Analytical SCUC/SCED Optimization Formulation for AMES V5.0*

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Abstract

U.S. centrally-managed wholesale power markets currently rely on *Security-Constrained Unit Commitment (SCUC)* and *Security Constrained Economic Dispatch (SCED)* optimizations to determine unit commitments, reserve, and scheduled dispatch levels for generating units during future operating periods. AMES V5.0 is an open source Java/Python platform that implements a combined SCUC/SCED optimization capturing salient features of these actual market SCUC/SCED optimizations. This report provides extensive documentation for the analytical formulation of the AMES V5.0 SCUC/SCED optimization.

1 Introduction

In an earlier report [1], an analytically-formulated combined SCUC/SCED optimization was presented that extends the well-known SCUC/SCED optimization model by Carrión and Arroyo [2] in five key ways:

- Inclusion of non-dispatchable generation
- Inclusion of energy storage units
- Inclusion of nodal power balance constraints with possible transmission congestion
- Inclusion of zonal as well as system-wide reserve requirements
- Inclusion of imbalance penalty terms in the objective function for slack in power balance constraints

The earlier report [1] also discusses a software implementation of this SCUC/SCED optimization by means of the *Python Optimization Modeling Objects (Pyomo)* package [3, 4, 5], an open-source tool for optimization applications. Hereafter this extended SCUC/SCED optimization formulation will be referred to as the Basic Extended Carrión-Arroyo Model, or the *Basic ECA Model* for short.

Modified versions of the Basic ECA Model with Pyomo implementation have been incorporated into the AMES[1] wholesale power market platform [6], starting with AMES V4.0 [7] and continuing

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[1]AMES is an acronym for *A*gent-based *M*odeling of *E*lectricity *S*ystems. The AMES wholesale power market platform is open source software specifically designed to function as a support tool for research, teaching, and training purposes. It is not intended for commercial-grade applications.

with AMES V5.0 [8]; see Fig. 1. For example, a modified version of the Basic ECA Model was used in the AMES V4.0 implementation of stochastic and deterministic SCUC/SCED optimizations for an ISO New England test system [9, 10] with all dispatchable generation, and for an extended ISO New England test system [11] that includes physically-modeled wind turbine agents. A generalized version of the Basic ECA Model was also used in the AMES V5.0 implementation of an ERCOT test system [12] modeling ERCOT day-ahead and real-time markets over successive days of operation.

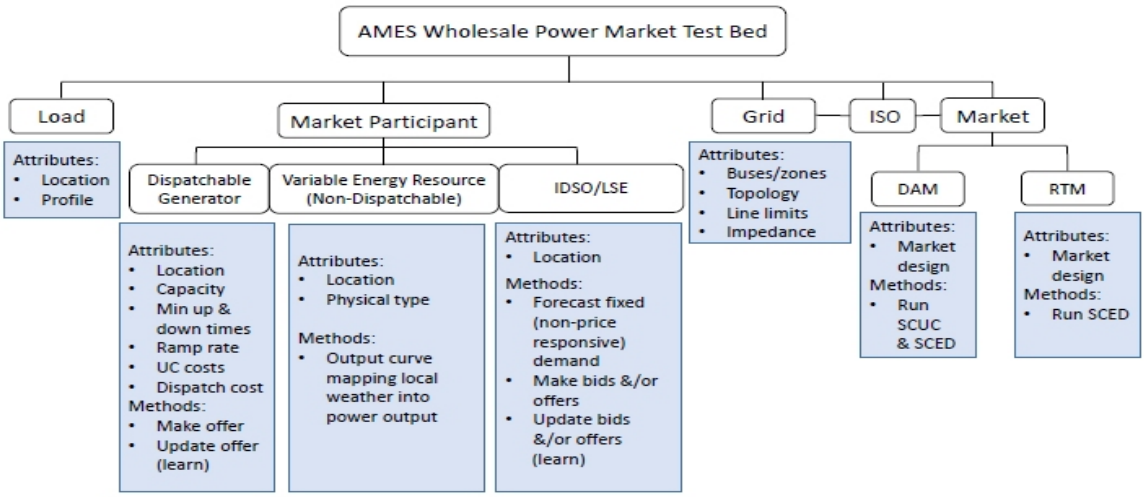

Figure 1: *Partial agent taxonomy for AMES V5.0.*

The current report provides careful documentation for the generalized version of the Basic ECA Model that has been incorporated into AMES V5.0. This *Generalized ECA Model* extends the Basic ECA Model presented in [1] in two important ways.

First, the SCUC/SCED optimization has been reformulated and generalized while still retaining a *Mixed Integer Linear Programming (MILP)* problem representation. For example, careful attention is paid to the appropriate scaling of this optimization for operating periods with different durations, and to the appropriate specification of initial state conditions for this optimization. Also, load-serving entities are permitted to submit price-sensitive demand bids, with a corresponding change in SCUC/SCED objective from constrained cost minimization to the constrained maximization of benefit net of cost.

Second, the analytical presentation of the SCUC/SCED optimization is accompanied by detailed explanations and justifications. The initial state conditions, objective function, decision variables, and system constraints for this optimization are carefully presented and motivated, and the nomenclature tables presented for this optimization include units of measurement and detailed verbal explanations to facilitate understanding.

The remainder of this report is organized as follows. Section 2 provides a broad overview of the Generalized ECA Model, and Section 3 describes the basic form of an AMES V5.0 simulation run. Section 4 provides complete nomenclature tables for the Generalized ECA Model with all fixed loads; these tables group similar terms with accompanying explanatory notes. The determination of piece-wise linear approximations for the total production cost functions of dispatchable generators is explained in Section 5. A complete analytical SCUC/SCED optimization formulation for the Generalized ECA Model with all fixed loads is provided in Section 6. This formulation is seen to be a MILP problem amenable to solution by means of standard MILP solvers.

Sections 7 and 8 then show how the SCUC/SCED optimization for the Generalized ECA Model with all fixed loads can be extended to include price-sensitive demand bids by load-serving entities while still retaining a MILP form. Three cases are considered: demand bids with time-of-use pricing; demand bids in the form of price-quantity demand schedules; and demand bids derived from general benefit functions. Section 9 explains how locational marginal price solutions are determined for the Generalized ECA Model. Concluding remarks are given in Section 10.

2 The Generalized ECA Model: Overview

The Generalized ECA Model provides a complete analytical formulation for a SCUC/SCED optimization undertaken by an *Independent System Operator (ISO)* tasked with ensuring the efficiency and reliability of wholesale power system operations. The participants in the optimization include dispatchable generators, non-dispatchable generation, energy storage units, and *Load-Serving Entities (LSEs)* functioning as intermediaries for retail power customers.

The objective of the ISO is to maximize the expected total net benefit of procuring sufficient resources to ensure the balancing of net load during a future operating period T. *Net load* is load net of non-dispatchable generation. *Total net benefit* is total benefit minus total cost. *Total cost* is the summation of production cost, start-up cost, shut-down cost, and imbalance cost. These costs are "expected" in the sense they are conditioned on period-T forecasts for fixed loads and non-dispatchable generation.[2]

The future operating period T is partitioned into $NK(\mathrm{T})$ time-steps k of equal duration. Given initial state conditions, together with period-T forecasts, the SCUC/SCED optimization jointly determines solution values for these $NK(\mathrm{T})$ time-steps, subject to system constraints. The solution values for each time-step k include: dispatchable generator unit commitments, energy storage unit commitments, dispatchable generator scheduled dispatch levels, energy storage discharge levels, energy storage charge levels, price-sensitive power usage levels, and locational marginal prices. The system constraints include:

- transmission line power flow constraints;
- power balance constraints (with slack variables);
- dispatchable generator capacity constraints;
- dispatchable generator ramp constraints for start-up, normal, and shut-down conditions;
- dispatchable generator minimum up-time/down-time constraints;
- dispatchable generator hot-start constraints;
- dispatchable generator start-up/shut-down cost constraints;
- storage unit limit constraints;
- storage unit charge/discharge constraints;
- storage unit ramping constraints;
- storage unit energy conservation constraints;
- storage unit end-point constraints;
- system-wide and zonal reserve requirement constraints.

[2]That is, certainty equivalence is used to approximate expectations.

The Generalized ECA Model formulation for power balance constraints relies on a standard DC Optimal Power Flow (DC-OPF) approximation. Consequently, it relies on the following three assumptions. First, the resistance for each transmission line is negligible compared to the reactance, hence the resistance for each transmission line is set to 0. Second, the voltage magnitude at each bus is equal to a common base voltage magnitude. Third, the voltage angle difference $\Delta\theta(\ell)$ across any line ℓ is sufficiently small that the following approximations can be used: $\cos(\Delta\theta(\ell)) \approx 1$ in size and $\sin(\Delta\theta(\ell)) \approx \Delta\theta(\ell)$ in size.

Finally, the following important terminological distinctions are used in this report. *Power output* refers to the amount of power (MW) that a resource is injecting into a transmission grid at a particular point in time. In contrast, *power generation* refers to the amount of power (MW) that a resource is producing at a particular point in time. This power generation can include local (behind-the-meter) power that the resource needs to produce and use locally in order to bring itself into a "synchronized state". A *synchronized state* is an operating state in which a resource is ready and able to inject power into the grid, even if no actual power injection is currently taking place.

3 Basic Form of an AMES V5.0 Simulation Run

3.1 Overview

AMES V5.0 [8] is an open-source Java/Python platform that captures salient features of the two-settlement system commonly implemented in all seven U.S. energy regions organized as RTO/ISO-managed wholesale power markets operating over high-voltage transmission grids.[3] Users can set a variety of parameter values to tailor these features to specific energy regions. Users can also readily extend the AMES V5.0 classes to model additional existing or envisioned features. Finally, users can activate the integration of AMES V5.0 within a higher-level co-simulation framework by the simple setting of a flag. For a detailed illustration of these capabilities, see Battula et al. [12].

This report focuses on the AMES V5.0 implementation of SCUC/SCED optimizations for carrying out daily wholesale power market operations. AMES V5.0 implements a daily ISO-managed SCUC/SCED optimization to clear bids and offers submitted into a *Day-Ahead Market (DAM)*; the purpose of a DAM held on day D is to determine unit commitments, scheduled dispatch levels, and price settlements to facilitate net load balancing on day D+1. AMES V5.0 also implements multiple daily ISO-managed SCED optimizations to clear bids and offers submitted into *Real-Time Market (RTM)* processes; the purpose of the RTM processes held on day D+1 is to adjust day-D DAM scheduled dispatch levels as needed in order to ensure net-load balancing during near-term operating periods during day D+1.

One particularly important feature of AMES V5.0 is that it permits users to simulate DAM/RTM operations over multiple days by making use of the Generalized ECA Model to express and implement successive SCUC/SCED optimizations with continually updated state conditions. The next two subsections explain the basic form of a typical AMES V5.0 simulation run, with careful attention paid to the distinction between parameters and state conditions.

[3] These seven energy regions are CAISO, ERCOT, ISO-NE, MISO, NYISO, PJM, and SPP. See Tesfatsion [13] for links to their homepages where training and business practice manuals explaining their market operations can be accessed.

3.2 A Typical AMES V5.0 Simulation Run

AMES V5.0 simulates ISO-managed wholesale power market operations over successive simulated days D0, ..., DMax. During the first day D0 the ISO conducts a DAM SCUC/SCED optimization to plan for operations during all twenty-four hours of day D1. On each subsequent day D, the ISO conducts a DAM SCUC/SCED optimization for day D+1 in parallel with a sequence of RTM SCED optimizations for near-term operating periods. These parallel DAM/RTM operations are depicted in Fig. 2.

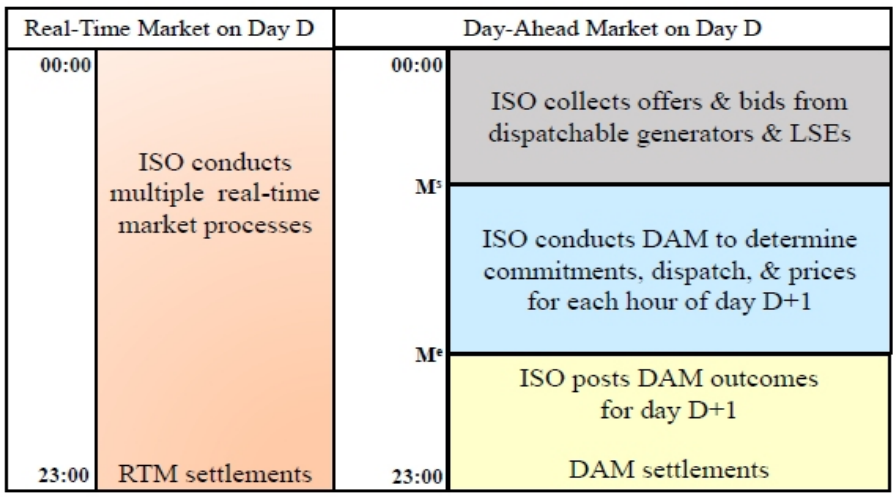

Figure 2: *AMES V5.0 simulation of DAM and RTM operations during a typical day D.*

Fig. 3 shows how AMES V5.0 implements timing for a general market M(T) corresponding to a future operating period T, with a look-ahead horizon LAH(T). Each operating period T is partitioned into time-steps $k = 1, \ldots NK(\mathrm{T})$, where each time-step k has a common duration Δt measured in hours (e.g., 1.0h, 0.25h). For any given T, *time-step 0* refers to the time-step with duration Δt that immediately precedes T.

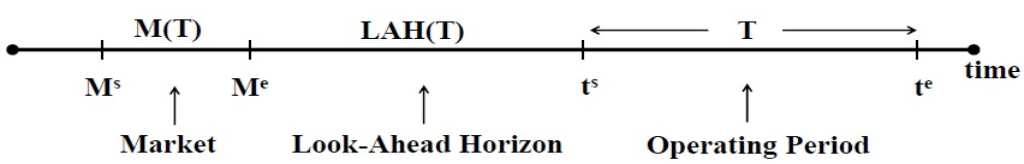

Figure 3: *AMES V5.0 timing for the simulation of a general market M(T) for a future operating period T with a look-ahead horizon LAH(T).*

For a DAM process M(T) held on any simulated day D, AMES V5.0 hardcodes the following timing aspects: the start-time for M(T) is set to 6:00 on day D; the end-time for M(T) is set to 13:30 on day D; and the operating period T is set to be the entire following day D+1 with duration 24h. These hardcoded settings imply that the DAM look-ahead horizon LAH(T) has duration 10.5h. In addition, AMES V5.0 sets the default value for the time-step duration Δt for T to be 1h. However, users can override this default time-step duration by setting Δt equal to $r \times 1h$ for some positive real-valued r subject to one constraint: namely, r must be an exact divisor of 24.

