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# Parameter Sensitivity Analysis of a Tractor and Single Axle Grain Cart Dynamic System Model

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**Abstract.** *Tractor and towed implement system models have become increasingly important for model-based guidance controller design, virtual prototyping, and operator-and-hardware-in-loop simulation. Various tractor and towed implement models have been proposed in the literature which contain uncertain or time-varying parameters. Sensitivity analysis was used to identify the effect of system parameter variation on system responses and to identify the most critical system parameters. Sensitivity analysis was performed with respect to three tire cornering stiffness parameters, three tire relaxation length parameters, and two implement inertial parameters. Overall, the system was most sensitive to the tire cornering stiffness parameters and least sensitive to the implement inertial parameters. In general, the variation in the input parameters and the system state variables were related in a non-linear fashion. With the nominal parameter values for a MFWD tractor, a single axle grain cart, and corn stubble surface conditions, a 10% variation in cornering stiffness parameters caused a 5% average variation in the system responses whereas an 80% change in cornering stiffness parameters caused an 80% average variation at 4.5 m/s forward velocity. If a 10% average variation in system responses is acceptable, the cornering stiffness parameters and implement inertial parameters must be estimated within 20% and 30% of actual/nominal values respectively. The relaxation length parameters have to be within 75% of the nominal values.*

**Keywords.** tractor and implement model, sensitivity analysis, sensitivity measures, parameter uncertainty

## 1.0 Introduction

Off-road vehicle system models are becoming increasingly important as mechatronic engineers increasingly rely on model-based controller design, virtual prototyping, and real-time hardware-and-operator-in-loop simulation (Karkee and Steward 2008). As a tremendous amount of computational power has become available for numerical simulation, vehicle models have become increasingly complex often possessing dozens of model parameters. Accurate estimation of those parameters is often difficult because of the high variability in field conditions. Because uncertain parameter estimates will have an effect on the model responses, the off-road vehicle simulations may often be unrealistic, which limits practioner's confidence in the models and in model-based studies (Kioustsioukis, et al. 2004). It is important to understand and quantify the effect of these parameter uncertainties or variations on the system response (Fales, 2004). Sensitivity analysis is one approach to identify and quantify these relationships (Xu and Gertner, 2007), which can improve our confidence in off-road vehicle models.

Sensitivity analysis evaluates the changes in dynamic model states and output with respect to (w.r.t.) changes in model parameters (Crosetto and Tarantola, 2001; Deif, 1986). Thus, sensitivity analysis can be used to perform uncertainty analysis, estimate model parameters, analyze experimental data, guide future data collection efforts, and suggest the accuracy to which the parameters must be estimated (Rodriguez-Fernandez and Banga, 2009).

Traditionally, sensitivity analysis has been used to optimize the vehicle system design (Jang and Han, 1995; Park et al., 2003). Jang and Han (1997) used a direct differentiation method for sensitivity analysis of vehicle lateral dynamics. They performed a state sensitivity analysis w.r.t. tire cornering stiffness, location of vehicle center of gravity (CG), vehicle mass, and vehicle moment of inertia (MI) using a bicycle model of a front wheel steering vehicle. The study was performed at typical on-road vehicle velocities ranging from 9 m/s (20 mph) to 53 m/s (120 mph). Park et al. (2003) performed a dynamic sensitivity analysis for a pantograph of a rail vehicle. Dominant design variables were identified using derivative-based state sensitivity measures and were modified for optimal design. Ruta and Wojciki (2003) also applied sensitivity analysis to a dynamic railroad model. They focused on developing an analytical solution for derivative-based sensitivity analysis of a system represented by a set of differential equations. Both first and second order derivatives were used to isolate a set of parameters to which the model outputs were the most sensitive. Eberhard (2007) performed a vehicle model sensitivity analysis w.r.t. various design variables. The vehicle dynamics was highly sensitive to MI and the CG location. The study was conducted at 10 m/s (~22mph) forward velocity. In off-road vehicle systems, the application of sensitivity analysis to understand the effect of uncertain parameters on a tractor and towed implement lateral dynamics is important to support the emerging farm automation techniques such as implement guidance and coordinated guidance.

Various tractor and towed implement steering models have been proposed in the literature for both on-road (Chen and Tomizuki, 1995; Dag and Kang, 2003; Kim et al., 2007) and off-road (Feng et al., 2005; Karkee and Steward, 2008) operations. As suggested by these models, the lateral dynamics of an off-road tractor and towed implement system depend on the lateral forces generated by soil-tire interactions. The responses may be different with different soil types and/or different soil moisture content. Other soil parameters including internal friction angle, cohesion and porosity may also affect the vehicle responses. Similarly, the responses may vary with the variation in tire construction, size, inflation pressure and normal load. It may be hard to estimate these variables accurately, and it is also hard to find a widely accepted model to relate these variables to the tire lateral force. The cornering stiffness coefficient, which is the initial sensitivity of tire force to side slip angle, is the lumped parameter used to represent the combined effect of these variables (Metz, 1993). Consequently, this parameter tends to be highly uncertain. Vehicle tire relaxation length, which is defined as the distance a tire rolls before the steady state side slip angle is reached, is another parameter that could be highly uncertain (Bevly et al., 2002). In the case of implements such as a grain cart or a sprayer, implement mass and MI may also be uncertain or may vary substantially over time.

