SCATTERING OF RAYLEIGH SURFACE WAVES FROM PARTLY-CLOSED SURFACE-BREAKING CRACKS[†]

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ABSTRACT

A method, involving the expansion in localized functions (ELF) of the crack-opening-displacement in a boundary integral representation of the elastic displacement in a flawed half-space, is applied to the calculation of elastic wave scattering from surface cracks in a two-dimensional geometry. The positions of the localized functions can be controlled in order to simulate cracks with various numbers of islands of closure. The Rayleigh backscattering from a surface-breaking crack changes dramatically at some frequencies as the crack is partially closed from the tip, consistent with recent observations. For the open surface-breaking crack, the calculations reproduce the positions of the known peaks in the reflection coefficient at kL = 1, π, 3π. The amplitude of the kL = 1 peak is sensitive to certain parameters of the model. The potential usefulness of this method lies in its flexibility and in the fact that it can be straightforwardly applied in 3d.

INTRODUCTION

It is important in NDE to be able to detect and characterize cracks in elastic materials, particularly surface cracks. In support of ultrasonic scattering measurements it would be desirable to have a reliable theoretical method to compute the effects of cracks on bulk and surface waves. Many calculations of surface crack scattering have been performed in recent years, but all of them (including the present one) have considered simplified geometries (2d) and are

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therefore not directly relevant to interpretation of data from real fatigue cracks. What distinguishes the theory which we describe here from others is that our method may be straightforwardly extended to 3d surface cracks, and that cracks which are not simply connected (e.g., have islands of closure) can be modeled.

Some experiments have been performed recently by Tittmann, Ahlberg and Buck,[1] which underline the importance of understanding the scattering from partially-closed surface cracks. They allow a Rayleigh surface wave to impinge on a surface-breaking fatigue crack and measure its reflection and conversion into bulk waves as functions of applied stress on the sample, which is supposed to open and close the crack. A surprising feature of their results is that Rayleigh backscatter (reflection coefficient) is much enhanced when the crack is not quite fully open over its value for an open crack or a more-closed crack. It has been suggested[2] that this enhancement could constitute a flag indicating the presence of cracks in a sample undergoing cycled applied stress. But a theoretical understanding of the mechanism is essential; the effects of partial closure of a given crack may be expected to depend strongly on the incident wave length relative to the size of the crack.

In this paper we will sketch the method and illustrate it by application to a 2d surface-breaking crack (normal to the surface) which is partially closed from the tip in successive stages by a sequence of islands of closure.

METHOD OF CALCULATION

The detailed equations of the theory have[3] or will[4] appear elsewhere; here we will only sketch the basic ideas of the method.

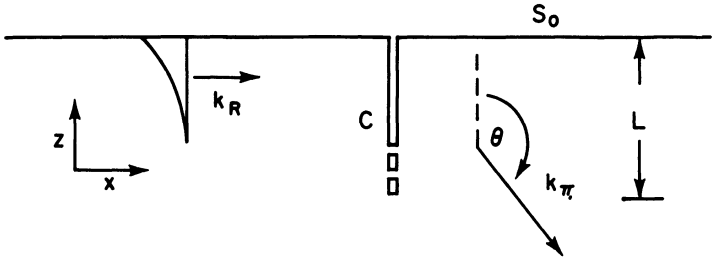

Fig. 1 Surface-breaking crack geometry. A Rayleigh wave with wavenumber k_R is incident from the left; it is reflected, transmitted, and converted into bulk waves k_π. The origin of the x-z coordinates is the crack mouth, C is the right-hand crack surface, and S_0 is the z = 0 plane. Overall crack length is L.