In contrast, AMES V5.0 gives users wide latitude to structure RTM operating periods T in ac-

cordance with their own particular research needs. Specifically, users can set the duration RTOPDur (in minutes) of each RTM operating period T subject only to one constraint: namely, RTOPDur must be an exact divisor of 24×60, the number of minutes in a day. The resultant of this division determines the number of RTM processes M(T) that take place during each simulated day. For any RTM process M(T), AMES V5.0 hardcodes the start-time for M(T) to be RTOPDur minutes before the start of T, and the end-time for M(T) to be one minute after this start-time. Consequently, LAH(T) = RTOPDur-1.

See, for example, the AMES V5.0 timings depicted in Figs. 4-5. These figures show how the ERCOT Test System [12], implemented by means of AMES V5.0, sets timings for modeled ERCOT DAM and RTM operations.

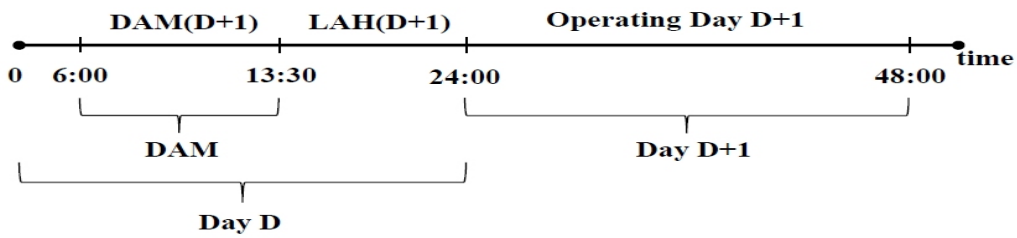

Figure 4: *AMES V5.0 timing for a DAM conducted on day D to facilitate net-load balancing on day D+1.*

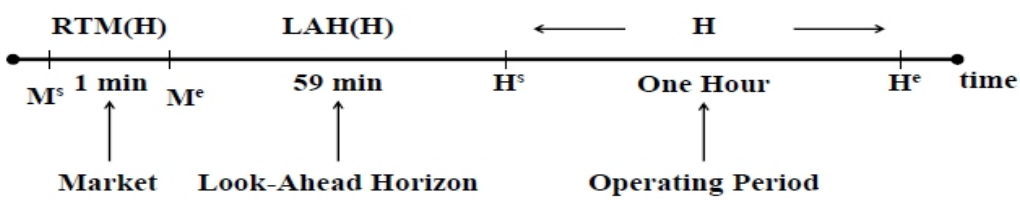

Figure 5: *Illustrative AMES V5.0 timing for an RTM conducted on day D to facilitate net-load balancing for a near-term operating hour H.*

3.3 AMES V5.0 Base Parameters and Initial State Conditions

Prior to conducting a simulation run with AMES V5.0, a user needs to set new or default values for all of the *base parameters* appearing in the AMES V5.0 code. The set $\mathbb{BP}$ of base parameters for AMES V5.0 is defined by the following three conditions: (i) Each element of $\mathbb{BP}$ is a parameter appearing in the AMES V5.0 code; (ii) Each parameter appearing in the AMES V5.0 code can be expressed as a function of one or more elements in $\mathbb{BP}$; (iii) No element in $\mathbb{BP}$ can be non-trivially expressed as a function of other elements in $\mathbb{BP}$.

A parameter appearing in the AMES V5.0 code that is not an element of $\mathbb{BP}$ is said to be a *derived parameter*. AMES V5.0 calculates derived parameters as needed during run-time. For example, AMES V5.0 automatically scales the base parameters for use in various SCUC/SCED optimizations M(T) whose future operating periods T have differently specified durations.

Also, prior to running AMES V5.0 a user needs to specify *base initial state conditions* for the dynamic variables appearing in AMES V5.0. For example, prior to each simulation run the user must specify the power level and up-time/down-time status for each dispatchable generator at the

start of day D1. These initial state conditions for day D1 are needed in order to implement the initial DAM SCUC/SCED optimization conducted on day D0. AMES V5.0 then automatically updates these state conditions as needed for subsequent DAM/RTM optimizations implemented during the course of the simulation run.

The following section describes the particular base parameters and base initial state conditions required for the AMES V5.0 implementation of SCUC/SCED optimizations in accordance with the Generalized ECA Model with all fixed loads. Three cautions are in order.

First, the symbols used below for these base terms are mathematical expressions selected to facilitate analytical representations; they differ from the names assigned to these base terms in the AMES V5.0 code. Second, these base terms are a proper subset of the full set of base terms required for the implementation of AMES V5.0 simulation runs. For example, these base terms do not include the AMES V5.0 base simulation control parameters, or the AMES V5.0 base initial state conditions characterizing the initial financial state of each wholesale power market participant. Third, the base parameters characterizing reserve requirements, generator ramp rates, generator power output limits, and generator production cost functions are expressed below as functions of time. However, to simplify user input requirements, AMES V5.0 currently implements a version of the Generalized ECA Model for which these attributes are assumed to be stationary over time.

4 Generalized ECA Model Nomenclature: Fixed Load Case

4.1 Overview

The Generalized ECA Model is a complete analytical formulation for an ISO-managed SCUC/SCED optimization M(T) whose purpose is to ensure net load balancing for a future operating period T. This complete analytical formulation is developed and presented in Sections 5-6 for the special case in which all LSE demand bids take the form of fixed (non-price-sensitive) loads. The more general case in which LSE demand bids are permitted to be price sensitive is covered in Sections 7-8.

This section provides annotated nomenclature tables for the Generalized ECA Model assuming all loads are fixed. These nomenclature tables are separated into the following categories: acronyms and generic terms; sets and subsets; base parameters for each operating period T; derived parameters for each operating period T; base initial state conditions for the start of day D1; updated state conditions for the start of a general operating period T; forecasts for a general operating period T; and ISO decision variables and derived solution variables for M(T).

4.2 Acronyms and Generic Terms

b	Transmission grid bus;
D	Generic symbol for a day;
DAM	Day-Ahead Market;
H	Generic symbol for an hour;

g	Dispatchable generator;
$k = [k^{\mathsf{s}}, k^{\mathsf{e}})$	A time-step for an operating period T;
$k^{\mathsf{T}}(0)$	Time-step 0 for T, i.e., the time-step of duration Δt that immediately precedes T;
LAH(T)	The Look-Ahead Horizon for a market M(T);
LMP	Locational Marginal Price;
LSE	Load-Serving Entity;
M(T)	A market held in advance of an operating period T.
n	Non-dispatchable generator;
RTM	Real-Time Market;
s	Energy storage unit;
T	An operating period partitioned into time-steps k;
z	Reserve zone (collection of grid buses);
Δt	Duration of a time-step k, measured in hours;
Δk	Duration of a time-step k, measured in minutes.

4.3 Sets and Subsets

$\mathbb{B} = \{1, \ldots, NB\}$	Index set for the buses b of a transmission grid;
$\mathbb{B}(z) \subseteq \mathbb{B}$	Subset of buses constituting reserve zone z;
$\mathbb{G}$	Index set for participant dispatchable generators g;
$\mathbb{G}(b) \subseteq \mathbb{G}$	Subset of dispatchable generators located at bus b;
$\mathbb{G}(z) \subseteq \mathbb{G}$	Subset of dispatchable generators located in zone z;
$\mathbb{K}(\mathrm{T}) = \{1, \ldots, NK(\mathrm{T})\}$	Index set for the time-steps k that form a partition of an operating period T;
$\mathbb{K}^0(\mathrm{T}) = \{0, 1, \ldots, NK(\mathrm{T})\}$	Index set $\mathbb{K}(\mathrm{T})$ augmented with time-step 0 for T;
$\mathbb{L} \subseteq \mathbb{B} \times \mathbb{B}$	Index set for the lines ℓ of a transmission grid;
$\mathbb{L}_{O(b)} \subseteq \mathbb{L}$	Subset of transmission lines originating at bus b;
$\mathbb{L}_{E(b)} \subseteq \mathbb{L}$	Subset of transmission lines ending at bus b;
$\mathbb{LS}$	Index set for participant LSEs j
$\mathbb{LS}(b) \subseteq \mathbb{LS}$	Subset of LSEs servicing customers at bus b;
$\mathbb{LS}(z) \subseteq \mathbb{LS}$	Subset of LSEs servicing customers in zone z;
$\mathbb{NG}$	Index set for participant non-dispatchable generators n;
$\mathbb{NG}(b) \subseteq \mathbb{NG}$	Subset of non-dispatchable generators located at bus b;
$\mathbb{NG}(z) \subseteq \mathbb{NG}$	Subset of non-dispatchable generators located in zone z;
$\mathbb{NS}_g(k) = \{1, \ldots, NS_g(k)\}$	Index set for the piecewise-linear approximation of g's total production cost function for time-step k;
$\mathbb{S}$	Index set for participant energy storage units s;
$\mathbb{S}(b) \subseteq \mathbb{S}$	Subset of energy storage units located at bus b;
$\mathbb{S}(z) \subseteq \mathbb{S}$	Subset of energy storage units located in zone z;
$\mathbb{Z}$	Set of indices $z = 1, \ldots, NZ$ for zones $\mathbb{B}(z)$, which form a partition of $\mathbb{B}$, i.e., $\cup_{z \in \mathbb{Z}} \mathbb{B}(z) = \mathbb{B}$, and $\mathbb{B}(z_i) \cap \mathbb{B}(z_j) = \emptyset$ for any z_i and z_j in $\mathbb{Z}$ with $i \neq j$.

4.4 Base Parameters for Each Operating Period T

Physical attributes of the dispatchable generators $g \in \mathbb{G}$ for each $k \in \mathbb{K}^0(\mathrm{T})$:

$DT_g \geq 0$	Minimum down-time for g, measured in hours;
$UT_g \geq 0$	Minimum up-time for g, measured in hours;
$NRD_g(k)$	Nominal ramp-down rate (MW/min) for g during time-step k;
$NRU_g(k)$	Nominal ramp-up rate (MW/min) for g during time-step k;
$NSD_g(k)$	Nominal shut-down ramp rate (MW/min) for g during time-step k;
$NSU_g(k)$	Nominal start-up ramp rate (MW/min) for g during time-step k;
$P_g^{\mathsf{max}}(k) \geq 0$	Max power output (MW) for a committed g at time k^{s} ;
$P_g^{\mathsf{min}}(k) \geq 0$	Min power output (MW) for a committed g at time k^{s}, which must satisfy $P_g^{\mathsf{min}}(k) \leq P_g^{\mathsf{max}}(k)$.

Remark: Note that the above base parameters for operating period T are defined for the augmented index set $\mathbb{K}^0(\mathrm{T})$ that includes time-step 0 for T, i.e., the time-step $k^{\mathsf{T}}(0)$ of duration Δt that immediately precedes T. Thus, for example, the maximum power output $P_g^{\mathsf{max}}(0)$ for a committed g specified above for the start of time-step 0 of operating period T can also be the designated maximum power output for a committed g at the start of the final time-step for an immediately preceding operating period T′; and similarly for all of the remaining base parameters that are functions of the time-step k.

Start-up/shut-down attributes of the dispatchable generators $g \in \mathbb{G}$:

$CSC_g \geq 0$	Cold-start cost (\$) for g;
$CST_g \geq 0$	Cold-start time for g, measured in hours;
$HSC_g \geq 0$	Hot-start cost (\$) for g, which must satisfy $HSC_g \leq CSC_g$;
$SDC_g \geq 0$	Shut-down cost (\$) for g .

Remark: The cold-start time CST_g has the following meaning. If g has been offline prior to a current time t for at least CST_g hours, then g is in a *cold-start state* at time t and any start-up of g at time t incurs the cold-start cost CSC_g. Otherwise, g is in a *hot-start state* at time t and any start-up of g at time t incurs the hot-start cost HSC_g.

Approximate total production cost function for each $g \in \mathbb{G}$ and $k \in \mathbb{K}(\mathrm{T})$:

$a_g(k) \geq 0$	Production cost function coefficient (\$/h) for g;
$b_g(k) \geq 0$	Production cost function coefficient (\$/MWh) for g ;
$c_g(k) > 0$	Production cost function coefficient (\$/[MW]^{2}h) for g;
$NS_g(k) \geq 1$	Number of segments i used for the piecewise-linear approximation of g's total production cost function for time-step k.

Remark: The construction of these approximate total production cost functions is explained in detail in Section 5, below.

Physical attributes of the energy storage units $s \in \mathbb{S}$:

$EPSOC_s \geq 0$	Target charge state (decimal %) for s ;
$ES_s^{\mathsf{max}} \geq 0$	Maximum energy storage capacity (MWh) of s ;
$NIRD_s$	Nominal input (charge) ramp-down rate (MW/min) for s ;
$NIRU_s$	Nominal input (charge) ramp-up rate (MW/min) for s ;
$NORD_s$	Nominal output (discharge) ramp-down rate (MW/min) for s ;
$NORU_s$	Nominal output (discharge) ramp-up rate (MW/min) for s ;
PI_s^{max}	Maximum input (charge) power (MW) for s ;
$PI_s^{\mathsf{min}} \geq 0$	Minimum input (charge) power (MW) for s ;
PO_s^{max}	Maximum output (discharge) power (MW) for s ;
$PO_s^{\mathsf{min}} \geq 0$	Minimum output (discharge) power (MW) for s ;
$SOC_s^{\mathsf{min}} \geq 0$	Minimum state of charge (decimal %) for s ;
$\eta_s \geq 0$	Round-trip efficiency (decimal %) for s .

Remark: AMES V5.0 permits users to implement a SCUC/SCED optimization either with or without energy storage units. This is done by setting a binary simulation control variable 'StorageFlag' either to 1 (*with* storage) or to 0 (*without* storage). This flag setting is used by AMES V5.0 either to include or exclude the appearance of storage variables and storage constraints in the implemented SCUC/SCED optimization.

System-wide and zonal down/up reserve requirements for each $k \in \mathbb{K}(\mathrm{T})$:

$RRD(k) \geq 0$	Reserve requirement (decimal %) for system-wide down-power at time k^{s} ;
$RRU(k) \geq 0$	Reserve requirement (decimal %) for system-wide up-power at time k^{s} ;
$RRD_z(k) \geq 0$	Reserve requirement (decimal %) for down-power in zone z at time k^{s} ;
$RRU_z(k) \geq 0$	Reserve requirement (decimal %) for up-power in zone z at time k^{s} .

Remark: The system-wide down/up reserve requirements $RRD(k)$ and $RRU(k)$ appear on the right-hand side of the system-wide down/up reserve requirement constraints (57) and (58) as decimal percentages of the forecasted system-wide net fixed load $\widehat{NL}^{\mathsf{f}}(k)$ at the start-time k^{s} for each time-step k. The zonal down/up reserve requirements $RRD_z(k)$ and $RRU_z(k)$ appear on the right-hand side of the zonal down/up reserve requirement constraints (59) and (60) for zone z as decimal percentages of the forecasted net fixed load $\widehat{NL}_z^{\mathsf{f}}(k)$ in zone z at the start-time k^{s} for each time-step k.

