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UMI®
AN ECONOMETRIC ANALYSIS OF THE DEMAND
FOR LIVESTOCK AND LIVESTOCK PRODUCTS

by

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Graduate Faculty in Partial Fulfillment of
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DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

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I. INTRODUCTION

With the existence of current farm legislation and the possibility that the Government will continue to formulate policy proposals to achieve certain desired results, the need arises for devising the best analytical tools to assist in attaining the desired goals. This thesis is an attempt to apply certain statistical procedures to a model designed to represent the main factors influencing production and prices of livestock and livestock products in the United States. The model specifies the relationships assumed to exist among the relevant variables. Some of the parameters contained in the model will be estimated and to the extent that the accuracy of such estimates can be established a useful aid for evaluating alternative agricultural policy proposals is obtained. In particular, accurate estimates of the structural parameters would be of importance to the economist in advising on those variations in structure that would produce the most desirable results, given the end in view. The model should be sufficiently broad in scope that questions as to the probable effect of a given policy should be answerable.

The estimates of the structural parameters may be subject to question; for example, mistakes in the specification of the model may be made or there may be sufficient shortcomings in the data to lead to inaccurate results. The estimates obtained may be disappointing, since there are still sufficient gaps in our knowledge of the small sample properties of estimates from multiple equations. However,
this study may have two remaining sources of value. In the first
place, the conduct of the study may uncover weaknesses in the method-
ology which can thereby be improved in future statistical research.
Secondly, the cooperative work of economists and commodity specialists
in formulating the model provides valuable insights into the functioning
of a particular market. The information so obtained may be of
value in practical situations where a decision based on existing
knowledge is required of an administrator.

In this study the parameters of one equation, a commercial demand
relation, will be estimated by two procedures. The statistical
techniques employed will be the standard single equation least squares
and a method due to Anderson and Rubin (3). This method is sometimes
known as the method of limited information, since only information
regarding the form of the one structural equation subject to estimation
is used. However, the method necessitates the specification
of a required minimum of information about the other structural
equations, viz., a knowledge of the list of predetermined variables
which occur in the system without occurring in the equation to be
estimated. At the present stage of development in the application
of estimation methods to systems of equations, the investigator
considers that problems in the specification of the model are pre-
eminent. At best, such models are a simplification, at worst they
are completely inaccurate descriptions. Although Anderson (1,
p. 317) has developed a technique for computing small sample confi-
dence regions, in this study the emphasis will be placed on specifica-
tion of the model and point estimation. The use of classical least
squares permits comparison with the results from limited information.

The theoretical considerations in economics are seldom numerous enough or specific enough to enable the research worker to derive unique expressions for the relationships in which he is interested. In view of this uncertainty it was decided to experiment with different functional forms of the demand equation. In the first instance, an equation linear in certain observed variables is assumed and subsequently an equation linear in the logarithms of these observed variables is assumed.
II. THE ECONOMIC MODEL

A. The Concepts Employed

The model itself represents a description of the assumed actual behavior of a particular sector of the economy, in this study, the livestock economy. The model is designed to explain annual fluctuations in the amounts of livestock and livestock products, as a group, produced and consumed in the United States, in the price of livestock products and in the amounts of feed fed and their prices. No definitive set of rules exists for the construction of economic models but the economist does have a fund of a priori information on which he may draw in making explicit the assumptions as to the way certain observed data are produced. One source of such information is the nature of supply and demand functions resulting from utility maximization by consumers and profit maximization by firms. The maximizing behavior attributed to these two groups might be regarded as idealized behavior. However, in a description of actual behavior other magnitudes which do not appear in the idealized supply and demand functions may be relevant. In order to incorporate these other magnitudes in the model a further source of information may be used, namely, the knowledge of existing production conditions and of the way in which farmers make decisions possessed by people in close contact with livestock producers. The experience of personnel
in the Bureau of Agricultural Economics, the American Meat Institute and Swift and Company has been drawn upon, and to the extent possible, their information has been incorporated in the model.

The sector of the economy with which we are concerned is viewed as describable by a set of simultaneous equations expressing the relationships among the economic variables. The variables in this system of equations are classified into two main types, jointly dependent and predetermined. Jointly dependent variables at time \( t \) are variables which are determined by a process of instantaneous interactions within the system, that is, variables whose formation is to be explained. A variable is classed as predetermined at time \( t \) on the basis of two principles, following Koopmans (19). With the departmental principle those variables outside the scope of economics are treated as predetermined. Using the causal principle, a variable is said to be predetermined at time \( t \) if it influences contemporaneous values of the remaining (jointly dependent) variables but is not influenced thereby. The approximate causal principle leads us to treat as predetermined those variables on which the influence of the jointly dependent variables is presumed to be small. The use of the verbal definitions of these two principles without considering the statistical requirements along with them provides insufficient criteria for classification of variables. The formal requirements for the treatment of variables as predetermined are based on some of the statistical properties of the model. These requirements will be discussed in Chapter III when the statistical model will be specified.
For the purposes of the present discussion we shall content ourselves with the use of the verbal definitions of the two principles mentioned above.

B. The Model Proper

The procedure followed in developing the model will be this. The variables which were assumed to enter the various relationships will be written down, the rationale for the inclusion of the variables in each relationship will be given and the specification (predetermined or otherwise) of each variable will be explained. Below each relation a brief definition of the symbols is given. A detailed description of the aggregates used will be given in the Appendix.

1. Livestock production relation

As a point of departure the following variables were assumed to appear in the production relationship:

2.1 \( y_{lt}, y_{st}, z_{lt}, z_{st} \)

where \( y_{lt} \) is the quantity of livestock and livestock products produced in the United States in a given year. The sample in this study consisted of observations on the variables denoted by \( y \) and \( z \) for the years 1920-49 inclusive. The components of \( y_{lt} \) were cattle and calves, hogs, sheep and lambs, chickens, turkeys,
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and fatting a more accurate measure of the

and lambs in the period both in parturition of foals and livers

period, during this period there have been considerable

practices which have taken place during the same

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is time. The use of a steed fodderable was so severe

vegetables. It is as it is.

mentioned in each category and vendor, the estimation of

an average potential production for an unimproved

age and sex), the number were combined to estimate

average, cattle, horses, sheep and lambs by age and

were fed to produce an output, the components of which were

measured to which inputs in the form of feed were

begunning of the year were considered as the "

the year. The livestock numbers on hand at the

is the number of animals on hand at the beginning of

nutrients

nutrients

compliance was measured in pounds of daily

is the total quantity of feed red to livestock.

worth at average prices.

form a single quantity of feed generated in daily

average from previous the observation period to

in pounds. The components of "ve" were estimated by

all in pounds live-weight, age in dozens and milk
The inclusion of a trend variable is unsatisfactory since it is in effect a catchall variable. Ideally, the investigator would seek to incorporate in the economic model specific variables such that the trend variable could be eliminated. If time is included in 2.1, then on logical grounds, time should also be included in 2.2, 2.3 and 2.4. However, it was felt that for these last three equations the catchall residual that might be attributed to time was small and it seemed preferable not to include a trend variable in 2.2, 2.3 and 2.4.

The y's were treated as jointly dependent variables, that is variables which were determined within the system of relationships which were assumed to represent the feed-livestock economy. The z's were predetermined variables. \( z_{1t} \) was treated as predetermined using the causal principle, \( z_{2t} \) using the departmental principle.

2. The demand for feed grains

Feed input has been divided into two categories, the feed grains and the protein feeds. The basis for the classification was that, in general, the feed grains are farm produced whereas the protein feeds are byproduct feeds from non-farm sources and it seemed desirable to recognize this difference in constructing the model.

Samuelson (24) has shown that given the demand conditions for
the finished product, the demand for a factor of production, as a result of profit maximizing behavior, is a function of the prices of all inputs. Given demand conditions mean that a producer knows at what price he may sell whatever output that is produced. When the demand conditions are themselves variable, the demand for an input is a function not only of the price of all inputs but also the price of the finished product. The demand for feed grains then becomes a function of the price of the finished product and the prices of all inputs. Inputs have been aggregated into feed grains and protein feeds so that the set of price factors is the price of livestock, the price of protein feeds and the price of feed grains. However, in a description of actual behavior other variables influence the decisions of farmers to feed livestock. Three such variables were assumed to enter the demand for feed grains equation. They were the number of animals on hand at the beginning of the year, the amount of pasture and roughage fed and the physical supply of feed grains at the beginning of the year. It was assumed that, other things being equal, the larger the numbers of animals on hand at the beginning of the period the more feed grains would be fed. Similarly, an abundance of good pasture was assumed to depress the quantity of feed grains fed. In terms of alternative opportunities, pasture represents a less costly feed to the farmer than feed grains. Finally, it was assumed that the physical supply of feed grains influenced the quantity of feed grains fed in the following manner. An abundant supply of feed grains was assumed to be accompanied by a heavy feeding rate and
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the quantity of seed grains red to destroy during

46

where

\[ 46 \times \frac{46}{46} = 46 \times \frac{46}{46} = 46 \]

46

considerably the demand for seed grains may then be written as

10
to harvest farmers can estimate what their crop is likely to be and the rate of feeding will be conditioned by the crop prospects. The fractions employed for the feed grains entering \( z_{st} \) will be given in the Appendix.

The price and quantity variables were taken as jointly dependent, \( z_{st} \) which consists mainly of pasture and hays was treated as predetermined on the following grounds. In the case of pasture, consumption in the \( t \)-th period was taken equal to production in the \( t \)-th period since storage of pasture is negligible. Pasture production was assumed influenced predominantly by the weather, a predetermined variable on the departmental principle. Pasture consumption may be influenced by the numbers of animals retained at the beginning of the period. That is, pasture consumption is dependent on predetermined variables only but influences the jointly dependent variables \( Y_{1t}, Y_{2t}, Y_{3t} \). Applying the casual principle pasture consumption was taken as predetermined. In the case of hay consumption, there is some storage of hay and this has to be considered. The consumption of hay, on a national basis, during the \( t \)-th period may be said to depend on the supply of hay during the \( t \)-th period and perhaps also on the livestock "raw material" at the beginning of the \( t \)-th period. The supply of hay during the \( t \)-th period is composed of production during the \( t \)-th period and carry-over from the \((t-j)\)th, \( j \neq 0 \), periods. That is,
\[ S_t = C_{t-j} + P_t \]

where

- \( S_t \) is the supply of hay during the \( t \)-th period.
- \( C_{t-j} \) is carry-over from previous period.
- \( P_t \) is production during the \( t \)-th period.

It was assumed that production in the \( t \)-th period is predominately influenced by weather, a predetermined variable. Hay consumption is then influenced by predetermined variables (\( C_{t-j} \), \( z_{1t} \) and weather) and applying the causal principle as in the case of pasture consumption, hay consumption was treated as a predetermined variable. Since the two major components of \( z_{3t} \) were predetermined, the latter variable itself was called predetermined. \( z_{9t} \) has as its components carry-over and a fraction of production. It was assumed that production in the \( t \)-th period was mainly influenced by weather, a predetermined variable. The carry-over from previous periods is dependent on decisions taken prior to the \( t \)-th period and consequently the physical supply of feed grains in the \( t \)-th period was treated as predetermined.

3. The demand for protein feeds

The rationale for the inclusion of the variables in this equation is the same as that given for the demand for feed grains equation. The quantity of protein feeds was substituted for the quantity of feed grains. The role of \( z_{9t} \) is the same as in equation 2.2 except
that heavy rates of feeding for the feed grains were assumed to be accompanied by lighter rates of feeding for the protein feeds. That is, there is a substitution of feed grains for protein in the ration. The demand for protein feeds may then be written as:

\[ y_{3t}, y_{5t}, y_{6t}, y_{7t}, z_{1t}, z_{3t}, z_{9t} \]

where

\[ y_{3t} \] is the quantity of protein concentrates fed to livestock. Quantity was measured as pounds of total digestible nutrients.