In this work, dynamic model sensitivity analysis was performed to quantify the degree of dependence of tractor and single axle grain cart system responses on the uncertain model parameters. This analysis will help us understand the relative significance of these parameters so that more resources can be allocated to accurately estimate those parameters to which the system is more sensitive. Specific objective will be:

- to evaluate the effect of parameter uncertainties on the system responses and
- to identify the parameters to which the model is most sensitive

## 2.0 Methods:

In this research, sensitivity of a tractor and single axle grain cart steering system w.r.t. variation in various model parameters was analyzed using derivative-based local sensitivity analysis method. A dynamic bicycle model of the system (Karkee and Steward, 2008) was adapted to perform the analysis. Grain cart mass and MI, tire cornering stiffness, and tire relaxation length parameters were assumed to have some level of uncertainty and/or variation over the field operation. A range of these uncertain parameter values were used to define a parameter space for the study. A numerical simulation-based approach was used to calculate local sensitivities w.r.t. those parameters.

### *2.1 Vehicle Model*

A tractor and single axle towed implement model was required to study the sensitivity of the system states and/or outputs w.r.t. various system parameters. Karkee and Steward (2008) studied various models of a tractor and single axle grain cart system. Among the three models used, a dynamic bicycle model with tire relaxation length dynamics represented the system most accurately. In this work, their dynamic model was adjusted as follows to study the sensitivity of the system w.r.t. eight system parameters, namely implement mass, implement MI, tractor front tire cornering stiffness, tractor rear tire cornering stiffness, implement tire cornering stiffness, tractor front tire relaxation length, tractor rear tire relaxation length, and implement tire relaxation length. Common vehicle dynamics symbols were used to describe the dynamics of the system (for notation, see the list of the variables). This model is described by Eqs. 1 to 8:

$$\left(m^t + m^i\right)\dot{v}_c^t - m^i c\dot{\gamma}^t - m^i d\dot{\gamma}^i = -(m^i + m^t)u_c^t\gamma^t - C_{\alpha,f}^t\alpha_f^t - C_{\alpha,r}^t\alpha_r^t - C_{\alpha,r}^i\alpha_r^i \quad (1)$$

$$\left(I_z^t + m^i c^2\right)\dot{\gamma}^t - m^i c\dot{v}_c^t + m^i cd\dot{\gamma}^i = m^i cu_c^t\gamma^t - aC_{\alpha,f}^t\alpha_f^t + bC_{\alpha,r}^t\alpha_r^t + cC_{\alpha,r}^i\alpha_r^i \quad (2)$$

$$\left(I_z^i + m^i d^2\right)\dot{\gamma}^i - m^i d\dot{v}_c^t + m^i cd\dot{\gamma}^t = m^i du_c^t\gamma^t + (d+e)C_{\alpha,r}^i\alpha_r^i \quad (3)$$

$$\dot{\alpha}_f^t = \frac{v_c^t}{\sigma_f^t} + \frac{a\gamma^t}{\sigma_f^t} - \frac{u_c^t}{\sigma_f^t}\delta - \frac{u_c^t}{\sigma_f^t}\alpha_f^t \quad (4)$$

$$\dot{\alpha}_r^t = \frac{v_c^t}{\sigma_r^t} - \frac{b\gamma^t}{\sigma_r^t} - \frac{u_c^t}{\sigma_r^t}\alpha_r^t \quad (5)$$

$$\dot{\alpha}_r^i = \frac{v_c^t}{\sigma^i} - \frac{c\gamma^t}{\sigma^i} - \frac{(d+e)\gamma^i}{\sigma^i} + \frac{u_c^t}{\sigma^i}\varphi^t - \frac{u_c^t}{\sigma^i}\varphi^i - \frac{u_c^t}{\sigma^i}\alpha_r^i \tag{6}$$

$$\dot{\psi}^t = \gamma^t \tag{7}$$

$$\dot{\psi}^i = \gamma^i \tag{8}$$

Eqs. 1 – 8 can be represented in matrix form as,

$$M\dot{X} = NX + PU \tag{9}$$

where, $\dot{X} = [\dot{v}_c^t \quad \dot{\gamma}^t \quad \dot{\gamma}^i \quad \dot{\alpha}_f^t \quad \dot{\alpha}_r^t \quad \dot{\alpha}_r^i \quad \dot{\varphi}^t \quad \dot{\varphi}^i]^T$,

$X = [v_c^t \quad \gamma^t \quad \gamma^i \quad \alpha_f^t \quad \alpha_r^t \quad \alpha_r^i \quad \varphi^t \quad \varphi^i]^T$ and $U = [\delta]$.

The parameter values suggested by Karkee and Steward (2008) were used as the nominal values in this study (Table 1). The parameters were based on a MFWD tractor (model 7930, Deere and Co., Moline, IL) and a single axle 500 bu grain cart (model 500, Alliance Product Group, Kalida, OH). An uncertainty region of ±100% was defined about the nominal value for each parameter used in the sensitivity analysis.

