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THERMOMAGNETIC AND GALVANOMAGNETIC EFFECTS

R. D. REDIN

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Ames Laboratory
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F. H. Spedding, Director
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ABSTRACT

A review is given of the various thermomagnetic and galvanomagnetic effects. The coefficients and relations between coefficients are listed for both metals and semiconductors. A quantitative analysis is given of the error in Hall effect measurements owing to non-isothermal conditions.

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II. THERMOMAGNETIC AND GALVANOMAGNETIC EFFECTS

I. INTRODUCTION

Theories of the Hall effect which relate the Hall coefficient to the number of carriers are generally worked out for the case of a sample with no temperature gradients. The Hall coefficient so obtained is called the isothermal Hall coefficient. Experimentally it is usually difficult to completely eliminate all temperature gradients and hence it is of interest to determine the effect of these gradients on the measured Hall voltages.

The Hall effect is one of a large number of effects known as galvanomagnetic and thermomagnetic effects. These effects concern the changes in electric potentials and temperature gradients brought about by the application of a magnetic field to a material carrying electric or heat currents. Comprehensive reviews of the effects have been given by Campbell (3) and Meissner (10).

The effects may be divided into two classes, depending upon whether the magnetic field is applied parallel to the primary current (heat or electric) or perpendicular to the primary current. The former condition leads to the longitudinal effects and the latter condition leads to the transverse effects. We shall consider only the transverse effects as this is the class that contains the Hall effect.

II. THE THERMOMAGNETIC AND GALVANOMAGNETIC COEFFICIENTS

A. Phenomenological Description

1. Experimental arrangement

A typical experimental arrangement for measuring the Hall

effect is shown in Fig. 1a. The sample A consists of a thin flat plate. The Hall probes B and current leads C are generally made of a different material than that of the sample. The circle with the arrow indicates the direction of the current producing the magnetic field; hence, the field is directed out of the paper. L, b, t are the length, width and thickness of the plate, respectively.

2. Equations of current flow

If we assume that the sample A is homogeneous, simple symmetry considerations lead to the following equations for the electric current density \vec{j} and the heat current density \vec{w} :

$$\vec{j} = A_1 \vec{\nabla}V + A_2 \vec{\nabla}V \times \vec{H} + A_3 \vec{\nabla}T + A_4 \vec{\nabla}T \times \vec{H} \quad (1)$$

$$\vec{w} = B_1 \vec{\nabla}V + B_2 \vec{\nabla}V \times \vec{H} + B_3 \vec{\nabla}T + B_4 \vec{\nabla}T \times \vec{H} \quad (2)$$

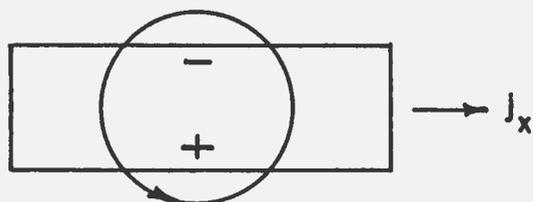
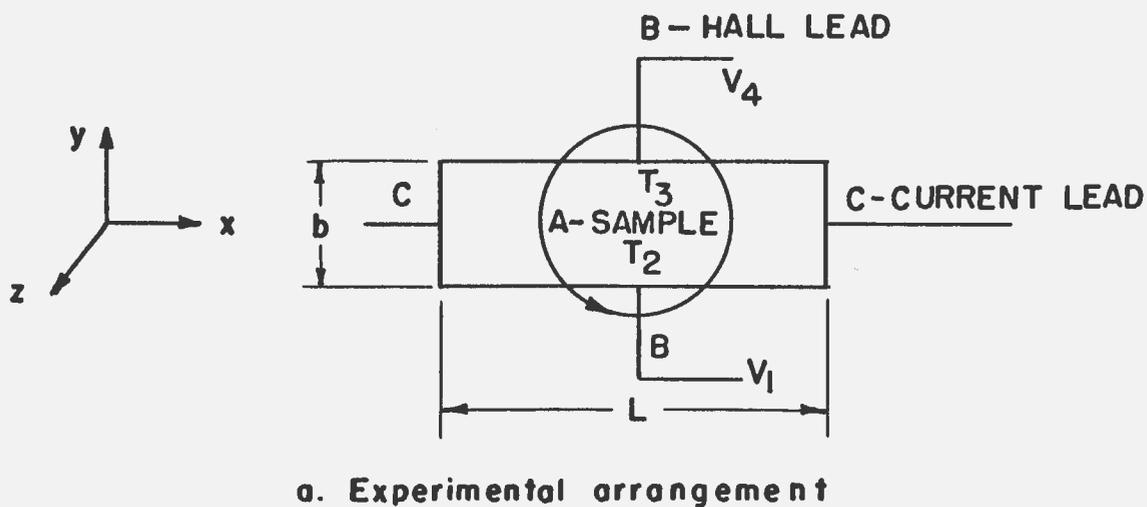
where $\vec{\nabla}V$ is the electric potential gradient, $\vec{\nabla}T$ is the temperature gradient, \vec{H} is the magnetic field strength and the A's and B's are coefficients which may depend on the temperature and magnetic field.