Figure 1 illustrates the geometry of the system we consider. The basic equation is the boundary-integral-representation (BIR) for the elastic displacement u or the stress tensor σ, which we schematically write as

$$\int_{S_0} dS'\ \Xi(r,r')\ u^{scatt}(r') + \int_C dS'\ \Xi(r,r')\ \Delta u(r')$$

$$= \begin{cases} \sigma^0(r) - \sigma(r) & z < 0 \qquad (1) \\ 0 & z > 0. \qquad (2) \end{cases}$$

Here $u^{scatt} = u - u^0$, Δu is the crack-opening displacement, σ^0 is the stress tensor associated with the incident wave u^0 which satisfies stress-free boundary conditions on S_0, and Ξ(r,r') is a Green's stress dyadic. Equation (2) is now used to analytically solve for $u^{scatt}(r)$ with r on S_0, the result is substituted into (2), which is then contracted with $\hat{n}$, the normal to C. This yields, on account of the boundary condition on the surface traction on C,

$$\int_C dS'\ [R(r,r') - Q(r,r')]\ \Delta u(r') = \begin{cases} \tau(r) - \tau^0(r) & z < 0 \qquad (3) \\ -\tau^0(r) & r \text{ on } C \qquad (4) \end{cases}$$

where $\tau = \hat{n}\,\sigma$ is the surface traction vector, $Q = \hat{n}\,\Xi$, and R is obtained from the solution of (2) for u^{scatt}. It describes the field at r reflected in S_0 of the COD at r'. Equation (4) may now be solved for Δu and the result may be substituted into (3) to get τ(r) for all r with z < 0. In particular, if one lets $|r| \to \infty$, then the asymptotic form of (3) will give us the scattered amplitudes and the cross-sections.

The actual strategy for solving (4) for Δu remains to be specified. The idea of our ELF (expansion in localized functions) method is as follows. Take a convenient set of localized functions $v_n(r)$, r on C, and write

$$\Delta u(r) = \sum_{n=1}^{N} C_n\ v_n(r). \qquad (5)$$

The v_n's could be step functions, triangle functions, or any of many other choices. Most convenient for our purposes, especially from the 3d perspective, is the gaussian function, because it allows the calculation in 3d to be pushed further analytically before a numerical procedure must be resorted to. Thus we choose, for the 2d case,

$$v_n(r) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(z-z_n)^2/2\sigma^2} \quad . \tag{6}$$

If (4) is now multiplied by $v_m(r)$ and integrated on s, then it becomes

$$\sum_{n=1}^{N} (R_{mn} - Q_{mn})C_n = -\tau_m^0 \quad , \tag{7}$$

where

$$R_{mn} = \int_C dS \int_C dS' \; v_m(r) \; R(r,r') \; v_n(r') \quad , \tag{8}$$

etc. In practice, in order to reap the benefits of our choice (6) of gaussian functions, it is necessary to replace in (8)

$$\int_C dS \to \int_{-\infty}^{\infty} dz \quad , \tag{9}$$

an approximation which, because of the rapid falloff of $v_n(z)$, should not be expected to induce much error. Another source of error is caused by the finiteness of N in (5), which is necessary in order to solve (7) on a computer. Eq. (7) is a linear system that may be straightforwardly solved for C_n, which, in turn, substituted into (3), will give scattered amplitudes and cross-sections.

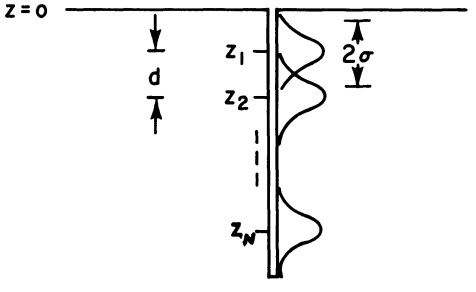

Fig. 2 The localized functions $v_n(r)$ in which the COD is expanded are gaussions of width 2σ centered at $z = z_1, z_2, \ldots, z_N$. The z's are equally spaced ($z_n - z_{n-1} = d$); the value of z_1 determines how deeply, if at all, the crack is buried.