Table 1: Dynamic bicycle model parameters and uncertainties for the JD 7930 tractor and Parker 500 grain cart system.

| Tractor | | Implement | |
|---|---|---|---|
| Parameters | Values ± Uncertainty | Parameters | Values ± Uncertainty |
| a | 1.7 m | d | 3.62 m |
| b | 1.2 m | e | 0.1 m |
| c | 2.1 m | | |
| $m^t$ | 9391 kg | $m^i$ | 2127 ± 2127 kg** |
| $I_z^t$ | 35709 kg-m$^2$ | $I_z^i$ | 6402 ± 6402 kg-m$^2$** |
| $C_{\alpha f}^t$ | 220* ± 220 KN/rad** | | |
| $C_{\alpha r}^t$ | 486* ± 486 KN/rad** | $C_{\alpha r}^i$ | 167* ± 167 KN/rad** |
| $\sigma_f^t$ | 1.1# ± 1.1 m** | $\sigma_r^i$ | 1.1# ± 1.1 m** |
| $\sigma_r^t$ | 1.5# ± 1.5 m** | | |

**Values based on the work of Metz (1993) and Schwanghart and Rott (1984).*

*#Values based on the work of Bevly et al. (2002).*

***Minimum values of zero for these parameters were not possible. Small positive numbers were used in the study instead of zeros.*

### *2.2 Sensitivity Analysis*

Sensitivity is the measure of the change in system responses w.r.t. to small changes in a system parameter. When using a dynamic model, the more a state variable changes with a small change in a model parameter, the more sensitive it is to this parameter (Rodriguez-Fernandez and Banga, 2009). The sensitivity measure calculated at a particular location in the parameter space is called the local sensitivity. This measure will be valuable in studying the parameter uncertainty about a nominal point or a point of interest. The cumulative sensitivity measure calculated over the entire parameter space is called the global sensitivity. This type of sensitivity is important to identify the overall ranking of parameters within the parameter space as the method accounts for the interaction between correlated parameters (Kucherenko et al., 2009).

Various sensitivity analysis techniques have been proposed in the literature to calculate both local and global sensitivity measures or indices. Primarily, these techniques can be classified into three groups, namely variance-based methods, screening methods, and derivative-based methods. In the variance-based methods, the contribution of a model parameter to the output variance is estimated (Kioutsioukis et al., 2004). Some of the widely used variance based methods are Fourier Amplitude Sensitivity Test (FAST) (Cukier et al., 1973; McRae et al., 1982), the extended FAST (Saltelli et al., 1999) and the method of Sobol (Sobol, 1993). The drawback of the variance based methods is their computational inefficiency requiring a large number of function evaluations to calculate the sensitivity with a reasonable accuracy (Kucherenko et al., 2009). Screening methods are used to qualitatively rank the model parameters from the most influential to the least influential order. These methods are computationally attractive, but are less accurate (Kucherenko et al., 2009). The method of Morris (Morris, 1991) is one of such methods.

Derivative-based methods take the first order differentiation of the model outputs with respect to the model parameters. Local and global sensitivity measures are defined based on those derivatives. Jang and Han (1995) and Kucherenko et al. (2009), among others, have proposed and used the derivative-based methods. Kucherenko et al. (2009) used the method to analyze sensitivity of seven algebraic equations and compared the results with those from the method of Sobol and Morris. This method was more accurate than the method of Morris and was computationally efficient than the Sobol’s method. Moreover, derivative-based sensitivity analysis method was preferred over other methods in analyzing dynamic models (Jang and Han, 1997; Park et al., 2003; Ruta and Wojcicki, 2003; Eberhard et al., 2007). In this work, some modifications were applied to the derivative-based formulation developed by Kucherenko et al. (2009) so that the sensitivity analysis of a complex dynamic model would be possible. Only the local sensitivity measures were used.

### 2.2.1 Sensitivity Measures

A multi-input multi-output (MIMO) dynamic system such as one presented in section 2.1 can be represented as,

$$\dot{X} = f\left(X, p, t\right) \tag{10}$$

where, $p$ is the parameter vector and $t$ is time.

A local sensitivity of the system to a parameter $p_i$ is defined as the partial derivative of the function $f$ w.r.t. the parameter. The local sensitivity function about a nominal point $p^*$ in the parameter space is then represented as,

$$S_i(p^*) = \left.\frac{\partial \dot{X}}{\partial p_i}\right|_{p=p^*} \tag{11}$$

where, $i$ = 1, …, $n$ and $n$ = size of parameter vector. As the point $p^*$ moves in the parameter space, the local sensitivity measures will also change.

Because the analytical solution to the local sensitivity measure (Eq. 11) was difficult to obtain for the tractor and implement system used in this study, a numerical simulation-based solution was used. Tractor lateral velocity and yaw-rate, implement yaw-rate, and three tire side slip angle states were used as output variables, which allowed us to use output sensitivities and state sensitivities interchangeably. The tractor and grain cart dynamic system (Eq. 9) was then represented in the state space form as,

$$\dot{X} = AX + BU \tag{12}$$

$$Y = CX + DU \tag{13}$$

where, $A = M^{-1}N$ , $B = M^{-1}P$ , $C$ = identity matrix of size 8, and $D$=[0]. $M$ and $N$ were defined in Eq. 9.

First, the model was simulated with a 10° step steering input at the nominal point $p^*$ in the parameter space. The simulation time was twice the settling times of the slowest non-zero eigenvalue of the system model, which were 30 s, 6 s and 3 s respectively at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities. With this simulation time, the sensitivity analysis accounted for both the transient and steady state sensitivities. The output responses from this simulation were used as the reference. Parameters of interest were then varied one at a time by a small amount to get the perturbed output responses. Finally, the local sensitivity of an output variable w.r.t. the parameters at the nominal point was calculated using the finite difference equation,

$$S_{ij}(p^*) = \frac{Y_j\left(p_1^*, \ldots\ldots\ldots, p_{i-1}^*, p_i^* + \Delta p_i^*, p_{i+1}^*, \ldots\ldots., p_n^*\right) - Y_j(p^*)}{\Delta p_i^*} \tag{14}$$

where, $j$ = 1,…..,$m$; $m$ = number of output or state variables; $\Delta p_i^*$= small change in the parameter of interest. The variable $Y_j$ represented a time series output data of the dynamic system. A mean of the absolute change in output variable over the simulation time was used to calculate the local sensitivity measure. This local sensitivity measure was calculated for all parameters of interest at various points in the parameter space. The local sensitivities at the nominal point were used to evaluate the effect of parameter variations to the system responses and model the relationships between the variation in the input parameters and the output variables (Grau, 2002).

