observe that the total motion energy, W_m,

of a electron is

$$W_m = \tfrac{1}{2} m v_e^2 + \frac{W_{Le}}{ZN} \tag{34}$$

where

$W_{Le} \equiv$ inductive energy of electrons.

Also, for a positive ion,

$$W_m = \tfrac{1}{2} M v_i^2 + \frac{W_{Li}}{N} . \tag{35}$$

Then

$$W_m = \tfrac{1}{2} m v_e^2 + \frac{L I_e^2}{2ZN} \tag{36}$$

$$= \tfrac{1}{2} m v_e^2 + \frac{L}{2ZN} \left[\rho_e \pi \frac{D^2}{4} v_e\right]^2 . \tag{37}$$

Therefore, the effective electron mass, m_e, is

$$m_e = m + \frac{L}{ZN} Z^2 e^2 \rho^2 \pi^2 \frac{D^4}{16} \tag{38}$$

$$= m + \frac{LZe^2}{16N} [\pi \rho D^2]^2 . \tag{39}$$

Similarly

$$M_i = M + \frac{L}{N} \left[Ze\rho\pi \frac{D^2}{4}\right]^2 \tag{40}$$

$$= M + \frac{LZ^2e^2}{16N} [\pi \rho D^2]^2 . \tag{41}$$

For each positive charge then, the mass appears to be

$$\frac{M_i}{Z} = \frac{M}{Z} + \frac{LZe^2}{16N} [\pi \rho D^2]^2 . \tag{42}$$

So the difference between the real mass and the effective mass of an electron is almost exactly equal to the difference between the real mass and the effective mass of a positive charge. A simple calculation shows that in both cases the real mass is of negligible size, compared to this "inductive mass".

At this point, it is important to recall that the plasma energy was obtained through impulse.

$$F\Delta t = \Delta(mv) \tag{43}$$

So, the positive charges tend to acquire about the same initial speed as the electrons, and therefore have as much inductive energy initially. Later, momentum transfer and other mechanisms will change this relationship.

<u>d. Particle velocities</u> Let us assume that effectively half of the plasma energy, other than ionization energy, is contained in electron flow. Let us further assume, as above, that this energy is in inductive form. Then, the average electron and positive ion speeds can be found (note that we must now eventually show that electron kinetic energy and positive ion kinetic energy are negligible). Since the number of atoms per volume under consideration are known, the electron speed is

$$v_e = \frac{\ell I}{2NZe} . \tag{44}$$

The particle velocities are rather small, and might be thought of as drift velocities. Now, such small velocities would not be sufficient to cause the avalanche ionization spoken of, but if we select our volumes to allow sufficiently large particle velocities, the resultant initial conditions would be much more selective. It is felt that the computer results will be more valid with a crude initial condition which, it is hoped, the computer will refine into the anticipated rod and ball

Table 4. Particle velocities

Z	ℓ=D (cm)	N	I (amperes)	$v_e = v_i$ (meters/second)
1	14	1.16×10^{23}	1.72×10^{7}	64.9
	12	7.30×10^{22}	1.88	96.6
	10	4.23	2.07	153.
	8	2.16	2.33	263.
2	14	1.16×10^{23}	1.62	30.6
	12	7.30×10^{22}	1.81	50.8
	10	4.30	2.03	75.0
	8	2.16	2.31	130.
3	12	7.30	1.69	29.0
	10	4.30	1.96	48.3
	8	2.16	2.27	85.5
4	10	4.30	1.84	34.0
	8	2.16	2.20	62.1
5	8	2.16	2.12	47.9
6	8	2.16	2.00	37.6

shape, utilizing the basic equations given.

3. Negligible energies

At this point, we can investigate the importance of electron kinetic energy, W_{eK}, and positive ion kinetic energy, W_{iK}. Each of them must be something less than 10^3 joules if they are to be negligible.

a. Electron kinetic energy   The total kinetic energy contained in the electron's motion is

$$W_{eK} = \tfrac{1}{2} Nmv_e^2 \ . \tag{45}$$

Now

$$v_e^2 < (264)^2 < 7 \times 10^4 \tag{46}$$

$$m = 9.1 \times 10^{-31} \text{ Kg}$$

$$N < 4 \times 10^{23}.$$

So

$$W_{eK} < \tfrac{1}{2}(4 \times 10^{23})(9.1 \times 10^{-31})(7 \times 10^4) \tag{47}$$

$$< 1.4 \times 10^{-2} \text{ joules } < 10^3 \text{ joules.} \tag{48}$$

Thus, electron kinetic energy is negligible.

b. Positive ion kinetic energy Similarly for positive ions

$$W_{iK} = \tfrac{1}{2}NMv_i^2. \tag{49}$$

Now

$$v_i^2 < 7 \times 10^4 \tag{50}$$

$$M = 25,600 \times 9.1 \times 10^{-31} \text{ Kg}$$

$$N < 4 \times 10^{23}$$

$$W_{iK} < \tfrac{1}{2}(4 \times 10^{23})(2.56 \times 10^4)(9.1 \times 10^{-31})(7 \times 10^4) \tag{51}$$

$$< 3.5 \times 10^2 \text{ joules } < 10^3 \text{ joules.} \tag{52}$$

Thus, positive ion kinetic energy is negligible.

4. Selected initial conditions

Rather than investigate each initial condition given in Table 4, a few will be extracted and the results used to indicate the proper type of initial condition needed. These few are listed below.

Table 5. Selected initial conditions

Z	$\frac{\ell}{2} = \frac{D}{2}$ (meters)	$\rho_e = \rho_i$ (coulombs/meter^3)	$\lvert v_e \rvert = \lvert v_i \rvert$ (meters/second)
1	0.06	8.61×10^6	96.6
	0.04		263.
3	0.06	2.58×10^7	29.0
	0.04		85.5

B. Reduction of Basic Equations

The first step is to express the vector equations involved in incremental form. Then the equations can be further changed into cylindrical form. Symmetry can be applied to reduce the number of terms involved; and the final listing can be made of applicable, cylindrical-coordinate equations. For this portion of the solution, only Part 1 of the initial condition will be used.

1. Vector equations

For computer purposes, it will also be necessary to express the relationships of Chapter III, Section D, in summation form wherever possible. Also, Maxwell's equations will be used in their integral (instead of differential) form since this will avoid the requirement of adding integration constants to the initial condition.

$\vec{E}_1$ can be expressed as

$$\vec{E}_1 = \frac{1}{4\pi\epsilon} \sum_V \frac{(\rho_i - \rho_e)\Delta V}{g^3} \vec{g} \tag{53}$$

where

$\vec{g} \equiv$ distance from ΔV to test charge.

$\vec{E}_2$ is more difficult to determine. First, the electromotive force, δ, around an incremental area must be found

$$\Delta\delta_r = -\mu \frac{\Delta\vec{H}_r}{\Delta t} \cdot \Delta\vec{S}_r \tag{54}$$

where

$\Delta\vec{S}_r$ = incremental vectorial area.

Then, the electromotive force on each side of $\Delta\vec{S}_r$ can be approximated as $\Delta\delta_r$ divided by the perimeter, $P_r = 2(\Delta r+\Delta z)$. (55)
Also, the same must be done for $\Delta\vec{S}_{r-\Delta r}$, which has a common side, ξ_r, with $\Delta\vec{S}_r$. These ($\Delta\delta_r$ and $\Delta\delta_{r-\Delta r}$) must be vectorially added. So, for the common side, ξ_r,

$$(\vec{E}_2)_\xi = \frac{\Delta\delta_{r+\Delta r}}{P_{r+\Delta r}} \left(\frac{\vec{\xi}_{r+\Delta r}}{|\vec{\xi}_{r+\Delta r}|}\right) + \frac{\Delta\delta_{r-\Delta r}}{P_{r-\Delta r}} \left(\frac{\vec{\xi}_{r-\Delta r}}{|\vec{\xi}_{r-\Delta r}|}\right) \tag{56}$$

where

$$\vec{\xi}_{r-\Delta r} = -\vec{\xi}_r$$

and

$$(\vec{E}_2)_\xi = \mu\left[\frac{\Delta\vec{H}_{r+\Delta r}}{\Delta t} \cdot \frac{\Delta\vec{S}_{r+\Delta r}}{P_{r+\Delta r}} - \frac{\Delta\vec{H}_{r-\Delta r}}{\Delta t} \cdot \frac{\Delta\vec{S}_{r-\Delta r}}{P_{r-\Delta r}}\right]\left(\frac{\vec{\xi}_{r-\Delta r}}{|\vec{\xi}_{r-\Delta r}|}\right). \tag{57}$$

When this is done for the three directions consistent with the coordinate system used and in the same location, the total $\vec{E}_2$ is approximately the vectorial sum in the three directions. This, of course, is only true if $\vec{E}_2$ does not vary rapidly with respect to the lengths of the incremental sides used. We assume here that $\vec{E}_2$ is slow-varying.

The electric field is

$$\vec{E}_\xi = (\vec{E}_1)_\xi + (\vec{E}_2)_\xi . \tag{58}$$

$\vec{H}$ can be found in straightforward fashion since

$$\vec{H} = \frac{1}{4\pi} \sum_{v} \Delta V \left[\frac{(\rho_i \vec{v}_i - \rho_e \vec{v}_e + \epsilon \dot{\vec{E}}) \times \vec{g}}{g^3} \right] . \tag{59}$$

In the later stages, provided the solutions approach steady state, $\vec{E}$ will approach a constant value. This suggests that the above equation could be simplified by assuming $\dot{\vec{E}} = \frac{\partial \vec{E}}{\partial t} = 0$. That will be done here.

The continuity equation has been broken into two equations. By applying the principle that the change in the number of charges in an incremental volume is the amount moving in per unit time minus the amounts moving out and lost per unit time, we have

$$\rho_e = (\rho_e)_0 + \Delta\rho_e \tag{60}$$

where

$$\Delta\rho_e = - \left[\frac{1}{r} \left\{ \frac{\Delta(r\rho_e v_{er})}{\Delta r} + \frac{\Delta(\rho_e v_{e\theta})}{\Delta\theta} \right\} + \frac{\Delta(\rho_e v_{ez})}{\Delta z} + \frac{\alpha}{e} \rho_e \rho_i \right] \Delta t. \tag{61}$$

Similarly for ρ_i

$$\rho_i = (\rho_i)_0 + \Delta\rho_i \tag{62}$$

where subscript 0 indicates time Δt prior to present, and

$$\Delta\rho_i = - \left[\frac{1}{r} \left\{ \frac{\Delta(r\rho_i v_{ir})}{\Delta r} + \frac{\Delta(\rho_i v_{i\theta})}{\Delta\theta} \right\} + \frac{\Delta(\rho_i v_{iz})}{\Delta z} + \frac{\alpha}{e} \rho_e \rho_i \right] \Delta t. \tag{63}$$

Overall charge must be neutral, so

$$\sum_{v} \rho_i \Delta V = \sum_{v} \rho_e \Delta V. \tag{64}$$

The electron transport equation becomes

$$\frac{\Delta \vec{v}_e}{\Delta t} = -\vec{v}_e \cdot \nabla \vec{v}_e - \frac{e}{m} \vec{E} - \frac{e\mu}{m} \vec{v}_e \times \vec{H}. \tag{65}$$

Similarly, for the positive ion equation

$$\frac{\Delta \vec{v}_i}{\Delta t} = -\vec{v}_i \cdot \nabla \vec{v}_i + \frac{eZ}{M} \vec{E} + \frac{Ze\mu}{M} \vec{v}_i \times \vec{H}. \tag{66}$$

2. Expression in cylindrical coordinates

The above vectorial equations must be rewritten in cylindrical form with provisions made for the fact that this three dimensional problem will be solved on a two dimensional grid. The orientation of the two dimensional grid is such that $\theta_j = 0$. Symmetry will be of great aid. The best way to provide for the three dimensional effect is to solve the grid separately for distance, g, and orthogonality factors, to place these values in storage, and call them from storage as needed. That will be done here.

To provide both reasonable computational time and adequate display of shape of solution, the grid will be limited to 50 points, 32 points of which carry active information.