PARAMETERS

The replacement (9), as conjectured above, causes no problems in the evaluation of Q_{nm}. But in R_{nm} it does, because our procedure involves using a complex Fourier representation for R(r,r'), and in R_{nm} (not in Q_{nm}) the extension of the z integral causes the contour integral in the Fourier representation to diverge. At the cost of a manyfold increase (in the 3d case) in numerical labor one could obviate this difficulty by renouncing (9) and probably choosing a different form for (6). Instead we choose to replace (6), as far as R_{nm} is concerned, with

$$v_n(r) = \delta(z-z_n) \quad , \tag{10}$$

and smearing the resultant too-sharp position dependence of R(r,r') for z and z' near the surface by putting an ad-hoc convergence factor $\exp[-(q\sigma')^2]$ parametrized by a length σ' into the Fourier integral on q im R_{nm}. This is a labor-saving substitute for a very difficult correct 3d treatment, which introduces an additional parameter σ' to be determined by comparison with experiment or other theories.

The other parameters of the theory are σ and z_1. As discussed in (4) (and for σ in (3)), σ may be fixed by requiring the exactly-known low-frequency scattering from an isolated crack (we call this condition I) to be reproduced, and the value of z_1 corresponding to a surface-breaking crack may be determined by requiring that the exactly-known SH scattering from a normal surface-breaking crack (condition II) compute. The former condition gives a lower limit on σ and the latter gives a very precise value for z_1, for any given σ' and N.

One more exactly-known experimental feature is still needed to fix σ' for a 2d surface-breaking crack. This feature is provided by the forward Rayleigh scattering amplitude at short wavelength, for a Rayleigh wave incident. It can easily be seen that this amplitude must, after averaging over a wavenumber range $\Delta k >> L^{-1}$, be equal to the negative of the incident amplitude (condition III) in order that the transmitted intensity should, in this limit, be frequency-independent. Thus, the determination of σ, z_1, σ' involves a search for numerical self-consistency, in which we have only taken the first step. What we have done up to now is to take $\sigma' = \sigma$ and satisfy conditions I and II by appropriately choosing z_1 and σ, then varying σ' to satisfy III. The next step will be to use this σ' and again vary σ and z to satisfy I and II, and so on for hopefully only a few iterations, until I, II, and III are simultaneously fulfilled.

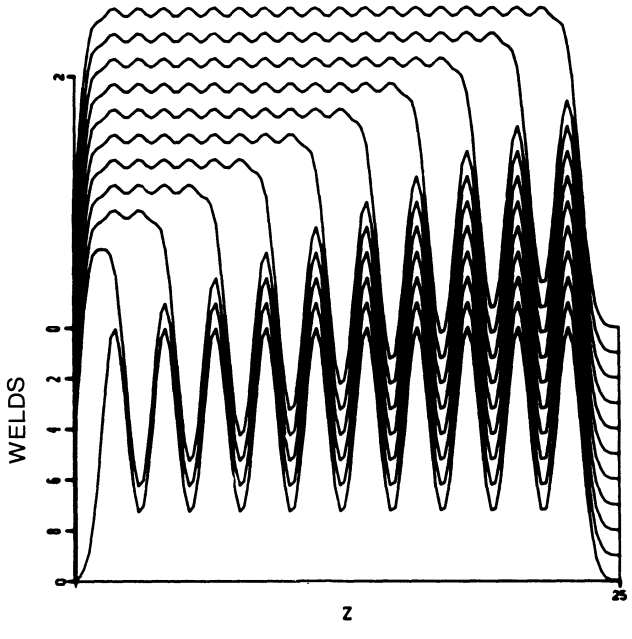

Fig. 3 Crack signatures (defined by $\sum_{n=1}^{N} v_n(r)$ for a sequence of cracks which are increasingly (back to front) partially closed from the tip (at the right).