The changes in input parameters were divided by the nominal values of the parameters to normalize the results as it was not appropriate to directly compare or average the output sensitivities w.r.t. various input parameters whose numerical ranges differed by several order of magnitude. Output variables were also of different orders of magnitudes. A good way to define the sensitivity in this case would also be to normalize the changes in the output variables by the nominal output values. However, because we were dealing with a dynamic system with zero crossings, the output variables were normalized only after the mean of the absolute changes in output variables were calculated.

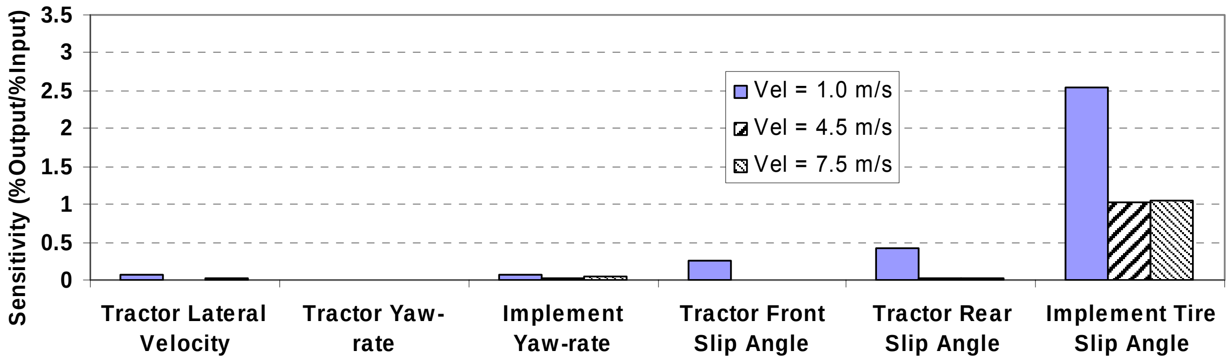
Finally, the overall local sensitivity to a parameter was calculated using a mean over the output variables,

$$S_i = \frac{1}{m}\sum_{j=1}^{m} S_{ij} \tag{15}$$

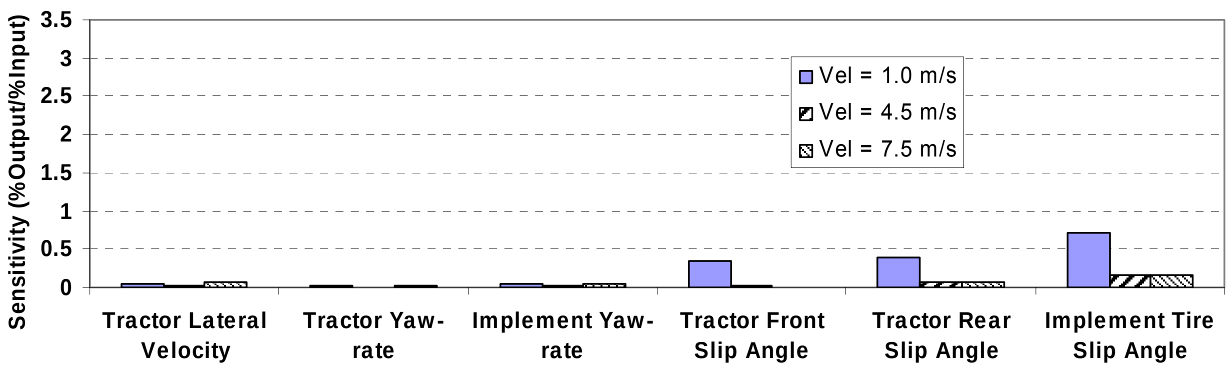
## 3.0 Results and Discussions:

The local sensitivities at the nominal point (Table 1) in the parameter space showed the effect of variations in the system parameters on the system states or outputs (Fig. 1- Fig. 3). At low forward velocities, tire side slip angles were highly sensitive to all input parameters. As expected, the implement tire side slip angle was most sensitive to the implement mass, implement MI, and implement tire cornering stiffness parameters. The tractor lateral velocity and yaw-rate states were most sensitive to the front and rear tire cornering stiffness parameters and so were the tractor and implement yaw rate parameters (Fig. 2).

In general, local state/output sensitivities w.r.t. implement inertial parameters decreased as the forward velocity was increased (Fig. 1). Implement tire side slip angle was the most sensitive output to both the implement mass and implement MI. This result was expected as the implement inertial parameters will have a higher influence on the implement dynamics. The implement tire side slip sensitivities w.r.t. implement mass were 2.5, 1.0 and 1.0 respectively at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities. The same w.r.t. implement MI was 0.7 at 1.0 m/s forward velocity. The sensitivities of all other output variables w.r.t. both implement mass and MI were less than 0.4 at all velocities.