3. The supply of livestock and livestock products

The supply of livestock products might be considered to contain the following variables:

\[ y_{1t}, y_{4t}, y_{5t}, y_{6t}, y_{7t}, z_{1t}, z_{3t} \]

The price of feed inputs and the price of the commodity is in line with the economists' common representation as to factors influencing supply. \[ z_{gt} \] the price of labor from \( (36, \ p. \ 12) \), is the price of an input and has been explicitly recognized following evidence presented by Roselitz \textit{et al.} (14) to the effect that the liquidation of sheep herds between 1940-45 was essentially due to the increased price of farm labor. \[ z_{1t} \] is included on the basis that, other things being equal, a larger number of animals retained at the beginning of the period will result in an increased supply during the \( t \)-th period and
conversely, $z_{gt}$ was treated as predetermined using the approximate causal principle. Similarly, the greater production is in the $t$-th period then the more likely, other things equal, is supply to increase in the $t$-th period and vice versa. It was assumed that little speculation in finished animals was possible for farmers.

In equation 2.4, $y_{lt}$ is the quantity of livestock products sold in the United States at time $t$. The components of $y_{lt}$ were cattle, calves, hogs, sheep and lambs, chickens, turkeys, broilers, all in pounds liveweight, eggs in dozens and milk in pounds. The components of $y_{lt}$ were weighted by average farm prices over the period of observation to form a single quantity aggregate measured in dollars worth at average prices. Home consumed products were treated as sold.

5. The demand for livestock and livestock products

The maximizing behavior of the individual consumer results in the demand for a commodity being expressed as a function of the price of the commodity, the prices of other commodities and the disposable income of the consumer. By analogy, the market demand for a commodity, being the sum of individual demands, may be considered a function of all prices and of the total disposable income of all consumers. The demand relationship for livestock and livestock products was assumed to contain the following variables:

2.5 \[ y_{lt}^1, y_{lt}^2, z_{4t}^1, z_{5t}^2, z_{6t}^3, z_{7t}^4. \]
where

$z_{4t}$ is $y_{4t-1}$. The use of lagged income as a way of expressing the influence of past behavior on current behavior has been common in many empirical demand studies. However, in this model it was assumed that consumption patterns are formed with respect to types of commodities and that lagged consumption seemed a more direct, and, it was hoped, a more accurate way of expressing the above influence.

$z_{5t}$ is disposable personal income ($32, (38)$.

$z_{6t}$ is population from ($38, p. 15$), an obvious influence on the total quantity of livestock and livestock products consumed.

$z_{7t}$ is the Bureau of Labor Statistics Index of Consumer Prices for Moderate Income Families in Large Cities ($42), (28, 1950)$. This variable was designed to represent the influence of the prices of other commodities in the demand equation. The effect of the non-exclusion of livestock and livestock product prices from $z_{7t}$ will be considered in Chapter V.

The price and quantity variables were treated as jointly dependent.$z_{6t}$ was treated as predetermined on the basis of the departmental principle. $z_{4t}, z_{5t}, z_{7t}$ were treated as predetermined using the causal principle which may only be approximate for $z_{5t}$ and $z_{7t}$. The
demand relation is for demand at the farm level. There is, however, a gap between the farm and the ultimate consumer, namely, the commercial sector represented by packers and wholesalers. In this model it was assumed that the commercial sector was passive and exactly reflected demand at retail. No behavior equation for the commercial sector was written down but in Chapter IV the effects of certain postulated behavior by the commercial sector on some of the estimates will be considered.

6. The supply of feed grains

The supply of feed grains for feeding was assumed to be a function of the price of feed grains, the physical supply of feed grains and certain unobservable factors. The supply of feed grains may be written as:

\[ Y_{2t}, Y_{6t}, g_t, \varepsilon_{1t} \]

where \( \varepsilon_{1t} \) represents the unobserved factors. The availability of feed grains for feeding is affected by government and private storage policies and it was difficult to describe the behavior of these two groups in terms of quantifiable variables. In addition, factors affecting foreign demand and supply of feed grains were also difficult to observe.
In both equations 2.6 and 2.7 there has been considerable over-
the most part, unobserved.

where

Supplies of protein seeds may be written as

The supply of protein seeds, quantity by product, was assumed

The supply of protein seeds, quantity by product, was assumed

the approach as component product

nutrients. ZT was bred as a predetermined weight
produced of total dry weight.

the domestic production of protein concentrates.

the apparent domestic production for domestic feedings.

the apparent domestic production for domestic feedings.
production policy of manufacturers of byproduct feeds. The importance of the unobserved factors $s_{1t}$ and $s_{2t}$ would preclude estimation of the parameters in these two equations. If $s_{1t}$ and $s_{2t}$ contain only predetermined variables then the model is complete, in the sense that there are as many equations as there are jointly dependent variables. If $s_{1t}$ and $s_{2t}$ contain jointly dependent variables then the model is incomplete and fewer claims can be made for the estimators using limited information. The properties of such estimators will be discussed in Chapter III.

8. The feed identity

Since the same measure was used for all types of feed the total feed input may be written:

$$y_{gt} = y_{2t} + y_{3t} + z_{3t}$$

9. A farm livestock relation

Farmers' decisions as to the number of animals to sell and the numbers to retain were regarded as simultaneous. Once the decision to sell so many animals was made then it was assumed that the farmer was restricted as to the number of animals he could retain. There is an accounting relation between sales, production and inventories which may be written as:

$$y_{gt} = s_{1t} + \lambda(y_{1t-1} - y_{ht-1}) + n_t$$

where
\[ \lambda \text{ is a constant.} \]
\[ v_t \text{ is a random disturbance.} \]
\[ y_{gt} \text{ is the number of animals on hand at the end of the } t\text{-th period.} \]

This relation expresses the fact that, in general, if sales are high relative to production, inventories will decline. If livestock and livestock products constituted an internally homogeneous quantity, this accounting relation would be exact. Since the weighting system used in constructing \( y_{1t} \) and \( y_{4t} \) differed from that used in constructing \( y_{gt} \) and \( z_{1t} \), the change in livestock inventory that accompanies a given difference between production and sales depends on the behavior of the components of the production and sales aggregates. In this context \( \lambda \) may be regarded as a conversion factor, converting units of measure of \( y_{1t} \) and \( y_{4t} \) into units of measure of \( y_{gt} \cdot v_t \); a disturbance, represents the fact the relationship among the aggregate variables is inexact, depending on the behavior of the components of the aggregates.
III. THE STATISTICAL MODEL

A. Properties of the Disturbances and Algebraic Form of the Equations

The economic model contained in Chapter II represented a description of the relationships assumed between various observable, and some nonobservable, economic variables. \( s_{1t} \) and \( s_{2t} \) contain a complex of variables which were not measurable. In constructing the economic model the relevance of the factors contained in \( s_{1t} \) and \( s_{2t} \) was given explicit recognition but it proved impossible to obtain from published data any measure of the effects of government storage, exports and imports of feed on the availability of feeds for feeding livestock. The investigator would be inclined to regard \( s_{1t} \) and \( s_{2t} \) as consisting of predetermined variables on the basis of the causal principle. If this assumption were correct then the relevance of \( s_{1t} \) and \( s_{2t} \) would be recognized in constructing the economic model but the information contained in \( s_{1t} \) and \( s_{2t} \) would be neglected in the estimation procedure. If \( s_{1t} \) and \( s_{2t} \) contain jointly dependent variables then the model is incomplete. In limited information, using an incomplete model is tantamount to neglecting certain predetermined variables outside the equation whose parameters are being estimated. Anderson and Rubin (3, p. 574) have shown that the omission of certain predetermined variables does not affect the consistency
property of the limited information estimators.

Even if \( s_{1t} \) and \( s_{2t} \) were observable an investigator would hardly maintain the proposition that the functions implicit in the symbols in Chapter II were exact relationships. The vagaries of human behavior would undoubtedly lead an investigator to state that not all the relevant variables influencing human behavior have been incorporated in the model. One way of visualizing the role of these excluded variables is that they are disturbances or shocks, and certain statistical properties are ascribed to them. The shocks represent the stochastic or random element in human behavior. The market relations given in Chapter II were derived from assumptions as to individual behavior patterns. Given the stochastic element in human activity, the market relations will be characterized by certain compound stochastical variables. While the idea of a disturbance has a certain intuitive appeal, in view of our uncertain knowledge of human motivation, the formal statistical properties ascribed to the disturbances are required for the use of existing estimation methods. The properties that can be claimed for the estimators are contingent on the assumptions made about the statistical properties of the disturbances.

The type of model considered in this thesis consists of disturbances in the equations but no errors of observation in variables appearing in the equations. This latter assumption may be questionable where aggregates are constructed from published data which are themselves subject to errors of observation. Models which assume errors of
observation in variables but no disturbances in equations have been considered by some economists, for example, Tintner (26). The estimation procedure employed for models of this type is weighted regression. Ideally, we would like to use an estimation procedure which would handle both errors of observation and disturbances.

Anderson and Hurwicz (2) have considered more general models involving both errors of observation and disturbances for simple models but as yet the more complex systems are not tractable.

The first part of the statistical model is then the assumptions one is prepared to make about the statistical properties of the disturbances in the market relations of Chapter II. The economic model contained in Chapter II may now be written as:

\[
\begin{align*}
3.1 & \quad y_{1t} & y_{gt} & z_{1t} & z_{2t} & u_{1t} \\
3.2 & \quad y_{2t} & y_{yt} & y_{6t} & z_{1t} & z_{3t} & z_{9t} & u_{2t} \\
3.3 & \quad y_{3t} & y_{5t} & y_{6t} & y_{7t} & z_{1t} & z_{3t} & z_{9t} & u_{3t} \\
3.4 & \quad y_{4t} & y_{yt} & y_{6t} & y_{6t} & y_{7t} & z_{1t} & z_{3t} & z_{9t} & u_{4t} \\
3.5 & \quad y_{4t} & y_{yt} & y_{4t} & y_{6t} & y_{6t} & y_{7t} & z_{1t} & z_{3t} & z_{9t} & u_{5t} \\
3.6 & \quad y_{2t} & y_{6t} & z_{9t} & z_{1t} & u_{6t} \\
3.7 & \quad y_{3t} & y_{7t} & z_{10t} & z_{2t} & u_{7t} \\
3.8 & \quad y_{9t} = y_{2t} + y_{3t} + z_{3t} \\
3.9 & \quad y_{9t} = z_{1t} + \lambda(y_{1t-1} - y_{4t-1}) + u_{9t}
\end{align*}
\]
In order to avoid confusion with the assumptions made in the previous chapter, it is important to note that the assumptions made in the current chapter are different. The assumptions made in the previous chapter are as follows:

1. The model is linear.
2. The errors are normally distributed.
3. The variance of the errors is constant.
4. The errors are independent.

In this chapter, we will focus on the assumptions made in the current chapter, which are as follows:

1. The model is non-linear.
2. The errors are not normally distributed.
3. The variance of the errors is not constant.
4. The errors are not independent.

However, even with these assumptions, the results obtained in this chapter are still valid. The assumptions made in this chapter are more realistic, and therefore the results obtained are more applicable to real-world situations.
Suppose the assumption which would not have been written down if the assumption were not accepted was shown in the previous and the previous assumption, which introduced the maximum of a different function, was based on the assumption that the maximum of the function was non-existent.

However, as proven in the previous assumption, if the assumption of the maximum being true leads to the vector that lies in the vector, the form of the previous assumption of the maximum being true as an assumption.

Furthermore, in other assumptions derived from the above assumption on the maximum of the same assumption or the same assumption of the maximum, which introduces the maximum of the function, in the parameters, the dependence on the assumption that is guaranteed to be true.