RESULTS

The results which we show here are for a crack normal to the free surface, described by a COD which is expanded into N = 20 localized functions. Partial closure of the crack is simulated by simply omitting every other v_n, starting with n = 19 (next to the crack tip), then n = 19, 17, then n = 19, 17, 15, continuing until all the odd n's are omitted. The signature of this sequence of cracks is illustrated in Fig. 3, where the crack tip is at the right and the crack closure (quantified by the number of "welds", i.e. omitted v_n's) decreases toward the back of the plot.

For values of σ = .5d, z_1 = -.59d, and σ' = .5σ as determined by the procedure explained above, results for the backscattered Rayleigh intensity are shown in Fig. 4. Zero wavenumber is at the right; the plotting variable in $k_s L$, where k_s is the shear wavenumber. For Poisson's ratio = 1/3, which is the value we always use, the Rayleigh wavenumber is $k_R = 1.072k_s$. One can see that, for a value of $k_R L$ = 10 (approximately that used in Ref. (1)) the Rayleigh backscatter does, in fact, increase as the crack tip opens, in agreement with experiment.

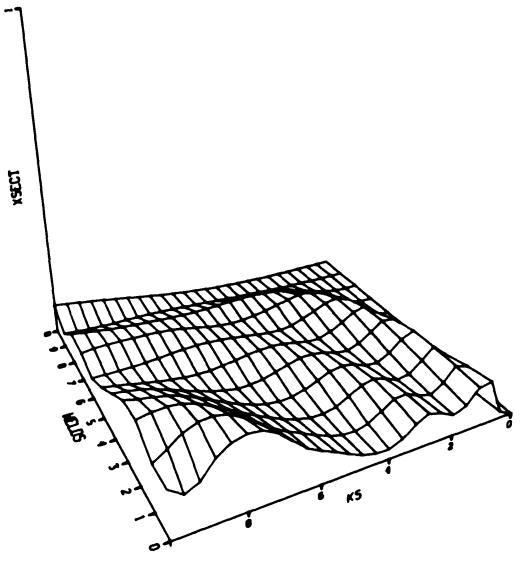

Fig. 4 Backscattered Rayleigh intensity (Rayleigh incidence) for the crack illustrated in Fig. 3. Closure increases front to back. Frequency increases from right ($k_s L = 0$) to left ($k_s L = 10$).

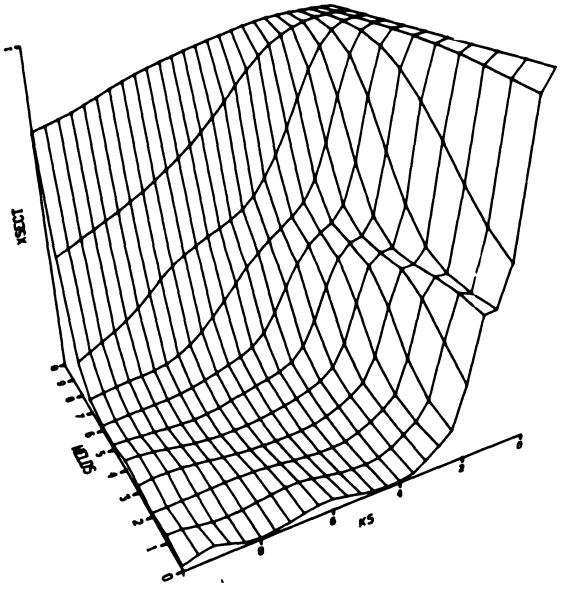

Fig. 5 Forward Rayleigh intensity for partly closing crack. The crack which is completely partially closed (10 welds) becomes nearly transparent to Rayleigh waves.