It is generally more convenient to have $\vec{\nabla}V$ and \vec{w} expressed in terms of \vec{j} and $\vec{\nabla}T$ since \vec{j} and $\vec{\nabla}T$ are more easily determined experimentally. Equations (1) and (2) are easily transformed to give

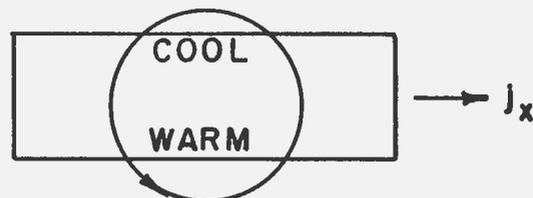
$$\vec{\nabla}V = A'_1 \vec{j} + A'_2 \vec{j} \times \vec{H} + A'_3 \vec{\nabla}T + A'_4 \vec{\nabla}T \times \vec{H} \quad (3)$$

$$\vec{w} = B'_1 \vec{j} + B'_2 \vec{j} \times \vec{H} + B'_3 \vec{\nabla}T + B'_4 \vec{\nabla}T \times \vec{H}. \quad (4)$$

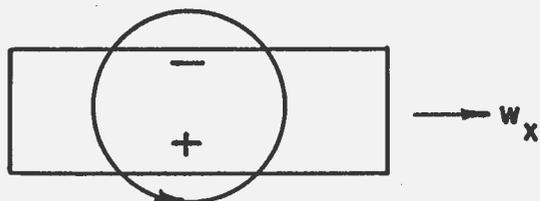
Equations for the primed coefficients in terms of the unprimed coefficients are easily found. However, since the equations are quite complicated and will not be used in the following discussion, they will not be given here.



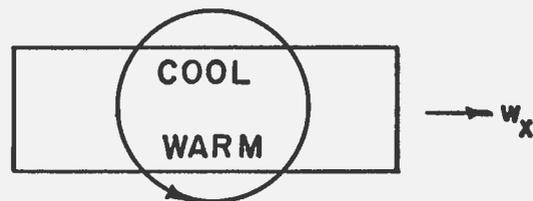
b. Hall effect



c. Ettingshausen effect



d. Nernst effect



e. Righi - Leduc effect

Fig. 1. Experimental arrangements representing positive galvanomagnetic and thermomagnetic coefficients.

3. Definitions of the coefficients

Coefficients for the effects of interest can now be defined.

The subscript i indicates an isothermal effect ($\partial T/\partial y = 0$).

The subscript a indicates an adiabatic effect ($w_y = 0$).