In line with that, the basic equations will be expressed in 50 point grid form. For the 50 point grid, each incremental volume will consist of

$$\Delta r = 0.020 \text{ meters}$$

$$\Delta z = 0.020 \text{ meters}$$

$$\Delta\theta = 1.2566 \text{ radians.}$$

The section of the plane selected will be 10 cm wide in the r direction and 20 cm long in the z direction. The volume will be divided into 5 equal sections in the θ direction with the first incremental volume having its center point at $\theta = 0$.

a. Distance and orthogonality factors Since the orthogonality factors will be used many times, they will be solved for separately. The distance, g_{jkl}, between the j^{th} point and k^{th} point is

$$g_{jkl} = [(r_j - r_k \cos\theta_l)^2 + (z_j - z_k)^2 + (r_k \sin\theta_l)^2]^{1/2}. \quad (67)$$

The orthogonality factors and inverse-square distance effects can be expressed as one term, G_{jk}.

$$(G_{jk})_r = \sum_l \frac{(r_j - r_k \cos\theta_l)}{g_{jkl}^3} \quad (68)$$

$$(G_{jk})_z = (z_j - z_k) \sum_l g_{jkl} \quad (69)$$

$$(G_{jk})_\theta = 0 \quad (70)$$

due to symmetry. Then,

$$(G_{jk})_r = \frac{(r_j - r_k)}{[(r_j - r_k)^2 + (z_j - z_k)^2]^{3/2}} \quad (71)$$
$$+ \frac{2(r_j - 0.30902r_k)}{[(r_j - 0.30902r_k)^2 + (z_j - z_k)^2 + (0.95106r_k)^2]^{3/2}}$$
$$+ \frac{2(r_j + 0.80902r_k)}{[(r_j + 0.80902r_k)^2 + (z_j - z_k)^2 + (0.58779r_k)^2]^{3/2}}$$

and

$$(G_{jk})_z = (z_j - z_k)\Big\{[(r_j - r_k)^2 + (z_j - z_k)^2]^{-3/2} + 2[(r_j - 0.30902r_k)^2 + (z_j - z_k)^2 + (0.95106r_k)^2]^{-3/2} + 2[(r_j + 0.80902r_k)^2 + (z_j - z_k)^2 + (0.58779r_k)^2]^{-3/2}\Big\}. \tag{72}$$

b. Electrostatic field vector The electrostatic field vector, $\vec{E}_1$, at the j^{th} point becomes

$$(E_{ij})_r \vec{a}_r + (E_{ij})_\theta \vec{a}_\theta + (E_{ij})_z \vec{a}_z$$
$$= \frac{1}{4\pi\epsilon} \sum_k (\rho_{ik} - \rho_{ek})\Delta r_k(r_k\Delta\theta_k)\Delta z_k[(G_{jk})_r \vec{a}_r + (G_{jk})_z\vec{a}_z]. \tag{73}$$

This equation is now broken into two equations.

$$(E_{ij})_r = 4.5239 \times 10^6 \sum_k (\rho_i - \rho_e)_k\, r_k\, (G_{jk})_r \tag{74}$$

$$(E_{ij})_z = 4.5239 \times 10^6 \sum_k (\rho_i - \rho_e)_k\, r_k\, (G_{jk})_z \tag{75}$$

c. Electromagnetic field vector The magnetically generated electric field is now not too difficult to obtain when one recalls that initially the only charge motion is in the z direction. Also, as seen from the motion equations, only motion in the r and z directions will occur. Thus, $\vec{H}$ must always have only a θ component because of symmetry. The field cancels itself in the other directions. Due to the selection of the grid, the normal to each grid surface element will have only a θ component. So

$$(E_{2j})_r = \frac{\mu}{2} \left(\frac{\Delta r_j \ \Delta z_j}{\Delta r_j + \Delta z_j} \right) \left[\frac{H_{j+\Delta z} - (H_{j+\Delta z})_0 - H_{j-\Delta z} + (H_{j-\Delta z})_0}{\Delta t} \right] \tag{76}$$

$$(E_{2j})_z = -\frac{\mu}{2} \left(\frac{\Delta r_j \ \Delta z_j}{\Delta r_j + \Delta z_j} \right) \left[\frac{H_{j+\Delta r} - (H_{j+\Delta r})_0 - H_{j-\Delta r} + (H_{j-\Delta r})_0}{\Delta t} \right] \tag{77}$$

with

$$(E_{2j})_\theta = 0 \tag{78}$$

from symmetry. Then,

$$(E_{2j})_r = 6.2832 \times 10^{-9} \left[\frac{H_{j+\Delta z} - (H_{j+\Delta z})_0 - H_{j-\Delta z} + (H_{j-\Delta z})_0}{\Delta t} \right] \tag{79}$$

$$(E_{2j})_z = -6.2832 \times 10^{-9} \left[\frac{H_{j+\Delta r} - (H_{j+\Delta r})_0 - H_{j-\Delta r} + (H_{j-\Delta r})_0}{\Delta t} \right]. \tag{80}$$

d. Electric field The electric field is

$$(E_j)_r = (E_{1j})_r + (E_{2j})_r \tag{81}$$

$$(E_j)_z = (E_{1j})_z + (E_{2j})_z. \tag{82}$$

e. Magnetic field vector The magnetic field can be resolved in one equation as pointed out earlier. By use of symmetry

$$\vec{H}_j = \frac{1}{4\pi} \sum_k r_k \Delta r_k \Delta\theta_k \Delta z_k \Big\{ [\rho_{ik}(v_{ikr}\vec{a}_r + v_{ikz}\vec{a}_z)$$

$$- \rho_{ek}(v_{ekr}\vec{a}_r + v_{ekz}\vec{a}_z)]^{\times}[(G_{jk})_r\vec{a}_r + (G_{jk})_z\vec{a}_z] \Big\} \tag{83}$$

and, since

$$(H_j)_r = (H_j)_z = 0, \tag{84}$$

$$(E_{2j})_r = \frac{\mu}{2} \left(\frac{\Delta r_j \ \Delta z_j}{\Delta r_j + \Delta z_j} \right) \left[\frac{H_{j+\Delta z} - (H_{j+\Delta z})_0 - H_{j-\Delta z} + (H_{j-\Delta z})_0}{\Delta t} \right] \tag{76}$$

$$(E_{2j})_z = -\frac{\mu}{2} \left(\frac{\Delta r_j \ \Delta z_j}{\Delta r_j + \Delta z_j} \right) \left[\frac{H_{j+\Delta r} - (H_{j+\Delta r})_0 - H_{j-\Delta r} + (H_{j-\Delta r})_0}{\Delta t} \right] \tag{77}$$

with

$$(E_{2j})_\theta = 0 \tag{78}$$

from symmetry. Then,

$$(E_{2j})_r = 6.2832 \times 10^{-9} \left[\frac{H_{j+\Delta z} - (H_{j+\Delta z})_0 - H_{j-\Delta z} + (H_{j-\Delta z})_0}{\Delta t} \right] \tag{79}$$

$$(E_{2j})_z = -6.2832 \times 10^{-9} \left[\frac{H_{j+\Delta r} - (H_{j+\Delta r})_0 - H_{j-\Delta r} + (H_{j-\Delta r})_0}{\Delta t} \right]. \tag{80}$$

d. <u>Electric field</u> The electric field is

$$(E_j)_r = (E_{1j})_r + (E_{2j})_r \tag{81}$$

$$(E_j)_z = (E_{1j})_z + (E_{2j})_z . \tag{82}$$

e. <u>Magnetic field vector</u> The magnetic field can be resolved in one equation as pointed out earlier. By use of symmetry

$$\vec{H}_j = \frac{1}{4\pi} \sum_k r_k \Delta r_k \Delta\theta_k \Delta z_k \Big\{ [\rho_{ik}(v_{ikr}\vec{a}_r + v_{ikz}\vec{a}_z)$$

$$- \rho_{ek}(v_{ekr}\vec{a}_r + v_{ekz}\vec{a}_z)]^{\times}[(G_{jk})_r\vec{a}_r + (G_{jk})_z\vec{a}_z] \Big\} \tag{83}$$

and, since

$$(H_j)_r = (H_j)_z = 0, \tag{84}$$

$$H_{j\theta} = \frac{\Delta r_k \Delta\theta_k \Delta z_k}{4\pi} \sum_k r_k \Big\{ [\rho_{ik} v_{ikz} - \rho_{ek} v_{ekz}] \, (G_{jk})_r$$

$$+ [\rho_{ek} v_{ekr} - \rho_{ik} v_{ikr}] \, (G_{jk})_z \Big\}. \tag{85}$$

So

$$H_{j\theta} = 4 \times 10^{-5} \sum_k r_k \Big\{ [\rho_{ik} v_{ikz} - \rho_{ek} v_{ekz}] (G_{jk})_r$$

$$+ [\rho_{ek} v_{ekr} - \rho_{ik} v_{ikr}] \, (G_{jk})_z \Big\}. \tag{86}$$

<u>f. Electron continuity equation</u> The electron continuity equation also is simplified through symmetry about the z axis.

$$\rho_{ej} = (\rho_{ej})_0 - \Big\{ \frac{1}{2r_j \, \Delta r_j} [r_{j+\Delta r} \, (\rho_{ej+\Delta r})_0 (v_{ej+\Delta r})_r$$

$$- r_{j-\Delta r} (\rho_{ej-\Delta r})_0 \, (v_{ej-\Delta r})_r]$$

$$+ \frac{1}{2\Delta z_j} [(\rho_{ej+\Delta z})_0 (v_{ej+\Delta z})_z - (\rho_{ej-\Delta z})_0 (v_{ej-\Delta z})_z] \Big\} \Delta t$$

$$- \frac{\alpha}{e} \, (\rho_e \rho_i)_0 \, \Delta t \tag{87}$$

So,

$$\rho_{ej} = (\rho_{ej})_0 - \Big\{ \frac{2.5 \times 10^1}{r_j} [r_{j+\Delta r} (\rho_{ej+\Delta r})_0 (v_{ej+\Delta r})_r$$

$$- r_{j-\Delta r} (\rho_{ej-\Delta r})_0 (v_{ej-\Delta r})_r] + 2.5 \times 10^1 [(\rho_{ej+\Delta z})_0 (v_{ej+\Delta z})_z$$

$$- (\rho_{ej-\Delta z})_0 (v_{ej-\Delta z})_z] \Big\} \Delta t - \frac{\alpha}{e} \, (\rho_e \rho_i)_0 \, \Delta t \tag{88}$$

where for $z = 1$, $\frac{\alpha}{e} = 6.242 \times 10^5$ for $v_{ej} < 3.05 \times 10^6$ meters/sec

$= 0$ for $v_{ej} \geq 3.05 \times 10^6$ meters/sec

and for $z = 3$, $\frac{\alpha}{e} = 6.242 \times 10^5$ for $v_{ej} < 4.35 \times 10^6$ meters/sec

$= 0$ for $v_{ej} \geq 4.35 \times 10^6$ meters/sec.

g. Positive ion continuity equation   Similarly for the positive ion continuity equation,

$$\rho_{ij} = (\rho_{ij})_0 - \Big\{ \frac{2.5 \times 10^1}{r_j} [r_{j+\Delta r}(\rho_{ij+\Delta r})_0 (v_{ij+\Delta r})_r$$

$$- r_{j-\Delta r}(\rho_{ij-\Delta r})_0 (v_{ij-\Delta r})_r] + 2.5 \times 10^1 [(\rho_{ij+\Delta z})_0 (v_{ij+\Delta z})_z$$

$$- (\rho_{ij-\Delta z})_0 (v_{ij-\Delta z})_z] \Big\} \Delta t - \frac{\alpha}{e} (\rho_{ej}\rho_{ij})_0 \Delta t. \qquad (89)$$

h. Overall charge   Overall charge must be ~~neutral~~.