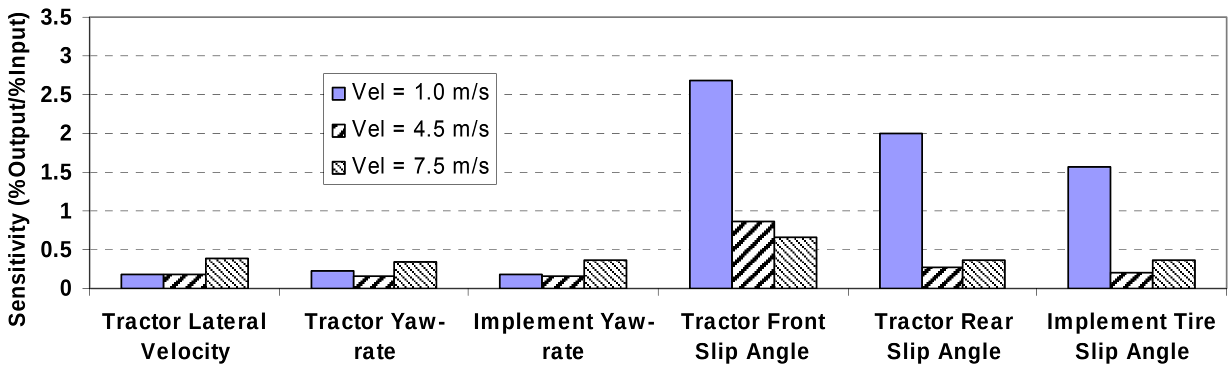

**a) Local sensitivities of state variables w.r.t. implement mass**



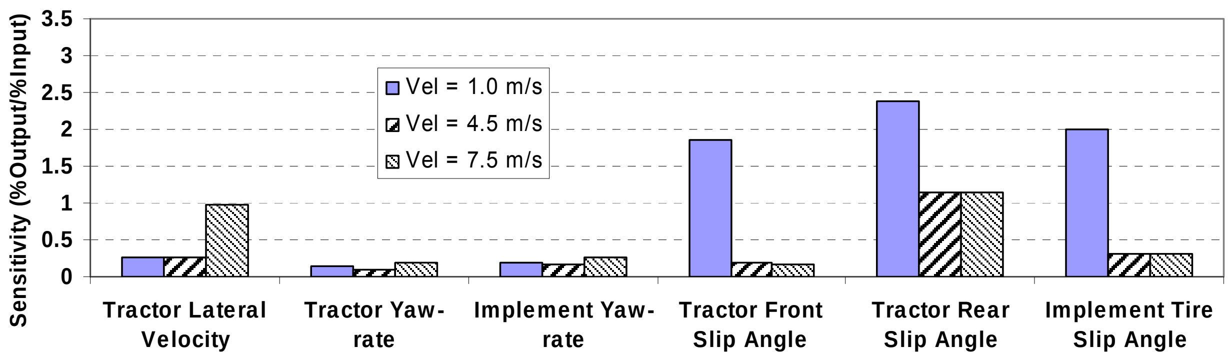

**b) Local sensitivities of state variables w.r.t. implement MI**

**Fig. 1: Local sensitivities at the nominal point in the parameter space w.r.t. implement inertial parameters; a) implement mass, and b) implement MI. The sensitivities were calculated at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**

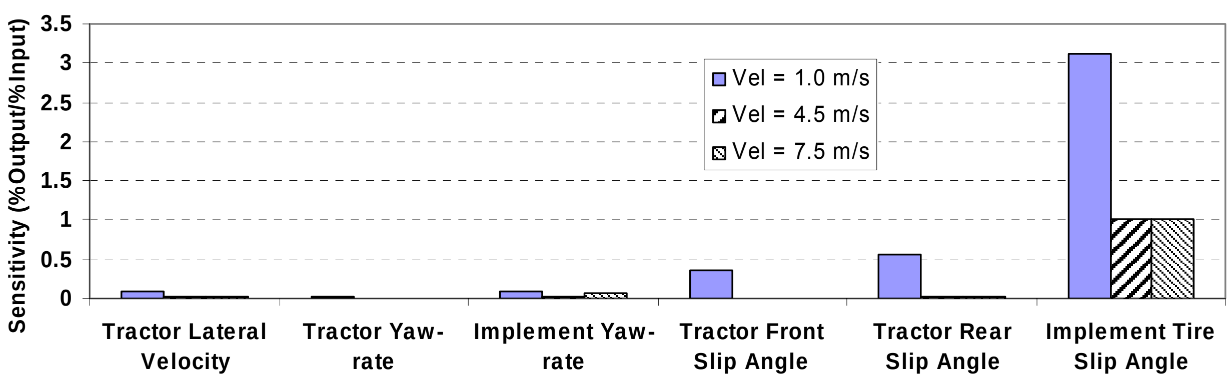
Sensitivities w.r.t. tire cornering stiffness parameters showed a mixed variation with increasing forward velocity (Fig. 2). In general, the tire side slip states were less sensitive to these parameters as the forward velocity was increased. The opposite trend was observed with the lateral and yaw velocity states. At all velocities, the tire side slip angles were the most sensitive state variable w.r.t the tire cornering stiffness parameters. The state sensitivities were as high as 2.6 for the front tire cornering stiffness, 2.4 for the rear tire cornering stiffness and 3.1 for the implement tire cornering stiffness. The system was more sensitive to the cornering stiffness parameters than to the implement inertial parameters and relaxation length parameters. Tire relaxation length parameters mostly influenced the corresponding tire side slip angles (Fig. 3).