Hall effect

$$\left. \begin{array}{l} R_i \\ R_a \end{array} \right\} = - \frac{\partial V/\partial y}{j_x H_z}, \quad j_y = \partial T/\partial x = \left\{ \begin{array}{l} \partial T/\partial y \\ w_y \end{array} \right\} = 0$$

Ettingshausen effect

$$P = - \frac{\partial T/\partial y}{j_x H_z}, \quad j_y = \partial T/\partial x = w_y = 0$$

Nernst effect

$$\left. \begin{array}{l} N_i \\ N_a \end{array} \right\} = \frac{\partial V/\partial y}{(\partial T/\partial x) H_z}, \quad j_y = j_x = \left\{ \begin{array}{l} \partial T/\partial y \\ w_y \end{array} \right\} = 0$$

Righi-Leduc effect

$$S = \frac{\partial T/\partial y}{(\partial T/\partial x) H_z}, \quad j_y = j_x = w_y = 0$$

Electrical resistivity

$$\rho_i = - \frac{\partial V/\partial x}{j_x}, \quad j_y = \partial T/\partial x = \partial T/\partial y = 0$$

Thermal conductivity

$$K_i = - \frac{w_x}{\partial T/\partial x}, \quad j_y = j_x = \partial T/\partial y = 0$$

Figures 1b, 1c, 1d, 1e are given to further clarify the sign conventions used. Note that in all cases for a positive effect, $(\vec{j}$ or $\vec{w})$, \vec{H} , $(\vec{v}\vec{v}$ or $\vec{v}\vec{T})$ form a right-handed coordinate system. The sign convention used by a particular author should be carefully noted since not all authors use the convention given above.

Equations (3) and (4) can be expressed in terms of the coefficients

defined above.

$$\vec{\nabla}V = -\rho_1 \vec{j} + R_1 \vec{j} \times \vec{H} + A_3' \vec{\nabla}T - N_1 \vec{\nabla}T \times \vec{H} \quad (5)$$

$$\vec{w} = B_1' \vec{j} + K_1 P \vec{j} \times \vec{H} - K_1 \vec{\nabla}T - SK_1 \vec{\nabla}T \times \vec{H} \quad (6)$$

Also the relations

$$R_a = R_1 + A_3' P \quad (7)$$

$$N_a = N_1 + A_3' S \quad (8)$$

are seen to hold.

4. Distinction between local and measured effects

The coefficients defined above describe local effects. Experimentally, one measures potential differences, temperature differences and total current rather than electric fields, temperature gradients and current densities. A set of measured coefficients for the geometry of Fig. 1a may be defined by replacing $\partial V / \partial y$ by $(V_4 - V_1)/b$; $\partial T / \partial y$ by $(T_3 - T_2)/b$; j_x by I_x/bt , etc. in the definitions given. The measured and the local coefficients will be the same except in the cases of the adiabatic Hall and adiabatic Nernst effects. In these cases, the measured voltage $(V_4 - V_1)$ is not given simply by $(\partial T / \partial y)b$ but contains other terms due to the temperature difference in the y-direction. This point will be discussed further in the next section.

B. The Adiabatic Hall Effect

1. General equation

Consider now the general adiabatic case ($j_y = w_y = 0$). Equation (5) gives for the transverse electric field in the sample

$$\partial V / \partial y = - R_1 H j_x + A_3 \partial T / \partial y + N_1 H \partial T / \partial x. \quad (9)$$

The transverse temperature gradient is obtained from Equation (6).

$$0 = - K_1 P H j_x - K_1 \partial T / \partial y + S K_1 H \partial T / \partial x$$

or

$$\partial T / \partial y = - P H j_x + S H \partial T / \partial x. \quad (10)$$

The measured potential difference between the ends of the Hall probe leads is obtained by integrating the electric field in the sample and the field in the probes and leads. The field in the probes and leads plus the second term on the right hand side of Equation (7) combine to give the ordinary Seebeck emf of the thermocouple B:A:B (see Fig. 1a).

$$(V_4 - V_1)_s = - Q_{AB}(T_3 - T_2) \quad (11)$$

Q_{AB} is the thermoelectric power of the thermocouple B:A:B. When Q_{AB} is positive, conventional current flow is from A to B at the cold junction.