Since any of the three conditions may be true, the assumption is not rejected, and the assumption on the condition that the approximation to the approximation to the approximation to the approximation to the approximation is not true.
estimation of the parameters was possible. This assumption was concerned with the algebraic form of the market relationships in Chapter II. Two courses were possible here. Firstly, the algebraic form of every equation in the model could have been specified. Secondly, the form of the demand equation, whose parameters were to be estimated, could have been specified without completely specifying the form of the other equations in the system. In this study the latter procedure was adopted. Anderson and Rubin (1, p. 574) have shown that the limited information estimators still possess the property of consistency even in mixed, linear and nonlinear, systems. The model in Chapter II is a mixed system so long as a nonlinear form is used for any of the first seven relations.

Of the various functional forms of the demand equation that were conceivable, only two were deemed statistically tractable. One form would be an equation linear in the observed variables and the other an equation linear in the logarithms of the observed variables. While a choice of the algebraic form of the function was to some extent arbitrary, apart from statistical manageability, some of the considerations which might enter into a preference for one form over the other were given attention. Economists, for example Tobin (27), often assume that the marginal propensity to consume food with respect to income decreases as income increases. In a relation containing an income variable, a relation linear in the observed variables would not permit a decreasing marginal propensity to consume.
However, in the case of "superior" foods like the livestock products, it seemed doubtful to the investigator to assume that there is also a decreasing marginal propensity to consume the livestock products in particular.

In the static theory of consumer's choice the demand functions resulting from maximizing behavior by the consumer are homogeneous of zero degree. By analogy, there could be a tendency to choose an algebraic form of the market demand function which permitted the sum of the elasticities with respect to variables of a monetary dimension to be zero. Such a condition is not permitted by functions linear in observed variables. There is a class of nonlinear functions which permits the zero-sum restriction on the elasticities and a function linear in the logarithms of the observed variables is one member of this class. However, although the homogeneity postulate results from maximizing behavior in the static theory of consumer's choice, it may be doubted whether the same postulate holds in dynamic theory. Tinnter (25) contains a treatment of the factors which result in the invalidity of the homogeneity postulate.

This invalidity may arise from the presence of prices and incomes in the utility functions of consumers, conspicuous consumption being a case in point. In addition, oligopolistic elements in the market may also result in the lack of homogeneity of zero degree in the demand functions. Although in Chapter II it was assumed that the wholesale sector of the livestock economy was passive and exactly reflected demand at retail it may be argued that the meat packers
constitute oligopolistic elements in the livestock economy.

In view of the uncertainty about the appropriate form of the demand relation it was decided that some experimenting might be in order. The assumption of constant elasticities in \( 3.5^\circ \) seemed about as extreme as that of constant slopes in \( 3.5^1 \). Constant elasticities permits recognition of the fact that the effect of a change in price of any commodity may depend on the heights of other prices. One could argue that there is a price above which none of the commodity will be bought and a price below which no additional increments of the commodity will be bought as the price falls. An assumption of constant elasticities does not permit this possible consumer behavior at extreme prices to manifest itself. An assumption of constant slopes does permit this possible consumer behavior at extreme prices but is inconsistent with consumer behavior based on the assumption that the effect of a change in price of any commodity depends on the height of other prices. Since the theory of consumer demand is based on the individual consumer, and market demand is the aggregate of consumer demand, it was decided to deflate consumption and income figures by population, rather than enter population explicitly. Two alternative forms for the demand relation were tried. The relation was first assumed to be linear in the deflated quantity variables and the price variables and was alternatively assumed to be linear in the logarithms of the same variables. The normalization requirement for the estimation procedure using limited information was carried out on \( y_{5t} \). This gave the following relations to be estimated.
\[ 3.5' \quad \beta \left( \frac{y_{h+1}}{z_{6t}} \right) + y_{5t} + \gamma_{54} \left( \frac{z_{h+1}}{z_{6t}} \right) + \gamma_{55} \left( \frac{z_{5t}}{z_{6t}} \right) + \gamma_{57} y_{7t} + \gamma_{50} = u_{5t} \]

\[ 3.5'' \quad \beta_{54} \log \left( \frac{y_{h+1}}{z_{6t}} \right) + \log y_{5t} + \gamma_{54} \log \left( \frac{z_{h+1}}{z_{6t}} \right) + \gamma_{55} \log \left( \frac{z_{5t}}{z_{6t}} \right) + \gamma_{50} = u_{5t} \]

where \( z_{6t}^* \) is population lagged one year, since \( z_{4t} \) is \( y_{h+1} \).

B. Identification of the Demand Relation

One additional problem arose before estimation of the parameters of the demand equation was possible. This problem is known as the problem of identification and occurs in equation systems where observations are produced through the simultaneous interaction of a number of relations. In the discussion of identification several concepts will be used and these concepts will first be elaborated. In the first instance, the question of identification will be considered in connection with linear models which can be written in matrix form as:

\[ 3.10 \quad By'(t) + \Gamma z'(t) = u'(t) \]

where

\[ B \quad \text{is a matrix of beta coefficients. A structural equation is not essentially altered if all its coefficients are multiplied by the same number (different from zero),} \]
provided corresponding adjustments are made to the
elements of the variance-covariance matrix of the u's.
To avoid this indeterminancy, we add to the a priori
restrictions on the system 3.10 a normalization rule
for each equation, for instance by prescribing the
value 1 or -1 for a given coefficient. S and S* are
to be regarded as normalized in this way.

\[ y'(t) \] is a column vector of jointly dependent variables at
time t.
\[ \Gamma \] is a matrix of gamma coefficients.

\[ z'(t) \] is a column vector of predetermined variables at time t.
\[ u'(t) \] is a column vector of disturbances at time t.

A structure S is given by specific numerical values of the
elements of the B and \( \Gamma \) matrices and a joint distribution of the u's.
This joint distribution is of known form, say, normal, and is charac-
terized by a set of parameters with known numerical values. Two
structures S and S* are called observationally equivalent if, and
only if, they are connected by a nonsingular linear transformation.
That is, \( S = \Gamma S^* \) where

\[
S = \left[ \begin{bmatrix} B \Gamma \varphi \{ u'(t) \} \end{bmatrix} \right]
\]
\[
S^* = \left[ \begin{bmatrix} B^* \Gamma^* \varphi \{ u^{*'}(t) \} \end{bmatrix} \right]
\]

\( \Gamma \) is nonsingular and transforms S into S*.

The proof of this theorem is due to Koopmans(18, p. 25). A brief
outline of this proof follows. Two structures are said to be observationally equivalent if they determine the same density function of the observations. Suppose we consider two structures S and S*, of which S is known to be the true structure. The equations that S and S* represent may be written as:

\[3.11 \quad B y'(t) = - \int_{t}^{\infty} z'(t) + u'(t) \cdot\]

\[3.12 \quad B^* y'(t) = - \int_{t}^{\infty} z'(t) + u^*(t) \cdot\]

We may premultiply 3.11 by \(B^{-1}\) and 3.12 by \(B^*^{-1}\) to obtain

\[3.13 \quad y'(t) = -B^{-1} \int_{t}^{\infty} z'(t) + B^{-1} u'(t) \cdot\]

\[3.14 \quad y'(t) = -B^*^{-1} z'(t) + B^*^{-1} u^*(t) \cdot\]

Taking conditional expectations of \(y'(t)\) given \(z'(t)\) and using the assumption that \(E\left[ u'(t)/z'(t) \right] = E\left[ u'(t) \right] = 0\), where \(E\) is the expectation symbol, we obtain from 3.13

\[3.15 \quad E\left[ y'(t) \right] = -B^{-1} z'(t) \cdot\]

Similarly, from 3.14 and using the assumption that \(E\left[ u^*(t)/z'(t) \right] = E\left[ u^*(t) \right] = 0\) we obtain from 3.14

\[3.16 \quad E\left[ y'(t) \right] = -B^*^{-1} \int_{t}^{\infty} z'(t) \cdot\]

Since it was assumed that 3.11 and 3.12 were observationally equivalent they determine the same conditional density function of the \(y'(t)\) and therefore the same conditional expected values. That is,
3.17 \[ B^{-1} \int z'(t) = B^* -1 \int * z'(t) \cdot \]

From 3.17 it may be shown that

3.18 \[ B^{-1} \int = B^* -1 \int * . \]

Premultiply 3.18 by \( B^* \) to obtain

3.19 \[ \int * = B^* B^{-1} \int * . \]

Now if we let

3.20 \[ \Upsilon = B^* B^{-1} \]

where \( \Upsilon \) is necessarily nonsingular since \( B \) and \( B^* \) were assumed nonsingular in 3.11 and 3.12 we may obtain from 3.19 and 3.20

3.21 \[ \int * = \Upsilon \int \]

3.22 \[ B^* = \Upsilon B^* \]

Substituting 3.21 and 3.22 into 3.12 we may show that

3.23 \[ u^*(t) = \Upsilon u'(t) \]

Combining 3.21, 3.22 and 3.23 we have

3.24 \[ S^* = S \]

that is, if two structures are observationally equivalent, then they are connected by a nonsingular linear transformation. The converse of this statement may be proved as follows.
\[(\text{1.1}) \cdot \text{1.1} = \text{1.1}\]

The matrix may be partitioned as follows:

\[(\text{1.2}) = \left( \begin{array}{cc} A & 0 \\ 0 & 0 \end{array} \right) \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right) = \text{1.1}\]

where

\[(\text{1.3}) A + (\text{1.4}) B = (\text{1.5}) C\]

5.2.56

Reduced form is obtained from 5.2.56 by premultiplication by \( B \) to give

The reduced form is obtained from the reduced form.

The expression is in a system of linear equations. These conditions are


are transformed within a model. If the same value for all

these equations are obtained by changing any parameter or

all the parameters within a given model.

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where $\Pi^*$ is the matrix of coefficients $\Pi^*_{ik}$, $i = 1, \ldots, G^*$, $k = 1, \ldots, K^*$, of those predetermined variables in 3.10 which appear in the structural equation being identified. $\Pi^{**}$ is the matrix of coefficients $\Pi^{**}_{ij}$, $j = K^*+1, \ldots, K$, which do not. $G^*$ is the number of jointly dependent variables appearing in the equation, $K$ is the total number of predetermined variables appearing in the system.

Suppose we write the one equation, say the first in the system, in which we are interested as:

$$3.27 \quad B^*y^*(t) + \sum z^*(t) = u^*(t)$$

where

$B^*$ is a row vector of the nonzero coefficients $\beta_{1i}$, $i = 1, 2, \ldots, G^*$.

$y^*(t)$ is a column vector of jointly dependent variables which appear in the first equation of the system.

$\sum$ is a row vector of the nonzero coefficients $\gamma_{1k}$, $k = 1, 2, \ldots, K^*$.

$z^*(t)$ is a column vector of predetermined variables which appear in the first equation of the system.

We may partition 3.26 as follows:

$$3.28 \quad y^*(t) = \Pi^*z^*(t) + \Pi^{**}z^{**}(t) + w^*(t)$$

$$3.29 \quad y^{**}(t) = \Pi^*z^{**}(t) + \Pi^{**}z^{***}(t) + w^{**}(t)$$

where
\[ y^{**}(t) \] is a column vector of jointly dependent variables which do not appear in the first equation of the system.

\[ z^{**}(t) \] is a column vector of predetermined variables which do not appear in the first equation of the system.

\[ w^*(t) \] is \( B^*-1 u^*(t) \).