**a) Local sensitivities of state variables w.r.t. front tire cornering stiffness**

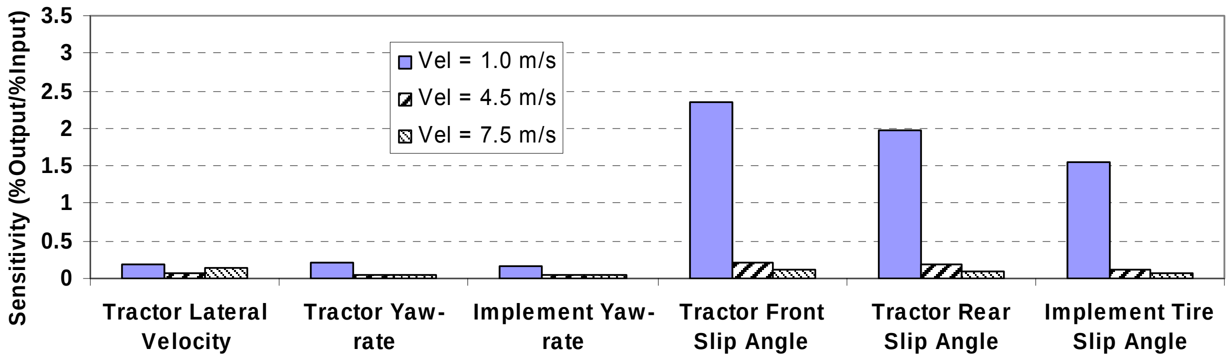



**b) Local sensitivities of state variables w.r.t. rear tire cornering stiffness**

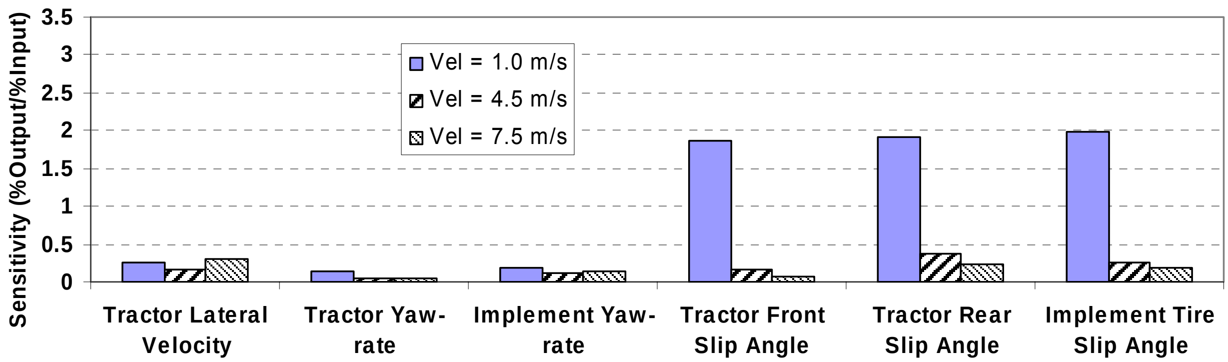



**c) Local sensitivities of state variables w.r.t. implement tire cornering stiffness**

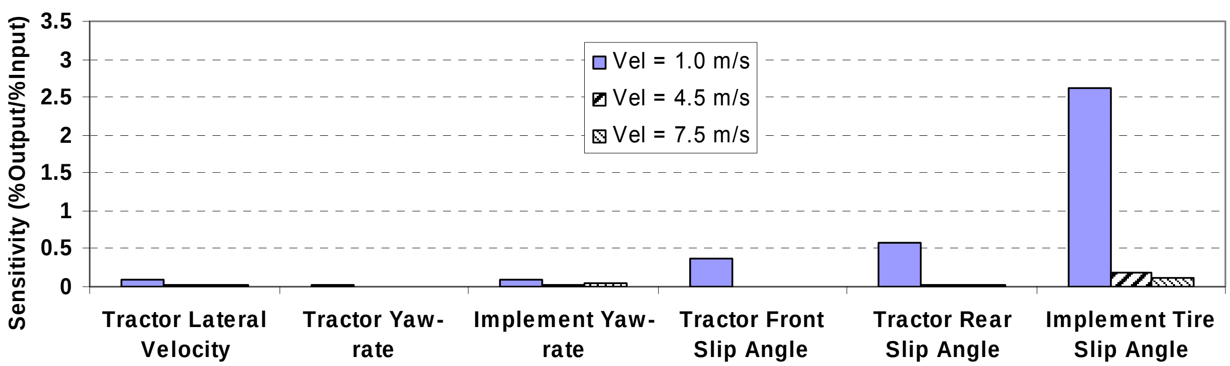
**Fig. 2: Local sensitivities at the nominal point in the parameter space w.r.t. cornering stiffness parameters; a) front tire cornering stiffness, b) rear tire cornering stiffness, and c) implement tire cornering stiffness. The sensitivities were calculated at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**