The temperature difference ($T_3 - T_2$) can be obtained by integrating Equation (10) across the width of the sample.

$$\begin{aligned} T_3 - T_2 &= \int (\partial T / \partial y) dy = - \int (P H j_x - S H \partial T / \partial x) dy \\ &= - P H I / t + S H b \partial T / \partial x \end{aligned} \quad (12)$$

where I is the total current. We have assumed that the integrand is independent of y .

The remainder of the measured voltage is obtained by integrating

the first and third terms of Equation (9) across the sample.

$$\begin{aligned} (V_4 - V_1)_H &= \int (-R_1 H j_x + N_1 H b \partial T / \partial x) dy \\ &= -R_1 H I / t + N_1 H b \partial T / \partial x. \end{aligned} \quad (13)$$

The measured potential difference is obtained by combining Equations (11), (12) and (13).

$$\begin{aligned} V_4 - V_1 &= - \left\{ (R_1 - Q_{AB} P) I / t - (N_1 - Q_{AB} S) b \partial T / \partial x \right\} H \\ &= - \left\{ \left[1 - Q_{AB} / (R_1 / P) \right] R_1 I H / t - \left[1 - Q_{AB} / (N_1 / S) \right] N_1 H b \partial T / \partial x \right\}. \end{aligned} \quad (14)$$

In general, then, the measured "Hall Voltage" contains terms which are due to the isothermal Hall effect, the Ettingshausen effect, the Nernst effect, and the Righi-Leduc effect.

From Equation (14) and the definition of the measured adiabatic Hall coefficient (R_a') we see that $R_a' = R_1 - Q_{AB} P$. When this expression is combined with Equation (7) we get

$$R_a' = R_a - (Q_{AB} + A_3') P. \quad (15)$$

Similarly,
$$N_a' = N_a - (Q_{AB} + A_3') S. \quad (16)$$

2. Experimental methods for eliminating the thermal effects

a. dc method. The usual dc method of measuring the Hall effect will often eliminate all of the disturbing effects except the Ettingshausen effect. This subject is discussed by Lindberg (8). It is assumed that the measured voltage V is given by

$$V = AIH + BH + CI + D. \quad (17)$$

The first two terms on the right hand side of this equation correspond to the terms of Equation (14). The term CI arises when the Hall probes are not placed directly opposite each other. The last term includes any voltages independent of I and H . If, now, V is measured

for both directions of I and both directions of H it is easily seen that

$$A = \frac{V(+I,+H) - V(-I,+H) + V(-I,-H) - V(+I,-H)}{4IH} . \quad (18)$$

A includes both the isothermal Hall effect and the Etingshausen effect and these cannot be further separated by this method.

This procedure assumes that $\partial T / \partial x$ (included in B) is independent of the sample current I. This will be true if $\partial T / \partial x$ is mainly due to an ambient temperature gradient. However, when a current passes through the sample, the Peltier effect will cause one end to heat and the other end to cool. For some materials, particularly semiconductors, this effect can produce an appreciable temperature gradient. This gradient will reverse with the current I and hence the term BH in Equation (17), which includes the Nernst and Righi-Leduc effects, will not be eliminated. Pugh and Allison (11) point out that in certain ferromagnetic materials, the Nernst potential resulting from the temperature gradient produced by the Peltier effect can be as much as six per cent of the Hall potential.

The Etingshausen effect produces a transverse voltage through the combination of the transverse temperature gradient and the Seebeck voltage produced at the contacts between the Hall probes and the sample. By making the Hall probes out of the same material as that of the sample, this voltage may be eliminated.

b. ac method. The transverse thermal gradient produced by the Etingshausen effect and the longitudinal thermal gradient produced by the Peltier effect will reverse when the sample current

is reversed. If the current is reversed rapidly enough, there will not be time for these gradients to build up in either direction and therefore all of the disturbing thermal effects will be eliminated. The rapid current reversal is usually achieved by using a sinusoidal alternating current. A frequency between 10 and 100 cycles per second is usually sufficient. By making measurements for both directions of the magnetic field, the voltage due to offset Hall probes (CI in Equation (17)) is also eliminated.