$$\sum_j (\rho_{ij} - \rho_{ej}) r_j \Delta r_j \Delta\theta_j \Delta z_j = 0 \qquad (90)$$

i. Electron transport equation   The electron transport equation is

$$\frac{\Delta v_{ejr}}{\Delta t} \vec{a}_r + \frac{\Delta v_{ej\theta}}{\Delta t} \vec{a}_\theta + \frac{\Delta v_{ejz}}{\Delta t} \vec{a}_z + v_{ejr} \frac{\Delta \vec{a}_r}{\Delta t} + v_{ej\theta} \frac{\Delta \vec{a}_\theta}{\Delta t} + v_{ejz} \frac{\Delta \vec{a}_z}{\Delta t}$$

$$= -(v_{ejr} \frac{\Delta}{\Delta r_j} + \frac{v_{ej\theta}}{r_j} \frac{\Delta}{\Delta\theta_j} + v_{ejz} \frac{\Delta}{\Delta z_j})(v_{ejr}\vec{a}_r + v_{ej\theta}\vec{a}_\theta$$

$$+ v_{ejz}\vec{a}_z) - \frac{e}{m} (E_{jr}\vec{a}_r + E_{j\theta}\vec{a}_\theta + E_{jz}\vec{a}_z)$$

$$- \frac{e\mu}{m} [(v_{ej\theta}H_{jz} - v_{ejz}H_{j\theta})\vec{a}_r + (v_{ejz}H_{jr} - v_{ejr}H_{jz})\vec{a}_\theta$$

$$+ (v_{ejr}H_{j\theta} - v_{ej\theta}H_{jr})\vec{a}_z]. \qquad (91)$$

Neglecting Part 2 of the initial condition for the present, there is no initial motion in the θ direction. So the final results will be symmetrical about the z axis with no θ direction motion. The equation becomes

$$v_{ejr} = (v_{ejr})_0 + \Delta v_{ejr}$$

where

$$\Delta v_{ejr} = \left\{ -\frac{v_{ejr}}{2\Delta r_j} \left[(v_{ej+\Delta r})_r - (v_{ej-\Delta r})_r \right] - \frac{v_{ejz}}{2\Delta z_j} \left[(v_{ej+\Delta z})_r - (v_{ej-\Delta z})_r \right] - \frac{e}{m} E_{jr} + \frac{e\mu}{m} v_{ejz} H_{j\theta} \right\} \Delta t \tag{92}$$

and

$$v_{ejz} = (v_{ejz})_0 + \Delta v_{ejz} \tag{93}$$

where

$$\Delta v_{ejz} = \left\{ -\frac{v_{ejr}}{2\Delta r_j} \left[(v_{ej+\Delta r})_z - (v_{ej-\Delta r})_z \right] - \frac{v_{ejz}}{2\Delta z_j} \left[(v_{ej+\Delta z})_z - (v_{ej-\Delta z})_z \right] - \frac{e}{m} E_{jz} - \frac{e\mu}{m} v_{ejr} H_{j\theta} \right\} \Delta t. \tag{94}$$

Now, if we let

$$\beta = \vec{v} \cdot \nabla \vec{v} \tag{95}$$

then

$$\Delta v_{ejr} = \left\{ -\beta_{ejr} - 1.759 \times 10^{11} E_{jr} + 2.210 \times 10^{5} v_{ejz} H_{j\theta} \right\} \Delta t \tag{96}$$

and

$$\Delta v_{ejz} = \left\{ -\beta_{ejz} - 1.759 \times 10^{11} E_{jz} - 2.210 \times 10^{5} v_{ejr} H_{j\theta} \right\} \Delta t \tag{97}$$

where

$$\beta_{ejr} = 2.5 \times 10^{1} \left\{ v_{ejr} \left[(v_{ej+\Delta r})_r - (v_{ej-\Delta r})_r \right] - v_{ejz} \left[(v_{ejK\Delta z})_r - (v_{ej-\Delta z})_r \right] \right\} \tag{98}$$

and

$$\beta_{ejz} = 2.5 \times 10^{1} \left\{ v_{ejr} \left[(v_{ej+\Delta r})_z - (v_{ej-\Delta r})_z \right] - v_{ejz} \left[(v_{ej+\Delta z})_z - (v_{ej-\Delta z})_z \right] \right\}. \tag{99}$$

j. Positive ion transport equation Similarly, the positive ion transport equation becomes

$$v_{ijr} = (v_{ijr})_0 + \Delta v_{ijr} \tag{100}$$

where

$$\Delta v_{ijr} = \left\{-\beta_{ijr} + 6.87 \times 10^6\ ZE_{jr} - 8.633\ Zv_{ijz}H_{j\theta}\right\} \Delta t \tag{101}$$

and

$$\Delta v_{ijz} = \left\{-\beta_{ijz} + 6.87 \times 10^6\ ZE_{jz} + 8.633\ Zv_{ijr}H_{j\theta}\right\} \Delta t \tag{102}$$

where

$$\beta_{ijr} = 2.5 \times 10^1 \left\{ v_{ijr}[(v_{ij+\Delta r})_r - (v_{ij-\Delta r})_r] - v_{ijz}\ [(v_{ij+\Delta z})_r - (v_{ij-\Delta z})_r\ ] \right\} \tag{103}$$

and

$$\beta_{ijz} = 2.5 \times 10^1 \left\{ v_{ijr}[(v_{ij+\Delta r})_z - (v_{ij-\Delta r})_z] - v_{ijz}[(v_{ij+\Delta z})_z - (v_{ij-\Delta z})_z] \right\}. \tag{104}$$

k. Time increments The time increments will be selected such that electric field build-up will not be excessive during any one time increment. The time increment used will be 2×10^{-15} seconds. The effect of retarded potentials will not be considered.

3. Sequence of solution

It is important that the above equations be solved in proper order. Otherwise, the equations will be utilizing information which is not from the correct time period. First, introduce the initial condition into the grid storage locations. Since two grid storage locations are needed to allow for calculations of incremental changes with respect to time, some of the initial condition information must be introduced into more than one grid storage location. Then, the sequence to use is as follows.

a) Print out t, Δt, ρ_e

b) Solve for G

c) Print out ρ_i

d) Jump to "o"

e) Solve for and print out t, Δt

f) Solve for $\frac{\alpha}{e} \rho_e \rho_i \, \Delta t$

g) Solve for ρ_e, ρ_i

h) Print out ρ_e

i) Solve for β_e, β_i

j) Solve for v_e, v_i

k) Print out ρ_i

l) Solve for E_1

m) Solve for E_2, E

n) Solve for $(H)_0$

o) Solve for H

p) Solve for and print out W_T, $\frac{\Delta W_T}{\Delta t}$

q) Sample for end; if end, jump to "s"

r) Cycle to "e"

s) Print out v_e, v_i, E, H

t) Stop

C. Digital Computer Program for Shape Solution

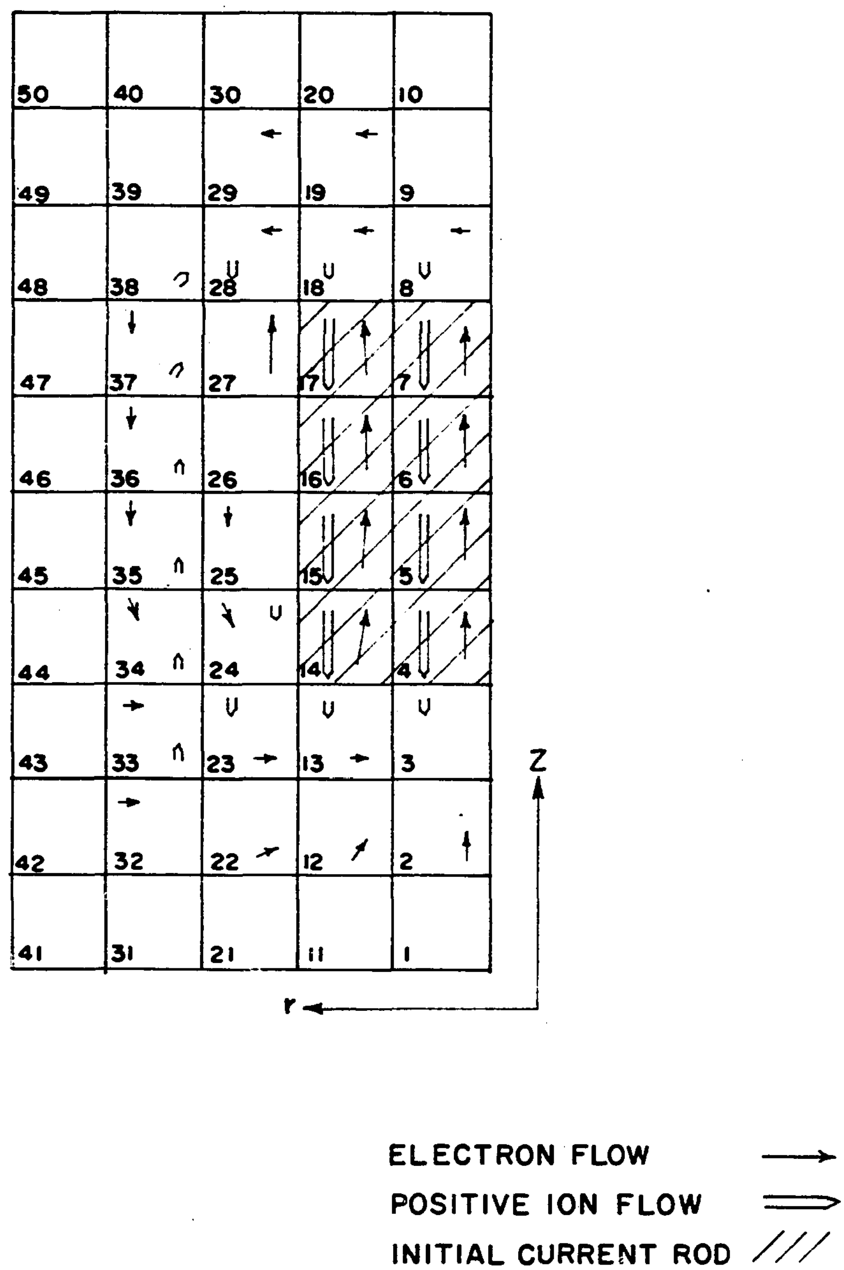
The digital computer used was the Cyclone of Iowa State University. It has 16,384 storage locations, a random access memory, and is very similar to the Illiac of the University of Illinois and the Mystic of Michigan State University. Interpretive codes are available for programs, and the present problem was translated by use of the Eerie interpretive code. The kugelblitz program and resulting data are contained in Carpenter (4).

D. Digital Computer Solution for Shape

The digital computer yielded information on particle velocities and directions, magnetic field strength, electric field strength, and charge distribution. The results were obtained at a time of 4×10^{-15} seconds after imposition of the initial condition in lightly ionized air, and did not take into account retarded potentials. Since the point of primary interest here is the initial shape of the solution, electron and positive ion flow directions are indicated on the 50 space grid in Figure 2.

The initial condition which yielded the best results was $Z=1$, $\ell=D=8$ cm, $\rho_e=\rho_i= 8.61 \times 10^6$ coulombs/meter^3, $v_e = v_i =$ 263 meters/second and field energy $= 4.95 \times 10^6$ joules. This suggests that the higher the charged particle velocities and magnetic field energy initially, the more readily the fireball

Figure 2. Fireball initial gross shape--based on an initial condition of $Z = 1$, $\ell = D = 8$ cm, $\rho_e = \rho_i = 8.61 \times 10^6$ coulombs/meter^3, $|v_e| = |v_i| = 263$ meters/second, field energy $= 4.95 \times 10^6$ joules, and with the current rod suspended in lightly ionized surrounding air; each square is 2 cm on a side in the grid shown; the grid is for positive r


50 40 30 20 10
49 39 29 19 9
48 38 28 18 8
47 37 27 17 7
46 36 26 16 6
45 35 25 15 5
44 34 24 14 4
43 33 23 13 3
42 32 22 12 2
41 31 21 11 1
Z
r
ELECTRON FLOW
POSITIVE ION FLOW
INITIAL CURRENT ROD

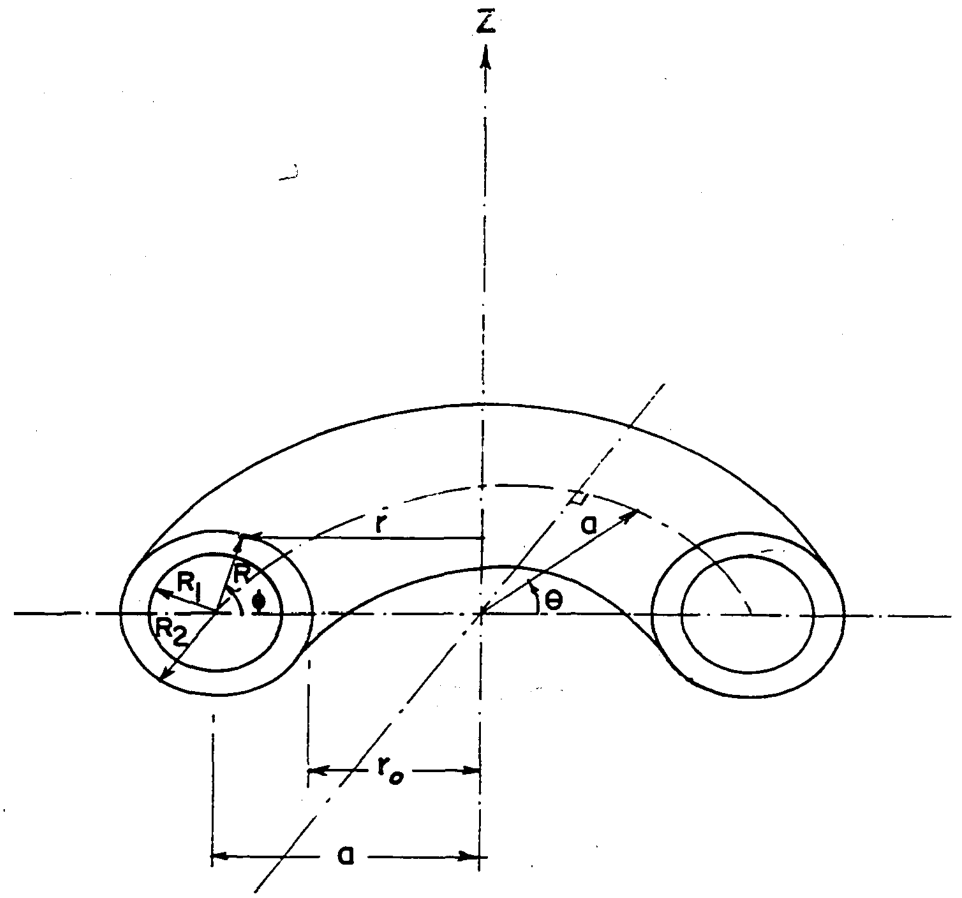
shape will result.