Other Base Parameters for Each Operating Period T:

$E(\ell)$	End bus for transmission line ℓ;
$F^{\mathsf{max}}(\ell) \geq 0$	Capacity limit (MW) for transmission line ℓ ;
$NK(T)$	Number of time-steps k forming a partition of operating period T ;
$O(\ell)$	Originating bus for transmission line ℓ ;
$S_o > 0$	Positive base power (MW) ;
$V_o > 0$	Base voltage magnitude (kV) ;
$X(\ell) > 0$	Reactance (ohms) on transmission line ℓ ;
$\Delta t > 0$	Common duration of each time-step k for T, measured in hours, i.e., $\Delta t = r \times 1\mathrm{h}$ for some positive real-valued r ;
$\Lambda^-, \Lambda^+ \geq 0$	Imbalance penalty weights (\$/MWh) for power-balance slack terms .

Remark on the settings for $NK(T)$ and Δt: As explained in Section 3, AMES V5.0 imposes admissibility restrictions on the duration of an operating period T and the duration Δt of the time-steps k that partition T; the exact form of these restrictions depends on whether the market process M(T) under consideration is a DAM or an RTM. These admissibility restrictions are not restated here.

4.5 Derived Parameters for Each Operating Period T

$B(\ell)$	Inverse of reactance (pu) on transmission line ℓ ;
$x(\ell)$	Reactance (pu) on transmission line ℓ ;
$SCST_g$	Scaled cold-start time for g, measured as integer number of time-steps k ;
SDT_g	Scaled min down-time for g, measured as integer number of time-steps k ;
SUT_g	Scaled min up-time for g, measured as integer number of time-steps k ;
$SIRD_s$	Scaled input (charge) ramp-down limit (MW) for s during each k ;
$SIRU_s$	Scaled input (charge) ramp-up limit (MW) for s during each k ;
$SORD_s$	Scaled output (discharge) ramp-down limit (MW) for s during each k ;
$SORU_s$	Scaled output (discharge) ramp-up limit (MW) for s during each k ;
$SRD_g(k)$	Scaled ramp-down limit (MW) for g during each k ;
$SRU_g(k)$	Scaled ramp-up limit (MW) for g during each k ;
$SSD_g(k)$	Scaled shut-down ramp limit (MW) for g during each k ;
$SSU_g(k)$	Scaled start-up ramp limit (MW) for g during each k ;
Z_o	Base impedance (ohms) ;
Δk	Common duration of each time-step k, measured in minutes .

Calculations for Derived Parameters:

- $B(\ell) = 1/x(\ell)$;
- $x(\ell) = X(\ell)/Z_o$;
- $SCST_g = \texttt{round}(CST_g/\Delta t)$;
- $SDT_g = \texttt{round}(DT_g/\Delta t)$;
- $SUT_g = \texttt{round}(UT_g/\Delta t)$;
- $SIRD_s = \Delta k \times NIRD_s$;
- $SIRU_s = \Delta k \times NIRU_s$;
- $SORD_s = \Delta k \times NORD_s$;
- $SORU_s = \Delta k \times NORU_s$;
- $SRD_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta k \times NRD_g(k)\}$;
- $SRU_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta k \times NRU_g(k)\}$;
- $SSD_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta k \times NSD_g(k)\}$;
- $SSU_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta k \times NSU_g(k)\}$;
- $Z_o = (V_o)^2/S_o$;
- $\Delta k = \Delta t \times [60\text{min}/1\text{h}]$.

Remark on the use of the `round` *function:* In the above calculations for $SCST_g$, SDT_g and SUT_g, "$\texttt{round}(r)$" denotes a round-off function that rounds off a real-number argument r to the nearest integer value. For example, round(15.52159) = 16 whereas round(15.49321) = 15.

4.6 Base Initial State Conditions for Day D1

$\bar{x}_s(0)$	Power output (discharge) (MW) for storage unit s at the start of time-step 0 for day D1 ;
$\underline{x}_s(0)$	Power input (charge) (MW) for s at the start of time-step 0 for day D1 ;
$SOC_s(0)$	State of charge (decimal %) for s at the start of time-step 0 for day D1 ;
$p_g(0)$	Power output (MW) for generator g at the start of time-step 0 for day D1 ;
$\hat{v}_g(0)$	Up-time/down-time status for g at the start of day D1, measured in hours .

Remark on the meaning of $\hat{v}_g(0)$*:* Recall that simulation runs for AMES V5.0 commence on the first simulated day D0 with an initial ISO-managed DAM SCUC/SCED optimization whose purpose is to ensure net-load balancing during the following day D1. The operating day D1 is partitioned into twenty-four time-steps, each with duration Δt = 1h. Suppose $\hat{v}_g(0)$ is set to a positive (negative) value for some dispatchable generator $g \in \mathbb{G}$ at the start of a simulation run. This sets the consecutive amount of time immediately prior to the start of day D1 that g is assumed to have been online (offline). Note that $\hat{v}_g(0)$ cannot be zero, by definition.

State Conditions Derived from Base Initial State Conditions

$v_g(0)$	ON/OFF (1/0) status for generator g at the start of day D1 ;
ITO_g	Initial time ON for g, i.e., the number of initial time-steps for day D1 during which g must be online ;
ITF_g	Initial time OFF for g, i.e., the number of initial time-steps for day D1 during which g must be offline;
$\widehat{sv}_g(0)$	Scaled up-time/down-time status for g at the start of day D1, measured as a non-zero number of hourly time-steps.

Calculations for the Derived Initial State Conditions:

- If $\hat{v}_g(0) > 0$, $v_g(0) = 1$; otherwise, $v_g(0) = 0$
- If $\hat{v}_g(0) > 0, ITO_g = \min\left(NK(\mathrm{T}), \max\left(0, \texttt{round}((UT_g - \hat{v}_g(0))/1h)\right)\right)$; else $ITO_g = 0$
- If $\hat{v}_g(0) < 0, ITF_g = \min\left(NK(\mathrm{T}), \max\left(0, \texttt{round}((DT_g + \hat{v}_g(0))/1h)\right)\right)$; else $ITF_g = 0$
- The non-zero scaled up-time/down-time status for g is derived as follows:

$$\widehat{sv}_g(0) = \left\{ \begin{array}{lll} \texttt{round}(\hat{v}_g(0)/1h) & \text{if} & 1 \leq |\widehat{v}_g(0)|; \\ 1 & \text{if} & 0 < \widehat{v}_g(0) < 1; \\ -1 & \text{if} & -1 < \widehat{v}_g(0) < 0. \end{array} \right\}$$

4.7 Initial State Conditions for a General Operating Period T

Let M(T) denote an ISO-managed SCUC/SCED optimization undertaken for a future operating period T during the course of an AMES V5.0 simulation run. The operating period T is partitioned

into $NK(\mathrm{T})$ time-steps k, each with duration Δt measured in hours. The time-step 0 for T, denoted by $k^{\mathsf{T}}(0)$, is the time-step of duration Δt that immediately precedes T.

The optimization M(T) must be conditioned on *period-T initial state conditions*, i.e., initial state conditions for the start of operating period T that are updated versions of the base initial state conditions set by a user for the start of day D1. The needed period-T initial state conditions are as follows:

$\bar{x}_s^{\mathsf{T}}(0)$	Power output (discharge) (MW) for storage unit s at the start of time-step 0 for T ;
$\underline{x}_s^{\mathsf{T}}(0)$	Power input (charge) (MW) for s at the start of time-step 0 for T ;
$SOC_s^{\mathsf{T}}(0)$	State of charge (decimal %) for s at the start of time-step 0 for T ;
$p_g^{\mathsf{T}}(0)$	Power output (MW) for generator g at the start of time-step 0 for T ;
$\hat{v}_g^{\mathsf{T}}(0)$	Up-time/down-time status for g at the start of T, measured in hours .

Remark on the meaning of $\hat{v}_g^{\mathsf{T}}(0)$: Suppose $\hat{v}_g^{\mathsf{T}}(0)$ has a positive (negative) value for some dispatchable generator $g \in \mathbb{G}$. This value indicates the consecutive amount of time immediately prior to the start of operating period T that g has been online (offline), given the initial up-time/down-time status $\hat{v}_g(0)$ assumed for g at the start of day D1. Thus, $\hat{v}_g^{\mathsf{T}}(0)$ is an updated version of $\hat{v}_g(0)$. Note that $\hat{v}_g^{\mathsf{T}}(0)$ cannot be zero.

Remark on the method used by AMES V5.0 to specify the remaining period-T initial state conditions: AMES V5.0 automatically generates these initial state conditions using the following simple method. If T = D1, the period-T initial state conditions reduce to the base initial state conditions. Suppose the operating period T occurs subsequent to the start of day D1. Let M(T') denote the most recent previous market process for which the operating period T' encompasses $k^{\mathsf{T}}(0)$, the time-step 0 for T. The solution determined by M(T') is used to specify the initial state conditions for period T.

Remark on additional period-T initial state conditions needed for RTM SCED optimizations: For an ISO-managed RTM SCED optimization M(T) held during some simulated day D subsequent to the initial simulated day D0, the Generalized ECA Model can be applied with all 1/0 unit commitment variables for dispatchable generators fixed at their optimal solution values determined in the ISO-managed DAM SCUC/SCED optimization conducted on the previous day D-1. These fixed unit commitment solution values then constitute additional period-T initial state conditions for M(T).

Derived Initial State Conditions for a General Operating Period T

$v_g^{\mathsf{T}}(0)$	ON/OFF (1/0) status for dispatchable generator g at the start of T;
ITO_g^{T}	Period-T initial time ON for g, i.e., the number of initial time-steps for T during which g must be online ;
ITF_g^{T}	Period-T initial time OFF for g, i.e., the number of initial time-steps for T during which g must be offline;
$\widehat{sv}_g^{\mathsf{T}}(0)$	Scaled up-time/down-time status for g at the start of T, measured as a non-zero number of time-steps .

Calculations for the Derived Initial State Conditions for a General Operating Period T:

- If $\hat{v}_g^{\mathsf{T}}(0) > 0$, $v_g^{\mathsf{T}}(0) = 1$; otherwise, $v_g^{\mathsf{T}}(0) = 0$.
- If $\hat{v}_g^{\mathsf{T}}(0) > 0$, $ITO_g^{\mathsf{T}} = \min\big(NK(\mathrm{T}), \max\big(0, \mathtt{round}((UT_g - \hat{v}_g^{\mathsf{T}}(0))/\Delta t)\big)\big)$; else $ITO_g^{\mathsf{T}} = 0$.

- If $\hat{v}_g^{\mathsf{T}}(0) < 0$, $ITF_g^{\mathsf{T}} = \min\big(NK(\mathrm{T}), \max\big(0, \mathtt{round}((DT_g + \hat{v}_g^{\mathsf{T}}(0))/\Delta t)\big)\big)$; else $ITF_g^{\mathsf{T}} = 0$.
- The non-zero scaled up-time/down-time status for g at the start of T is derived as follows:

$$\widehat{sv}_g^{\mathsf{T}}(0) = \left\{ \begin{array}{lll} \mathtt{round}(\hat{v}_g^{\mathsf{T}}(0)/\Delta t) & \text{if} & 1 \leq |\widehat{v}_g^{\mathsf{T}}(0)|; \\ 1 & \text{if} & 0 < \widehat{v}_g^{\mathsf{T}}(0) < 1; \\ -1 & \text{if} & -1 < \widehat{v}_g^{\mathsf{T}}(0) < 0. \end{array} \right\} \tag{1}$$

4.8 Period-T Forecasts: Fixed Load & Non-Dispatchable Generation

Step 1: Specify Fixed Load Forecasts $\forall b \in \mathbb{B}, i \in \mathbb{LS}, k \in \mathbb{K}(\mathrm{T})$, and $z \in \mathbb{Z}$:

$\hat{p}_i^{\mathsf{f}}(k) \geq 0$	Forecast (MW) by load-serving entity i for the fixed power usage of its customers at time k^{s} ;
$\widehat{L}_b^{\mathsf{f}}(k) = \sum_{i \in \mathbb{LS}(b)} \widehat{p}_i^{\mathsf{f}}(k)$	Derived forecast (MW) for fixed load at bus b at time k^{s} ;
$\widehat{L}_z^{\mathsf{f}}(k) = \sum_{i \in \mathbb{LS}(z)} \widehat{p}_i^{\mathsf{f}}(k)$	Derived forecast (MW) for fixed load in zone z at time k^{s} ;
$\widehat{L}^{\mathsf{f}}(k) = \sum_{i \in \mathbb{LS}} \widehat{p}_i^{\mathsf{f}}(k)$	Derived forecast (MW) for system-wide fixed load at time k^{s} .

Step 2: Specify Non-Dispatchable Generation Forecasts $\forall b \in \mathbb{B}, k \in \mathbb{K}(\mathrm{T}), n \in \mathbb{NG}$, $z \in \mathbb{Z}$:

$\widehat{p}_n(k) \geq 0$	Forecast (MW) for power output of n at time k^{s} ;
$\widehat{NG}_b(k) = \sum_{n \in \mathbb{NG}(b)} \widehat{p}_n(k)$	Derived forecast (MW) for non-dispatchable generation at bus b at time k^{s} ;
$\widehat{NG}_z(k) = \sum_{n \in \mathbb{NG}(z)} \widehat{p}_n(k)$	Derived forecast (MW) for non-dispatchable generation in zone z at time k^{s} ;
$\widehat{NG}(k) = \sum_{n \in \mathbb{NG}} \widehat{p}_n(k)$	Derived forecast (MW) for non-dispatchable generation system-wide at time k^{s} .

Step 3: Specify Net Fixed Load Forecasts $\forall b \in \mathbb{B}, i \in \mathbb{LS}, k \in \mathbb{K}(\mathrm{T}), n \in \mathbb{NG}$, $z \in \mathbb{Z}$:

$\widehat{NL}_b^{\mathsf{f}}(k) = [\widehat{L}_{\mathsf{b}}^{\mathsf{f}}(k) - \widehat{NG}_b(k)]$	Derived forecast (MW) for net fixed load at bus b at time k^{s} ;
$\widehat{NL}_z^{\mathsf{f}}(k) = [\widehat{L}_z^{\mathsf{f}}(k) - \widehat{NG}_z(k)]$	Derived forecast (MW) for net fixed load in zone z at time k^{s} ;
$\widehat{NL}^{\mathsf{f}}(k) = [\widehat{L}^{\mathsf{f}}(k) - \widehat{NG}(k)]$	Derived forecast (MW) for system-wide net fixed load at time k^{s} .