III. RESULTS OF THEORY AND EXPERIMENT

A. Free Electron Theory

1. Qualitative description

A qualitative description of the galvanomagnetic and thermomagnetic effects can be given based on the free electron model. The Lorentz force on a particle of charge q and velocity \vec{v} moving in a magnetic field \vec{B} is given by $\vec{F} = q\vec{v} \times \vec{B}$. If we consider the geometry of Fig. 1a we see that this force tends to produce a current at right angles to the primary current. This transverse current quickly charges the sides of the sample until the process is stopped by the resulting transverse electric field. The transverse electric field produces the isothermal Hall voltage.

All of the electrons comprising the primary current do not have the same velocity and, owing to the fact that the Lorentz force is velocity dependent, there will be some separation of the fast electrons from the slow ones. The Hall field insures that there will be no net transverse electric current but there will still be a transverse heat current consisting of faster electrons moving in one direction

and slower electrons moving in the other direction. This transverse heat current produces the Ettingshausen transverse temperature gradient.

When a longitudinal temperature gradient is applied to a sample there is a flow of heat and, initially, an electric current. A longitudinal electric field develops which stops the electric current. When a magnetic field is applied a transverse electric field will appear since the Lorentz force is velocity dependent and hence the distribution of transverse velocities will be different than that of longitudinal velocities. The appearance of this transverse electric field is the Nernst effect.

The magnetic field also acts on the longitudinal heat current to produce a transverse heat current which results in the Righi-Leduc effect.

2. Metals

Sommerfeld and Frank (14) calculated the galvanomagnetic and thermomagnetic coefficients by using Fermi-Dirac statistics. They also gave the results obtained by using Boltzmann statistics. They assumed that the mean free path was independent of energy and considered only electronic conduction. A weak magnetic field approximation was used. The expressions they obtained are given in Table I.

Table I. Expressions for coefficients for energy independent mean free path.

| | Fermi-Dirac statistics | Boltzmann statistics |
|---------|------------------------------|---------------------------|
| R_i | $-1/(ne)$ | $-3\pi/(8ne)$ |
| R_a | $-1/(ne)$ | $-27\pi/(64ne)$ |
| P | $-(T/T_f)/(2nk)$ | $-3\pi/(32nk)$ |
| N_i | $-(\pi^2/6)(k/e)(T/T_f)\mu$ | $-(3\pi^2/16)(k/e)\mu$ |
| N_a | $-(\pi^2/3)(k/e)(T/T_f)\mu$ | $-(45\pi/128)(k/e)\mu$ |
| S | $-\mu$ | $-(21\pi/64)\mu$ |
| C_i | $1/(ne\mu)$ | $1/(ne\mu)$ |
| K_i | $(\pi^2/3)(k^2 T_n \mu / e)$ | $2k^2 T_n \mu / e$ |
| A_3 | $eL(2mkT_f)^{-1/2}$ | $(4/3)eL(27mkT_f)^{-1/2}$ |
| R_i/P | $2(k/e)(T_f/T)$ | $4(k/e)$ |
| N_i/S | $(\pi^2/6)(k/e)(T/T_f)$ | $(4/7)(k/e)$ |
| A'_3 | $(\pi^2/6)(k/e)(T/T_f)$ | $(1/2)(k/e)$ |

In Table I, n is the electron concentration, e is the electronic charge, m is the electron mass, k is Boltzmann's constant, L is the mean free path, T is the absolute temperature and kT_f is the Fermi energy.

Sommerfeld and Frank also noted the following relations which hold for Fermi-Dirac statistics:

- (a) $R_a = R_i = R$
- (b) $N_a - N_i = A'_3 S = N_i$
- (c) $T N_i = K_i P$
- (d) $S = R/\rho_i$
- (e) $(N_i R)/(PS) = K_i \rho_i / T$.

Tables of experimental values of the coefficients for many metals are given by Campbell (3), Meissner (10) and the International Critical Tables (6). Most of the data are old and not very accurate. Wilson (16) gives tables which compare relations c, d and e with experiment. The order of magnitude agreement is indicated.