E. Kugelblitz Intermediate Structure

The fireball will now be investigated at some time between initial formation and achievement of a stable condition. For purposes of clarification, the coordinates R and $\emptyset$ will be introduced as indicated in Figure 3. All future reference to coordinates will be in terms of the system illustrated.

At this point, Part 2 of the initial condition is of value. The initial charged particle motion in the Θ direction yielded no electric or magnetic fields since the plasma was originally homogeneous. However, as charge separation occurs due to magnetic constriction of charge motion in the z direction, an electric field appears in the R direction, and a magnetic field appears in the $\emptyset$ direction. These fields exist between the electron and positive ion streams. From minimum energy considerations, the positive ions tend to form a thin shell located at R_1 and travelling in the Θ and $-\emptyset$ directions. Similarly, the electrons tend to form a thin shell at R_2, travelling in the Θ and $\emptyset$ directions. Initially the charges and velocities in the Θ direction of the two shells are about the same, but as the electron shell loses energy in interaction against air, a continually larger percentage of the total electrons leave the electron shell and drift into the positive ion shell. This means that at first the $\emptyset$ magnetic field is

Figure 3. Coordinate system--symmetry exists within each of the mutually orthogonal θ and ϕ directions


Z
r
a
R
R1
R2
ϕ
θ
ro
a

contained between the two shells, but later the $\emptyset$ magnetic field extends outside of the electron shell as indicated in Figure 4.

Thus, at this intermediate stage we have a positive-ion, thin shell toroid (located at R_1) nested in an electron, thin shell toroid (located at R_2). This structure is beginning to look like Ladikov's postulated magneto-vortex ring (11). Although Ladikov's model differs from this one at this point in that he treated a toroid with one homogeneous shell, his derived stability criterion should apply. This criterion is that the density of the plasma must be less than the density of the surrounding atmosphere.

F. Kugelblitz Intermediate Characteristics

Various intermediate characteristics of the kugelblitz can now be determined. They are: shell surface charge distribution, electric field, magnetic field, particle velocities, fireball energy, and electron shell thickness.

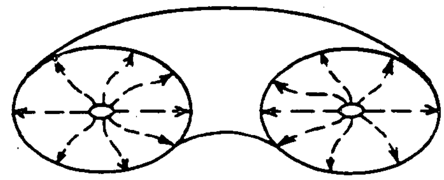
1. Shell surface charge distribution

Each shell effectively forms a toroidal conductor. The distribution of charge, σ, on the surface of a conductor tends to be proportional to the curvature of the surface. So

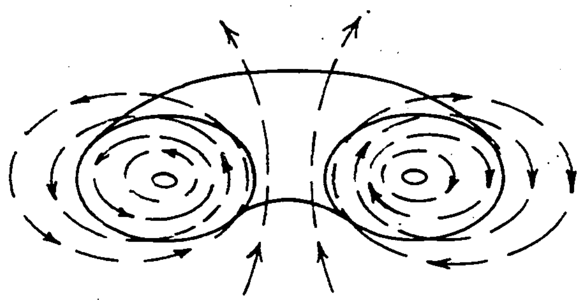
$$\sigma = \frac{q(a^2 - R^2)^{1/2}}{A(a - R\cos\emptyset)} \tag{105}$$

where

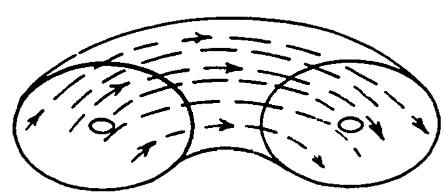
Figure 4. Fireball intermediate model



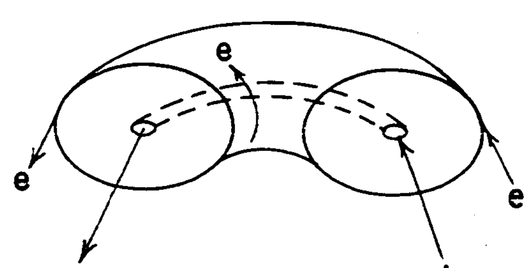
ELECTRIC FIELD



MAGNETIC FIELD DUE TO θ MOTION



MAGNETIC FIELD DUE TO ϕ MOTION



CHARGED PARTICLE MOTION

$$A = 4\pi^2 aR. \tag{106}$$

Therefore, the surface charge distribution of the electrons and positive ions are respectively

$$\sigma_e = \frac{q_e(a^2 - R_2^2)^{1/2}}{4\pi^2 aR_2(a - R_2 \cos \phi)} \tag{107}$$

and

$$\sigma_i = \frac{q_i(a^2 - R_1^2)^{1/2}}{4\pi^2 aR_1(a - R_1 \cos \phi)} . \tag{108}$$

2. Electric field

The electric field in the R direction can be found by an extension of Gauss's law. Inside of R_1 and outside of R_2 the field is essentially zero. At the outer surface of R_1

$$E_R = \frac{\sigma_i}{\epsilon} .$$

However, E_R will vary between R_1 and R_2. Considering only R geometry,

$$E_R = \frac{\sigma_i R_1}{\epsilon R} . \tag{109}$$

Considering only r geometry,

$$E_R = \frac{\sigma_i(a - R_1 \cos \phi)}{\epsilon(a - R \cos \phi)} . \tag{110}$$

Therefore, the total E_R near R_1 is

$$E_R = \frac{\sigma_i R_1(a - R_1 \cos \phi)}{\epsilon R(a - R \cos \phi)} \qquad \text{for } R_1 \leq R < R_2 \tag{111}$$

while the total E_R near R_2 is

$$E_R = \frac{\sigma_e R_2(a - R_2 \cos \phi)}{\epsilon R(a - R \cos \phi)} \qquad \text{for } R_1 < R \leq R_2 . \tag{112}$$

If $R_2 - R_1 \ll R_2$,

$$E_R = \frac{q_i(a^2 - R_1^2)^{1/2}}{4\pi^2 \epsilon a R(a - R\cos\phi)} \qquad \text{for } R_1 \leq R \leq R_2. \tag{113}$$

3. Magnetic field

First, the magnetic field in the ϕ direction (H_ϕ) will be determined. Now,

$$\int \vec{H} \cdot d\vec{\ell} = I \tag{114}$$

so

$$R\int_0^{2\pi} H_\phi d\phi = \int_0^{2\pi} \sigma_i \, v_{i\theta} \, R_1 \, d\phi. \tag{115}$$

Thus

$$H_\phi = \frac{q_i(a^2 - R_1^2)^{1/2} \, v_{i\theta}}{4\pi^2 a R(a - R_1 \cos\phi)} \quad \text{for } R_1 \leq R \leq R_2 \tag{116}$$

and

$$H_\phi = 0 \qquad \text{for } R < R_1 \text{ and } R > R_2. \tag{117}$$

Next, the magnetic field in the θ direction (H_θ) will be determined. Now, at a fixed distance, r, H_θ must be a constant due to symmetry. So for r and z such that $R_1 < R < R_2$

$$(a - R\cos\phi)H_\theta = -\sigma_e \, v_{e\phi}(a - R_2\cos\phi) \tag{118}$$

where

$$\sigma_e \, v_{e\phi}(a - R_2\cos\phi) = \text{constant} \tag{119}$$

so

$$H_\theta = \frac{-q_e(a^2 - R_2^2)^{1/2} \, v_{e\phi}}{4\pi^2 a R_2(a - R\cos\phi)} \qquad \text{for } R_1 \leq R \leq R_2 \tag{120}$$

and

$$H_\theta = 0 \qquad \text{for } R > R_2. \tag{121}$$

The case where $R < R_1$ must be considered separately.

$$H_\theta = \frac{-q_e(a^2 - R_2^2)^{1/2} v_{e\phi}}{4\pi^2 a R_2 (a - R\cos\phi)} + \frac{q_i (a^2 - R_1^2)^{1/2} v_{i\phi}}{4\pi^2 a R_1 (a - R\cos\phi)}, \quad \text{for } R < R_1. \tag{122}$$

In addition, any free electrons in the positive ion shell will yield an extra term.

4. Particle velocities

For stability of orbit, a positive ion on the outside edge of the positive ion shell must have its electric field force and centrifugal force balanced by its magnetic field force. In addition, let us assume that a fraction, $1 - U$, of the total electrons are in the outer shell, and that a fraction, U, are in the volume $R \leq R_1$. Neglecting ϕ direction motion, and assuming that the electron fraction U has no net magnetic field effect at this time

$$\frac{e q_i (1-U)(a^2 - R_1^2)^{1/2}}{4\pi^2 \epsilon a R_1 (a - R_1 \cos\phi)} + \frac{M v_{i\theta}^2}{(a - R_1\cos\phi)} = \frac{\mu q_i v_{i\theta}^2 \, e (a^2 - R_1^2)^{1/2}}{4\pi^2 a R_1 (a - R_1\cos\phi)} \tag{123}$$

where

$$\phi = \pi.$$

So

$$v_{i\theta} = c(1-U)^{1/2} \left[1 - \frac{4\pi^2 a R_1 M}{\mu q_i e (a^2 - R_1^2)^{1/2}}\right]^{-1/2} \tag{124}$$

where

$$c = 3 \times 10^8 \text{ meters/second.}$$

Therefore

$$U > \frac{4\pi^2 aR_1 M}{\mu q_i e(a^2-R_1^2)^{1/2}} . \tag{125}$$

Since

$$1 > U$$

$$1 \gg \frac{4\pi^2 aR_1 M}{\mu q_i e(a^2-R_1^2)^{1/2}} . \tag{126}$$

For the fireball under question

$$M = 2.33 \times 10^{-26} \text{ Kg}$$

$$a = 3.75 \times 10^{-2} \text{ meters}$$

$$R_2 = 3.65 \times 10^{-2} \text{ meters.}$$

Also, if we assume the magnitude of current flow would tend to be maintained, that the original velocity was 263 meters/second, and that the initial charge density was 8.61×10^6 coulombs/meter^3, then an increase in velocity to 3×10^7 meters/second would yield a charge density of 75.5 coulombs/meter^3. Since the original volume was 4×10^{-4} meters^3, q_i should be on the order of 3×10^{-2} coulombs. Then

$$\frac{4\pi^2 aR_1 M}{\mu q_i e(a^2-R_1^2)^{1/2}} = \frac{5.7 \times 10^2 R_1}{[(3.75)^2-(100R_1)^2]^{1/2}} . \tag{127}$$

At this point, Ladikov's density criterion (11, p. 59) will be used with the interpretation that the mass density of the shell cannot exceed atmospheric density. Then, assuming the inner shell has become a rod of charge $q_i = 3 \times 10^{-2}$ coulombs, $R_1 = 6.85 \times 10^{-5}$ meters.

So

$$U \geq 10^{-2}. \tag{128}$$

But

$$v_{i\theta} = 0.1\ c$$

and, approximately

$$v_{i\theta} = c[1-U]^{1/2}. \tag{129}$$

So

$$U = 0.99. \tag{130}$$

This appears to be a large number, and is indicative that the composition to be expected in steady state will be a homogeneous one.