Remarks on net fixed load forecasts:

- Load is said to be *fixed* if it is not sensitive to price.
- *Net (fixed) load* for a designated region (e.g., bus, zone, entire system) is defined to be (fixed) load for this region minus non-dispatchable generation for this region. Thus, non-dispatchable generation is treated as negative load.
- A SCUC/SCED optimization is a forward-market planning tool for ensuring resource availability for net load balancing in subsequent real-time operations. If the optimization is conducted several hours in advance of real-time operations, it would generally not be credible to assume real-time fixed loads and non-dispatchable generation are known with certainty at the time of this optimization.
- In U.S. RTO/ISO-managed day-ahead markets, each LSE's demand bid for next-day operations is permitted to include both a 24-hour fixed load profile and 24 hourly price sensitive demand schedules. The 24-hour fixed load profile is generally interpreted to be the LSE's forecast for the next-day power usage of its customers.

- Forecasts for non-dispatchable generation are typically formulated by the RTO/ISO.
- The RTO/ISO must use LSE demand bids to determine forecasted next-day loads.
- The RTO/ISO includes reserve requirements in its SCUC/SCED optimization constraints to protect against the possibility of net load forecast errors.
- The SCUC/SCED optimization formulated by Carrión and Arroyo [2] for a day-ahead wholesale power market does not include non-dispatchable generation and does not consider transmission congestion. Consequently, the only external forcing term for each hour k for next-day operations is "demand" $D(k)$, where $D(k)$ denotes forecasted system-wide fixed load for hour k.

4.9 ISO Decision Variables and Derived Solution Variables for M(T)

Binary-Valued ISO Decision Variables For Each $k \in \mathbb{K}(\mathrm{T})$:

$v_g(k)$	1 if dispatchable generator g is committed for time-step k; else 0
$\bar{u}_s(k)$	1 if storage unit s is committed for k for power output (discharge); else 0
$\underline{u}_s(k)$	1 if storage unit s is committed for k for power absorption (charging); else 0

Continuously-Valued ISO Decision Variables for Each $k \in \mathbb{K}(\mathrm{T})$:

$\bar{x}_s(k)$	Power output (discharge) (MW) for storage unit s at time k^{s}
$\underline{x}_s(k)$	Power input (charge) (MW) for storage unit s at time k^{s}
$\delta_{i,g}(k)$	Variable (MW) used to determine the power output $p_g(k)$ of generator g at time k^{s} in the total production cost approximation method for g
$\theta_b(k)$	Voltage angle (radians) for bus $b \in \mathbb{B}/\{1\}$ at time k^{s}

Solution Variables Derived from Decisions and Constraints for Each $k \in \mathbb{K}(\mathrm{T})$:

$c_g^{\mathsf{p}}(k)$	Total production cost (\$) for dispatchable generator g for time-step k
$c_g^{\mathsf{u}}(k)$	Start-up cost (\$) for dispatchable generator g for time-step k
$c_g^{\mathsf{d}}(k)$	Shut-down cost (\$) for dispatchable generator g for time-step k
$hs_g(k)$	1 if dispatchable generator g is in a hot-start state at time k^{s}; else 0
$p_g(k)$	Power output (MW) for dispatchable generator g at time k^{s}
$\underline{p}_g(k)$	Run-time determined lower limit on $p_g(k)$ at time k^{s}
$\overline{p}_g(k)$	Run-time determined upper limit on $p_g(k)$ at time k^{s}
$w_\ell(k)$	Power flow (MW) on transmission line ℓ at time k^{s}
$z_s(k)$	State of charge (decimal %) for storage unit s at time k^{s}
$\beta_b^-(k), \beta_b^+(k)$	Power-imbalance slack terms (MW) for bus b at time k^{s}
$\beta_b(k)$	Power-imbalance slack variable (MW) for bus b at time k^{s}
$\theta_1(k)$	Voltage angle (radians) for angle reference bus 1 at time k^{s}

Remark on the slack variable terms: For any real variable y, there exist unique non-negative values $y^- = \max\{0, -y\}$ and $y^+ = \max\{0, y\}$ satisfying $y^+ - y^- = y$ and $y^+ + y^- = |y|$. As will be seen below in (10), the objective function for the Generalized ECA Model decomposes the slack variable $\beta_b(k)$ into $(\beta_b^-(k), \beta_b^+(k))$ in order to permit the imposition of different penalties on negative and positive deviations from power balance at bus b at time k^{s}.

5 Approximation of Generator Total Production Costs

Let M(T) denote an ISO-managed SCUC/SCED optimization undertaken for a future operating period T during the course of an AMES V5.0 simulation run, and let $\mathbb{K}(\mathrm{T}) = \{1, \ldots, NK(\mathrm{T})\}$ denote the index set for the time-steps k that form a partition of T. As will be seen in Section 6, the total production cost (\$) incurred by each dispatchable generator g during each time-step k appears in the objective function for M(T) in approximate form as $c_g^{\mathsf{p}}(k)$ (\$). This approximation relies on the following four assumptions:

- Each time-step $k \in \mathbb{K}(\mathrm{T})$ has an equal user-set duration Δt, measured in hours.
- The set of feasible power levels for g at time k^{s} is given by an interval $\mathbb{P}_g(k) = [P_g^{\mathsf{min}}(k), P_g^{\mathsf{max}}(k)]$ with $0 \leq P_g^{\mathsf{min}}(k)$.
- The ISO dispatches generation for operating period T by means of dispatch set points. Specifically, the ISO's dispatch instruction conveyed to a committed generator g for each time-step $k \in \mathbb{K}(\mathrm{T})$ consists of a single dispatch set point $p_g(k) \in \mathbb{P}_g(k)$ signaled to g at the start-time k^{s} for time-step k, which indicates the required power level for g at k^{s}.
- The production cost function (\$/h) of each dispatchable generator g for each time-step k can be expressed as a non-decreasing convex function of the dispatch set point p taking the following quadratic form:

$$\mathrm{C}_{g,k}(p) \;=\; a_g(k) + b_g(k)p + c_g(k)p^2 \;, \quad \forall p \in \mathbb{P}_g(k) \;, \tag{2}$$

  where the cost coefficient $a_g(k) \geq 0$ has units \$/h, the cost coefficient $b_g(k) \geq 0$ has units \$/MWh, and the cost coefficient $c_g(k) > 0$ has units \$/[MW]^{2}h.

Given these four assumptions, the version of the Pyomo Model incorporated into AMES V5.0 constructs a piecewise-linear approximation for g's production cost function (2) for any time-step k by connecting finitely many power-cost points $\{(P_i, C_i) \mid i = 0, \ldots, NS_g(k)\}$ in the power-cost plane satisfying

$$P_g^{\mathsf{min}}(k) = P_0 \;<\; P_1 \;<\; P_2 \;\ldots\; < P_{NS_g(k)} \;=\; P_g^{\mathsf{max}}(k) \tag{3}$$

and $C_i = \mathrm{C}_{g,k}(P_i)$ for each i. See, for example, Fig. 6. As will be clarified below, the user can permit the AMES V5.0 Pyomo Model to automatically set these power-cost points by internal calculations as a function of the user-designated number $NS_g(k)$ of line segments to be used in the approximation.

Suppose for the moment that these power-cost points have been set. The approximate total production cost $c_g^{\mathsf{p}}(k)$ (\$) incurred by g during time-step k is then determined by the system of equations (4)-(8), below, as a function of the ISO's optimal selection of the continuously-valued decision variables $\{\delta_{i,g}(k) \mid i = 1, \ldots NS_g(k)\}$:[4]

[4]The following Total Production Cost Approximation Method is adapted from [2, Sec. II.A].

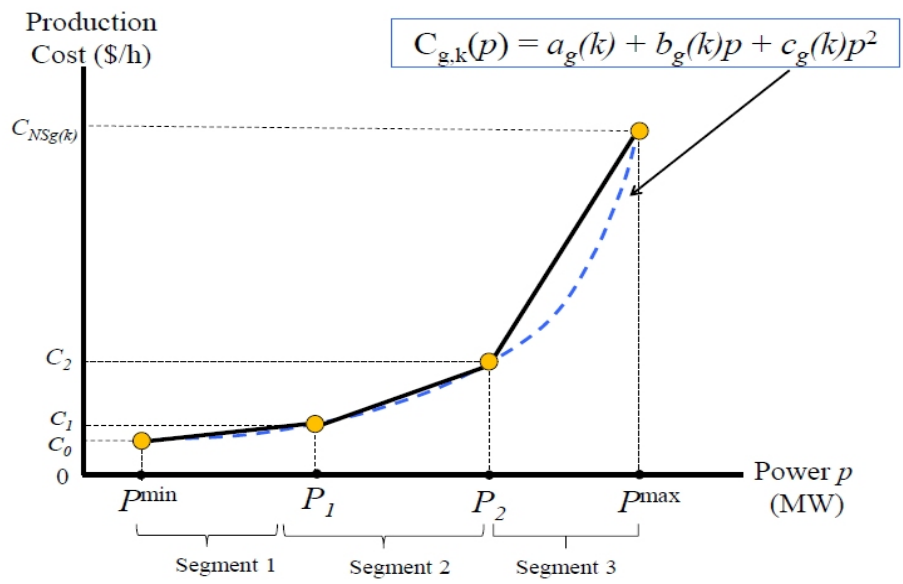

Figure 6: *Illustration of the piecewise-linear approximation for the per-hour production cost function (2) of a dispatchable generator g for a time-step $k \in \mathbb{K}(\mathrm{T})$. The depicted approximation uses $NS_g(k) = 3$ line segments.*

Total Production Cost ($) Approximation for Dispatchable Generator g for any $k \in \mathbb{K}(\mathrm{T})$:

$$c_g^{\mathsf{p}}(k) = C_0 \Delta t v_g(k) + \sum_{i=1}^{NS_g(k)} \big(MC_i \cdot \delta_{i,g}(k)\big)\Delta t\,; \tag{4}$$

$$p_g(k) = P_0 v_g(k) + \sum_{i=1}^{NS_g(k)} \delta_{i,g}(k)\,; \tag{5}$$

$$\delta_{i,g}(k) \le P_i - P_{i-1}\,,\ \forall i = 1, \cdots, NS_g(k)\,; \tag{6}$$

$$\delta_{i,g}(k) \ge 0,\ \forall i = 1, \cdots, NS_g(k)\,, \tag{7}$$

where:

$$MC_i \ = \ \frac{C_i - C_{i-1}}{P_i - P_{i-1}},\ \forall i = 1, \ldots, NS_g(k)\,. \tag{8}$$

The unit commitment indicator $v_g(k)$ in (4) indicates whether (1) or not (0) the ISO commits generator g for time-step k, and Δt denotes the duration of time-step k measured in hours. If the ISO commits g for time-step k, the power level $p_g(k)$ in (5) denotes the ISO's choice of a dispatch set point for g at time k^{s}.[5]

For each segment $i \in \{1, ..., NS_g(k)\}$, the marginal cost ($/MWh) of generator g evaluated at any power level between P_{i-1} and P_i is approximated by MC_i in (8). The variables $\delta_{i,g}(k)$ (MW) appearing in constraints (4)-(7) are incorporated into the SCUC/SCED optimization as continuously-valued ISO decision variables. For example, suppose $v_g(k) = 1$ and there exists a

[5]The SCUC/SCED constraints presented in Section 6 imply that $c_g^{\mathsf{p}}(k)$ is zero for any time-step $k \in \mathbb{K}(\mathrm{T})$ for which g is not committed. That is, if $v_g(k) = 0$, then $p_g(k) = 0$; hence, constraints (4), (5), and (7) together with the assumption $P_0 = P_g^{\mathsf{min}}(k) \ge 0$ imply that $c_g^{\mathsf{p}}(k) = 0$ as well.

segment $n \in \{1, \ldots, NS_g(k)\}$ such that $\delta_{i,g}(k)$ takes on its maximum possible value for $i = 1, \ldots, n$ and $\delta_{i,g}(k) = 0$ for $i = n+1, \ldots, NS_g(k)$. Then $p_g(k) = P_n$ and $c_g^P(k) = C_n \Delta t = C_{g,k}(P_n)$. On the other hand, suppose $v_g(k) = 1$ but $\delta_{i,g}(k) = 0$ for all $i = 1, \ldots, NS_g(k)$. Then $p_g(k) = P_0$ and $c_g^P(k) = C_0 \Delta t \equiv \mathrm{C}_{g,k}(P_0)\Delta t \equiv \mathrm{C}_{g,k}(P_g^{\mathsf{min}}(k))\Delta t$.

Finally, the following simple method can be used to construct a collection of power-cost points $\{(P_0, C_0), \ldots, (P_{NS_g(k)}, C_{NS_g(k)})\}$ satisfying restrictions (3) with $C_i = \mathrm{C}_{g,k}(P_i)$ for each i. The method partitions generator g's range $[P_g^{\mathsf{min}}(k), P_g^{\mathsf{max}}(k)]$ of feasible power output levels at time k^{s} into segments having equal lengths. The resulting power-cost points are treated as exogenous inputs to the ISO's SCUC/SCED optimization.

Power-Cost Point Setting Method for a Dispatchable Generator g at a Time-Step k:

The method requires the following inputs: (i) admissible values for the production cost coefficients $(a_g(k), b_g(k), c_g(k))$ in (2); (ii) the minimum and maximum sustainable power output levels $P_g^{\mathsf{min}}(k)$ and $P_g^{\mathsf{max}}(k)$; and (iii) a positive integer value $NS_g(k)$ for the total number of line segments i to be used in the approximation. The AMES V5.0 Pyomo Model uses these inputs to compute[6] power-cost points $\{(P_i, C_i) \mid i = 0, \ldots NS_g k\}$ for use in the Total Production Cost Approximation Method, as follows:

(a) The initial power-cost points are set to $P_0 = P_g^{\mathsf{min}}(k)$ and $C_0 = \mathrm{C}_{g,k}(P_0)$;

(b) The power-width of each segment $i = 1, \ldots, NS_g(k)$ is set equal to

$$w_g(k) \;\equiv\; \frac{[P_g^{\mathsf{max}}(k) - P_g^{\mathsf{min}}(k)]}{NS_g(k)}\,; \tag{9}$$

(c) For each segment $i = 1, \ldots NS_g(k)$, the power point P_i is set equal to $P_0 + i w_g(k)$;

(d) For each segment $i = 1, \ldots, NS_g(k)$, the cost point C_i is set equal to $\mathrm{C}_{g,k}(P_i)$.

Figure 7 illustrates the energy requirements for a dispatchable generator g that maintains a constant power output $p_g(k) = P_2$ during a time-step $k \in \mathbb{K}(\mathrm{T})$. In this figure, g^{sync} denotes the non-negative power generation level at which the generator attains a synchronized state, ready to inject power into the grid but not yet injecting any power into the grid. The generator's total power generation level sustained during time-step k is thus given by $g(t) = P_2 + g^{\mathsf{sync}}$ for all $t \in k$.