Premultiply 3.28 by \( B^* \) to obtain

\[ 3.30 \quad B^* y^{**}(t) = B^* \Pi^* z^{**}(t) + B^* \Pi^* z^{**}(t) + B^* w^*(t) \]

Since 3.30 must be identical with 3.27 we must have

\[ 3.31 \quad -B^* \Pi^* = \Gamma^* \]

\[ 3.32 \quad B^* \Pi^* = 0 \]

To establish the necessary and sufficient conditions for the identifiability of a single equation in the system it is necessary to assume that all the elements of the matrix \( \Pi \) are known exactly. Under this assumption it is obvious from 3.31 and 3.32 that once the elements of \( B^* \) are known we can calculate the elements of \( \Gamma^* \). Therefore, the question of the determination of the parameters of the first structural equation from a knowledge of \( \Pi \) reduces to the question of the solvability of 3.32 for the elements of \( B^* \). We know that there exists at least one nonvanishing solution \( B^* \) of 3.32 given by the first
elements of the first row of the matrix \( B \) in the system 3.10.  
3.27 is assumed part of 3.10 and at least one of the first \( G^* \) elements of the first row of \( B \) is not zero, otherwise \( B \) would be singular which would be contrary to the implicit assumption made in obtaining 3.26 from 3.10. Koopmans (18, p. 18) had previously established theorem 73. This theorem states that for systems of linear homogeneous equations of the form \( Ax^i = 0 \), where \( A \) is a matrix of coefficients and \( x^i \) is a column vector with \( n \) elements, the existence of a nonvanishing solution implies that the rank of \( A < n \). Applying this theorem to 3.32 we may state that

\[ 3.33 \quad \Psi( \Pi^{**} ) < G^* \]

where

\[ \Psi( \Pi^{**} ) \] is the rank of \( \Pi^{**} \).

\[ G^* \] is the number of elements in \( B^* \).

In addition, if it is possible to prove that,

\[ 3.34 \quad \Psi( \Pi^{**} ) = G^* - 1 \]

then the ratios of the \( \beta_{11} \) are uniquely determined, and the addition to 3.34 of a suitable normalization rule will determine the \( \beta_{11} \) uniquely. Theorem 85, the converse of theorem 73, had been previously established by Koopmans (18, p. 22). Applying theorem 85 to 3.32 it follows that at least one nonvanishing solution of 3.32 exists, which we shall call \( B^0 \). It follows that if 3.34 is true then \( \Pi^{**} \) contains
at least one nonsingular matrix of order $G^* - 1$. We may permute the
elements of $y^*(t)$ and $z^*(t)$ and then partition $\Pi^*$ as follows:

$$
\Pi^* = \begin{pmatrix}
\Pi_{11} & \Pi_{11}^#
\Pi_{11} & \Pi_{11}^#
\end{pmatrix}
$$

so that the determinant of $\Pi_{11}^#$ is not equal to zero.

$\Pi_{11}^#$ is a row vector with $G^* - 1$ columns and $\Pi_{11}^#$ is a square
matrix of order $G^* - 1$. Similarly, we may partition $B^*$ as follows:

$$
B^* = \begin{pmatrix}
\beta_{11} & \beta_{11}^#
\end{pmatrix}
$$

where $\beta_{11}$ is a scalar and $\beta_{11}^#$ is a row vector with $G^* - 1$ columns.

Any solution of 3.32 must be a solution of 3.37.

$$
\beta_{11} \Pi_{11}^# + \beta_{11}^# \Pi_{11}^# = 0.
$$

It follows for any nonvanishing solution of 3.32 $\beta_{11}$ is not equal
to zero, because otherwise, from 3.37 we would have

$$
\beta_{11}^# \Pi_{11}^# = 0.
$$

Since the determinant of $\Pi_{11}^#$ is not equal to zero, 3.38 would
imply the vanishing of all components of $B^*$, contrary to the assump-
tions. Therefore, in particular $\beta_{11}^0$ is not equal to zero and it
is compatible with 3.32 to introduce a normalization rule
3.39 \[ \beta_{11} = 1 \]

and we shall assume such a rule in force for both \( B^0 \) and \( B^* \) in general. Then 3.37 becomes

3.40 \[ \beta_{11} \# \Pi_{11} \# = -\Pi_{1} \# \]

Koopmans (18, p. 16) in theorem 63 had previously shown for systems of the form 3.38 that if the determinant \( \Pi_{11} \# \) was not equal to zero then there existed one and only one solution \( \beta_{11} \# \). Supplemented by 3.39 this solution must coincide with the solution \( B^0 \) found earlier to be a solution of 3.32. That is, if 3.34 is true, 3.32 and 3.39 have a unique solution \( B^0 = B^* \).

Subsequent to theorem 85 Koopmans (18, p. 23) had established that if \( \varphi (\Pi^{**}) < 0^* - 1 \) there is an infinity of possible solutions \( B^* \) of 3.32, normalized in a suitable manner. Since 3.33 excludes the rank of \( \Pi^{**} \) exceeding \( 0^* - 1 \), the conclusion is that 3.34 is a necessary and sufficient condition for the unique determination of \( B^* \) and \( \Pi^* \) from a knowledge of \( \Pi \). A necessary condition for 3.34 is that there be at least \( 0^* - 1 \) columns in \( \Pi^{**} \), which means that there be at least \( 0^* - 1 \) predetermined variables excluded from the first equation. If the \( \Pi \) matrix is not known but subject to estimation, 3.33 remains true, but we do not know whether or not 3.34 is satisfied. We do know, however, the number of columns in \( \Pi^{**} \), that is, the number of predetermined variables which are excluded from the first equation. The number of columns in \( \Pi^{**} \) is given by
the demand equation

The demand equations were used as the dependent variables considered as a predetermined variable. In the model presented in this paper, the demand for many agricultural products would be expected to increase, for example, as some stock products could be considered as a predetermined variable. Therefore, the quantity of the other variables as independent variables, the choice of the parameter was treated as the dependent variable and the logarithm of the growth in the agricultural sector was treated as the dependent variable in equation 3.5. The least squares estimation of the coefficients of the variables in equations 3.4 and 3.5 were also

for purposes of comparison. 

THE EMPIRICAL EQUATION

$\frac{49}{40} \frac{49}{40} \frac{49}{40} \frac{49}{40}$
reduces the efficiency of the limited information estimators.

The computational methods used in limited information were those given in Bronfembrrenner and Chernoff (5). The results of the least squares and limited information estimation of the parameters in the demand equation are shown in Tables I and II. The calculated standard errors of the estimates are shown in Tables III and IV. The calculated standard errors using limited information should be interpreted with some caution since they were calculated from formulae for asymptotic variances of estimates with unknown parameters replaced by their estimated values. The multiple correlation coefficients for the least squares calculations of 3.5' and 3.5'' were respectively 0.986 and 0.988.

Interpretation of the results and comparisons with other studies and with a priori ideas of what is reasonable can probably be facilitated by computing from the estimated relations some of the elasticities that economists commonly use to characterize demand relations.

To reduce the number of ratios and subscripts that need to be written, the notation will be changed. It will be convenient to rewrite equations corresponding to 3.5' and 3.5'' by placing the quantity variable on the left, all other variables on the right and dividing through the expressions by the coefficient on the quantity variable. With the disturbances set equal to zero, 3.5' and 3.5'' would appear as—
### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
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### Table I

<table>
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</table>
### Table III

**Standard Errors of Estimates of Parameters of 3.5°**

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameter</th>
<th>( \beta_{54} )</th>
<th>( \gamma_{54} )</th>
<th>( \delta_{55} )</th>
<th>( \delta_{57} )</th>
<th>( \delta_{50} )</th>
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<tr>
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<td>.0065</td>
<td>.00015</td>
<td>.0013</td>
<td>.376</td>
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<tr>
<td>Limited Information</td>
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<td>.0065</td>
<td>.00015</td>
<td>.0013</td>
<td>.379</td>
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### Table IV

**Standard Errors of Estimates of Parameters of 3.5°**

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameter</th>
<th>( \beta_{54} )</th>
<th>( \gamma_{54} )</th>
<th>( \delta_{55} )</th>
<th>( \delta_{57} )</th>
<th>( \delta_{50} )</th>
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<tr>
<td>Least Squares</td>
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<td>.327</td>
<td>.354</td>
<td>.099</td>
<td>.157</td>
<td>2.80</td>
</tr>
<tr>
<td>Limited Information</td>
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<td>.559</td>
<td>.426</td>
<td>.128</td>
<td>.165</td>
<td>3.62</td>
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</table>
4.1 $c_t = \xi c_0 + \xi_1 y_t + \xi_2 p_t + \xi_3 y_t^* + \xi_4 c_{t-1}$

4.2 $c_t^* = \eta_0 + \eta_1 y_t^* + \eta_2 p_t^* + \eta_3 y_t^* + \eta_4 c_{t-1}^*$

where

$c_t$ is the per capita consumption of livestock products in period $t$.

$y_t$ is per capita disposable income in period $t$ in current dollars.

$p_t$ is the price of livestock products.

$r_t$ is the Index of Consumer Prices for Moderate Income Families in Large Cities.

An asterisk attached to any variable denotes the logarithm of that variable. Thus $c_t^*$ is the logarithm of $c_t$ and so on. The $\xi$'s and $\eta$'s are functions of the coefficients of 3.5' and 3.5". The calculated values of coefficients of 4.1 and 4.2 corresponding to the estimates given in Tables I and II are given below in Tables V and VI.

The estimates of $\eta_1$, $\eta_2$, $\eta_3$, $\eta_4$ which appear in Table VI are estimated elasticities of quantity with respect to the other variables in the relation. Since they pertain to year to year changes in the variables, they will be called short-run elasticities to distinguish them from the elasticities that would characterize demand if prices and incomes remained constant long enough for consumers to fully adjust to them. The estimates of $\xi_1$, $\xi_2$, $\xi_3$, $\xi_4$, which appear in Table V are estimated rates of change of quantity with respect
### Table V
Calculated Values of Coefficients of \( 4.1 \)

<table>
<thead>
<tr>
<th>Method</th>
<th>( \xi_0 )</th>
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<th>( \xi_2 )</th>
<th>( \xi_3 )</th>
<th>( \xi_4 )</th>
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<td>0.41</td>
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<td>Limited Information</td>
<td>10.54</td>
<td>0.07</td>
<td>-60.24</td>
<td>0.20</td>
<td>0.42</td>
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### Table VI
Calculated Values of Coefficients of \( 4.2 \)

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<th>Method</th>
<th>( \eta_0 )</th>
<th>( \eta_1 )</th>
<th>( \eta_2 )</th>
<th>( \eta_3 )</th>
<th>( \eta_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least Squares</td>
<td>-1.039</td>
<td>0.753</td>
<td>-0.674</td>
<td>0.303</td>
<td>-0.013</td>
</tr>
<tr>
<td>Limited Information</td>
<td>-0.428</td>
<td>0.565</td>
<td>-0.462</td>
<td>0.160</td>
<td>0.119</td>
</tr>
</tbody>
</table>
The standard error of the estimate in Tables III and IV and the standard error of the estimate in Tables I and II, were estimated. The standard error is not the same as the standard error of estimate in Tables I and II, since both have been computed over one period to the next. However, it has been agreed with the allergists that consumption patterns tend to repeat themselves as shown in Table VI. A negative test would not confirm the theory of the percentages appearing reasonably with the expectation of the allergists.

Table III shows the percentages in Table II.

The value used for 1937 is the average of the 1937 and the 1938 observed values. The values used for 1938 are other than 1937.

The second computation was made on the observed 1937 values for all variables and the third computation was made on the average observed values for all variables.

The percentages have been computed for three points in the estimated income, price, and general price level and the effect of consumption respectively. General price level and the effect of consumption are represented by the general price level with respect to the observed price level. The general price level is represented by the general price level with respect to the observed price level. The general price level is represented by the general price level with respect to the observed price level.
# Table VII

Values of Variables Used in Elasticity Computations

<table>
<thead>
<tr>
<th></th>
<th>$e_t$</th>
<th>$y_t$</th>
<th>$p_t$</th>
<th>$r_t$</th>
<th>$e_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Computed</td>
<td></td>
<td></td>
<td>Observed</td>
</tr>
<tr>
<td>1933</td>
<td>55.07</td>
<td>53.77</td>
<td>54.67</td>
<td>358</td>
<td>.385</td>
</tr>
<tr>
<td>Average</td>
<td>55.89</td>
<td>55.89</td>
<td>55.89</td>
<td>643</td>
<td>.798</td>
</tr>
<tr>
<td>1949</td>
<td>61.74</td>
<td>69.97</td>
<td>73.07</td>
<td>1249</td>
<td>1.454</td>
</tr>
</tbody>
</table>

In Table VII, L.S. refers to least squares and L.I. to limited information.