The numbers differ slightly for $v_{e\theta}$, but the results are about the same. Maximum $v_{e\theta}$ occurs at $\phi = 0$, where

$$v_{e\theta} = c(1-U)^{1/2}[1 - \frac{4\pi^2 aR_2 m}{\mu q_i e(a^2 - R_1^2)}]^{-1/2}. \tag{131}$$

So

$$v_{e\theta} = 0.1\ c \qquad \text{for } R = R_2. \tag{132}$$

Actually, the magnitude of $v_{e\theta}$ will depend on many factors. In the absence of accurate information regarding these other factors, assume $v_{e\theta} = 0.1$ c at $\phi = 0$ and $R = R_2$. Then, when conservation of momentum is considered, $v_{e\theta}$ tends to follow the relationship

$$mv_{e\theta}r = \text{constant}. \tag{133}$$

The variation in $v_{i\theta}$ is small since the variation in r is small.

Now $v_{e\phi}$ is more difficult to determine. However, the velocity distribution can be found. Since

$$\sigma_e\ v_{e\phi}(a - R_2 \cos \phi) = \text{constant} \tag{134}$$

and since

$$\sigma_e(a - R_2 \cos \phi) = \text{constant} \tag{135}$$

then

$$v_{e\phi} = \text{constant for all } \phi. \tag{136}$$

5. Fireball energy

The field energy due to θ direction particle motion, $W_{i\theta}$, can be found by use of the theorem of Pappas for the volume of rotational solids, and the relationships obtained above. For $R_1 < R < R_2$

$$W_{i\theta} = 2\int_{R_1}^{R_2} \int_0^{\pi} \frac{(\vec{B}_\phi \cdot \vec{H}_\phi)}{2} (2\pi)(a - R \cos \phi) R d\phi dR \tag{137}$$

$$= 2\pi\mu \int_{R_1}^{R_2} R \int^{\pi} [\frac{q_i (a^2-R_1^2)^{1/2} \ v_{i\theta}}{4\pi^2 Ra(a-R_1 \cos \phi)}]^2 (a-R \cos \phi) d\phi dR. \tag{138}$$

Since $R_1 \ll a$ and $v_{i\theta}$ is approximately 0.1 c,

$$W_{i\theta} = \frac{10^{-2} \ q_i^2 \ c^2\mu}{8\pi^2 a} \quad \ln \frac{R_2}{R_1} \ . \tag{139}$$

So

$$W_{i\theta} = 2.16 \times 10^6 \text{ joules.} \tag{140}$$

Now the shell at R_2 is composed of about only 0.01 of the total electrons, so the magnetic field actually extends beyond R_2. The energy in the total field can be closely estimated by use of the equation

$$W_{i\theta} = \tfrac{1}{2} LI^2 \tag{141}$$

where

$$L_\theta = 1.257\ a[\ln \frac{8a}{R_1} - 1.75] \times 10^{-6} \quad (9,\ p.\ 3260). \tag{142}$$

Since

$$I_\theta = \frac{q_i\ v_{i\theta}}{2\pi a} \tag{143}$$

then

$$\tfrac{1}{2} L_\theta I_\theta^{\ 2} = 2.88 \times 10^6 \text{ joules.} \tag{144}$$

The particle kinetic energy in the θ direction must also be considered. Since $v_{i\theta} = 0.1\ c$, and the effective mass involved is

$$\frac{Mq_i}{e} = 4.37 \times 10^{-9}\ \text{Kg},$$

then

$$W_{i\theta K} = \frac{M}{2} v_{i\theta}^{\ 2} = 1.97 \times 10^6 \text{ joules.} \tag{145}$$

The kinetic energy of the electrons in the θ direction will be smaller by a factor of about 25,600, and can be neglected.

The field energy from ϕ direction motion can be found. Since the electrons produce a toroidal magnetic field,

$$W_{e\phi} = 2 \int_0^{R_2} \int_0^{\pi} \left(\frac{\vec{B}_\theta \cdot \vec{H}_\theta}{2} \right) 2\pi\ (a - R \cos \phi) R d\phi dR \tag{146}$$

$$= 2\pi\mu \int_0^{R_2} R \int_0^{\pi} \left[\frac{-q_e (a^2 - R_2^2)^{1/2}\ v_{e\phi}}{4\pi^2\ R_2 a (a - R \cos \phi)} \right]^2 (a - R \cos \phi)\, d\phi dR. \tag{147}$$

So

$$W_{e\phi} = \frac{q_e^2\, \mu\, v_{e\phi}^{\ 2}\ (a^2 - R_2^2)\,[a - (a^2 - R_2^2)^{1/2}]}{8\pi^2 a^2 R_2^2} \tag{148}$$

where, in this case, $q_e = q_i(1-U) = 3 \times 10^{-4}$ coulombs, and

$R_2 = 3.65 \times 10^{-2}$ meters. Then

$$W_{e\phi} = 1.68 \times 10^{-15}\, v_{e\phi}^2 \; . \tag{149}$$

Even if $v_{e\phi}$ were to approach the speed of light

$$W_{e\phi} < 1.51 \times 10^2 \text{ joules.} \tag{150}$$

However, this value is sensitive to selection of R_2 and can become larger by a factor of 11.4 for maximum $W_{e\phi}$ at the limit of $R_2 = 0$ meters. If the fraction $(1 - U)$ of the electrons which are in the volume $R < R_1$ have velocity near c, and the positive ions also rotate with $(-\phi)$ motion in a fashion which contributes to the field, the maximum

$$W_{e\phi} < 3.5 \times 10^5 \text{ joules.} \tag{151}$$

This suggests that even in the stable state only a small portion of the total energy will be contained in the W_ϕ magnetic field.

The kinetic energy due to ϕ direction particle motion is another matter. However, since $v_{i\phi}$ and $v_{e\phi}$ are not known, it will be assumed that the sum of the energies involved in ϕ direction motion is negligible.

Electric field energy is found to be

$$W_R = 2\pi\epsilon \int_{R_1}^{R_2} R \int_0^{\pi} \left[\frac{(1-U)\,q_i\,(a^2 - R_1^2)^{1/2}}{4\pi^2 \epsilon a R (a - R\cos\phi)}\right]^2 (a-R\cos\phi)\,d\phi\,dR. \tag{152}$$

So

$$W_R = \frac{q_i^2(1-U)^2}{8\pi^2\epsilon a} \ln \frac{R_2(a + \sqrt{a^2 - R_1^2}}{R_1(a + \sqrt{a^2 - R_2^2}} \tag{153}$$

and

$$W_R = 2.4 \times 10^4 \text{ joules.} \tag{154}$$

Then, the total energy is approximately 4.87×10^6 joules. This is somewhat less than 4.95×10^6 joules initially in the magnetic field, but the results are well within reasonable limits and are quite encouraging.

6. Thickness of electron shell

The electron shell has a total charge of 3×10^{-4} coulombs. From the area theorem of Pappas, the shell area is $4\pi^2 aR_2$. The shell thickness, ΔR_2, can now be found by applying the interpretation of Ladikov's density criterion used earlier.

$$\Delta R_2 = \frac{\sigma_e}{8.61 \times 10^6} \tag{155}$$

$$= \frac{q_e(a^2 - R_2^2)^{1/2}}{3.44 \times 10^7 \pi^2 a(a - R_2 \cos \phi)} \tag{156}$$

so

$$(\Delta R_2)_{min} = 2.77 \times 10^{-12} \text{ meters} \tag{157}$$

This is on the order of nuclear dimensions, and coupled with the relatively low energy content indicated earlier, points up the fact that this is an intermediate stage which transforms into a more stable condition.

G. Kugelblitz Stable Structure

The fireball stable structure which has resulted from the preceding considerations is a homogeneous plasma shell with ring current in the θ direction and circle currents in the $-\phi$ direction (see Figure 5). This is essentially the model Ladikov (11) investigated for stability. However, a number of details remain to be clarified regarding the functioning of this model.

H. Kugelblitz Stable Characteristics

Characteristics of interest in the stable state can now be determined. They are: magnetic field, fireball energy, particle velocities, level of ionization, toroidal dimensions, and fireball dimensions.

1. Magnetic field

Assume the θ motion of the electrons to be of low energy, then the magnetic field at the shell surface will be

$$H_{\phi} = \frac{q_i (a^2 - R_1^2)^{1/2} v_{i\theta}}{4\pi^2 a R_1 (a - R_1 \cos \phi)}. \tag{158}$$

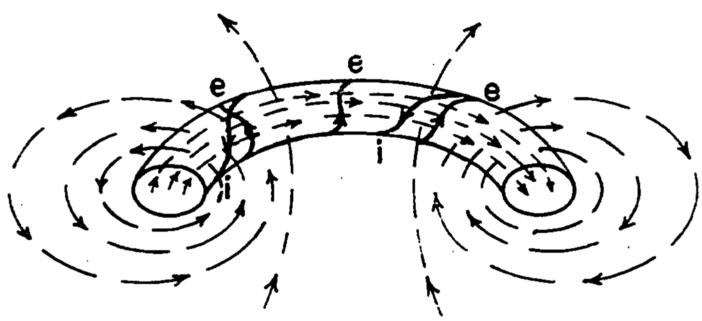
The magnetic field (within the shell) due to ϕ motion will be treated from the viewpoint that the positive ion and electron gases have the same mechanical energy content in the ϕ direction, and have opposite directions of motion. Then

$$H_{\theta} = \frac{q_i (a^2 - R_1^2)^{1/2}}{4\pi^2 a R_1 (a - R_1 \cos \phi)} [v_{i\phi} - v_{e\phi}]. \tag{159}$$

So

$$H_{\theta} = \frac{+q_i v_{i\phi} (a^2 - R_1^2)^{1/2}}{4\pi^2 a R_1 (a - R_1 \cos \phi)} [1 + (\frac{M}{m})^{1/2}]. \tag{160}$$

Figure 5. Fireball stable model--the toroidal plasma shell exposed cross section is the same as a fine wire's cross section



ELECTRON FLOW —→ e

POSITIVE ION FLOW i ←—

MAGNETIC FIELD — →

TOROIDAL PLASMA SHELL

2. Fireball energy

Since the stored electron Θ motion energy and the stored ionization energy have been assumed negligible, the remaining fireball energy will be stored in the Θ and ϕ magnetic fields and in the kinetic energy of the particles. Then the fireball energy will be something less than the 4.95×10^6 joules contained in the magnetic field at the start. Using the inductive energy expression of $\frac{1}{2}LI^2$ and observing that

$$L_\Theta = 1.257 \times 10^6 a[\ln \frac{8a}{R_1} - 1.75] \quad (9, \text{p. } 3260) \qquad (142)$$

$$I_\Theta = (\frac{q_i\ v_{i\Theta}}{2\pi a}) \qquad (143)$$

$$L_\phi = 1.257 \times 10^{-6}[a-(a^2-R_1^2)^{1/2}] \qquad (161)$$

$$I_\phi = \frac{+q_i\ v_{i\phi}}{2\pi R_1}\ [1 + (\frac{M}{m})^{1/2}]. \qquad (162)$$

Then, approximately,

$$4.95 \times 10^6 = 6.285 \times 10^{-7} a[\ln \frac{8a}{R_1} - 1.75][\frac{q_i\ v_{i\Theta}}{2\pi a}]^2$$
$$+ 6.285 \times 10^{-7}[a-(a^2-R_1^2)^{1/2}][\frac{q_i\ v_{i\phi}}{2\pi R_1}]^2[1 + (\frac{M}{m})^{1/2}]^2$$
$$+ \frac{q_i\ M}{2e}\ v_{i\Theta}^2 + \frac{q_i\ M}{2e}\ v_{i\phi}^2 + \frac{q_i\ M}{2e}\ v_{e\phi}^2. \qquad (163)$$

To remain consistent with previous work let

$$q_i = 3 \times 10^{-2} \text{ coulombs}$$

and

$$a = 3.75 \times 10^{-2} \text{ meters.}$$

The exact distribution of energy will be determined when the particle velocities are found.