To estimate the magnitudes of the terms in Equation (14) for a metal, we shall consider the case of silver. Experimental values for silver are $R = -8.32(10)^{-13}(\text{volt-cm})/(\text{amp-gauss})$, $P = -1.65(10)^{-10}(\text{°C-cm})/(\text{amp-gauss})$, $N_a = -4.30(10)^{-12}\text{volts}/(\text{gauss-°C})$ and $S = -4.04(10)^{-7}\text{ gauss}^{-1}$.

According to relation (a) above, the Etingshausen effect should cause negligible error in a Hall measurement. The error can be estimated by computing the second term in the first square bracket of Equation (14). We find $R/P = 5.04(10)^{-3}(\text{volts/°C})$. For Q_{AB} we take the thermoelectric power of silver (relative to lead) which is $2.5(10)^{-6}\text{ volts/°C}$. Therefore, $Q_{AB}/(R/P) = 4.97(10)^{-4}$. Therefore, the Etingshausen correction is 0.05 per cent and is negligible.

According to relation (b), the adiabatic Nernst effect should be twice the isothermal Nernst effect. To check this fact the second term in the second square bracket of Equation (14) can be calculated. We note that it is always the adiabatic Hall effect which is measured experimentally. $N_1/S = N_a/2S = 5.3(10)^{-6}\text{ volts/°C}$. Therefore, $Q_{AB}/(N_1/S) = 0.47$. Therefore, the Righi-Leduc effect would decrease the isothermal Nernst effect by 47 per cent. Part of the discrepancy between this result and that predicted by relation (b) above is due to the fact that relation (b) concerns local effects while Equation

(14) is concerned with measured effects.

Finally, it is desirable to compare the terms $R_1 I H / t$ and $N_1 H b \partial T / \partial x$ of Equation (14). Typical values for a measurement on silver might be $I = 1$ amp, $t = 0.01$ cm, $b = 1$ cm, $\partial T / \partial x = 2^\circ\text{C}/4\text{cm}$ so that $R_1 I H / t = -1.66(10)^{-11} \text{H volts}$ and $N_1 H b \partial T / \partial x = -0.55(10)^{-11} \text{H volts}$. Therefore, the voltage due to the Nernst effect is 1.3 per cent of the Hall voltage.

It can be concluded that the Etingshausen effect has a negligible effect on a Hall measurement on metals while the Nernst effect may lead to an error of a few per cent. A measurement of the Nernst effect would be seriously affected by the Righi-Leduc effect.

3. Semiconductors

An analysis of the magnitudes of the galvanomagnetic and thermomagnetic coefficients for semiconductors is more difficult because of the strong dependence of the various parameters on temperature and purity. Also, in general, both holes and electrons will contribute to the coefficients. As will be shown later, it is essential to take the lattice thermal conductivity into account.

A glance at Table I would indicate that for semiconductors, where Boltzmann statistics apply, all of the coefficients except K_1 would be much larger than for metals because of the fewer number of carriers and the higher mobilities in semiconductors. Putley (13) gives for PbSe at 300°K , $\rho = 6.67(10)^{-3}$ ohm-cm, $R_1 = -8(10)^{-8}$ (volt-cm)/(amp-gauss), $P = 4.4(10)^{-6}$ ($^\circ\text{C-cm}$)/(amp-gauss), $N_1 = 2.3(10)^{-10}$ volt/(gauss- $^\circ\text{C}$) and $S = 2.5(10)^{-7}$ gauss $^{-1}$.

All the coefficients except S are higher than those given for silver. There are two considerations which may modify the expressions given in Table I. One is the effect of the energy dependence of the mean free path; the other is the effect of the lattice thermal conductivity.

Wright (17) discusses the effect of different choices of the energy dependence of the mean free path. He shows that if the mean free path is proportional to the velocity of the electron the Etingshausen and Nernst effects vanish. If the mean free path is proportional to the energy, then the Etingshausen and Nernst coefficients are positive. As can be seen from Table I these coefficients are negative for an energy independent mean free path.