# Table VIII

Short-run Elasticities Based on Estimates of $3.5^t$

<table>
<thead>
<tr>
<th>Method</th>
<th>Values of Variables</th>
<th>$e_y$</th>
<th>$e_p$</th>
<th>$e_r$</th>
<th>$e_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1933</td>
<td>.44</td>
<td>-.40</td>
<td>.30</td>
<td>.41</td>
<td></td>
</tr>
<tr>
<td>Least Average</td>
<td>.76</td>
<td>-.79</td>
<td>.38</td>
<td>.41</td>
<td></td>
</tr>
<tr>
<td>Squares</td>
<td>1949</td>
<td>1.18</td>
<td>-1.15</td>
<td>.43</td>
<td>.35</td>
</tr>
<tr>
<td>1933</td>
<td>.47</td>
<td>-.42</td>
<td>.47</td>
<td>.41</td>
<td></td>
</tr>
<tr>
<td>Limited Average</td>
<td>.83</td>
<td>-.86</td>
<td>.43</td>
<td>.42</td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td>1949</td>
<td>1.24</td>
<td>-1.20</td>
<td>.47</td>
<td>.33</td>
</tr>
</tbody>
</table>
have from equation 1.3:

Under the assumption of a constant percentage markup we would

\[ p + \frac{pd}{100} = \frac{3p}{2} \]

"That is, related to zero price by a constant dollar markup, k. Hence in the second instance, it will be assumed that the real price is

\[ \frac{p'}{p} = \frac{3}{2} \]

Where

\[ \frac{100}{100 + x} \frac{p'}{p} = \frac{3}{2} \]

That is related to a constant percentage mark-up.

In the first instance, it will be assumed that the real price is

By the wholesale sector of the livestock industry. In the first

the retail level and therefore assumption will be made as to behavior

the comparison of price gross margin at the farm level with those at

with this present study had been directed at demand at retail. For

most of the available studies which were comparable on some basis

proceeded did not carry over into the livestock case.

Barrett form is very close. This agreement between the two correlation

the agreement between least squares and intended information in the

condition. One of the interesting results was that in Table L where

estimations based on the estimated which

-47-
\[
\frac{\partial c_t}{\partial p_t^{**}} \cdot \frac{p_t^{**}}{c_t} = \xi_2 \left( \frac{100}{100 + \alpha} \right) \cdot \frac{100 + \alpha}{100} \cdot \frac{p_t}{c_t}
\]

\[
\frac{\xi_2 p_t}{c_t} = E_p
\]

That is, the price elasticity at retail is equal to the price elasticity at the farm level, under this assumption. Under the assumption of a constant percentage markup, equation 4.2 may be rewritten as:

\[
c_t = \xi p_t \left( \frac{100 + \alpha}{100} \right) \eta_2
\]

where \( \xi \) refers to all terms not containing \( p_t^{**} \). It is immediately obvious that \( \eta_2 \) is the same as in 4.2, so that the price elasticity at retail is equal to the price elasticity at the farm level, under the assumption of a constant percentage markup.

If equation 4.1 were written out using retail price of livestock products in place of farm price and with the assumption of a constant dollar margin we would have:

\[
c_t = \xi c_o + \xi_1 y_t + \xi_2 p_t^{**} + \xi_3 y_t + \xi_4 a_{t-1}
\]

\[
\frac{\partial c_t}{\partial p_t^{**}} \cdot \frac{p_t^{**}}{c_t} = \xi_2 (p_t^{**}) \cdot \frac{1}{c_t} = \frac{\xi_2 p_t}{c_t} + \frac{\xi_2 k}{c_t}
\]

That is, the price elasticity at retail is greater than the price elasticity at the farm level by an amount \( \frac{\xi_2 k}{c_t} \). Similarly, equation 4.2 may be rewritten as:
\[ c_t = K(p_t^{**} - k) \]

where

\[ K \text{ refers to all terms not containing } p_t^{**}. \]

\[ \frac{\partial c_t}{\partial p^{**}} = \eta_2 \frac{K(p_t^{**} - k)}{n_2 - 1} \]

\[ E_{p^{**}} = \frac{p_t^{**}}{K(p_t^{**} - k)} \eta_2 \cdot \eta_2 \frac{K(p_t^{**} - k)}{n_2 - 1} \]

\[ = \frac{\eta_2 p_t^{**}}{K(p_t^{**} - k)} = \eta_2 \cdot \frac{p_t^{**}}{p_t} \]

Since, in general, \( p_t^{**} \) is greater than \( p_t \), then \( E_{p^{**}} \) is greater than \( \eta_2 \), or, the price elasticity at retail is greater than the price elasticity at the farm level by an amount \( \frac{p_t^{**}}{p_t} \).

In comparison with other studies at retail, the numerical estimates of price elasticities in Tables VI and VIII will be directly comparable under the assumption of a constant percentage markup, or will be adjusted upwards under the assumption of a constant dollar margin.

Fox (10, p. 76) in two time series studies on the retail demand for all food livestock products and using first differences of logarithms obtained price elasticities of \(-.56\) and \(-.52\) and income elasticities of \(.47\) and \(.40\). The estimation method used was single equation least squares but the above estimates are close to those obtained in this study using limited information in Table VI.
French (11) in a study based on the demand for meat at retail obtained a price elasticity of \(-.238\) and an income elasticity of \(.505\) using limited information on a linear system of equations. Foote (9), using first differences of logarithms, in a study of the retail demand for all food livestock products obtained a price elasticity of \(-.481\) and an income elasticity of \(.697\). In this latter study the observation period was the seven months November through May. Fox (10, p. 80) in a budget study for the year 1948 found an income elasticity with respect to the quantity of all food livestock products of \(.23\). The estimates of \(\beta_v\) in Table VIII for 1949, a high income year like 1948, were not consistent with this budget study result.

In both least squares and limited information it is assumed that the disturbances are non-autocorrelated. Work by Cochrane and Orcutt (8), (23) has thrown some doubt on the validity of this assumption. These two authors have suggested that the shock term may possess a simple autoregressive structure of the form

\[ u_t + \alpha u_{t-1} = \nu_t \]

where \(\nu_t\) is non-autocorrelated. In particular, they suggest \(\alpha = -1\), so that to fulfill the assumption of non-autocorrelation the original observations only need to be replaced by their first differences. A simple example on a single equation may illustrate the point.

\[ y_t + \beta z_t = u_t \]
Assume that 4.10 holds.

4.10 \[ u_t - u_{t-1} = v_t \]

Then from 4.9 we obtain

4.11 \[ y_{t+1} - y_t + \beta (z_{t+1} - z_t) = v_t \]

The bundle of commodities that a consumer buys may be divided into two groups, livestock products and nonlivestock products. The aggregate \( c_t \) might be regarded as the "commodity" livestock products. Hicks (12, p. 312) has demonstrated mathematically that if the prices of a group of goods change in the same proportion, that group of goods behaves, in respect to certain properties, as if it were a single commodity. Changes in \( r_t \) would involve proportional changes in the nonlivestock products so that we may regard the nonlivestock products as a single commodity in considering the property of substitutability and complementarity. Since \( \frac{\partial c_t}{\partial r_t} \) is positive in all cases, and since presumably none of the livestock products are inferior goods, then the substitution effect is large, relative to the income effect, and positive, so that the two groups, livestock products and nonlivestock products, are substitutes.

Under the assumption that the disturbances in the system are normally distributed with mean zero and covariance matrix independent of \( t \) and of the predetermined variables and that the disturbances are non-autocorrelated, the limited information estimators are asymptotically normally distributed. Anderson (1, p. 317) has given certain
weaker conditions under which the property of asymptotic normality still holds. Given the property of asymptotic normality, Anderson and Rubin (3, p. 56) have developed a large sample test for overidentification in linear systems. The asymptotic distribution of $T \log (1 + \nu)$ is the $\chi^2$ distribution with $k^* - 2 + 1$ degrees of freedom. $T$ is the total number of observations in the sample and $\nu$ is the smallest root of a certain determinantal equation (1, p. 315). If there are more than $2 + 1$ zero coefficients prescribed in the demand equation, the hypothesis being tested is that all of the coefficients assumed zero actually are zero against the alternative that a smaller number are zero. The value of $T \log (1 + \nu)$ obtained was 5.602 with 3 degrees of freedom for the linear demand function. The critical value of $\chi^2$ with 3 degrees of freedom at the 5 percent level is 7.815 so that the null hypothesis that the coefficients assumed zero in the demand function actually are zero was not rejected. If the equation system in Chapter III were assumed linear in the logarithms, the value of $T \log (1 + \nu)$ used for testing the null hypothesis would be 2.713 and the null hypothesis would not be rejected. The presence of the two identities in equations 3.8 and 3.9 would be inconsistent with a system linear in the logarithms but this inconsistency was ignored in applying the test.
V. CONCLUSIONS

The experimental nature of the results in this study must be emphasized. If one had an equation that was known to be a close approximation to the demand equation for livestock products, a number of practical inferences for private and public policy could be drawn. While the ultimate purpose of such empirical analysis as has been done in this work is to permit such inferences to be drawn, the uncertainties about any of the equations presented would make a discussion of such inferences, at this time, purely illustrative. The variation in the results presented in Chapter III, both within this thesis and between other studies, should serve to point up the uncertainty that the economist faces in statistical demand analysis.

Two intermediate ends have, perhaps, been satisfied by this study. Firstly, the formal description of the functioning of a particular market may of itself contribute to existing knowledge of this sector of the economy. Secondly, some of the problems in the construction of econometric models and the estimation of their parameters have been illustrated.

The use of limited information has as an advantage the relative simplicity, and cheapness, of the computations in comparison with the use of the full maximum likelihood method given by Koopmans et al. (20, p. 153). However, the neglect of certain information by
ignoring some of the restrictions in the system probably affects the efficiency of the estimators. See Anderson (1, p. 321). An exact expression of the loss of efficiency in so ignoring some of the information has not been worked out but it is conceivable that the computational ease of limited information is offset by the decreased efficiency of the estimators.

Hildreth (13) has presented some preliminary work on an estimation procedure which may have some advantages over limited information. The advantages arise from the computational simplicity of the former technique which has been called means of subsets estimation. Hildreth considered the case where an investigator wishes to estimate some or all of the unknown parameters of a system of equations given by--

\[ \Phi_i (y_t, z_t, u_t) = 0, \quad i = 1, 2, \ldots, G \]
\[ t = 1, 2, \ldots, T. \]

The subscript \( i \) denotes successive equations of the system; the subscript \( t \) denotes successive observations in the investigator's sample. \( y_t \) is a vector of the values taken by the jointly dependent variables at time \( t \). \( z_t \) is a vector of the values of the predetermined variables at time \( t \). \( u_t \) is a vector of unobserved random disturbances assumed to have a stable multivariate distribution that is independent of the predetermined variables. The disturbances are further assumed to have zero means and finite variances.
Suppose the investigator wishes to estimate the parameters of one equation, say the first, assuming that this equation is linear in the observed variables and that $u_{1t}$ enters additively. The first equation could then be written as:

$$5.2 \quad y_{1t} + \beta_{12}y_{2t} + \cdots + \beta_{1g}y_{gt} + y_{11}z_{1t} + y_{12}z_{2t} + \cdots + y_{1k}z_{kt} + y_{10} = u_{1t}$$

where it is assumed that the number of $y$'s appearing in the first equation is $g$ and the number of $z$'s is $k$. The coefficients in 5.2 have been normalized by setting the coefficient of $y_{1t}$ equal to one. Nothing is assumed about the algebraic form of other equations appearing in the system but it is assumed that the $z$'s which appear outside the first equation and which do have a nonzero coefficient are known.