3. Particle velocities

For a positive ion to be stable at the shell surface where $\phi = \pi$, the magnetic force inward must be balanced by the centrifugal forces and magnetic force outward.

$$ev_{i\theta}H_{\phi} = +ev_{i\phi}H_{\theta} + \frac{Mv_{i\theta}^2}{(a+R_1)} + \frac{Mv_{i\phi}^2}{R_1} + ev_{e\phi}H_{\theta} + \frac{mv_{e\phi}^2}{R_1} \tag{164}$$

Assume $R_1 \ll a$, and

$$v_{e\phi} = -\left(\frac{M}{m}\right)^{1/2} v_{i\phi}. \tag{165}$$

Then, if $v_{i\phi} \ll v_{i\theta}$, and H_{θ} is of the same order of magnitude as H_{ϕ}, effectively

$$v_{i\theta} = \left(\frac{M}{m}\right)^{1/2} v_{i\phi}. \tag{166}$$

So

$$v_{e\phi} = v_{i\theta}. \tag{167}$$

When these relationships are substituted into the fireball energy equation and minor terms dropped

$$4.95 \times 10^6 = \Big\{6.285 \times 10^{-7} a\left[\ln \frac{8a}{R_1} - 1.75\right]\left[\frac{q_i}{2\pi a}\right]^2 + 6.285 \times 10^{-7}\left[a-(a^2 - R_1^2)^{1/2}\right]\left[\frac{q_i}{2\pi R_1}\right]^2 + \frac{q_i M}{2e}\Big\} v_{i\theta}^2. \tag{168}$$

Application of the binomial theorem yields

$$4.95 \times 10^6 = \Big\{6.285 \times 10^{-7}\left[\ln \frac{8a}{R_1} - 1.25\right]\frac{1}{a}\left(\frac{q_i}{2\pi}\right)^2 + \frac{q_i M}{2e}\Big\} v_{i\theta}^2 \tag{169}$$

$$v_{i\theta} = 1.135 \times 10^8\left[4.45 + \ln \frac{0.3}{R_1}\right]^{-1/2}. \tag{170}$$

Since $R_1 \ll a$, let $R_1 = 10^{-2}a = 3.75 \times 10^{-4}$ meters.

$$v_{i\theta} = 3.41 \times 10^7 \text{ meters/second} \quad (171)$$

$$v_{e\phi} = 3.41 \times 10^7 \text{ meters/second} \quad (172)$$

$$v_{i\phi} = -2.12 \times 10^5 \text{ meters/second} \quad (173)$$

And we assumed earlier $v_{e\theta} = 0$. From these values, we find the energies to be

$$W_{i\theta} = 2.19 \times 10^6 \text{ joules} \quad (174)$$

$$W_{e\theta} = 0 \quad (175)$$

$$W_{i\phi} = 9.6 \times 10^0 \text{ joules} \quad (176)$$

$$W_{e\phi} = 2.19 \times 10^5 \text{ joules} \quad (177)$$

$$W_{i\theta K} = 2.51 \times 10^6 \text{ joules} \quad (178)$$

$$W_{e\theta K} = 0 \quad (179)$$

$$W_{i\phi K} = 1 \times 10^2 \text{ joules} \quad (180)$$

$$W_{e\phi K} = 1 \times 10^2 \text{ joules.} \quad (181)$$

4. Level of ionization

In the present case it is difficult to determine the exact level of ionization of the plasma. Work will proceed on the computer-indicated single ionization basis.

5. Toroidal dimensions

The thickness, ΔR, of the plasma shell can be found by use of Ladikov's density criterion (11, p. 59). Assuming the plasma density to be smaller than atmospheric density by 10^{-2},

$$\Delta R = \frac{8.61 \times 10^4 \, e}{4\pi^2 a R_1 q_i} \quad (182)$$

$$= 8.3 \times 10^{-6} \text{ meters.} \quad (183)$$

This confirms the assumption that the shell was thin compared to the value of R_1.

The dimensions of the toroidal shell are now:

Toroidal radius	3.75×10^{-2} meters
Shell radius	3.75×10^{-4} meters
Shell thickness	8.3×10^{-6} meters.

There are many values of particle velocities and toroidal dimensions which would satisfy the necessary conditions. The ones selected above are only intended as representative values. Decrease in plasma energy will lead to increase in shell radius, and lead to decrease in toroidal radius and shell thickness.

6. Fireball dimensions

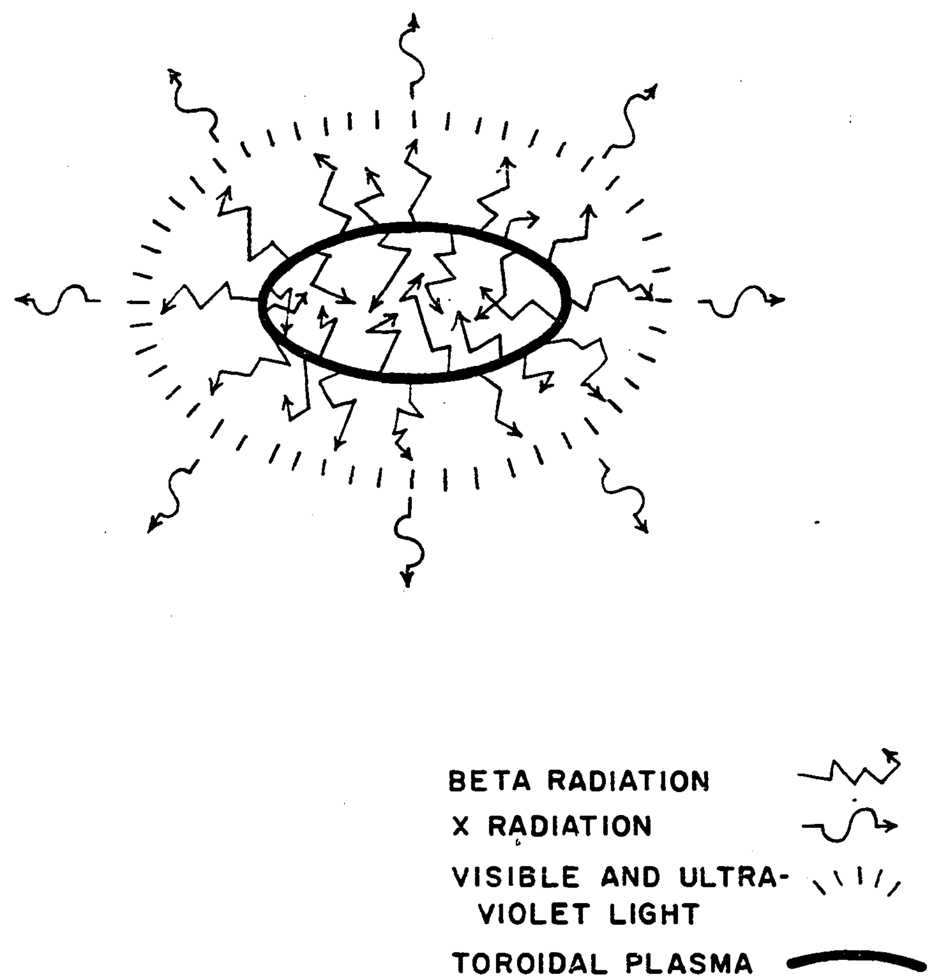
The toroidal dimensions given above do not conform to the size of the observed phenomenon. Indeed, the toroidal dimensions appear to be energy dependent in contrast to the almost constant size of a fireball during its existence.

However, all optical observation of this phenomenon has been confined to the visual portion of the spectrum. Therefore, the observed dimensions depend upon the region in which the fireball energy is converted to visual photon energy.

In all plasmas, scattering takes place. In this particular plasma, scattering occurs from a highly oriented flow pattern, resulting in many of the scattered particles leaving the torus. The scattered particles of primary interest are the electrons. Due to collisions, the plasma electrons will attempt to thermalize to the positive ion temperature. However, as the electrons become relativistic the magnetic field will no longer be sufficient to contain them. This failure of containment should occur in the velocity region about 0.5 c, which yields electrons of 1.2×10^{-14} joules. Electrons of this energy ionize air along a track of approximately 4.8×10^{-2} meters (13, p. 18). The ionized air then extends in a rough ellipsoid of axis 17×10^{-2} and 10×10^{-2} meters about the center of the torus (see Figure 6). The high energy electrons scattered from the toroidal shell are replaced by low energy electrons from the ionized air. Thus the toroidal electrical charge remains neutral. As the toroidal energy decreases, the change in toroidal dimensions therefore has little effect on the observable fireball dimensions. This readily explains why fireballs can pass through small openings and apparently shrink and swell in the process. The shrinking is actually accomplished by absorption of the scattered charged particles in the solid material around the small opening.

The apparent shape of the fireball will vary with angle of observation and beta particle energy. Viewed from the edge of the toroid, the fireball will appear ellipsoidal. Viewed from the top or bottom of the toroid, the fireball will appear to be a ball for high beta particle energy cases. For low beta particle energy cases and large toroidal diameter, the fireball will appear to be a toroid.

Figure 6. Fireball radiation field--primary and secondary radiation produced by the toroidal plasma


BETA RADIATION
X RADIATION
VISIBLE AND ULTRA-VIOLET LIGHT
TOROIDAL PLASMA

V. ENERGY LOSSES

Now that the plasma state and motion are somewhat better defined, we must investigate the fireball's rate of energy loss. It is this which will determine whether or not the vortex theory suggested here avoids the rapid recombination problem of previous plasma theories.

A. Energy Loss Mechanisms

There are many mechanisms for energy loss from a vortex plasma. Some of the better known ones are cyclotron radiation, bremsstrahlung, heat conduction, diffusion, charge exchange, excitation, ionization, recombination and Joules heating. These will be discussed in more detail, with energy losses estimated for some of the more important ones.

1. Cyclotron radiation

Accelerated charged bodies radiate energy at the rate of

$$\frac{dW}{dt} = \frac{6 \times 10^9 (ae)^2}{c^3} \qquad (17, \text{ p. } 99) \qquad (184)$$

where e = charge of accelerated body

a = acceleration of charged body

c = speed of light

$\frac{dW}{dt}$ = cyclotron radiation energy per charged body per unit time.

The radius of motion, X, of positive ions can be considered to be 3.75×10^{-2} meters in the θ direction and 3.75×10^{-4} meters in the ϕ direction. Also, the total number of positive ions in the plasma equals the total number of electrons and is on the order of 1.87×10^{17} particles.

<u>a. Positive ions</u> The velocity, v_i, of a positive ion in the plasma vortex is about 3.41×10^7 meters/sec in the θ direction and 2.12×10^5 meters/sec in the ϕ direction, so the acceleration of any one particle is

$$a_i = \frac{v_i^2}{x} = 3.1 \times 10^{16} \text{ meters/sec}^2 \text{ for } \theta \tag{185}$$

and

$$a_i = 1.2 \times 10^{14} \text{ meters/sec}^2 \text{ for } \phi. \tag{186}$$

Thus the total rate of ion energy lost through cyclotron radiation is around 10^{-3} joules/sec. If the positive ions travelled in clusters of 10, the rate of ion energy loss would be 10^{-2} joules/sec. However, it is felt that 10^{-3} joules/sec is closer to the correct maximum rate. This power should radiate at a frequency of about 150 Mc.

<u>b. Electrons</u> The velocity, v_e, of an electron in the plasma is about 3.41×10^7 meters/sec, and the radius of motion is 3.75×10^{-4} meters, so the acceleration of any one particle is

$$a_e = \frac{v_e^2}{x} = 3.1 \times 10^{18} \text{ meters/sec}^2. \tag{187}$$

Thus, the total rate of electron energy lost through cyclotron radiation is 10 joules/sec. If the electrons travelled in clusters of 10, the rate of electron energy loss would be 10^2 joules/sec. Again, it is felt that 10 joules/sec is closer to the correct rate. This power should radiate at a frequency of about 15,000 Mc.

2. Bremsstrahlung

Bremsstrahlung is produced by accelerated charged bodies according to the same law as cyclotron radiation. However, in this case the acceleration is produced by charged body encounters rather than motion through an electromagnetic field. Since the rate of encounter and magnitude of acceleration produced are not well known for this case, it will be assumed that the most important location for production of bremsstrahlung is along the toroidal plasma shell due to the intermixing of electron and positive ion gases. The rate of energy loss per unit volume is

$$\frac{dW}{dt} = 3 \times 10^{31} \frac{Z^2 e^6}{mc^3 h} n_e^2 s \quad (12, \text{ p. } 42) \qquad (188)$$

where

$\frac{dW}{dt}$ = radiated energy in joules/sec-meter^3

Z = 1 atomic charge/atom

e = 1.6×10^{-19} coulombs

m = 9.1085×10^{-31} kilograms

c = 3×10^8 meters/sec

h = 6.624×10^{-34} joule-sec

s = 5×10^7 meters/sec (estimated)

n_e = 5.35×10^{23} electrons/meter^3.

So

$$\frac{dW}{dt} = 4.4 \times 10^{11} \text{ joules/sec.} \tag{189}$$

Now, the volume of interaction is

$$V = (4\pi^2 R_1 a)\Delta R \tag{190}$$

where $R_1 = 3.75 \times 10^{-4}$ meters

$a = 3.75 \times 10^{-2}$ meters

$\Delta R = 8.3 \times 10^{-6}$ meters.