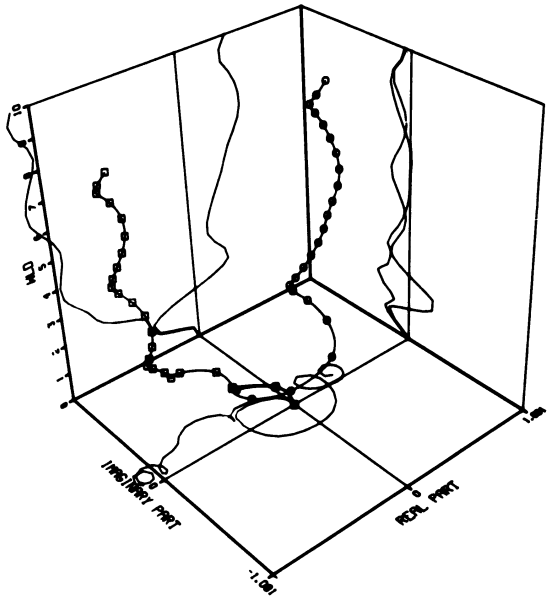

Fig. 6 Real and imaginary parts of the forward (squares) and backward (circles) Rayleigh amplitudes as functions of frequency (vertical coordinate) expressed as $k_s L$. The limits of the x, y axis are ± (incident amplitude); thus condition III is nearly satisfied for the transmitted amplitude.

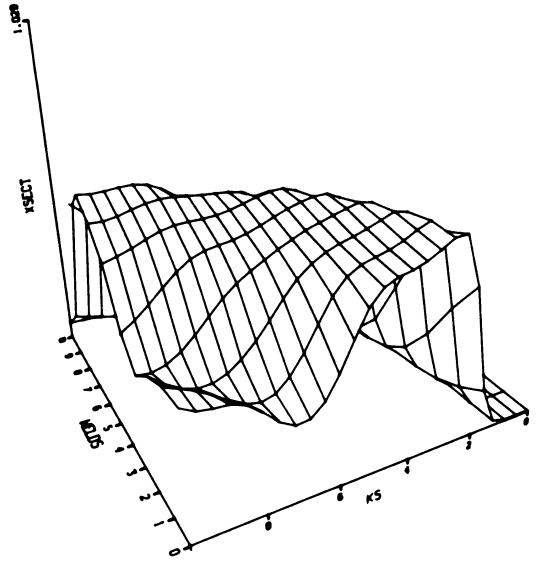

Fig. 7 SV intensity downward from partially closing crack, for incident Rayleigh wave. P scattering in this direction is negligible.

The forward Rayleigh intensity is shown in Fig. 5, and the forward and backward Rayleigh scattered amplitudes for this 2d crack are shown in Fig. 6. Finally, the converted SV radiation intensity is shown in Fig. 7 for $\Theta = 180°$ (directly down). Here, too, depending on the frequency, the intensity may be dramatically enhanced or attenuated as the crack closes.

DISCUSSION

Our results are consistent with the experiments of Tittmann et al.[1], which nurtures the hope that NDE involving ultrasonic Rayleigh waves concurrent with cycled applied stress may be a sensitive tool for crack detection and characterization. The shape of the scattered intensities as exemplified in Figs. 4, 5, and 7 are determined, of course, by the crack closure signature shown on Fig. 3. To determine the latter from the former (especially for a real 3d crack) adds another complication to the already very difficult inverse problem, and is unlikely to be accomplished soon. But we have at least established the qualitative result that, depending on the frequency, scattering can be increased or decreased by partial closure of the crack tip. Other scenarios for crack closure have also been computed (partial or complete closure from the center or from the root); they yield scattering characteristics which are different from those illustrated here.

The curve of Fig. 4 - Rayleigh backscatter - should, for the open crack (no "welds") agree with the Rayleigh backscatter results which appear in the literature.[5] They do agree reasonably well. Both have peaks at $k_R L \cong 1$, $k_R L \cong \pi$, and $k_R L \cong 3\pi$; the height of the peaks at π and 3π are about right, but the $k_R L = 1$ peak is too high by a factor of 2. Its height is sensitive to the z_1, and σ' parameters in our theory; hopefully this discrepancy will disappear if the parameters are determined iteratively as we discussed above.

As we have implied by labelling them in units of π, the π and 3π resonances can be understood in terms of standing Rayleigh waves on the crack. But the $k_R L = 1$ peak must be interpreted differently, perhaps as follows.