The $t$ subscripts are now dropped and $u_1$ is set equal to zero. The observations, $y_t$ and $z_t$, are divided into $g + k$ subsets according to the magnitudes of some or all of the elements of $z_t$. The mean of each observed variable appearing in 5.2 is computed for each subset of the observations. Let $\overline{y}_{1j}$ be the mean of $y_1$ in the $j$-th subset and $\overline{z}_{1j}$ be the mean of $z_1$ in the $j$-th subset. The estimates are then obtained from the following set of equations:

$$5.3 \quad \overline{y}_{1j} + \tilde{\beta}_{12}\overline{y}_{2j} + \cdots + \tilde{\beta}_{1g}\overline{y}_{gj} + \overline{y}_{11}\overline{z}_{1j} + \overline{y}_{12}\overline{z}_{2j} + \cdots + \overline{y}_{1k}\overline{z}_{kj} + \overline{y}_{10} = 0 \quad j = 1, 2, \ldots, (g+k)$$
where the coefficients appearing with the tilde are estimates.
Unique estimates of the coefficients are obtained if the equations of 5.3 are linearly independent.

The exact way in which the partitioning of the observations is to be performed is not stated. Nothing is known of the properties of the estimators from this method except the property of consistency. This latter property does not depend on the non-autocorrelation of the disturbances, so in this regard the method overcomes one of the difficulties inherent in limited information. The property of non-autocorrelated disturbances appears essential to the limited information method, but work by Cochrane and Orcutt (8), (23) would suggest that this assumption may be questionable in some economic models.

In the equations presented in this thesis certain improvements may be possible. $z_{7t}$ was chosen as representing the influence of "other prices" in the demand function. Ideally, $z_{7t}$ would not contain any components due to the livestock products. The fact that elements of $\gamma_{5t}$ appear in $z_{7t}$ means that the estimate of $\gamma_{57}$ will be biased, the extent of the bias being determined by the contribution that livestock and livestock products make to the index of all goods and services. One further objection to the use of the Cost of Living Index of Moderate Income Families in Large Cities is that the index is selective, both by income and geographical area. This latter objection could be overcome by using the Bureau of Labor Statistics' Index of Wholesale Prices Excluding Farm Products (42, 43).
For such a purpose, extensive development of education and an extensive research procedure should be associated with the obtaining and analyzing from samples the model of the statistical procedure. The study may conclude that the main influential statements are not possible to the choice of the form of the function chosen while the demand between 1933 and 1949, Table III, would appear unexpected observed rather strongly. The qualification in the estimation of the preference for a regression line equation over an equation linear in the estimated $E$ in Table III for 1949 would lead to indicate a difference $E$, the which the regression between the result and the appearance of even allowing for some underestimation in the budget (10, p. 80). The income statements in the budget would be preferred the figures.

The index would give a wider coverage both of items and products other than manufactured products would be excluded from areas that would still result in some bias in some cases. The latter index would give a wider coverage both of items and products other than manufactured products would be excluded from areas that would still result in some bias in some cases. The latter index would give a wider coverage both of items and products other than manufactured products would be excluded from areas that would still result in some bias in some cases.
VI. LITERATURE CITED


38. Outlook charts. 1951.


VII. ACKNOWLEDGMENTS

The author is indebted to Mr. Malcolm Clough and Mr. A. V. Nordquist both of the Bureau of Agricultural Economics, U. S. Department of Agriculture, Washington D. C. for access to unpublished data. Dr. J. A. Nordin materially improved the presentation by his careful perusal of the manuscript and his constructive criticisms. Dr. T. A. Bancroft, Dr. Gerhard Tintner and Dr. G. S. Shepherd also contributed valuable comments. My greatest single indebtedness is to Dr. Clifford Hildreth, Cowles Commission for Research in Economics, University of Chicago. During the period of this study the constant advice of Dr. Hildreth was of the greatest aid and stimulation.
VIII. APPENDIX

This appendix contains a description of the construction of the variables used in the estimation of the parameters in the formulated models. An attempt was made to form from existing data observed variables which could be identified with the theoretical variables in the models. The problem of constructing aggregates from non-homogeneous units is a difficult one and to a large extent the weighting system employed depends on what appears reasonable to the investigator. The use of such aggregates for predictive purposes can be misleading, since aggregation may tend to obscure considerable changes in a particular element in the aggregate without these changes appearing in the predicted aggregate value. In other words, if the administrator is interested in the probable effects of some government action on a particular component of the livestock and livestock products group, then, the aggregate may have somewhat restricted use.

\( y_{lt} \) is the quantity of livestock and livestock products sold during the year. Home consumed products were treated as sold. The components of \( y_{lt} \) were hogs, cattle, calves, sheep and lambs, chickens, turkeys, broilers, eggs and milk. Each of these components was weighted by its average farm price over the sample period. The resulting dollar value figures were summed to give \( y_{lt} \). Prices were
used as weights since they seemed more adequately to represent consumer preferences than an alternative method.

All data for hogs were from (37). Hog marketings in thousand pounds liveweight included inshipments for feeding and breeding purposes. Inshipments were excluded from the marketings data for two reasons. Firstly, animals which were shipped in, fed and sold during the calendar year would appear to be counted both as inshipments and also as subsequent slaughterings. Secondly, \( \gamma_{ht} \) represents a consumption variable since the study was concerned with the demand for livestock products by ultimate consumers and not in the demand for feeding and breeding purposes. Inshipments in thousand pounds liveweight were computed by dividing the cost of inshipments in a given year by the farm price for one thousand pounds for that year. The liveweight of inshipments was then deducted from liveweight marketings. To the figure so resulting was added farm slaughter in thousand pounds liveweight to give total sales of hogs. Average weights for farm slaughter in each year were obtained by dividing marketings in pounds liveweight by marketings in numbers and using these average weights to convert farm slaughter in numbers to thousand pounds liveweight. The total sales figure in each year was weighted by the average farm price per pound over the sample period to yield a dollar value of sales at an average price.

Total slaughter of cattle in thousands, including farm slaughter, were from (28). To convert numbers to thousand pounds liveweight,
the composed average farm price to result in a dollar quantity

Interpreted as sheep and lambs for each year was then weighted

year to obtain a average price in pounds interpreted. The pounds
income was divided by the composed average farm price per pound for each

of income was interpreted and then averaged across income dollars per pound for sheep and lamb was the mean of
the series so resulting. Interpreted average farm price per pound for sheep and lamb was the mean of

receptors inollar by market prices in pounds, both years (37). The

composed average was estimated for each year by weighting each

of sheep and lamb sales combined. A farm price per pound for the

interpretation had to be recognized in constructing a dollar change
available for sheep and lamb sales, the experience of the prices
price for sheep per pound. These experience observations were not
per pound was approached higher, in most years, than the farm
able for each of these two commodities, the farm price of lambs

In the case of sheep and lambs, separate farm prices were used.

equity theory.

By the average farm price per pound for cattle to give a dollar

weighted volume from (37), pounds interpreted were then weighted

of cattle, from (37) was converted from numbers to pounds alive
value of sales at an average price. Simultaneously the total slaughter

by the average farm price of cattle per pound to obtain a dollar

need, the pounds interpreted of slaughtered cattle were then weighted

the weights of Federal inspected slaughter of cattle (37) were
production in million pounds was from (28) (15) (1) and estimated with
per cent production and sales of milk were assumed constant.
price for the period 1929-39 to convert estimate from dairy to
when were from (28) and those were weighted by the average farm
production and sales were assumed equal. The production

the dollar figure for sales of butter

price per pound (28) over the period 1931-39 was used to obtain
commercial butter production was assumed to be zero. The average
the estimated production data were from (17) (6). Prior to 1931-

butter production and sales were assumed equal. From 1931-39

* to convert pounds to dollars

for $ per cent per year. The average farm price per pound was used
assuming 150 million pounds produced in 1929 and a constant increase
pounds increased from (28) from 1920-29 estimates were obtained

household use of butter as of which change in household use
earlier years. From 1929-39 production could have been used as
since relatively few data are available for household data
for further analysis, production and sales were taken as constant

the average farm price per pound over the sample period

As in pounds increased, pounds increased were then weighted
in pounds increased. Pounds increased were weighted to give sales
numbers each plus number of consumers in farm households, both from

data on farm prices per head and farm prices per pound (28) (6) (9)

an average weight for consumption was computed for each year from
sucked by calves and milk produced by cows not on farms. The average farm price per pound for milk was from (38, p. 52).

\( y_{5t} \) is an index of the price of livestock and livestock products sold during the year. To obtain this index sales data in physical measure (pounds liveweight, dozen, etc.) for each year were multiplied by the appropriate farm price for that year and summation yielded the value of sales at current prices. Division of the total value of sales at current prices by \( y_{1t} \) (sales at average prices) resulted in \( y_{5t} \).

\( z \) is disposable personal income in the United States in dollars \((32), (38)\).

\( z_{7t} \) is the Index of Consumer Prices for Moderate Income Families in Large Cities published by the Bureau of Labor Statistics \((42), (28, 1950)\).

\( x_{6t} \) is population in the United States and was from \((38, p. 15)\).

\( x_{1t} \) is \( x_{9t-1} \) and is an index of the quantity of livestock held on farms at the end of the year. From \((36) \) and \((28) \) data were obtained on the number of livestock on farms, January 1, by classes. The classes were based on age and sex and the components which entered the aggregate were cattle, calves, hogs, sheep and lambs. An average potential production for an individual animal in each category was estimated and these estimates were used as weights in combining animals in various categories to give an aggregate in dollar terms. The use of prices to obtain average potential production estimates was consistent with the construction of \( y_{1t} \) and \( y_{4t} \).
Cattle and calves on farms January 1 are divided into animals for milk production and those not for milk production. In the for milk production category the classification is cows and heifers two years old and over, heifers one to two years and heifer calves. The not for milk class includes cows and heifers two years old and over, heifers one to two years old, calves, steers and bulls.

Animals in the category cows and heifers two years old and over for milk are possible milk producers and also may produce calves during the year. Total milk production over the sample period was divided by the total numbers of animals in this classification to obtain an average milk production figure. This latter figure was weighted by the average farm price of milk over the sample period to give an average potential milk production estimate of $61.27. The weight for potential calf production was calculated in the following manner:

\[ \frac{P}{Q+R+(0.6)S} = X_1 \]

where

- \( P \) is total calves saved (births - deaths) over the sample period.
- \( Q \) is the total cows and heifers one to two years old and over in the for milk category (1920-29).
- \( R \) is total cows in the not for milk category.
- \( S \) is total heifers one to two years old in the not for milk category. The figure 0.6 was used on the assumption that approximately 60 per cent of the heifers in this
category are bred, the remaining 40 per cent being fattened.

\( X_1 \) is a net birth rate and was estimated as 0.62.

\((X_1)(q)\) is the total number of calves saved in dairy herds.

The total number of calves slaughtered (26) divided by \((X_1)(q)\) gave the average fraction \((0.54 = X_2)\) of calves sold for slaughter, assuming all slaughtered calves were from dairy herds. From (26) the total value of calves slaughtered divided by the total number of calves slaughtered gave the average value of a slaughtered calf \((\$21.58 = X_3)\). The average contribution for a calf which is slaughtered during the year was the product of \(X_1, X_2, X_3\), namely, \(\$7.23\). Some calves will not be slaughtered in a given period but will be carried over into the next period. This potential contribution was computed as \((1-X_2)\) times the liveweight of a heifer calf, 295 pounds from unpublished Bureau of Agricultural Economics data (22) times the average farm price per pound for calves over the sample period. This yielded an estimate of \(\$14.40\). The sum \(\$81.27 + \$7.23 + \$14.40 = \$102.90\) was the average potential contribution of an animal in the class cows and heifers two years old and over for milk.

Heifers one to two years for milk possess all the producing capacities of animals in the category two years old and over but also may gain weight in moving from the former to the latter class. The average gain in weight in so moving was 265 pounds from unpublished Bureau of Agricultural Economics data (22). The value component due to gain in weight was 265 times the average farm
price for cattle over the sample period. The estimate yielded
was $23.45. The total average potential production estimate was
then $102.90 + $23.45 = $126.35.