So

$$V = 4.6 \times 10^{-9} \text{ meters}^3$$

and the total rate of energy loss due to electron-positive ion bremsstrahlung is

$$\frac{dW_b}{dt} = V\frac{dW}{dt} \tag{191}$$

$$= 2.0 \times 10^3 \text{ joules/sec.} \tag{192}$$

The actual total bremsstrahlung loss rate is slightly larger than this since the electron-electron and positive ion-positive ion interactions were not included in the above total. Bremsstrahlung power loss is important in the plasma vortex.

3. Heat conduction

Heat conduction will not be important here due to the laminar nature of the flow.

4. Diffusion

Diffusion is very important in that the high energy electron component which diffuses (or scatters) out of the toroidal shell produces the observable fireball. The exact rate of diffusion from the surface is unknown though, and so no values will be given here.

5. Charge exchange

It is possible for a rapidly moving positive ion to accept an electron from a less energetic positive ion (or neutral atom) through an exchange process. This would permit the now neutral (or less positively charged) particle to leave the plasma carrying its energy with it. The importance of this is not fully known. Charge exchange probably does not contribute significantly to energy loss.

6. Excitation and ionization

Excitation and ionization losses should be important in the toroidal plasma. However, they cannot be well estimated here. Their effect should be covered by use of the conductivity concept which will be discussed later.

7. Recombination

Another mechanism for energy loss is recombination. As electrons and positive ions recombine, they radiate energy in the form of X-rays. However, in this case, only a small amount of toroidal energy is in ionized form. So the recombination losses are limited to this small pool of energy plus the ionization losses just discussed.

8. Joule's heating

Standard Joule's heating plays an important part in plasma energy loss. It converts oriented particle energy into random motion energy. This random motion increases losses due to all of the other mechanisms, and weakens the electrostatic and electromagnetic fields which hold together the plasma. The rate at which Joule's heating occurs is very dependent upon the ionization level of the plasma, the randomness of particle motion within the plasma, and the magnitude of the viscous energy losses along the plasma boundaries. The magnitude of the energy loss rate due to Joule's heating can be estimated through use of the conductivity concept.

The conductivity of the toroidal shell can be found from

$$\sigma = 6.4 \times 10^{-10} \frac{mv_e^3}{e^2 \ln\nu} \text{ mhos/meter} \quad (12, \text{p. } 172) \qquad (193)$$

where $m = 9.1 \times 10^{-31}$ kilograms

$e = 1.6 \times 10^{-19}$ coulombs

$v_e = 3.41 \times 10^7$ meters/second

$\ln\nu = 13$ (approximate) (19, p. 73).

So

$$\sigma = 2.2 \times 10^{18} \text{ mhos/meter.}$$

The time constant for the e-folding time of an inductor is the inductance in henries divided by the path resistance in ohms. Thus, the time it takes for the θ motion energy of the plasma to decrease to 0.368 of its original value is

$$T_\theta = \frac{2\pi R_1 \Delta R \sigma L_\theta}{2\pi a} \qquad (194)$$

where $R_1 = 3.75 \times 10^{-4}$ meters

$\Delta R = 8.3 \times 10^{-6}$ meters

$a = 3.75 \times 10^{-2}$ meters

and

$$L_\theta = 1.257 \times 10^{-6} a[\ln \frac{8a}{R_1} - 1.75] \tag{142}$$

$$= 2.32 \times 10^{-7} \text{ henries.}$$

So

$$T_\theta = 4.29 \times 10^{4} \text{ seconds.} \tag{195}$$

The time for the ϕ motion energy of the plasma to decrease to 0.368 of its original value is

$$T_\phi = \frac{2\pi a \Delta R \sigma L_\phi}{2\pi R_1} \tag{196}$$

where $L_\phi = 1.257 \times 10^{-6}[a-(a^2 - R_1^2)^{1/2}]$ (161)

$= 2.36 \times 10^{-12}$ henries.

So

$$T_\phi = 4.36 \times 10^{3} \text{ seconds.} \tag{197}$$

Since the energy in θ motion is 4.7×10^6 joules and the energy in ϕ motion is 2.19×10^5 joules, the conductivity energy loss rate is 1.6×10^2 joules/second. This is a rather low value, and could be in error by a large amount in view of its cubic dependence upon the relative electron velocity. However, it does suggest that Joule's heating is not as important as other mechanisms of energy loss.

B. Predominant Energy Loss Mechanism

There is still some question as to whether bremsstrahlung or beta particle scattering out of the toroid is the predominant energy loss mechanism. The bremsstrahlung loss rate is about 2×10^3 joules/second, and the energy loss rate due to particle scattering should also have approximately that value.

Now, a beta particle gives up about 32.5 ev for each ion pair formed, but the ionization potential in air is on the order of 14 ev. This indicates that as much as 57 percent of the total beta particle kinetic energy can go into excitation. Excitation shows up primarily as visible light, so approximately half of the scattered particle energy could be in visible form. This suggests a light source of around 1,000 watts distributed over the beta particle radiation region. It also suggests that the total energy loss rate is approximately four or five times the visible light energy loss rate.

C. Time Dependence

The time duration for stable decay of this phenomenon will depend on the total energy and the sum of the rates of the various types of energy losses. In no way are the approaches used above for approximating these energy losses meant to suggest that either the absolute or relative rates of energy losses are constant. These are just crude methods intended to yield some indication of the orders of magnitude involved.

1. Total rate of energy loss

The total rate of energy loss can be approximated as the sum of the Joule's heating loss, the cyclotron losses, the beta particle scattering loss, and the bremsstrahlung loss. The result is

$$\frac{dW_T}{dt} = 160 + 10 + 2{,}000 + 2{,}000$$
$$= 4.2 \times 10^3 \text{ joules/second.} \tag{198}$$

2. Time duration

Since recombination was not considered in the total rate of energy loss above, the total ionization energy will not be considered here. Only inductive energy, W_L, will be used. Then, assuming no instabilities and assuming constant energy loss rate, the total time duration of the plasma can be estimated as

$$T = W_L \frac{dt}{dW_T} \tag{199}$$
$$= (4.95 \times 10^6)(2.4 \times 10^{-4})$$
$$= 1.2 \times 10^3 \text{ seconds}$$
$$= 20 \text{ minutes.} \tag{200}$$

This time is very significant when the observed time duration of kugelblitz is considered.

3. Decay rate

Based on the results of Section A of this chapter, the rate of change of total energy contained in the plasma appears to be approximately proportional to the total energy. Thus, estimates of the total energy and rate of change of total energy can be made for any time according to the relationship

$$W_T = W_0 e^{-\omega t} \tag{201}$$

where W_0 = total energy at start of "stable state"

ω = decay coefficient.

Now

$$\frac{dW_T}{dt} = \left(\frac{dW_T}{dt} \right)_0 e^{-\omega t} \tag{202}$$

$$= -\omega W_0 e^{-\omega t} \tag{203}$$

where $\left(\frac{dW_T}{dt}\right)_0$ = total energy change rate at start of "stable state".

So

$$\omega = - \left(\frac{dW_T}{dt}\right)_0 W_0^{-1} \tag{204}$$

$$= 8.4 \times 10^{-4} \tag{205}$$

and

$$W_T = 5 \times 10^6 \; e^{-8.4 \times 10^{-4} t}. \tag{206}$$

Thus, the decay rate is

$$-\frac{dW_T}{dt} = 4.2 \times 10^3 \; e^{-8.4 \times 10^{-4} t} \text{ watts.} \tag{207}$$

4. Stability of plasma vortex

The plasma vortex appears stable when symmetrical as evidenced by both vortex formation from a crude initial condition and the force equations of the last chapter. When asymmetry is introduced, the situation becomes more complicated. However, Ladikov's work (11) shows the system is stable under most conditions, and indications are that a large amount of asymmetry would be necessary to make the vortex unstable.

Perhaps the most important condition of stability in the plasma occurs in the relationship of mechanical to magnetic field force. As long as the system's mechanical force and expansive magnetic field force are the same as the constrictive magnetic field force, the system will remain stable. But, when the system's mechanical force and expansive magnetic field force exceed its constrictive magnetic force, the system will explode, dumping all of its energy in the order of milliseconds.

Now recombination removes charges, causing the remaining particles to be accelerated. Unless the kinetic energy loss rate is high enough, the particles will eventually become relativistic and the system explode. It appears that this occurs in a large number of cases.

One other point should be considered regarding system stability. This plasma is essentially a tube of fluid flowing through a surrounding medium. For every fluid there exists a Reynold's number beyond which the fluid flow changes from laminar to turbulent. For a fluid of this type the Reynold's number is not known. It is probable that instability due to this type of mechanism would not occur until a much higher relative velocity than that which causes explosion due to mechanical expansion. It is interesting that both of these mechanisms depend on increasing particle velocity.

It appears that every kugelblitz has a time duration limit for stability which depends upon vortex size, magnetic field strength, level of ionization, plasma density, particle velocities, and perhaps upon the core's Reynold's number. If this limit is achieved while the plasma still has a large content of energy, an "explosion" results. If the limit occurs at low plasma energy content, the plasma "fades" away.

D. Radiation Effects

The kugelblitz is assumed to be an ionized-gas vortex, and will therefore have certain noticeable effects associated with it. These effects are primarily radiation oriented and will be treated here from that viewpoint. The following will be discussed: radiation level, electric power, electromagnetic radiation (without considering ultraviolet radiation or heat), heat, and sound.

1. Radiation level

Since the vortex consists of ionized particles moving at high velocity, radiation is present in particle form within the toroid. Radiation is present also in X-ray and ultraviolet form due to bremsstrahlung, recombination and excitation.

a. Positive ions When the vortex first reaches steady state, it consists of about 1.9×10^{17} positive ions (Chapter IV). The average ion energy is 2.5×10^{-11} joules. This yields a radiation total of about 4.7×10^{11} rads-gms, spread over a volume of 4.6×10^{-9} meters^3. (The term "rads-gms" is defined as 100 ergs of any type of radiation.)

b. Beta rays Similarly, the radiation total for beta rays in the toroid is about 2.2×10^{10} rads-gms, spread over a volume of 4.6×10^{-9} meters^3 within the toroid. External to the toroid, the beta radiation level is 2×10^{8} rads-gms/second spread over a volume of about 1.4×10^{-2} meters^3. The radiation density is highest near the center of the ellipsoidal type volume due to both inverse square and absorptive type attenuation.

c. X-rays X-rays are produced throughout the vortex, primarily through bremsstrahlung. When steady state is first reached, approximately 2×10^{3} joules/sec are lost through bremsstrahlung. Since the maximum electron kinetic energy is about 1.8×10^{-14} joules, and since X-rays have between 2×10^{-14} and 2×10^{-17} joules/photon, X-ray production is probable at a very noticeable power level. If we assume that the X-rays are produced at an energy of 2.7×10^{-15} joules, and that ten percent of the bremsstrahlung energy appears as X-rays, then the X-radiation level at the surface of the fireball is 7 roentgen/second. At a distance of 1.5 meters from the center of the fireball, the radiation level is 15 milliroentgen/second.

d. Ultraviolet rays Bremsstrahlung is also responsible for some ultraviolet ray production. Ultraviolet rays have between 2×10^{-17} and 2×10^{-19} joules/photon. Assume about 70 percent production rate for an ultraviolet production rate of 7×10^{20} photons/sec. This is a large value. In addition, ion recombination and excitation produces ultraviolet photons. So the recombination rate and the excitation rate yield around 3×10^{20} photons/sec. This gives a total ultraviolet production of about 10^{21} photons/sec. Some of the bremsstrahlung energy is distributed across the visible spectrum and thus appears as white light.

e. Radiation distribution The toroid itself was found to contain radiation in particle form on the order of 4.9×10^{11} rads-gms. Upon contact with a non-conducting object, the kugelblitz would deliver much (perhaps nearly all) of this radiation to the region of contact in a short period of time. The mean lethal dose to humans is 500 rads delivered to the whole body. The above particle dose would be 6×10^{6} rads delivered to the whole body of a 180 pound man. If the object were a good conductor, electric and magnetic fields might be set up which would restrict the contact and decrease the radiation delivery rate to a very low value. External to the toroid is a high level particle radiation field which extends approximately 0.075 meters from the center of the toroid. The radiation in this field is 2×10^8 rads-gms/second.