The amount of energy that generator g expends locally (behind-the-meter) during time-step k in order to maintain itself in a synchronized state is denoted by NoLoad in Fig. 7. The amount of energy that g injects into the grid during time-step k in order to support its minimum feasible power output P^{min} during time-step k is denoted by MinRun. The remaining amount of energy that g injects into the grid during time-step k is denoted by NetDispatch.

Generator g's *no-load cost* (\$) for time-step k is the cost g incurs in order to maintain itself in a synchronized state during k. Generator g's *opportunity cost* (\$) for time-step k is the net earnings that g could have obtained from the deployment of its generation capacity in a next-best alternative use during k. Finally, g's *dispatch cost* (\$) for time-step k is the cost that g incurs for the dispatched delivery of power during k.

For the case depicted in Fig. 7, generator g's no-load cost for time-step k includes the energy cost incurred for NoLoad. The production cost function (2) can account for this no-load cost for time-step k, along with any opportunity cost for time-step k, by appropriate specification of

[6]This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

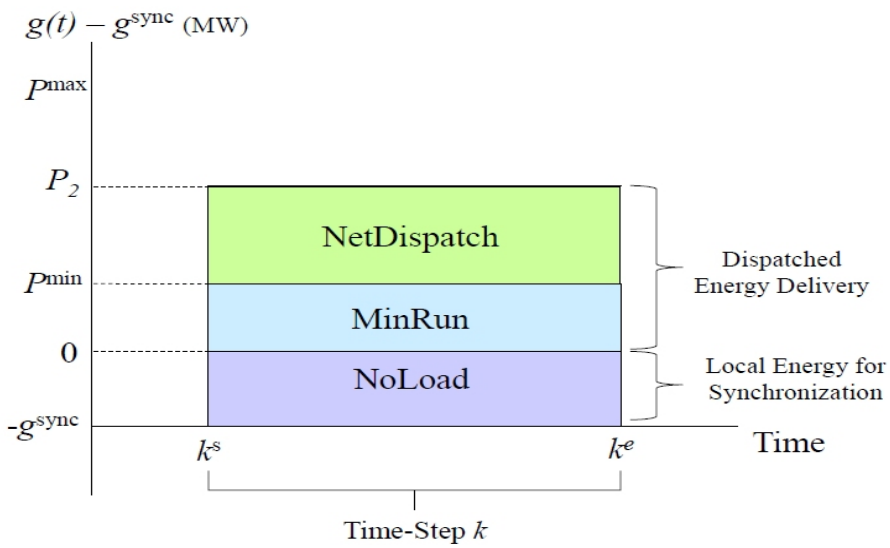

Figure 7: *Illustrative depiction of the energy requirements for a dispatchable generator g that maintains a constant power output P_2 during a time-step $k \in \mathbb{K}(\mathrm{T})$.*

$\mathrm{C}_{g,k}(0)\Delta t = a_g(k)\Delta t$. Generator g's dispatch cost for time-step k includes the energy cost incurred for MinRun and NetDispatch. These costs can be accounted for by appropriate specification of $[\mathrm{C}_{g,k}(P_2)$ - $\mathrm{C}_{g,k}(0)]\Delta t$.

6 Generalized ECA Model with All Fixed Loads

6.1 Overview

The Generalized ECA Model with all fixed loads provides a complete analytical MILP modeling M(T) of an ISO-managed SCUC/SCED optimization for a future operating period T. The objective of the ISO is to select admissible decision variables to minimize the expected total cost ($) of achieving a balancing of net fixed load during period T, given period-T initial state conditions, and subject to system constraints.

The future operating period T is partitioned into $NK(\mathrm{T})$ consecutive time-steps k of equal duration Δt. Total cost is the summation of production cost $c_g^{\mathsf{p}}(k)$, start-up cost $c_g^{\mathsf{u}}(k)$, shut-down cost $c_g^{\mathsf{d}}(k)$, and imbalance cost summed over all dispatchable generators $g \in \mathbb{G}$ and all time-steps $k \in \mathbb{K}(\mathrm{T}) = \{1, \ldots, NK(\mathrm{T}\}$. These costs are "expected" costs in the sense that they are conditioned on forecasts for next-day net fixed loads.[7]

All notation for this Generalized ECA Model with all fixed loads is carefully explained in Sections 4 and 5. For ease of notation, throughout the remainder of this section the period-T index set $\mathbb{K}(\mathrm{T})$, index set cardinality $NK(\mathrm{T})$, and period-T initial state variables $\bar{x}_s^{\mathsf{T}}(0)$, $\underline{x}_s^{\mathsf{T}}(0)$, $SOC_s^{\mathsf{T}}(0)$, $p_g^{\mathsf{T}}(0)$, $\hat{v}_g^{\mathsf{T}}(0)$, $\acute{s}v_g^{\mathsf{T}}(0)$,$v_g^{\mathsf{T}}(0)$, ITF_g^{T}, and ITO_g^{T} are expressed without reference to T: namely, as $\mathbb{K}$, NK, $\bar{x}_s(0)$, $\underline{x}_s(0)$, $SOC_s(0)$, $p_g(0)$, $\hat{v}_g(0)$, $\acute{s}v_g(0)$, $v_g(0)$, ITF_g, and ITO_g.

[7]That is, certainty equivalence is used to approximate expectations.

6.2 Complete Analytical SCUC/SCED Optimization Formulation

ISO Objective:

Select decision variables to minimize expected total cost, given initial state conditions, and subject to system constraints, where expected total cost ($) is given by:

$$\widehat{\mathrm{C}}(\mathrm{T}) \;=\; \sum_{k\in\mathbb{K}}\sum_{g\in\mathbb{G}}\Big[c_g^{\mathsf{p}}(k) \;+\; c_g^{\mathsf{u}}(k) \;+\; c_g^{\mathsf{d}}(k)\Big] \;+\; \sum_{b\in\mathbb{B}}\sum_{k\in\mathbb{K}}\big[\Lambda^-\beta_b^-(k) \;+\; \Lambda^+\beta_b^+(k)\big]\Delta t \tag{10}$$

ISO Decision Variables:

ISO Binary-Valued Decision Variables: $\forall g \in \mathbb{G}, k \in \mathbb{K},$ and $s \in \mathbb{S}$,

$$v_g(k) \in \{0,1\} \tag{11}$$
$$\bar{u}_s(k) \in \{0,1\} \tag{12}$$
$$\underline{u}_s(k) \in \{0,1\} \tag{13}$$

ISO Continuously-Valued Decision Variables: For all $b \in \mathbb{B}, g \in \mathbb{G}, i \in \mathbb{NS}_g(k), k \in \mathbb{K},$ and $s \in \mathbb{S}$,

$$\bar{x}_s(k) \tag{14}$$
$$\underline{x}_s(k) \tag{15}$$
$$\delta_{i,g}(k) \tag{16}$$
$$\theta_b(k) \tag{17}$$

Solution Values Determined by ISO Decision Variables and System Constraints:
$\forall b \in \mathbb{B}, g \in \mathbb{G}, k \in \mathbb{K}, \ell \in \mathbb{L},$ and $s \in \mathbb{S}$,

$$c_g^{\mathsf{p}}(k),\, c_g^{\mathsf{u}}(k),\, c_g^{\mathsf{d}}(k) \tag{18}$$
$$hs_g(k) \in \{0,1\} \tag{19}$$
$$p_g(k),\, \overline{p}_g(k),\, \underline{p}_g(k) \tag{20}$$
$$w_\ell(k) \tag{21}$$
$$z_s(k) \tag{22}$$
$$\beta_b(k),\, \beta_b^-(k),\, \beta_b^+(k) \tag{23}$$
$$\theta_1(k) \tag{24}$$

System Constraints:

Transmission line power flow constraints: For all $k \in \mathbb{K}$ and $\ell \in \mathbb{L}$,

$$w_\ell(k) = S_0 B(\ell)\left[\theta_{O(\ell)}(k) - \theta_{E(\ell)}(k)\right]\;; \tag{25}$$
$$-F^{\mathsf{max}}(\ell) \le w_\ell(k) \le F^{\mathsf{max}}(\ell)\,. \tag{26}$$

Power balance constraints (with slack variables): For all $b \in \mathbb{B}$ and $k \in \mathbb{K}$,

$$\sum_{g\in\mathbb{G}(b)} p_g(k) + \Big[\sum_{\ell\in\mathbb{L}_{\mathsf{E}}(b)} w_\ell(k) - \sum_{\ell\in\mathbb{L}_{\mathsf{O}}(b)} w_\ell(k)\Big] + \Big[\sum_{s\in\mathbb{S}(b)}\big(\overline{x}_s(k) - \underline{x}_s(k)\big)\Big] \;=\; \widehat{NL}_b^{\mathsf{f}}(k) + \beta_b(k)\,. \tag{27}$$

Slack variable constraints: For all $b \in \mathbb{B}$ and $k \in \mathbb{K}$,

$$\beta_b^-(k) = \max\{0, -\beta_b(k)\}\,; \tag{28}$$

$$\beta_b^+(k) = \max\{0,\, \beta_b(k)\}\,. \tag{29}$$

Dispatchable generator capacity constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\underline{p}_g(k) \;\le\; p_g(k) \;\le\; \overline{p}_g(k)\,; \tag{30}$$

$$\overline{p}_g(k) \;\le\; P_g^{\mathsf{max}}(k) v_g(k)\,; \tag{31}$$

$$\underline{p}_g(k) \;\ge\; P_g^{\mathsf{min}}(k) v_g(k)\,. \tag{32}$$

Dispatchable generator ramping constraints for start-up, normal, & shut-down:

$$\begin{aligned} \bar{p}_g(k) \;\le\;& p_g(k-1) + SRU_g(k-1)v_g(k-1) + SSU_g(k-1)[v_g(k) - v_g(k-1)] \\ &+ P_g^{\mathsf{max}}(k-1)[1 - v_g(k)], \quad \forall g \in \mathbb{G},\ \forall k \in \mathbb{K}\,; \end{aligned} \tag{33}$$

$$\begin{aligned} \bar{p}_g(k) \;\le\; P_g^{\mathsf{max}}(k)v_g(k+1) \;+\; SSD_g(k)[v_g(k) - v_g(k+1)]\,, \\ \forall g \in \mathbb{G},\ \forall k = 1 \cdots NK - 1\,; \end{aligned} \tag{34}$$

$$\begin{aligned} p_g(k-1) \;\le\;& \underline{p}_g(k) + \; SRD_g(k-1)v_g(k) \;+\; SSD_g(k-1)[v_g(k-1) - v_g(k)] \\ &+ P_g^{\mathsf{max}}(k-1)[1 - v_g(k-1)], \quad \forall g \in \mathbb{G},\ \forall k \in \mathbb{K}\,. \end{aligned} \tag{35}$$

Remarks on the ramping constraints: See Section 6.3 below for detailed explanations of these ramping constraints.

Dispatchable generator minimum up-time constraints:

$$\sum_{k=1}^{ITO_g} [1 - v_g(k)] = 0 \;\text{ for all } g \in \mathbb{G} \text{ with } ITO_g \ge 1\,; \tag{36}$$

$$\sum_{n=k}^{k+SUT_g-1} v_g(n) \ge SUT_g[v_g(k) - v_g(k-1)], \;\; \forall g \in \mathbb{G},\ \forall k = ITO_g + 1, \cdots, NK - SUT_g + 1\,; \tag{37}$$

$$\sum_{n=k}^{NK} \big(v_g(n) - [v_g(k) - v_g(k-1)]\big) \;\ge\; 0, \;\; \forall g \in \mathbb{G},\ \forall k = NK - SUT_g + 2, \cdots, NK\,. \tag{38}$$

Remarks on the minimum up-time constraints: To derive these constraints, consider the following. If $ITO_g \ge 1$ for generator g, then by definition of ITO_g it must hold that $v_g(k) = 1$ for all time-steps k satisfying $1 \le k \le ITO_g$. For $ITO_g + 1 \le k$, suppose a start-up event occurs for generator g at the start of time-step k; i.e., suppose $v_g(k-1) = 0$ and $v_g(k) = 1$, implying generator g is OFF in time-step $k-1$ and ON in time-step k. Then, by definition of SUT_g, generator g must remain ON for $SUT_g - 1$ additional time-steps, or until the end of the final modeled time-step NK if $NK \le k + SUT_g - 1$. The above minimum up-time constraints express these requirements in concise form.

Dispatchable generator minimum down-time constraints:

$$\sum_{k=1}^{ITF_g} v_g(k) = 0 \;\; \forall g \in \mathbb{G} \text{ with } ITF_g \geq 1\,; \tag{39}$$

$$\sum_{n=k}^{k+SDT_g-1} [1 - v_g(n)] \geq SDT_g[v_g(k-1) - v_g(k)], \;\; \forall g \in \mathbb{G}, \; k = ITF_g + 1, \cdots, NK - SDT_g + 1; \tag{40}$$

$$\sum_{n=k}^{NK} \big[1 - v_g(n) - [v_g(k-1) - v_g(k)]\big] \geq 0, \;\; \forall g \in \mathbb{G}, \; \forall k = NK - SDT_g + 2, \cdots, NK\,. \tag{41}$$

Remarks on the minimum down-time constraints: The derivation of the above minimum down-time constraints is similar to the derivation of the minimum up-time constraints, except that one considers shut-down events with $v_g(k-1) = 1$ and $v_g(k) = 0$ rather than start-up events.

Dispatchable generator hot-start constraints:

$$hs_g(k) = 1, \qquad \forall g \in \mathbb{G}, 1 \leq k \leq SCST_g : k - SCST_g \leq \widehat{sv}_g(0)\,; \tag{42}$$

$$hs_g(k) \leq \sum_{n=1}^{k-1} v_g(n), \qquad \forall g \in \mathbb{G}, 1 \leq k \leq SCST_g : k - SCST_g > \widehat{sv}_g(0)\,; \tag{43}$$

$$hs_g(k) \leq \sum_{n=k-SCST_g}^{k-1} v_g(n), \qquad \forall g \in \mathbb{G}, k = SCST_g + 1, \ldots, NK\,. \tag{44}$$

Remarks on Generator Hot-Start Constraints:

By construction (1), the scaled up-time/down-time status indicator $\widehat{sv}_g(0)$ for a dispatchable generator $g \in \mathbb{G}$ at the start of an operating period T is a non-zero integer. A positive (negative) value for $\widehat{sv}_g(0)$ approximates the number of consecutive time-steps prior to *and including* time-step 0 for T that g was ON (OFF), where each of these prior time-steps has a common duration Δt measured in hours.

By definition, g is in a hot-start state ($hs_g(k) = 1$) at the start of a time-step k if g was ON during any of the $SCST_g$ consecutive time-steps immediately preceding time-step k. Constraints (42) and (43) give the defining conditions for g to be in a hot-start state at the start of a time-step k when k *does not* exceed $SCST_g$. In this case, $\widehat{sv}_g(0)$ must be used to check the ON/OFF status of g for time-step 0, and possibly also for time-steps prior to time-step 0.