**a) Local sensitivities of state variables w.r.t. front tire relaxation length**



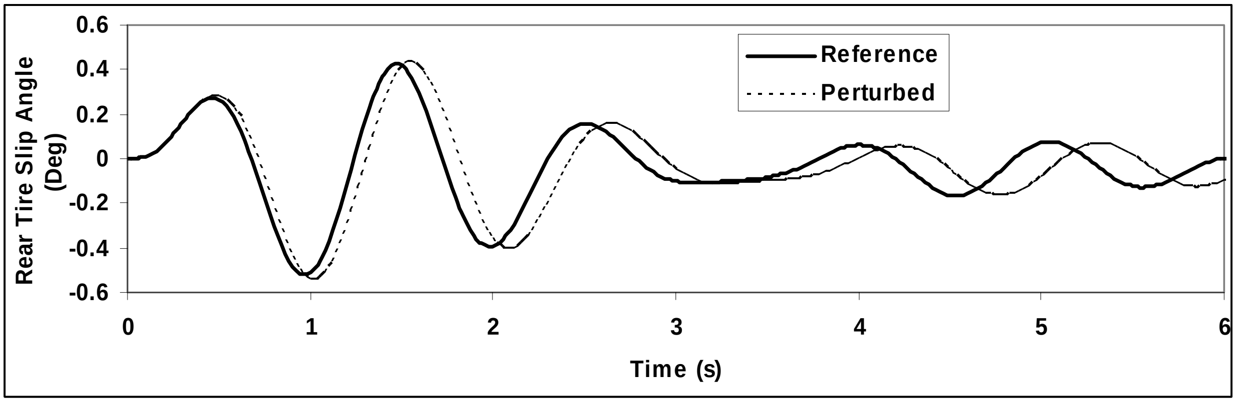

**b) Local sensitivities of state variables w.r.t. rear tire relaxation length**



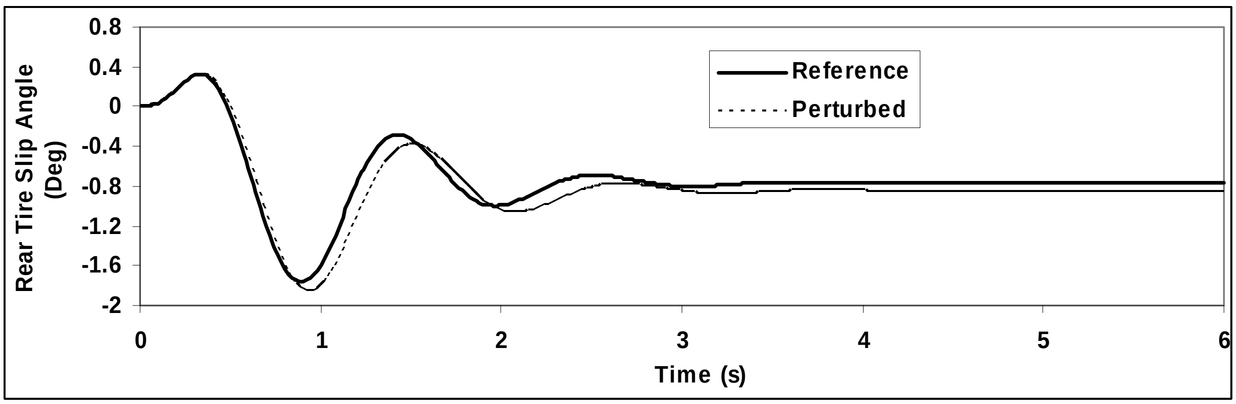

**c) Local sensitivities of state variables w.r.t implement tire relaxation length**

**Fig. 3: Local sensitivities at the nominal point in the parameter space w.r.t. to relaxation length parameters; a) front tire relaxation length, b) rear tire relaxation length, and c) implement tire relaxation length. The sensitivities were calculated at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**

The system was highly sensitive to all input parameters at 1.0 m/s forward velocity. Because, the system had more oscillatory transient behavior at 1.0 m/s than at 4.5 m/s and 7.5 m/s forward velocities (Fig. 4), the interaction between tire relaxation length and other parameters caused a larger phase shift between the reference and perturbed output responses at 1.0 m/s. The increased phase shift resulted to the higher system sensitivity at the low forward velocity.




**a) Time history of rear tire relaxation length at 1.0 m/s**




**a) Time history of rear tire relaxation length at 4.5 m/s**

**Fig. 4: Time history of rear tire side slip angle for the nominal parameters (reference) and 10% change in the cornering stiffness parameters (perturbed); a) at 1.0 m/s forward velocity, b) at 4.5 m/s forward velocity.**

At the nominal point, the overall system was most sensitive to the front and rear tire cornering stiffness parameters at all three forward velocities (Fig. 5). The front and rear tire relaxation lengths were the next two influential parameters. The system was least sensitive to the implement mass and MI. At 1.0 m/s, the system was almost equally sensitive to cornering stiffness and corresponding relaxation length parameters. At 4.5 m/s and 7.5 m/s, however, the tractor-and-implement system was twice as sensitive to cornering stiffness parameters as it was to relaxation length parameters. The system became less sensitive to relaxation length parameters as the forward velocity was increased. This result was expected because the tires travelled the relaxation length equivalent distance sooner as the forward velocity increased.

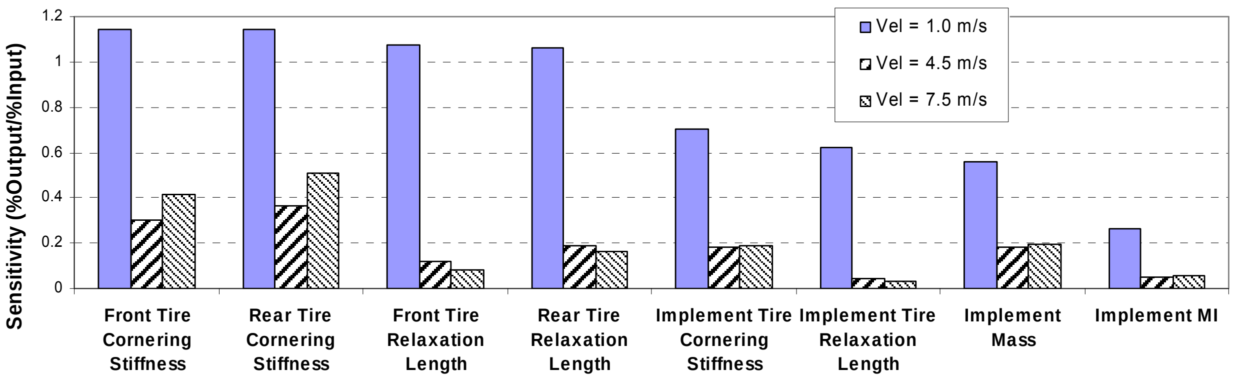



**Fig. 5: Local sensitivities at nominal point in the parameter space with respect to various system parameters at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**