Ultraviolet radiation may be at the rate of uv = 10^{21} photons/sec and will obey the inverse square law external to the kugelblitz. Therefore, disregarding buildup factors, the intensity, I_{uv}, will be loosely

$$I_{uv} = \frac{uv}{4\pi x^2} e^{-\lambda x} \qquad (208)$$

external to the kugelblitz where

x = distance from the center of the kugelblitz

λ = atmospheric ultraviolet absorption coefficient.

2. Electric power

The energy release of kugelblitz inductive and kinetic energy can be viewed in another fashion. Upon contacting another body, the charged particle flow could be looked upon as electric current flow. Since the positive ion kinetic energy is about 85 Mev per particle, it can be estimated that the potential difference between two points of contact would be at most 8.5×10^7 volts. If discharged through a resistance of 10^6 ohms, this would yield 85 amperes for 7×10^{-4} seconds, or 7.2×10^9 watts for 7×10^{-4} seconds (the available energy at the start of the "steady state" condition). This is sufficient to be fatal to organisms. It is apparent that the degree of damage done electrically (if any) to an object would depend upon the resistance of the object and the strength of the electric potential which is set up.

3. Electromagnetic radiation

Electromagnetic radiation is produced over a broad spectrum. X and ultraviolet radiation have already been considered, and infrared radiation will be considered under "Heat" below. Here will be covered light and radio waves.

a. Light   If we assume all ionization, excitation, and recombination occurs through mechanisms similar to those of a spark gap, we can use spark gap information to determine the color of the kugelblitz. Due to the broad range of frequencies produced, the predominent color will be white. However, overtones of color will be present because of the energy distribution within the radiation spectrum of each element. The principal overtones associated with the constituents of air (9, p. 2755), and the relative numbers of atoms of each type present are (9, p. 2546) given in Table 6.

Table 6. Principal overtones of light

Element	Color of light	Relative number
nitrogen	blue-red	0.807
oxygen	blue-red	0.190
argon	blue	0.0034
hydrogen	red	0.00145
carbon	green	0.0000905
neon	green	0.0000861
helium	orange	0.0000145

This means that the kugelblitz should appear to be basically white, with red and blue overtones. Due to the more efficient scattering of blue light by air, the kugelblitz should appear to have a blue fringe; in contrast then, the body of the ball should appear more light red or orange in color. This agrees with observation. The intensity of the light, I_L, can be estimated loosely as 20 percent of the bremsstrahlung rate and 50 percent of the excitation rate (about 10^3 watts), and with inverse square diminuation.

$$I_{Lu} = \frac{\ell u}{4\pi x^2} \tag{209}$$

where ℓu = photons produced/sec

$$I_{Lu} = \frac{5.5 \times 10^{19}}{x^2} \tag{210}$$

It would appear that close observation of a high energy fireball could be injurious to the eyes.

b. Radio waves Radio waves are produced through cyclotron type action. The positive ions have a velocity of about 3.41×10^7 meters/sec and an average path length of 2.35×10^{-1} meters, so the radiated frequency is about 150 Mc. Actually, clumps of positive ions will be travelling at slightly different frequencies from each other and thus produce a spectrum of radiation through heterodyne action. The power output is 10^{-3} joules/sec, and the radiation should be observable as radio static. Electrons will also radiate in this fashion. The average frequency will thus be about 15,000 Mc at a total power output of around 10 joules/sec. This should be observable.

4. Heat

Heat will be given off from the kugelblitz through both radiation and conduction. It will be produced through Joule's heating, bremsstrahlung, ionization, excitation, and recombination. Since Joule's heating is approximately 160 watts, total heat production should be on the order of 200 watts.

When stable, the fireball would not feel warm at any reasonable distance unless it possessed a great amount of energy. It would not usually burn an object through heat conduction, but could burn or melt the object due to beta ray or electric current effects.

5. Sound

The ionization and excitation of air should produce broad band static throughout the auditory range. Any low frequency fluctuations in toroidal shape would contribute to this.

VI. SUMMARY

Kugelblitz has been observed for over 2,000 years, and is usually described as a brightly glowing ball about 20 cm in diameter. The color is reddish during existence and the ball often explodes at the end of 3 to 5 seconds. Its motion appears to be governed by electric or magnetic forces, rather than mechanical forces such as that produced by wind. One estimate of kugelblitz energy content was about 5×10^6 joules.

A specific occurrence of kugelblitz on 13 October 1960 suggests the possibility of inadvertant man-made ball lightning. It also tends to disprove Kapitza's standing wave theory.

The postulate is made that initially a rod of electrical current exists in the center of a swirl of turbulent air. No sources of electrical potential exist. It is assumed that the following relations apply: Maxwell's equations, conservation of relative charge, Boltzmann's transport equation, the plasma is two-fluid, and all atoms are nitrogen. The equations are solved by digital computer for a specific case of energy 5×10^6 joules. A 16 cm diameter ball resulted as the initial shape solution.

Analysis of forces involved indicated that the initial shape would evolve into a stable shape within approximately 1 millisecond. This stable configuration was a cylindrical plasma shell in the form of a torus. The toridal diameter was

7.5 cm with cylindrical diameter of 0.075 cm, and shell thickness of 8.3×10^{-4} cm. Electron current flow was essentially around the cylinder while positive ion current flow was essentially around the torus. The system was stable while the flow velocities were non-relativistic. The system became unstable when the flow velocities became relativistic. There was a strong external magnetic field.

The primary energy loss mechanism was found to be either bremsstrahlung or particle scattering. An exact evaluation of which was the more important was not made. However, for the case examined, each had an energy loss rate of about 2,000 watts. The conductivity concept was used, and the Joule's heating loss rate was found to be about 160 watts.

Use of the energy loss rates led to a rough estimate of time duration as about 20 minutes. This is sufficiently close to the maximum observed time of 15 minutes to be significant.

The scattered particles leaving the toroidal shell were found to be primarily electrons. These high energy electrons form a dense beta particle field about the torus. The excitation and ionization in this field produce a region of light about 15 cm in diameter. This light has an energy rate of about 1,000 watts and is primarily reddish in color. When the toroid passes through a crack, the surrounding radiation field is absorbed by the solid material forming the crack, and the ball appears to shrink. A decrease in beta particle energy scattered out of the toroidal shell would allow the toroidal structure of the kugelblitz to be observed.

The kugelblitz was found to contain 4.7×10^{11} rads-gms of positive ion radiation and 2.2×10^{10} rads-gms of beta radiation. The following radiation was given off in steady state: 2×10^{8} rads-gms/sec of 0.082 Mev beta particles, 7 roentgens/second of 0.026 Mev X-rays at the surface of the fireball, about 1,000 watts of ultraviolet radiation, about 1,000 watts of visible light, about 1 milliwatt of 150 Mc radiation, and about 10 watts of 15,000 Mc radiation.

The above values were only for the 5×10^{6} joules total energy case.

VII. CONCLUSIONS

The postulated initial condition of an electrical current rod with both axial motion and rotation about the axis, and with no sources of electrical potential, appears to lead to a valid solution. It is probable that other initial conditions could be found which lead to the same solution.

Due to the crudeness of the initial condition, the resulting magneto-vortex ring is probably the correct solution for kugelblitz.

The kugelblitz is then composed of a magneto-vortex ring surrounded by a radiation field composed primarily of beta rays. This enveloping radiation field is the region observed during occurrences of this phenomenon.

All kugelblitz characteristics in information available to the author can be explained by this model of a core ring and a unique, enveloping radiation field.

The kugelblitz examined was extremely dangerous. Not only did it appear to contain particle radiation in excess of 10^{11} rads-gms, but it was surrounded by a 15 cm diameter field of 0.082 Mev beta particles at 2 x 10^8 rads-gms/sec and a field of 0.026 Mev X-rays at a 7 roentgen/sec radiation level at a radius of 7.5 cm.

In addition, ultraviolet radiation occurred at a power level of about 1,000 watts, and visible light radiation also occurred at a power level of 1,000 watts. This brilliance could be injurious to vision.

The high energy content of the kugelblitz examined could lead to a very forceful explosion if instability occurred. In this case at the start of the stable state, the explosion would be equivalent to 240 pounds of TNT. However, it is doubtful that ball lightning will have large scale military applications. Its use by an Aero-Space force is limited due to the apparent instability of the phenomenon when the density of the surrounding medium is exceeded by the density of the plasma; and its use as an atmospheric weapon is limited by its susceptibility to stray electric and magnetic fields.

One last point should be made! With the possible exception of lightning strokes and aurora borealis, ball lightning is probably the most concentrated form of naturally occurring radiation which exists within a planetary atmosphere.

VIII. RECOMMENDATIONS FOR FURTHER STUDY

In view of the apparent success obtained with the approach used in this investigation, the following recommendations are made for further study:

1) the level of ionization of the plasma be determined accurately so that more exact kugelblitz characteristics can be found;

2) a more detailed study be made of stability to determine which mechanism is responsible for kugelblitz explosion, and what the probability for explosion is;

3) a detailed analysis of the beta particle scattering mechanism be made so that both scattered particle energy distribution and the relative importance of the scattering energy loss can be found;

4) a more detailed analysis of the distribution of radiated energy be made so that the radiation pattern can be better defined;

5) an estimate be made of the energy levels this phenomenon could attain under varying planetary conditions. In particular, the atmospheres of Ju' ter and Saturn might provide environments where an encounter with ball lightning could be disasterous to an exploratory Aero-Space force;

6) experimentation be carried out using the postulated initial condition to verify both the production of kugelblitz and the resulting kugelblitz characteristics.

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XI. APPENDIX A

A. List of Symbols

Due to the large number of symbols required in this investigation, it is useful to group these symbols and their definitions. The list below covers all of the symbols used.

Symbol	Definition
a	1) Unit length 2) Acceleration 3) Toroidal radius
b	Subscript meaning bremsstrahlung
c	Speed of light
d	Derivative
e	1) Subscript meaning electron 2) Magnitude of charge on electron
g	Distance from $k\ell^{th}$ grid point to j^{th} grid point
i	1) Subscript meaning positive ion 2) Subscript meaning ionization for one atom
j	Subscript meaning grid point position j
k	Subscript meaning grid point position k
ℓ	1) Length of current rod at start 2) Subscript meaning angular position ℓ
ℓu	Production rate of photons of light
m	1) Electron mass 2) Particle mass
n	Particle density
p	Pressure
q	Electrical charge on body

r	1) Radial coordinate 2) Subscript meaning radial component
s	Magnitude of estimated electron velocity for bremsstrahlung
t	Time
u	Mean distance between electrons
uv	Production rate of ultraviolet photons
v	Charged particle velocity
x	Distance from center of kugelblitz
z	1) Transverse coordinate 2) Subscript meaning transverse component
A	Area
B	Magnetic induction
D	1) Total electric displacement 2) Diameter of current rod at start
D_1	Electric displacement produced by charge distribution
D_2	Electric displacement produced by motion of magnetic field
E	Total electric intensity
E_1	Electric intensity produced by charge distribution
E_2	Electric intensity produced by motion of magnetic field
F	Force
G	Multiplicative term combining orthogonality and inverse square distance factors
H	Magnetic intensity

I	1)	Current of plasma rod at start
	2)	Subscript meaning ionization of total nitrogen plasma
I_{Lu}		Intensity of light
I_{uv}		Intensity of ultraviolet radiation
J		Current density
K		Subscript meaning kinetic
L	1)	Inductance of current rod at start
	2)	Subscript meaning inductive
M		Mass of positive ion
N		Atoms in current rod at start
P		Perimeter
R	1)	Recombination rate of charged particles
	2)	Radial coordinate
	3)	Subscript meaning radial component
S	1)	Production rate of charged particles
	2)	Incremental surface area
T	1)	Time
	2)	Subscript meaning total plasma
U		Fraction of electrons in positive ion region
V		Volume
W		Energy
X		Cyclotron radius
Z		Magnitude of charge on positive ion in electron units
α		Recombination coefficient
β		Dot product of the vector velocity and the gradient of the vector velocity
δ		Electromotive force

ϵ		Numerical coefficient
θ	1)	Angular coordinate
	2)	Subscript indicating angular component
λ		Absorption coefficient
μ		Numerical coefficient
ν		Variable from a table of values
ξ	1)	Side
	2)	Subscript meaning side
ρ		Density of charge
σ	1)	Conductivity
	2)	Density surface charge
τ		Subscript meaning total field
ϕ	1)	Angular coordinate
	2)	Subscript meaning angular component
ω		Decay coefficient
Δ		Incremental
Σ		Summation
∇		Gradient
∂		Partial derivative
$\int$		Integral sign
$\longrightarrow$		Vector
$-$		Average value
0	1)	Subscript meaning incremental time prior to present
	2)	Subscript meaning time at start