Suppose k *does not* exceed $SCST_g$. If g was ON for time-step 0, i.e., if $\widehat{sv}_g(0) > 0$, then g must be in a hot-start state at the start of time-step k. Conversely, suppose g has been OFF for a maximum of $m > 0$ consecutive time-steps prior to and including time-step 0 (i.e., $\widehat{sv}_g(0) = -m$). Then g must be in a hot-start state at the start of time-step k if m plus the number $k-1$ of time-steps between the start of time-step 1 and the start of time-step k is strictly less than $SCST_g$; that is, if $[k - SCST_g] \leq \widehat{sv}_g(0)$. This establishes constraint (42). On the other hand, g must be in a cold-start state ($hs_g(k) = 0$) at the start of k if g has been OFF for time-steps $n = 1, \ldots, k-1$ and g was also OFF for a number of time-steps immediately preceding time-step 1 that exceeds

$SCST_g - k$; that is, if $\widehat{sv}_g(0) < k - SCST_g \leq 0$. This establishes constraint (43).

Suppose k *does* exceed $SCST_g$. In this case the ON/OFF status of g only needs to be checked for the $SCST_g$ time-steps n that precede k, where each n is subsequent to time-step 0. For each of these time-steps n, the ON/OFF status of g can be determined from g's unit commitment indicator $v_g(n)$. This establishes constraint (44).

For reasons explained in the remarks following the next set of constraints (i.e., the generator start-up cost constraints), if generator g's cold-start cost parameter CSC_g is strictly positive, then the cost-minimizing ISO will set $hs_g(k) = 1$ unless the generator hot-start constraints (42) through (44) force the ISO to set $hs_g(k) = 0$.

Dispatchable generator start-up cost constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\begin{aligned} c_g^{\mathsf{u}}(k) &= \max\{0, U_g(k)\}\,; \\ U_g(k) &= CSC_g - [CSC_g - HSC_g]hs_g(k) - CSC_g\big[1 - [v_g(k) - v_g(k-1)]\big]\,. \end{aligned} \tag{45}$$

Remarks on Dispatchable Generator Start-Up Cost: Definitions of a cold-start state versus a hot-start state for any dispatchable generator g are provided in Section 4.4. Also recall from this previous section that the user-specified costs CSC_g and HSC_g are required to satisfy $\text{CSC}_g \geq \text{HSC}_g$. Consequently, (45) implies: (i) $c_g^{\mathsf{u}}(k) = \text{CSC}_g$ if g starts up at time k^{s} (i.e., $v_g(k-1) = 0$ and $v_g(k) = 1$) while in a cold-start state ($hs_g(k) = 0$); and (ii) $c_g^{\mathsf{u}}(k) = \text{HSC}_g$ if g starts up at time k^{s} while in a hot-start state ($hs_g(k) = 1$). Otherwise, $c_g^{\mathsf{u}}(k) = 0$.

Thus, assuming $\text{CSC}_g > 0$, in order to minimize expected total cost the ISO will strive to avoid starting up generator g in a cold-start state, all else equal. In particular, unless ruled out by the hot-start constraints (42)-(44), the cost-minimizing ISO will set $hs_g(k) = 1$ if it commits generator g for time-step k.

Dispatchable generator shut-down cost constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\begin{aligned} c_g^{\mathsf{d}}(k) &= \max\{0, D_g(k)\}\,; \\ D_g(k) &= SDC_g[v_g(k-1) - v_g(k)]\,. \end{aligned} \tag{46}$$

Storage unit limit constraints: For all $k \in \mathbb{K}$ and $s \in \mathbb{S}$,

$$\underline{u}_s(k)PI_s^{\mathsf{min}} \leq \underline{x}_s(k) \leq \underline{u}_s(k)PI_s^{\mathsf{max}}\,; \tag{47}$$

$$\bar{u}_s(k)PO_s^{\mathsf{min}} \leq \bar{x}_s(k) \leq \bar{u}_s(k)PO_s^{\mathsf{max}}\,. \tag{48}$$

Storage unit charge/discharge constraint (cannot charge and discharge at same time):

$$\underline{u}_s(k) + \bar{u}_s(k) \;\leq\; 1,\ \forall k \in \mathbb{K}, s \in \mathbb{S}\,. \tag{49}$$

Storage unit ramping constraints: For all $k \in \mathbb{K}$ and $s \in \mathbb{S}$,

$$\bar{x}_s(k) \;\leq\; \bar{x}_s(k-1) + SORU_s\,; \tag{50}$$

$$\bar{x}_s(k) \;\geq\; \bar{x}_s(k-1) - SORD_s\,; \tag{51}$$

$$\underline{x}_s(k) \;\leq\; \underline{x}_s(k-1) + SIRU_s\,; \tag{52}$$

$$\underline{x}_s(k) \;\geq\; \underline{x}_s(k-1) - SIRD_s. \tag{53}$$

Storage unit energy conservation constraints: For all $k \in \mathbb{K}$ and $s \in \mathbb{S}$,

$$z_s(k) \;=\; z_s(k-1) + \frac{[-\bar{x}_s(k) + \eta_s \underline{x}_s(k)] \cdot \Delta t}{ES_s^{\mathsf{max}}}\,; \tag{54}$$

$$z_s(0) \;=\; SOC_s(0)\,. \tag{55}$$

Storage unit end-point constraints: For all $s \in \mathbb{S}$,

$$z_s(NK) = EPSOC_s\,. \tag{56}$$

System-wide down/up reserve requirement constraints: For all $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}} \underline{p}_g(k) \;\leq\; [1 - RRD(k)] \cdot \widehat{NL}^{\mathsf{f}}(k)\,; \tag{57}$$

$$\sum_{g \in \mathbb{G}} \overline{p}_g(k) \;\geq\; [1 + RRU(k)] \cdot \widehat{NL}^{\mathsf{f}}(k)\,. \tag{58}$$

Zonal down/up reserve requirement constraints: For all $k \in \mathbb{K}$ and $z \in \mathbb{Z}$,

$$\sum_{g \in \mathbb{G}(z)} \underline{p}_g(k) \;\leq\; [1 - RRD_z(k)] \cdot \widehat{NL}_z^{\mathsf{f}}(k)\,; \tag{59}$$

$$\sum_{g \in \mathbb{G}(z)} \overline{p}_g(k) \;\geq\; [1 + RRU_z(k)] \cdot \widehat{NL}_z^{\mathsf{f}}(k)\,. \tag{60}$$

Voltage angle specifications: For all $b \in \mathbb{B}/\{1\}$ and $k \in \mathbb{K}$,

$$\theta_b(k) \in [-\pi, \pi]\,; \tag{61}$$

$$\theta_1(k) = 0\,. \tag{62}$$

Total Production Cost Approximation Constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$c_g^{\mathsf{p}}(k) = C_0 \Delta t v_g(k) + \sum_{i=1}^{NS_g(k)} \big(MC_i \cdot \delta_{i,g}(k)\big)\Delta t\,; \tag{63}$$

$$p_g(k) = P_0(k) v_g(k) + \sum_{i=1}^{NS_g(k)} \delta_{i,g}(k)\,; \tag{64}$$

$$\delta_{i,g}(k) \leq P_{i,g}(k) - P_{i-1,g}(k)\,,\ \forall i = 1, \cdots, NS_g(k) - 1\,; \tag{65}$$

$$\delta_{NS_g(k),g}(k) \leq P_g^{\mathsf{max}}(k) - P_{NS_g(k)-1,g}(k)\,; \tag{66}$$

$$\delta_{i,g}(k) \geq 0,\ \forall i = 1 \cdots NS_g(k)\,; \tag{67}$$

$$MC_i \ = \ \frac{C_i - C_{i-1}}{P_i - P_{i-1}},\ i = 1, \ldots, NS_g(k)\,. \tag{68}$$

6.3 Ramping Constraint Tables

Tables 1-5 show the reduced form of each ramping constraint (33)-(35), conditional on the generation capacity constraints (30)-(32), for each possible setting of $\{v_g(k-1), v_g(k), v_g(k+1)\}$ for any $k = 1, \ldots, NK - 1$. A reduced-form ramping constraint is said to *hold automatically* if it is an implication of other imposed system constraints and/or parameter sign admissibility constraints. Otherwise, the constraint is said to be *consequential*.

Table 1: All three ramping constraints (33)-(35) hold automatically for the depicted cases.

$v_g(k-1)$	0	0
$v_g(k)$	0	0
$v_g(k+1)$	0	1
(30)-(32)	$p_g(k-1) = \overline{p}_g(k) = p_g(k) = \underline{p}_g(k) = 0$	
(33)	$0 \ \leq \ P_g^{\mathsf{max}}(k-1)$	
(34)	$0 \ \leq \ 0$	$0 \ \leq \ P_g^{\mathsf{max}}(k) - SSD_g(k)$
(35)	$0 \ \leq \ P_g^{\mathsf{max}}(k-1)$	

Table 2: The ramping constraint (35) is consequential for the depicted cases; the ramping constraints (33) and (34) hold automatically.

$v_g(k-1)$	1	1
$v_g(k)$	0	0
$v_g(k+1)$	0	1
(30)-(32)	$P_g^{\mathsf{min}}(k-1) \leq p_g(k-1); \quad \overline{p}_g(k) = p_g(k) = \underline{p}_g(k) = 0$	
(33)	$0 \;\leq\; p_g(k-1) + SRU_g(k-1) - SSU_g(k-1) + P_g^{\mathsf{max}}(k-1)$	
(34)	$0 \;\leq\; 0$	$0 \;\leq\; P_g^{\mathsf{max}}(k) - SSD_g(k)$
(35)	$p_g(k-1) \;\leq\; SSD_g(k-1)$	

Table 3: The ramping constraints (33) and (34) are consequential for the depicted cases; the ramping constraint (35) holds automatically.

$v_g(k-1)$	0	0
$v_g(k)$	1	1
$v_g(k+1)$	0	1
(30)-(32)	$p_g(k-1) = 0,\ P_g^{\mathsf{min}}(k) \leq \underline{p}_g(k) \leq p_g(k) \leq \overline{p}_g(k) \leq P_g^{\mathsf{max}}(k)$	
(33)	$\overline{p}_g(k) \;\leq\; SSU_g(k-1)$	
(34)	$\overline{p}_g(k) \;\leq\; SSD_g(k)$	$\overline{p}_g(k) \leq P_g^{\mathsf{max}}(k)$
(35)	$0 \;\leq\; \underline{p}_g(k) + SRD_g(k-1) - SSD_g(k-1) + P_g^{\mathsf{max}}(k-1)$	

Table 4: All three ramping constraints (33)-(35) are consequential for the depicted case.

$v_g(k-1)$	1
$v_g(k)$	1
$v_g(k+1)$	0
(30)-(32)	$P_g^{\mathsf{min}}(k) \leq \underline{p}_g(k) \leq p_g(k) \leq \overline{p}_g(k) \leq P_g^{\mathsf{max}}(k)$
(33)	$\overline{p}_g(k) \;\leq\; p_g(k-1) + SRU_g(k-1)$
(34)	$\overline{p}_g(k) \;\leq\; SSD_g(k)$
(35)	$p_g(k-1) - SRD_g(k-1) \;\leq\; \underline{p}_g(k)$

Table 5: Ramping constraints (33) and (35) are consequential for the depicted case; ramping constraint (34) holds automatically.

$v_g(k-1)$	1
$v_g(k)$	1
$v_g(k+1)$	1
(30)-(32)	$P_g^{\mathsf{min}}(k) \leq \underline{p}_g(k) \leq p_g(k) \leq \overline{p}_g(k) \leq P_g^{\mathsf{max}}(k)$
(33)	$\overline{p}_g(k) \leq p_g(k-1) + SRU_g(k-1)$
(34)	$\overline{p}_g(k) \leq P_g^{\mathsf{max}}(k)$
(35)	$p_g(k-1) - SRD_g(k-1) \ \leq \ \underline{p}_g(k)$

7 Incorporation of Price-Sensitive Demand Bids

7.1 Overview

Demand bids submitted by LSEs into current U.S. RTO/ISO-managed Day-Ahead Markets (DAMs) are demands for the delivery of power for retail customers, with or without accompanying price information indicating willingness to pay. If a demand bid submitted by an LSE into a SCUC/SCED optimization for a day-ahead market is cleared, the LSE must compensate the ISO for the resulting delivery of power to its customers. These LSE payments are determined in part through locational marginal price assessments (which take into account any LSE submitted price information) and in part through subsequent RTO/ISO allocations of its net costs across LSEs on the basis of their load shares.[8]

Currently in these DAMs, most LSE demand bids take a fixed form. A *fixed demand bid* submitted into a day-ahead market held on day D is a load profile designating a forecasted demand $\widehat{p}^{\mathsf{f}}(k)$ (MW) for power usage at the start of each hour k for day D+1, with no accompanying price information indicating willingness to pay.

In economic terms, a forecasted demand $\widehat{p}^{\mathsf{f}}(k)$ for power usage effectively represents a vertical demand curve in a power-price plane, as if customers had an infinite willingness to pay for this power usage. Although the effective[9] maximum willingness to pay for this power usage is necessarily finite, it cannot be determined from this fixed-bid form. Consequently, the presence of fixed demand bids hinders an RTO/ISO's ability to ensure that SCUC/SCED optimization solutions result in an efficient allocation of resources.

These concerns arise for the Generalized ECA Model of a SCUC/SCED optimization presented in Section 6 for the special case in which all LSE demand bids take a fixed-load form and hence are entered into power balance constraints as must-meet load obligations. Since benefits cannot be measured, the usually stated SCUC/SCED optimization objective, maximization of expected total net benefit, is replaced by the goal of minimizing expected total cost.

[8] To preserve its independent status, an RTO/ISO cannot have a financial stake in the market operations it manages. Thus, an RTO/ISO must pass through to market participants any net costs resulting from these market operations.

[9] *Effective* willingness to pay is willingness to pay back-stopped by actual purchasing power.

This section discusses how the fixed-load version of the Generalized ECA Model presented in Section 6 can be modified to permit LSEs to submit demand bids that include price-sensitive demand bids as well as fixed load forecasts. Three types of price-sensitive demand bids are considered: demand bids with *Time-of-Use (TOU)* pricing; demand bids in the form of price-quantity demand schedules; and demand bids derived from general benefit functions.

AMES V5.0 is able to implement the Generalized ECA Model of a SCUC/SCED optimization with price-sensitive LSE demand bids derived from general benefit functions. To activate this functionality, users of AMES V5.0 simply need to set a 1/0 binary simulation control variable (PriceSensitiveDemandFlag) to 1.

7.2 Incorporation of Benefits into the Generalized ECA Model

The Generalized ECA Model assumes that an ISO-managed SCUC/SCED optimization M(T) is conducted in order to secure net-load balancing resources for a future operating period T. The operating period T is partitioned into time-steps $k \in \mathbb{K}(\mathrm{T}) = \{1, \ldots, NK(\mathrm{T})\}$, where each time-step k has equal duration Δt measured in hours.