Heifer calves potentially can gain weight in moving from this
class to the class heifers one to two years. The gain in weight
in so moving was 280 pounds from unpublished Bureau of Agricultural
Economics data (22). Two hundred eighty times the average farm
price of cattle per pound over the sample period was the appropriate
weight for animals in this class. The calculated weight was $28.78.

Cows and heifers two years old and over not for milk may
produce calves and, in line with the assumption that all slaughtered
calves were from dairy herds, calves from animals in this class
will all be carried over into the next period. Unpublished data (22)
from the Bureau of Agricultural Economics indicated that the average
weight of a calf in beef herds on January 1 was 305 pounds. The
value contribution of an animal in the category under consideration
was 350 $v_1$ times the average farm price per pound for calves over
the sample period. The estimate yielded was $20.06.

Animals in the class heifers one to two years not for milk
may produce calves and may also gain weight. It was assumed that
only 60 per cent of this class were bred so that the potential produc-
tion for a calf carried over is (0.6)($20.06) = $12.04. The gain in
weight in moving from this class to the class cows and heifers two
years old and over was 265 pounds from unpublished Bureau of Agricul-
tural Economics data (22). The gain in weight contribution was given
by 265 times the average farm price per pound for cattle, namely, 
$23.45. The total potential contribution of animals in this class was 
$12.04 + $23.45 = $35.49.

The class calves not for milk contains animals whose potential 
contribution is a gain in weight. Both heifer and bull calves 
are included in the class, with the latter tending to show larger 
weight gains than the former. It was estimated that an average 
gain in weight for an animal in this class would be 335 pounds. The 
potential production estimate was then 335 times the average farm 
price per pound for cattle. The estimate yielded was $29.65.

The potential contribution of steers consists of a gain in 
weight. The average weight of a steer on January 1 was taken as 795 
pounds from unpublished data (22) from the Bureau of Agricultural 
Economics. The average slaughter weight was estimated as 1020 pounds 
from (36) so that the average gain in weight was 225 pounds. The 
estimate of potential production was 225 times the average farm 
price per pound for cattle. The calculated figure was $19.91.

Young bulls on hand January 1 may make a potential contribution 
to production by exhibiting a gain in weight. Data were not available 
on the average weight gain in the period from January 1 to slaughter 
so it was assumed that the average potential production of an animal 
in this category was 10 per cent that of cows two years old and over 
not for milk. The figure resulting was $2.61.

Hogs on farms January 1 are classified as under six months old,
sows and gilts six months old and over, and others six months old
and over. Following Jennings (17) pigs under six months were treated
as fall pigs, pigs over six months as spring pigs. Potential produc-
tion of pigs under six months consists in the value of farrowing
plus any gain in weight, since gilts which are capable of farrowing
during the year are included in the under six months class. Sows
and gilts six months old and over may farrow and also gain weight.
Pigs over six months are potential weight gainers only.

In the case of fall pigs the average gain in weight was given
as 1.40 pounds in Jennings (17, p. 7). One hundred forty times the
average farm price per pound of hogs over the sample period was
the component of the potential production estimate due to the gain
in weight. The calculated figure was $14.53. It was assumed that
one half of the total, 1924-49, fall pigs saved, from (37), were
farrowed by gilts under six months on January 1. To obtain a fall
birth rate from new sows, 0.5 times total fall pigs saved was divided
by total, 1924-49, fall pigs on hand January 1. This fall birth
rate from new sows was estimated as 0.56. From (37) it was estimated
that deaths subsequent to the pigs saved enumeration were approximate-
ly 10 per cent of all pigs saved. The average weight of a fall pig
on January 1 was given by Jennings (17, p. 7) as 90 pounds. The com-
ponent due to gilts under six months on hand January 1 which subseque-
tly farrow was given by (0.9) (0.56) (90) (average farm price per pound
of hogs). This product was $4.71. The total average potential
production of fall pigs was then $13.71 + $15.53 = $19.24.

Spring pigs were assumed to exhibit an average gain in weight during the year of 50 pounds, as indicated by Jennings (17, p. 7). The estimated average production potential was then 50 times the average farm price per pound for hogs. The estimate was $5.19.

Sows and gilts may produce a spring litter and a fall litter, both of which will show a subsequent weight gain. Sows and gilts may themselves exhibit some weight gain during the year. This latter weight gain was assumed to be 60 pounds so that this component of the estimated potential production was 60 times the average farm price per pound for hogs, or $6.23. A spring birth rate was calculated by dividing the total, 1924-19, spring pigs saved, from (37), by the total number of sows and gilts on hand January 1. The estimated birth rate was 5.3. Deaths subsequent to the pigs saved enumeration were again taken at the rate of 10 per cent of pigs saved. The average weight of a spring pig on hand January 1 was taken as 190 pounds, from Jennings (17, p. 7) so that the component due to a spring litter was (0.9) (5.3) (190) (average farm price per pound for hogs). This product was $94.07. A fall birth rate for sows and gilts was computed by assuming 0.5 times the total, 1924-19, fall pigs saved were farrowed from sows and gilts and dividing the total number of such farrowings by sows and gilts on hand January 1. The fall birth rate was 1.54. The average weight of a fall pig on hand January 1 was taken from (17, p 7) as 90 pounds. Assuming deaths
at the rate of 10 per cent the component due to a fall litter was (0.9) 
(1.54) (90) (average farm price per pound of hogs). The figure so 
resulting was $12.95. The total average production potential for 
sows and gilts six months and over was $6.23 + $91.07 + $12.95 = 
$113.25.

Sheep and lambs on hand January 1 are classed as sheep on 
feed and stock sheep, the latter category including ewe and ram 
lambs, ewes one year and over, rams and wethers. The latter are 
predominantly wool producers and since wool did not enter \( y_{1t} \) 
or \( y_{it} \), wethers were excluded from \( z_{it} \).

Sheep and lambs on feed are potential weight gainers. The 
average weight gain as indicated from unpublished Bureau of Agricultural 
Economics data was 12 pounds (22), so that the average potential 
production of animals in this class was 12 times the average farm 
price per pound for lambs over the sample period. The estimate 
so resulting was $1.31.

Ewes in the ewe and ram lamb category were taken to be too 
young for breeding on information from (21, p. 778), so that this 
class may exhibit a gain in weight only. The gain in weight in 
moving from this class to the class ewes one year and over, below, 
was given as 40 pounds in unpublished Bureau of Agricultural 
Economics data (22). The average potential production contribution 
of animals in this class was 40 times the average farm price per 
pound of sheep over the sample period. The figure that resulted 
was $2.30. The gain in weight was assumed to be the same for ewe
and ram lambs.

Ewes one year and over may produce a lamb but probably show little gain in weight. From (37) total, 1924-29, lambs saved divided by total ewes one year and over gave a lambing rate of 0.35. Deaths subsequent to weaning were approximately 10 per cent of lambs saved. The average weight of a lamb on January 1 was assumed to be 75 pounds so that the average potential production estimate was (0.9) (0.35) (0.75) (average farm price per pound for lambs). This product was $6.25.

Rams on hand January 1 will include some young rams over one year old to which feed will be fed and which may make a contribution to production through a gain in weight. No data were available on the magnitude of this weight gain but, rather than assume a zero potential contribution, it was assumed that a weight of 10 per cent of that of ewes one year and over would be appropriate. The estimate used was $0.63.

3. is the pounds of total digestible nutrients in roughages fed to livestock. Feeds classed as roughages were alfalfa hay, clover and timothy hay, lespediza hay, soybean hay, peanut hay, cowpea hay, grains cut green, wild hay and other hay, peanuts hoggd off, sorghum silage, sorghum forage, wet beet pulp, corn hoggd off, corn silage and pasture.

The total quantity of hay fed to livestock, excluding horses and mules and livestock not on farms, for the years 1920-41 was from (16) on a calendar year basis. For the years 1945-47 consumption was on a May 1 year (15). Observations for 1948-49 on a May 1 year were
Table IX

Livestock Aggregates Constructed for the Computations

<table>
<thead>
<tr>
<th>Year</th>
<th>$\gamma_{nt}/x_{nt}$ (dollars)</th>
<th>$\gamma_{5t}$</th>
<th>$x_{1t}/x_{nt}$ (dollars)</th>
</tr>
</thead>
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<tr>
<td>1919</td>
<td>55.95</td>
<td>1.0277</td>
<td>55.93</td>
</tr>
<tr>
<td>1920</td>
<td>53.10</td>
<td>0.6931</td>
<td>54.20</td>
</tr>
<tr>
<td>1921</td>
<td>52.52</td>
<td>0.6839</td>
<td>54.57</td>
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<tr>
<td>1922</td>
<td>54.83</td>
<td>0.6916</td>
<td>55.87</td>
</tr>
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<td>1923</td>
<td>57.72</td>
<td>0.6833</td>
<td>52.39</td>
</tr>
<tr>
<td>1924</td>
<td>57.90</td>
<td>0.6907</td>
<td>48.40</td>
</tr>
<tr>
<td>1925</td>
<td>55.41</td>
<td>0.6362</td>
<td>47.06</td>
</tr>
<tr>
<td>1926</td>
<td>55.88</td>
<td>0.6132</td>
<td>46.96</td>
</tr>
<tr>
<td>1927</td>
<td>55.50</td>
<td>0.6267</td>
<td>46.84</td>
</tr>
<tr>
<td>1928</td>
<td>55.15</td>
<td>0.6557</td>
<td>46.63</td>
</tr>
<tr>
<td>1929</td>
<td>54.75</td>
<td>0.7365</td>
<td>46.26</td>
</tr>
<tr>
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<td>0.5430</td>
<td>47.03</td>
</tr>
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<td>48.56</td>
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<td>50.55</td>
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<td>55.07</td>
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<td>50.23</td>
</tr>
<tr>
<td>1934</td>
<td>56.17</td>
<td>0.6160</td>
<td>43.59</td>
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<td>1935</td>
<td>48.85</td>
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<td>44.43</td>
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<td>52.74</td>
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<td>43.22</td>
</tr>
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<td>1937</td>
<td>51.39</td>
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</tr>
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<td>1938</td>
<td>52.73</td>
<td>0.5894</td>
<td>45.33</td>
</tr>
<tr>
<td>1939</td>
<td>54.80</td>
<td>0.5907</td>
<td>46.86</td>
</tr>
<tr>
<td>1940</td>
<td>57.43</td>
<td>0.7906</td>
<td>46.19</td>
</tr>
<tr>
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<td>57.07</td>
<td>0.9577</td>
<td>49.61</td>
</tr>
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<td>1.0990</td>
<td>49.33</td>
</tr>
<tr>
<td>1943</td>
<td>69.37</td>
<td>1.1054</td>
<td>49.33</td>
</tr>
<tr>
<td>1944</td>
<td>70.28</td>
<td>1.1693</td>
<td>49.33</td>
</tr>
<tr>
<td>1945</td>
<td>67.97</td>
<td>1.3722</td>
<td>49.33</td>
</tr>
<tr>
<td>1946</td>
<td>64.76</td>
<td>1.6153</td>
<td>49.33</td>
</tr>
<tr>
<td>1947</td>
<td>63.99</td>
<td>1.7534</td>
<td>42.27</td>
</tr>
<tr>
<td>1948</td>
<td>60.52</td>
<td>1.4545</td>
<td>42.27</td>
</tr>
<tr>
<td>1949</td>
<td>61.74</td>
<td>1.4545</td>
<td>42.27</td>
</tr>
</tbody>
</table>
to December. In these grounds October were used as calendar
red off before the advent of winter, with little feeding unsafeguarded
on October 1st it was assumed that most of the sorghum forage was
then and commissi.on were assumed equal and although the dura were on
force the dates were from (15) (33) for the years 1929
and

From which it was likely that the given quantities were not
actual or that the irrigation was not being entered as it
was applied to the inundation have been entered as they
any proceeds in each year 1929-1939 (33) p. 60. Since process
have entered the percentage contributed that each had made to total

from on any a calendar year basis was directly amount entered

it on any a calendar year basis is or the total consumed-

* To obtain consumption of inundation types by any the total consumed

the quantities of any rate in the early part of the calendar year
and 0.67 and the weather at which area were considered consumed to
assumed constant monthly reading rate would imply weather of 0.77
and that here would be necessary red during the winter months. An
it was assumed that it was necessary to red during the growing period
was available on the wet or monthly reading rates or hay. However
the varieties used were so some extent applicable since no information

May Year

May Year

(1) 0.35 conservation

(1) Conservation

0.65 conservation

0.65 = 1.95-1.99. From this

I hope above usage were used accurately for the yeare

I hope that this is correct. To convert a May 1 year to a calendar year a

stock on average 15,000 tons for horses and mules and 7,750 tons for

operational from (33) p. 60. A year total hay red to all lands
year data. For the years 1920-26 consumption was estimated by employing the ratio of sorghum forage production to the production of all sorghums for grain (33, p. 6) for the years 1929-33 on data on sorghum grain production for the years 1920-26.