Stilbans (15) and Putley (12) have pointed out that in carrying out the calculation for semiconductors it is important to take the lattice thermal conductivity into account. The lattice thermal conductivity reduces the Etingshausen and Righi-Leduc effects by the factor $K_e/(K_e + K_L)$, where K_e is the thermal conductivity due to electrons and K_L is the thermal conductivity due to the lattice. For Boltzmann statistics $K_e = 2(k/e)^2 T/\rho$. For high purity semiconductors this may be very small compared to K_L . Putley(12) gives the following values for PbSe:

$$T = 300^\circ\text{K}$$

$$\rho = 6.67(10)^{-3} \text{ ohm-cm}$$

$$K_e + K_L = 4.2(10)^{-2} \text{ watts cm}^{-1} \text{ deg}^{-1}.$$

Therefore, $K_e/(K_e + K_L) = 1.6(10)^{-2}$.

Grieco and Montgomery (5) give the following values for Ge:

$$T = 298^\circ\text{K}$$

$$\rho = 10 \text{ ohm-cm}$$

$$K_e + K_L = 0.586 \text{ watt cm}^{-1} \text{ deg}^{-1}.$$

Therefore, $K_e / (K_e + K_L) = 7.6(10)^{-7}$.

This factor may often make the correction of the Hall effect for the Ettingshausen effect negligible.

The isothermal Hall coefficient and the Righi-Leduc coefficient are proportional to the charge of the carrier and will therefore be negative for conduction by electrons and positive for conduction by holes. The Nernst and Ettingshausen coefficients are independent of the sign of the carriers. When both types of carriers are present the expressions for the coefficients become quite complicated. Such expressions are given by Putley (13). Under these conditions the Ettingshausen and Nernst coefficients may change sign when the the numbers of electrons and holes become nearly equal.

Perhaps the most complete expressions for the coefficients are given by Madelung (9). He gives tables and formulas from which the various coefficients can be evaluated for mixed conduction or single carrier, degenerate or non-degenerate conditions, thermal scattering, ionized impurity scattering and mixed scattering. These formulas can be exceedingly complex.

Johnson and Shipley (7) calculated the quantity $(R_a - R_i)/R_i$ under various conditions and applied the results to germanium. The highest value obtained was 1.3 per cent at 900°K. In applying their results it must be remembered that they have treated only local effects. As pointed out above, the measured adiabatic Hall coefficient is different from the local adiabatic Hall coefficient. It can be shown (Redin, unpublished work, see also Fieschi (4)) that for a

semiconductor with a thermoelectric power much greater than that of the Hall probes, Equation (15) becomes

$$R_a' = R_a - Pd(\mathcal{E}/e)/dT \quad (19)$$

where \mathcal{E} is the Fermi energy, so that

$$(R_a' - R_i)/R_i = (R_a - R_i)/R_i - (P/R_i)d(\mathcal{E}/e)/dT. \quad (20)$$

Johnson and Shipley's (7) treatment can be changed to apply to measured quantities by replacing the term $(T/k)d(\mathcal{E}/T)/dT$ in their Equation (14) by $-\mathcal{E}/(kT)$. Their succeeding equations must be changed accordingly. This change has the greatest effect in the saturation region where $d\mathcal{E}/dT$ is greatest. In the intrinsic region this change reduces the values given by Johnson and Shipley.

We conclude that for semiconductors it is difficult to give a general rule for the error in a Hall effect measurement due to non-isothermal conditions. Each individual case must be examined separately. The possibility of such an error should be considered especially in the case of a semiconductor with a low thermal conductivity.

B. Results from Thermodynamics

The thermodynamics of irreversible processes has been applied to the thermomagnetic and galvanomagnetic effects by Fiesche (4) and Callen (2). It is shown that although it is possible to define $8!/(2!3!3!) = 560$ effects similar to the 8 defined above, only 6 of these will be independent. A particularly important relation between the coefficients that is obtained by these methods is the Bridgman relation

$$K_1 P = TN_1. \quad (21)$$

This relation, which was first obtained by Bridgman (1) by a different method, may be considered to have the same general validity as the Kelvin relations.

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