During each time-step k, each LSE j services the power usage of retail customers at its bus location b_j. In Section 6 this servicing was assumed to take the following fixed-load form: each LSE j submits a fixed demand bid into the SCUC/SCED optimization that is expressed as a forecasted power-usage amount $\widehat{p}^{\mathsf{f}}_j(k)$ (MW) for its retail customers at bus b_j, with no accompanying price information.

Suppose, instead, that that each LSE j also submits into this SCUC/SCED optimization a price-sensitive demand bid for each time-step k on behalf of a collection $\mathbb{C}_j$ of retail customers. This bid expresses possible aggregate power-usage levels $p^{\mathsf{s}}_j(k)$ (MW) that the customers in $\mathbb{C}_j$ could maintain at bus b_j during time-step k together with some type of metric for measuring the expected benefit to these customers of each of these power-usage levels.

Let the expected benefit assigned by LSE_j to each possible power-usage sequence

$$\mathbf{p}^{\mathsf{s}}_j \;=\; \{p^{\mathsf{s}}_j(k) \mid k \in \mathbb{K}(\mathrm{T})\} \tag{69}$$

be denoted by $\mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j)$ (\$). The expected total benefit (\$) from these price-sensitive demand bids is then given by

$$\widehat{\mathrm{B}}(\mathrm{T}) \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j)\,. \tag{70}$$

In order to incorporate these price-sensitive demand bids and expected benefit calculations into the ISO-managed SCUC/SCED optimization presented in Section 6 for the Generalized ECA Model with all fixed loads, the following modifications must be made.

Required ISO Modifications:

First, the ISO objective function (10), expressed solely in terms of expected total cost $\widehat{\mathrm{C}}(\mathrm{T})$ for operating period T, needs to be modified to represent expected total net benefit for T. This expected total net benefit (\$) is expressed as

$$\widehat{\mathrm{NB}}(\mathrm{T}) \;=\; \widehat{\mathrm{B}}(\mathrm{T}) - \widehat{\mathrm{C}}(\mathrm{T}) \tag{71}$$

Second, the objective of the ISO needs to be changed from the minimization of period-T expected

total cost to the maximization of period-T expected total net benefit (71). Third, the price-sensitive power-usage amounts $\{p_j^{\mathsf{s}}(k) \mid j \in \mathbb{LS}, k \in \mathbb{K}(\mathrm{T})\}$ need to be included among the ISO's continuously-valued decision variables, subject to domain constraints of the form

$$p_j^{\mathsf{s}}(k) \in \mathbb{P}_j(k)\,,\ \forall j \in \mathbb{LS}, k \in \mathbb{K}(\mathrm{T})\,, \tag{72}$$

where each domain $\mathbb{P}_j(k)$ includes 0, i.e., the zero-power option.[10]

Required System Constraint Modifications:

Fourth, the price-sensitive power-usage amounts $p_j^{\mathsf{s}}(k)$ need to be appropriately entered into the power balance constraints (27). The resulting generalized constraints take the following form:

Generalized power balance constraints (with slack variables): For all $b \in \mathbb{B}$ and $k \in \mathbb{K}(\mathrm{T})$,

$$\sum_{g \in \mathbb{G}(b)} p_g(k) + \Big[\sum_{\ell \in \mathbb{L}_{\mathsf{E}}(b)} w_\ell(k) - \sum_{\ell \in \mathbb{L}_{\mathsf{O}}(b)} w_\ell(k) \Big] + \Big[\sum_{s \in \mathbb{S}(b)} \big(\overline{x}_s(k) - \underline{x}_s(k) \big) \Big] \;=\; \widehat{NL}_b(k) + \beta_b(k)\,, \tag{73}$$

where

$$\widehat{NL}_b(k) \;=\; \widehat{NL}_b^{\mathsf{f}}(k) + \sum_{j \in \mathbb{LS}(b)} p_j^{\mathsf{s}}(k)\,. \tag{74}$$

Fifth, the price-sensitive power-usage amounts $p_j^{\mathsf{s}}(k)$ need to be appropriately entered into the reserve requirement constraints (57) through (60). The resulting generalized constraints take the following form:

Generalized system-wide down/up reserve requirement constraints: For all $k \in \mathbb{K}(\mathrm{T})$,

$$\sum_{g \in \mathbb{G}} \underline{p}_g(k) \;\le\; [1 - RRD(k)] \cdot \widehat{NL}(k)\,; \tag{75}$$

$$\sum_{g \in \mathbb{G}} \overline{p}_g(k) \;\ge\; [1 + RRU(k)] \cdot \widehat{NL}(k)\,, \tag{76}$$

where

$$\widehat{NL}(k) \;=\; \widehat{NL}^{\mathsf{f}}(k) + \sum_{j \in \mathbb{LS}} p_j^{\mathsf{s}}(k)\,. \tag{77}$$

Generalized zonal down/up reserve requirement constraints: For all $k \in \mathbb{K}(\mathrm{T})$ and $z \in \mathbb{Z}$,

$$\sum_{g \in \mathbb{G}(z)} \underline{p}_g(k) \;\le\; [1 - RRD_z(k)] \cdot \widehat{NL}_z(k)\,; \tag{78}$$

$$\sum_{g \in \mathbb{G}(z)} \overline{p}_g(k) \;\ge\; [1 + RRU_z(k)] \cdot \widehat{NL}_z(k)\,, \tag{79}$$

[10] If the domains $\mathbb{P}_j(k)$ are not required to include a zero-power option, the ISO would have no way of refusing to service price-sensitive demand bids at LSE-set prices whose servicing results in a lowering of total net benefit. To avoid this forced servicing, it would be necessary to modify further the form of the SCUC/SCED optimization to include an ISO binary 1/0 unit commitment decision with regard to each submitted price-sensitive demand bid, thus enlarging the number of ISO binary-valued decision variables.

where

$$\widehat{NL}_z(k) \;=\; \widehat{NL}^{\mathsf{f}}_z(k) + \sum_{j \in \mathbb{LS}(z)} p^{\mathsf{s}}_j(k)\,. \tag{80}$$

7.3 Modeling of Price-Sensitive Demand Bids

7.3.1 Standard Demand Function Formulation

In general economic terms, a *price-sensitive demand bid* submitted by an LSE j for the start of a time-step k on behalf of a collection $\mathbb{C}_j$ of retail customers serviced at a bus b_j can be represented as a non-increasing demand function

$$D_{j,k} : \mathbb{P}_j(k) \to R_+\,, \tag{81}$$

where $0 \in \mathbb{P}_j(k) \subset R_+$. The power-usage domain $\mathbb{P}_j(k)$ denotes the possible aggregate power-usage levels $p^{\mathsf{s}}_j(k)$ that the customers in $\mathbb{C}_j$ could maintain at bus b_j during time-step k.

Given (81), the price-sensitive demand schedule submitted by LSE j for the start of time-step k consists of all power-price combinations (p, π) satisfying

$$\pi \;=\; D_{j,k}(p)\,, \quad p \in \mathbb{P}_j(k)\,. \tag{82}$$

Each power-price combination (p, π) has the following interpretation: π (\$/MWh) is the maximum price that LSE j is willing to pay at the start of time-step k for an incremental increase in the aggregate power usage of the customers in $\mathbb{C}_j$, given that the current aggregate power usage of these customers is at level p (MW). For example, the demand schedule (82) could take the simple linear form

$$\pi \;=\; e_j(k) - 2f_j(k) \cdot p\,, \qquad 0 \;\le\; p \;\le\; e_j(k)/2f_j(k)\,, \tag{83}$$

where the coefficients $e_j(k)$ (\$/MWh) and $f_j(k)$ (\$/[MW]^{2}h) are positively valued.

7.3.2 Price-sensitive Demand Bids with Time-of-Use Pricing

Consider, first, the case in which the demand schedule (82) for each LSE j designates a single price $\pi_j(k)$ (\$/MWh) for each time-step k, regardless of the power-usage level p (MW). In this case the expected benefit (\$) that an LSE j assigns to a price-sensitive power-usage sequence $\mathbf{p}^{\mathsf{s}}_j$ in (69) takes an extremely simple form, as follows:

$$\mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j) \;=\; \sum_{k \in \mathbb{K}(\mathsf{T})} \pi_j(k) p^{\mathsf{s}}_j(k) \Delta t\,, \tag{84}$$

where Δt denotes the duration of each time-step k measured in hours. The expected total benefit (\$) to be included in the ISO's objective function (71) for the entire future operating period T then takes the form:

$$\widehat{\mathrm{B}}(\mathrm{T}) \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j) \tag{85}$$

7.3.3 Price-Sensitive Demand Bids as Price-Quantity Schedules

Consider, instead, the case in which each LSE j submits a price-sensitive demand bid consisting of a demand schedule (82) for each time-step k such that the price π is sensitive to changes in the power-usage level p. How will this affect the expression for expected total benefit in the ISO's net-benefit objective function (71)?

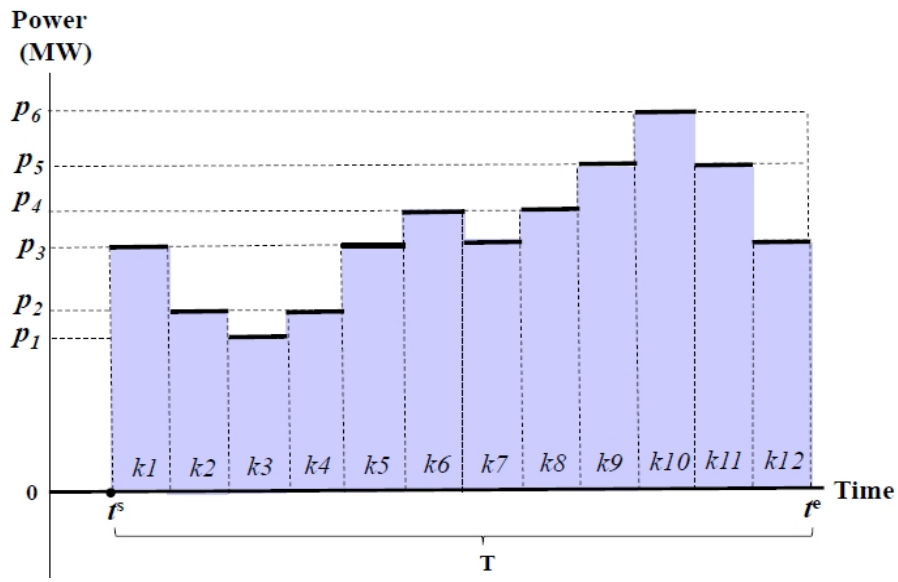

Figure 8: *Illustration of the physical aspects of a price-sensitive demand bid submitted by an LSE into an ISO-managed SCUC/SCED optimization for a future operating period T partitioned into 12 time-steps* $k1, \ldots, k12$. *The same seven possible power demands* $0, p_1, \ldots, p_6$ *are specified for each time-step* k. *The shaded region denotes one possible load profile the ISO could clear for period T.*

For example, Figure 8 illustrates the physical aspects of such a price-sensitive demand bid for an operating period T partitioned into twelve time-steps $k1, \ldots, k12$. Each time-step k has a common duration Δt, measured in hours; and the demand-function domain $\mathbb{P}_j(k)$ for each time-step k is a finite set consisting of seven possible power levels $\{0, p_1, \ldots, p_6\}$. Figure 9 depicts a power-price demand schedule (82) that LSE j could designate for a particular time-step k.[11]

Given this form of price-sensitive demand bid, the expected benefit (\$) that LSE j assigns to a power-usage sequence $\mathbf{p}_j^{\mathsf{s}}$ in (69) takes the following form:

$$\text{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \;=\; \sum_{k \in \mathbb{K}(\mathsf{T})} \pi_{j,n(k)}(k) p_{j,n(k)}^{\mathsf{s}}(k) \Delta t \,, \tag{86}$$

where $n(k)$ is an index that denotes the particular power-usage level $p_{j,n(k)}^{\mathsf{s}}(k)$ (MW) selected from $\mathbb{P}_j(k)$ for time-step k, and $\pi_{j,n(k)}$ (\$/MWh) denotes the corresponding price for power usage during time-step k. The expected total benefit (\$) to be included in the ISO's objective function (71) for

[11] This type of price-sensitive demand schedule has the form commonly required in U.S. RTO/ISO-managed DAMs. See, for example, the form required by the ERCOT DAM [14, Module 4].

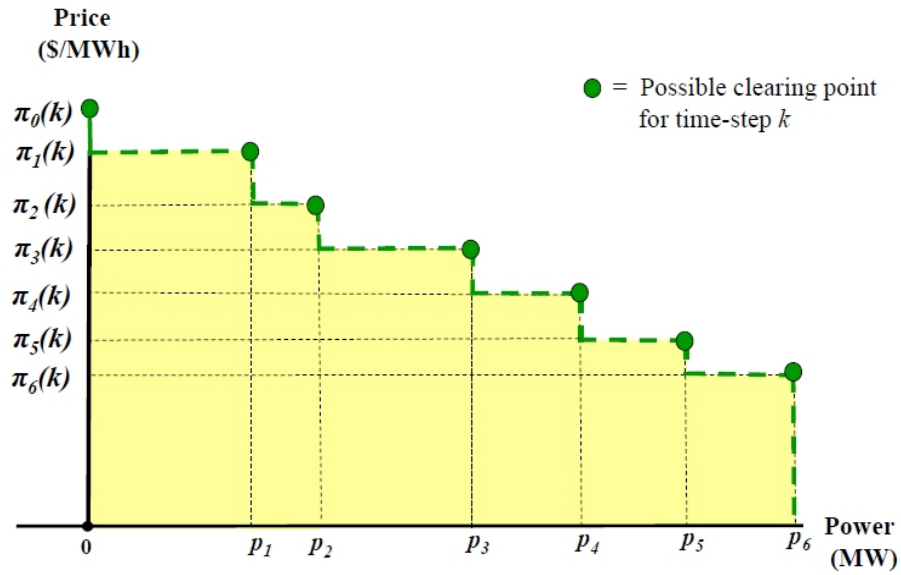

Figure 9: *A possible demand schedule for some time-step k that could be designated by the price-sensitive demand bid whose physical aspects are depicted in Fig. 8.*

the entire operating period T then takes the form

$$\widehat{\mathrm{B}}(\mathrm{T}) \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \tag{87}$$

7.4 Demand Bids Derived from General Benefit Functions

More generally, suppose each LSE $j \in \mathbb{LS}$ assigns an expected benefit (\$/h) to each possible power-usage level $p_j^{\mathsf{s}}(k) \in \mathbb{P}_j(k)$ for each time-step $k \in \mathbb{K}(\mathrm{T})$ by means of a non-decreasing concave benefit function

$$\mathrm{B}_{j,k} : \mathbb{P}_j(k) \to R \;, \tag{88}$$

where $\mathbb{P}_j(k) = [0, P_j^{\mathsf{max}}(k)]$. For example, (88) might take the quadratic form

$$\mathrm{B}_{j,k}(p) \;=\; d_j(k) \;+\; e_j(k) \cdot p \;-\; f_j(k) \cdot p^2 \tag{89}$$

with ordinate $d_j(k)$ (\$/h), positive coefficients $e_j(k)$ (\$/MWh) and $f_j(k)$ (\$/[MW]^{2}h), and a function domain given by

$$\mathbb{P}_j(k) \;=\; [0,\, e_j(k)/2f_j(k)]\,. \tag{90}$$

If the benefit function (88) is differentiable, the maximum willingness of LSE j to pay for an incremental increase in the aggregate power usage of its customers at the start of time-step k, given that the current aggregate power usage of these customers is at level p, can be expressed by the

marginal benefit function[12]

$$\pi_{j,k}(p) \;\equiv\; \frac{\partial B_{j,k}(p)}{\partial p} \;\geq\; 0\,. \tag{91}$$

Note that the price $\pi_{j,k}(p)$ (\$/MWh) depends on the power usage level p.