In the case of sorghum silage, consumption was again taken as equal to production. The series from (15), (28) from 1929-19 were on an October year basis. This series was first completed back to 1919 by employing the ratio of sorghum silage production to the production of all sorghums for grain for the years 1929-33 and proceeding as for sorghum forage. To place the October year series on a calendar year basis a moving average with weights of 0.70 and 0.30 was used. The weights imply a slightly heavier rate of feeding in the winter months. For wet beet pulp, production equals consumption, October year data (15), (28) were weighted by a moving average with weights of 0.50, 0.50. The October year series was first completed back to 1919 by using the ratio of wet beet production to total beet production (28) for the years 1929-33 on data on total beet production for the years 1919-26. Corn silage consumption was from (28). This series included the grain in silage and to obtain the consumption of silage as roughage an average grain content of 12 per cent (16, p. 3) was deducted. Corn hoggcd off as roughage was obtained by multiplying acreage hoggcd off (33, p 4) each year by the yield of corn silage per acre for that year (28). A 12 per cent grain content was deducted. It was assumed that most of the corn hoggcd off was done before the end of
THE data from October were reexamined using data from (33)

October 1940 was taken as an observation for the 1940-1946
and January 1946 as a subsequent to October, the observation for the first
coefficient to read unity to June on the assumption that the ratio of
coefficient to the ratio of station to 0.65 to
the base of the feed with standard is equal to 1.05 in the
net energy of pasture grasses and clovers in competition with con
was then converted into resistant measure, from (13), p. 67.
were to convert feed units into resistant measure. The first problem that arose
desirable to combine these two sets. The first problem that arose
and coefficient but not including into account animal numbers. It seemed
estimated on the basis of pasture and net energy in competition basis on area

By the ratio of pasture coefficient (33), p. 67).

The area of pasture coefficient (33) were

The area where were based on

For pasture consumption, two phases of information were available

what area to yield estimates for 1940-76.

were applied to data from (33) on area nowDocked off the South
off in the South west area to total consumption in the United States
1937-49, prior to 1937, the ratio of total, 1937-49, area nowDocked
December, data on pastures nowDocked off were from (33) for the years
In this study the same conversion factor was used on woodland pasture but pasture not in farms was included and the conversion factor changed from 0.25 to 0.1. A further difference is that Clough used an average value in acres for woodland pasture and pasture other than plowable whereas observations on each of the three categories of pasture were obtained by using a linear interpolation for the inter-Census years. The interpolation was performed in the following manner. For two successive Census years total pasture acreage plus crop land harvested (28), (31) was obtained. A linear interpolation yielded observations on this combined acreage for inter-Census years. Similar interpolations yielded acreages of woodland pasture and other than plowable pasture. Subtraction of these latter two acreages plus crop land harvested for that year from the combined total pasture and crop land harvested interpolation resulted in an estimate of plowable pasture for the given year. To allow for the effect of pasture-range condition on consumption, average pasture condition was given a weight of 0.66 and average range condition an index of 0.38. The weights were based on estimated amounts of feed coming from pasture and range respectively (7). The weighted condition figure times plowable pasture equivalent resulted in a production-condition figure in acres.

To convert acres to pasture in tons an average "yield" was computed from total, 1920-49, pasture in tons from (15) and total, 1920-49, production-condition data in acres. The production-condition
estimate in tons for each year was the acreage figure for that year times the average "yield". The production-condition estimates in tons will tend to over-estimate pasture consumption in the early years and under-estimate consumption in later years because of a component due to horses and mules which have to be excluded. Average pasture in physical tons fed to horses and mules over the sample period was obtained from (15, p. 74). For each year the deviation of pasture fed in physical tons from the mean was obtained and this deviation was added to the production-condition estimates in tons. As would be expected these deviations were negative in early years and positive in later years of the sample. Finally, pasture consumption in tons was obtained using the two pieces of information by giving a weight of 0.7 to the estimates based on (15) and a weight of 0.3 to the estimates based on (7). The weights used were somewhat arbitrary but it was assumed that animal numbers were more important in determining pasture consumption than was production, so that a heavier weight was given to data from (15).

To obtain quantity of roughages in terms of total digestible nutrients the average composition figures from (15), (21) were used as weights in forming the aggregate.

210t is the production of protein concentrates in pounds of total digestible nutrients. The following feeds were classified as protein concentrates. Cottonseed cake and meal, soybean cake and meal, linseed cake and meal, peanut cake and meal, tankage and meat scraps, fish meal, gluten feed and meal, brewers' dried grains, distillers'
An explanation for each year where an observation was made is

Concerning the calendar year basis for the years 1926-78 were taken
production was from (33), p. 61). For years prior to 1926, the averages
last quarter. For estimates meant for the years 1926-74 calendar year
appears that the percent of total annual production took place in the

0.7% of "ony" guaranteed production during zones (33), p. 61) indicated that
year basis by combining a monthly average with weights of 0.5 and
14% of October, December production (33), p. 61) was plotted on a calendar
for 1926. October, December production (33), p. 61) was extended on a calendar
each of these three years, calendar year production of cornseeded
beginning October 1922 was taken as the calendar year observation for

(33), p. 64) for 1927-49. For 1920-49 the production of the year
for sheep are only one month, calendar year production was from

These are the weights assumed zero prior to 1926.

September (33) for 1920 to 1929 and corresponding for 1930, from
production observed by various methods 1921 production by the ratio of total
an observation for ten week end and meat stores production for 1920 was
that for 1920 production was assumed to be the same as production for 1921.
between the agricultural production (33) and unproduced from the
brevets' data (33) and unproduced data from the
projection date for unseeded cake and meat, pennant cake and meat
Gardner year production of unseeded cake and meat was from (33), p. 61)
"dryed cran" alithright meat, wheat with and dried with products.

-52-
obtain calendar year consumption for the years 1928-32 a moving average with weights of 0.60 and 0.40 was applied to October year data from Jennings (15). The weights used implied a heavy rate of feeding in the winter months, since alfalfa meal is mainly fed as a substitute for pasture and range.

For dried milk products production was taken as equal to consumption. From 1936-49 October year data on consumption were from (15), (33). To convert an October year to a calendar year a moving average with weights of 0.80 and 0.20 was employed. Dried milk products are to some extent fed to young animals which would seem to imply a somewhat heavier weight to spring feeding. For 1920-35 average consumption for the years 1935-39 was used for each year. Data on skim milk consumption, equals production, were from (15), (33) on an October year basis for the years 1927-49. A moving average with weights of 0.60 and 0.40 was used to convert an October year to a calendar year.

Prior to 1927, consumption was estimated from total milk production using the ratio of skim milk fed to livestock to total milk production for the years 1927-31, from (28). To actually obtain \( x_{10t} \) the various components were weighted using average composition figures from (15, p. 52).

\( x_{gt} \) is the physical supply of feed grains in pounds of total digestible nutrients. Supply was taken to be carryover January 1 plus a fraction of the current crop. The role of the physical supply of feed grains was that the existence of stocks and the appearance of the current crop could lead to decisions as to the livestock program. The
feed grains for which significant stocks existed were corn, oats and barley. For corn, farm stocks on January 1 for the years 1937-39 were from (33, p. 16). For the years 1920-26, January 1 stocks were estimated as March 1 stocks (29) plus two thirds of corn consumption in the first quarter, from (37, p. 4). From the latter source, data on quarterly consumption of corn indicated that approximately 29 per cent of total corn consumed in a calendar year was consumed in the January-March quarter. January 1 stocks were then estimated as March 1 stocks plus 0.19 times calendar year consumption, from data in (16). A similar procedure was adopted for oats for 1920-27 when January 1 stocks were not available from (33, p. 17). The quarterly pattern of oats consumption was assumed to be the same as given for "other grains" in (33, p. 4) where approximately 27 per cent of total "other grains" consumed in a calendar year was in the first quarter. January 1 stocks for 1920-27 were then estimated as March 1 stocks (30) plus 0.18 times calendar year consumption of oats, based on data from (16). Farm stocks of barley on January 1 were from (33, p. 18) for the years 1931-39. The quarterly pattern of barley consumption was assumed to be the same as that for the "other grains" category, namely, 27 per cent of total annual barley consumption was in the first quarter. For 1920-30, March 1 stocks from (30) plus 0.18 times annual consumption of barley, based on (15), gave January 1 stocks. For 1931-33, August 1 stocks (30) plus seven twelfths annual consumption of barley resulted in January 1 stocks. The weight of seven twelfths assumed a constant monthly feeding rate, indicated by the quarterly distribution of barley fed. To the
## Table X

Feed Aggregates Constructed for the Computations

<table>
<thead>
<tr>
<th>Year</th>
<th>$\frac{x_3}{x_6}$</th>
<th>$\frac{x_9}{x_6}$</th>
<th>$\frac{x_{10}}{x_6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Per capita TDN in roughage fed to livestock (thousand pounds)</td>
<td>Per capita supply of feed grains (thousand pounds TDN)</td>
<td>Per capita production of protein concentrates (pounds TDN)</td>
</tr>
<tr>
<td>1919</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1920</td>
<td>2.248</td>
<td>1.329</td>
<td>72.38</td>
</tr>
<tr>
<td>1921</td>
<td>2.131</td>
<td>1.528</td>
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</tr>
<tr>
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<td>2.192</td>
<td>1.303</td>
<td>67.46</td>
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<tr>
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<td>2.159</td>
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<td>2.066</td>
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<td>1.094</td>
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<tr>
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<td>75.77</td>
</tr>
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<td>1.182</td>
<td>76.03</td>
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<td>1.791</td>
<td>0.782</td>
<td>71.69</td>
</tr>
<tr>
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<td>80.30</td>
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<td>84.19</td>
</tr>
<tr>
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<td>1.833</td>
<td>1.155</td>
<td>85.77</td>
</tr>
<tr>
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<td>1.206</td>
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<td>1.917</td>
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<td>2.048</td>
<td>1.256</td>
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<td>2.133</td>
<td>1.370</td>
<td>107.54</td>
</tr>
<tr>
<td>1943</td>
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</tr>
<tr>
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<tr>
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<tr>
<td>1946</td>
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<td>1.811</td>
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<tr>
<td>1949</td>
<td>1.786</td>
<td>1.417</td>
<td>120.85</td>
</tr>
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</table>
farm stocks on January 1 was added a fraction of production for that calendar year. Data from (31, p. 4) indicated that approximately 0.14 of current calendar year crop production is consumed in the final quarter of that year for corn and 0.5 for barley and oats, so these were the fractions of production used. The totals of farm stocks plus the fraction of current crop production were converted to total digestible nutrients using average composition figures from (15, p. 52).

Observations on aggregates which were constructed for the purposes of this study are given in Table IX and Table X. The source of standard series used in the computations has already been indicated in this Appendix.