In a Rayleigh wave, the material near the surface moves in elliptical paths,[6] counterclockwise if the wave propagates to the right. The major axis of the ellipses is vertical. As one goes deeper into the material the ellipses become more and more eccentric. At a depth given by $k_s |z| = 1.016$ (for Poisson's ratio = 1/3) the motion degenerates into vertical linear motion, and at greater depths it becomes elliptical again, but with a phase reversal (i.e. the material moves clockwise). This suggests that a crack with length $k_s L \cong 1.016$ (or $k_R L \cong 1.1$) should couple to Rayleigh waves most effectively, and provides a qualitative understanding for the presence of a peak near the corresponding frequency in Fig. 4.

OUTLOOK

Although there are easier and more accurate methods for calculating the effects of simple 2d surface cracks on elastic waves, the ELF method with gaussians seems to be unique in its flexibility and generalizability to 3d. It can, without essential modification, be used to compute, for example, scattering from partly-closed halfpenny cracks. The algebra involved in deducing the R matrix becomes quite tedious in 3d, and will be aided with a computerized symbol manipulation program. This work is underway.

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2. James H. Rose (private communication).

3. "Theory of Scattering of Elastic Waves from Flat Cracks of Arbitrary Shape", William M. Visscher, Wave Motion 5 (1983) 15-32.

4. "Elastic Wave Scattering by a Surface-Breaking or Subsurface Planar Crack", William M. Visscher, Los Alamos Report, LA-UR-83-750, (to be published).

5. See, for example, "Scattering of Elastic Waves by a Surface-Breaking Crack", D. A. Mendelsohn, J. D. Achenbach, and L. M. Keer, Wave Motion 2 (1980) 277-292.

6. "Rayleigh and Lamb Waves", I. A. Viktorov, (Plenum 1967), p.5.

DISCUSSION

J.H. Rose (Ames Laboratory): I'll start the questions. One would think that one would put such a thing in the boundary conditions rather than the variational wave function, or am I confusing what you did? It seemed to me that you have a set of variational wave functions some of which you arbitrarily set equal to zero. One would think that you would have very general wave functions but you would change the boundary conditions.

W.M. Visscher: Yes. Well, the frequency that I used on this fixes the Rayleigh wave function, the signal, I call it, I never changed that again. It describes one of the elements of the crack surface, so I then have the freedom in solving this linear system. I find the amplitude of that wavelength,

that is the normal Gaussian, which simulates a step function. If I did this with step functions, you would have problems with it. Well, this Gaussian is just what I substituted as a step function because it is much more convenient numerically.

B. Cox (Rockwell International Science Center): At the moment, I suppose you invert the matrix directly, do you?

W.M. Visscher: Oh, yes.

B. Cox: Would you be able to continue to do that when you go to fully three-dimensional problems?

W.M. Visscher: You mean because it would be a bigger matrix?

B. Cox: Yes.

W.M. Visscher: There might be some problems there. So far, I have only gone up to a 20-by-20 matrix, but in three dimensions, in order to do complicated shapes, you have to go to much bigger matrices, and I might need to use sophisticated techniques which I haven't really thought about yet.

J.H. Rose: In regards to that, I've always wondered why people have not tried non-linear variations with using a priori information to get what the wave function is and then having many fewer variational parameters.

W.M. Visscher: Say that again.

J.H. Rose: Well, you use a linear method there.

W.M. Visscher: Right.

J.H. Rose: Now, it would be quite possible in quantum mechanics to choose a basis set which is non-linear and much more complicated but where you can build in what you expect the wave function to look like. In such cases, you reduce the number of variational parameters tremendously at the cost of having to do a search for the minimum of the energy, and I was wondering if you considered such a method and why they haven't done it here?

W.M. Visscher: No, I haven't considered such a method. In fact, I think this nuclear method is the simplest that I can imagine. There is some advantage to keeping it simple.