The expected benefit (\$) assigned by LSE j to each possible power-usage sequence $\mathbf{p}_j^{\mathsf{s}} = \{p_j^{\mathsf{s}}(k) \mid k \in \mathbb{K}(\mathrm{T})\}$ then takes the form

$$\mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \;=\; \sum_{k\in\mathbb{K}(\mathsf{T})} \mathrm{B}_{j,k}(p_j^{\mathsf{s}}(k))\Delta t\,. \tag{92}$$

Consequently, the expected total benefit (\$) to be included in the ISO's objective function (71) for the entire future operating period T takes the form:

$$\widehat{\mathrm{B}}(\mathrm{T}) \;=\; \sum_{j\in\mathbb{LS}} \mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \tag{93}$$

8 MILP Tractable Approximation of Benefit Functions

The technique described in Section 5 for obtaining piecewise-linear approximations for non-decreasing *convex* production cost functions $\mathrm{C}_{g,k}(p)$ (\$/h) can similarly be applied to obtain piecewise-linear approximations for non-decreasing *concave* benefit functions $\mathrm{B}_{j,k}(p)$ (\$/h) taking form (88). For completeness, this section presents the latter method in full analytical form.

Suppose the benefit function used by each LSE $j \in \mathbb{LS}$ at each time-step $k \in \mathbb{K}(\mathrm{T})$ to measure the expected benefit (\$/h) to its customers of different possible price-sensitive power usage levels $p_j^{\mathsf{s}}(k)$ is given by a non-decreasing concave function $\mathrm{B}_{j,k}{:}\mathbb{P}_j(k) \to R$, where $\mathbb{P}_j(k) = [0, P_j^{\mathsf{max}}(k)]$. A piecewise linear approximation for this benefit function can then be obtained in three simple steps.

First, select points $\{P_0, P_1, \ldots, P_{NS_j(k)}\}$ from the domain $[0, P_j^{\mathsf{max}}(k)]$, subject to the following restriction:

$$0 = P_0 < P_1 < P_2 < \ldots < P_{NS_j(k)} = P_j^{\mathsf{max}}(k)\;. \tag{94}$$

Second, plot the power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots, NS_j(k)\}$ in the power-benefit plane, where $B_i = \mathrm{B}_{j,k}(P_i)$. Third, connect these power-benefit points by line segments whose slopes, by construction, are non-increasing in i. For example, Fig. 10 illustrates a five-segment piecewise-linear approximation for a non-decreasing concave benefit function taking the quadratic form (89).

Given these power-benefit points (P_i, B_i), the expected benefit (\$) attained by the customers of each LSE j during each time-step k can be incorporated into the Generalized ECA Model SCUC/SCED formulation in an approximate piecewise-linear form, $\mathrm{ben}_j(k)$ (\$), that preserves the MILP form of this optimization. This incorporation is similar to the incorporation of total production costs explained in Section 5. It proceeds as follows.

- The ISO's optimal selection of a price-sensitive power usage level $p_j^{\mathsf{s}}(k)$ for the customers of LSE j at time-step k, with corresponding expected benefit approximation $\mathrm{ben}_j(k)$ (\$), is determined for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}(\mathrm{T})$ by means of the linear constraints (96) through

[12] In economics, a benefit function $\mathrm{U}(q)$ measuring the benefit of consuming a good q in terms of utliity (utils) is referred to as a *utility function*. Standard budget-constrained utility-maximization problems include as first-order necessary conditions the requirement that $\lambda\pi = \partial\mathrm{U}(q)/\partial q$, where λ (utils/\$) denotes the marginal utility of money and π denotes the price of q measured in dollars per unit of q. In short, at optimal solution points, prices converted into utils per unit of good are expressed as rates of change for benefit functions.

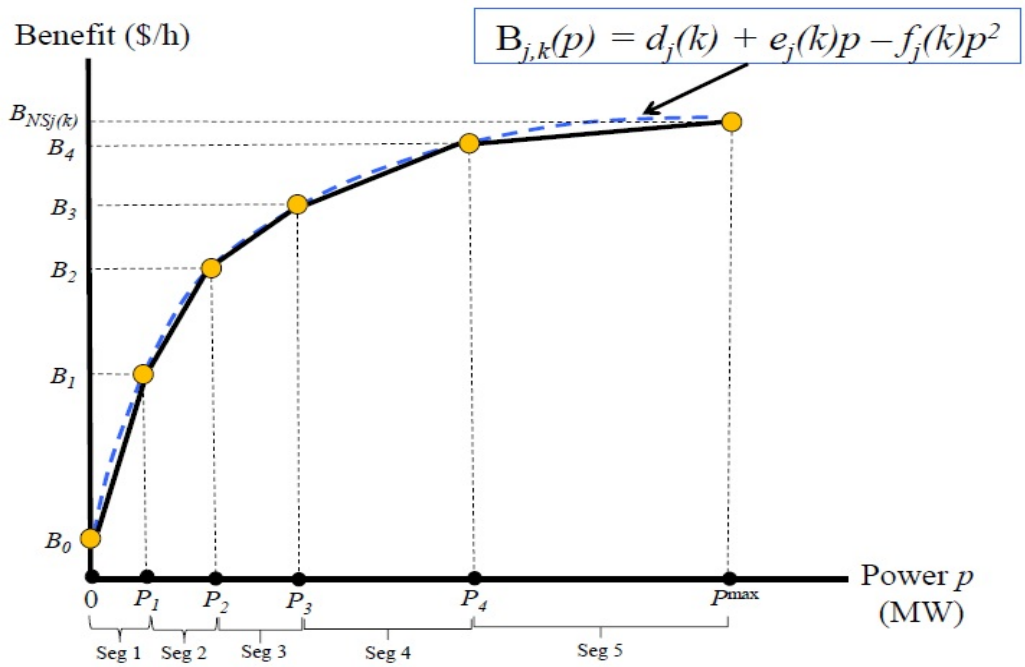

Figure 10: *Piecewise-linear approximation of a benefit function (89) used by an LSE j to measure the expected benefit of its customers at some time-step $k \in \mathbb{K}(\mathsf{T})$. The number of line segments specified for this approximation is $NS_j(k) = 5$.*

(100), below. These constraints must be incorporated into the system constraints for the ISO's optimization problem.

- The variables $\{\delta_{i,j}(k) \mid i = 1, \ldots, NS_j(k)\}$ that appear in the constraints (96) through (100) must be incorporated into the ISO's optimization problem as continuously-valued ISO decision variables, for each $j \in \mathbb{LS}$ and each $k \in \mathbb{K}(\mathrm{T})$.
- The price-sensitive power usage levels $\{p_j^{\mathsf{s}}(k) \mid j \in \mathbb{LS}, k \in \mathbb{K}(\mathrm{T})\}$ must be included among the derived variables determined by the ISO's decision variables and the system constraints.
- As explained with care in Section 7.2, the power usage level $p_j^{\mathsf{s}}(k)$ must be incorporated into the power balance constraint at LSE j's bus location b_j and into the forecasted net loads appearing in the system-wide and zonal reserve constraints, for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}(\mathrm{T})$.
- The ISO's objective function (10) must be extended to include expected total benefit ($) for operating period T, calculated as the summation of approximate expected benefits across all LSEs $j \in \mathbb{LS}$ and all time-steps $k \in \mathbb{K}(\mathrm{T})$ as follows:

$$\widehat{\mathrm{B}}(\mathrm{T}) \;=\; \sum_{j \in \mathbb{LS}} \sum_{k \in \mathbb{K}(\mathsf{T})} \mathrm{ben}_j(k) \tag{95}$$

As will be shown, below, given a user-set value for the number $NS_j(k)$ of line segments to be used for the approximation of the benefit function $\mathrm{B}_{j,k}(p)$ in (88) for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}(\mathrm{T})$, plus analytical forms for these benefit functions, the AMES V5.0 Pyomo Model can automatically construct power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j(k)\}$ satisfying restriction (94) with $B_i = \mathrm{B}_{j,k}(P_i)$ for each i. Suppose, for the moment, that these power-benefit points have already been

constructed.

The approximate expected benefit $\text{ben}_j(k)$ (\$/h) attained by the customers of LSE j during time-step k is then determined by the system of equations (96) - (100), below, as a function of the ISO's optimal selection of the continuously-valued decision variables $\{\delta_{i,j}(k) \mid i = 1, \ldots, NS_j(k)\}$:

Benefit Approximation Method for an LSE j at a Time-Step k:

$$\text{ben}_j(k) = B_0 \Delta t \; + \sum_{i=1}^{NS_j(k)} \big(MB_i \cdot \delta_{i,j}(k)\big) \cdot \Delta t \,; \tag{96}$$

$$p_j^{\mathsf{s}}(k) \;=\; \sum_{i=1}^{NS_j(k)} \delta_{i,j}(k) \,; \tag{97}$$

$$\delta_{i,j}(k) \;\leq\; P_i - P_{i-1} \,,\; \forall i = 1, \cdots, NS_j(k) \,; \tag{98}$$

$$\delta_{i,j}(k) \;\geq\; 0,\; \forall i = 1, \cdots, NS_j(k) \,; \tag{99}$$

$$MB_i \;=\; \frac{B_i - B_{i-1}}{P_i - P_{i-1}},\; \forall i = 1, \ldots, NS_j(k) \,. \tag{100}$$

The marginal benefit (\$/MWh) of LSE j's customers, evaluated at any power-usage level in the interval from P_{i-1} to P_i, is approximated by MB_i in (100). The expected benefit (\$) attained by LSE j's customers at the ISO-cleared price-sensitive power usage level $p_j^{\mathsf{s}}(k)$ in (97) is approximated by $\text{ben}_j(k)$ in (96). For example, suppose there exists a segment $n \in \{1, \ldots, NS_j(k)\}$ such that each $\delta_{i,j}(k)$ takes on its maximum possible value for $i = 1, \ldots, n$ and $\delta_{i,j}(k) = 0$ for $i = n+1, \ldots, NS_j(k)$. Then $p_j^{\mathsf{s}}(k) = P_n$ and $\text{ben}_j(k) = B_n \Delta t = \text{B}_{j,k}(P_n)\Delta t$. On the other hand, suppose $\delta_{i,j}(k) = 0$ for all $i = 1, \ldots, NS_j(k)$. Then $p_j^{\mathsf{s}}(k) = 0$ and $\text{ben}_j(k) = B_0 \Delta t \equiv \text{B}_{j,k}(0)\Delta t$.

Finally, the following simple method can be used to construct a collection of power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j(k)\}$ satisfying restriction (94) with $B_i = \text{B}_{j,k}(P_i)$ for each i. The method partitions LSE j's benefit function domain $[0, P_j^{\mathsf{max}}(k)]$ into power segments having equal lengths. The resulting power-benefit points are treated as exogenous inputs to the ISO's SCUC/SCED optimization.

Power-Benefit Point Setting Method for an LSE j at a Time-Step k:

The method requires the following inputs: (i) the analytical form of the benefit function (88); (ii) the maximum power-usage level $P_j^{\mathsf{max}}(k)$; and (iii) a positive integer value $NS_j(k)$ for the total number of line segments i to be used in the approximation. The AMES V5.0 Pyomo Model uses these inputs to compute[13] power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j(k)\}$ for use in the Benefit Approximation Method, as follows:

(a) The initial power-benefit points are set to $P_0 = 0$ and $B_0 = \text{B}_{j,k}(0)$;

(b) The power-width of each segment $i = 1, \ldots, NS_j(k)$ is set equal to

$$w_j(k) \;\equiv\; \frac{P_j^{\mathsf{max}}(k)}{NS_j(k)} \,; \tag{101}$$

(c) For each segment $i = 1, \ldots NS_j(k)$, the power point P_i is set equal to $P_0 + i w_j(k)$;

(d) For each segment $i = 1, \ldots, NS_j(k)$, the benefit point B_i is set equal to $\text{B}_{j,k}(P_i)$.

[13] This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

9 Derivation of Locational Marginal Prices

The Generalized ECA Model presented in previous sections models an ISO-managed SCUC/SCED optimization for a future operating period T partitioned into time-steps $k = 1, \ldots, NK(\mathrm{T})$. The outcomes of this optimization include unit commitments and scheduled dispatch levels for each time-step k. Consistent with actual practice, settlements for these scheduled dispatch levels can be determined in accordance with locational marginal pricing, i.e., the pricing of power in accordance with the location and timing of its injection into, or withdrawal from, a physical grid.

Specifically, *Locational Marginal Prices (LMPs)* can be derived as follows from the SCUC/SCED optimal solution. First, fix all unit commitment variables at their optimal binary (1/0) solution values. Second, re-run the optimization as a pure SCED optimization, conditional on these optimal unit commitment solution values. Third, calculate the LMP for each bus b at each time-step k as the dual variable for the power balance constraint (27) corresponding to this b and k.

The dual variable for a power balance constraint measures the change in the optimized value of the SCED objective function with respect to a change in the constraint constant for this power balance constraint. This constraint constant is typically taken to be the forecasted amount of fixed load appearing in the power balance constraint. A unique dual variable solution exists for a power balance constraint with constraint constant cc if the optimized SCED objective function is a differentiable function of cc at the optimal SCED solution point. A range of dual variable solutions exists if the optimized SCED objective function is right and left differentiable with respect to cc at the optimal SCED solution point but not differentiable with respect to cc.[14]

10 Concluding Remarks

This report provides detailed documentation for the ISO-managed SCUC/SCED optimization formulation used by AMES V5.0 to simulate U.S. RTO/ISO-managed day-ahead and real-time markets operating in tandem over a high-voltage transmission grid during successive days. Extensive additional AMES documentation can be found at the AMES homepage [6].

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