The system became less sensitive to the cornering stiffness parameters at 4.5 m/s as compared to the sensitivities at 1.0 m/s forward velocity. The average state sensitivity at 1.0 m/s was 1.18 w.r.t. the front and rear tire cornering stiffness parameters. However, it dropped to 0.35 at 4.5 m/s forward velocity. When the forward velocity was increased to 7.5 m/s, the system became more sensitive to all three cornering stiffness parameters. At this velocity, the average state sensitivity increased to 0.4 w.r.t. the front tire cornering stiffness and to 0.5 w.r.t. the rear tire cornering stiffness. This result can be explained because the cornering stiffness parameters are linked to the higher order system dynamics which become significant at higher velocities (Karkee and Steward, 2008). However, because the interaction between cornering stiffness and relaxation length parameters was substantial at lower velocities (Fig. 2), the system was highly sensitive to cornering stiffness parameters at 1.0 m/s forward velocity. Similar trends were observed with the implement inertial parameters. With respect to implement mass and MI, the system was most sensitive at 1.0 m/s forward velocity. The average state sensitivity was about 0.6 w.r.t. implement mass at 1.0 m/s. The sensitivity dropped to 0.2 at 4.5 m/s and increased slightly at 7.5 m/s. The sensitivity w.r.t. the implement MI was 0.25 at 1.0 m/s and less than 0.1 at 4.5 m/s and 7.5 m/s. In general, the system was highly sensitive to all system parameters at 1.0 m/s forward velocity. As discussed before (Fig. 4), higher system sensitivity at the low forward velocity was caused by the larger phase shift between the system transient responses.

The interactions between input parameters were observed by computing local sensitivities at various points in the parameter space. At a sample point with small relaxation length parameters (10% of the nominal values), large implement inertial parameters (200% of the nominal values), and nominal cornering stiffness parameters, the interaction between relaxation length and other input parameters became insignificant even at 1.0 m/s forward velocity (Fig. 6). As expected, the sensitivities at 1.0 m/s w.r.t. cornering stiffness and implement inertial parameters were smaller at this point. The system sensitivities w.r.t. cornering stiffness parameters increased consistently as the forward velocity was increased. In general, the system was most sensitive to cornering stiffness parameters, followed by the implement inertial parameters and then the relaxation length parameters.

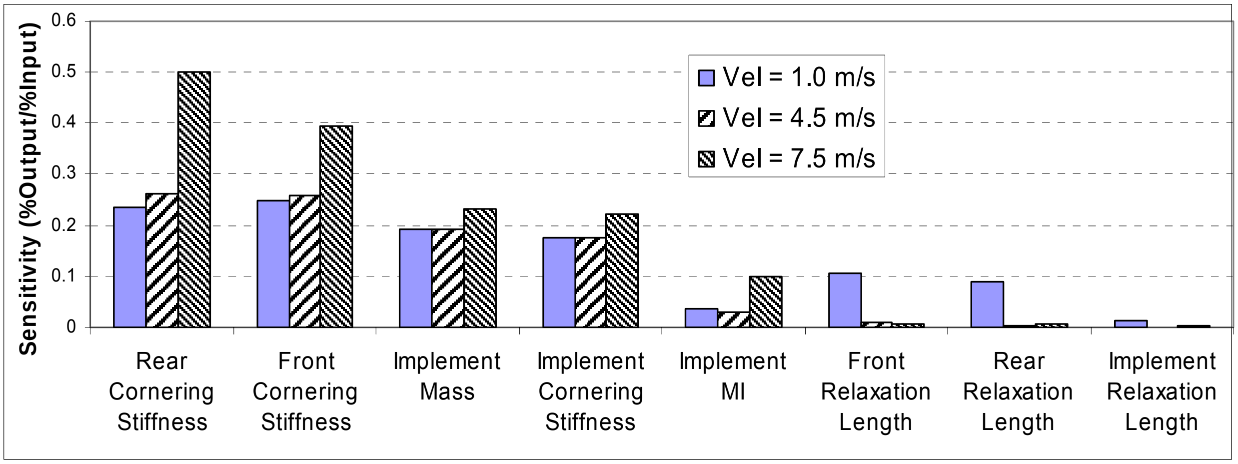



**Fig. 6: Local sensitivities at a sample point in the parameter space with respect to various system parameters at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**

The local sensitivity analysis method was used to study the relationship between system parameter variations and system state/output variations (Fig. 7 and 8). The variations in output variables were not symmetric about the nominal point. The outputs varied by a different amount about the output with nominal parameter values when a 50% changes were applied to the cornering stiffness parameters in the positive and negative direction (Fig. 7). In general, the change in output variables increased as the forward velocity was increased. The changes in output variables at 1.0 m/s were very small and were not apparent in the diagram. But the percentage changes about the reference values were still substantial as we can see in the local sensitivity plots (Fig. 5).

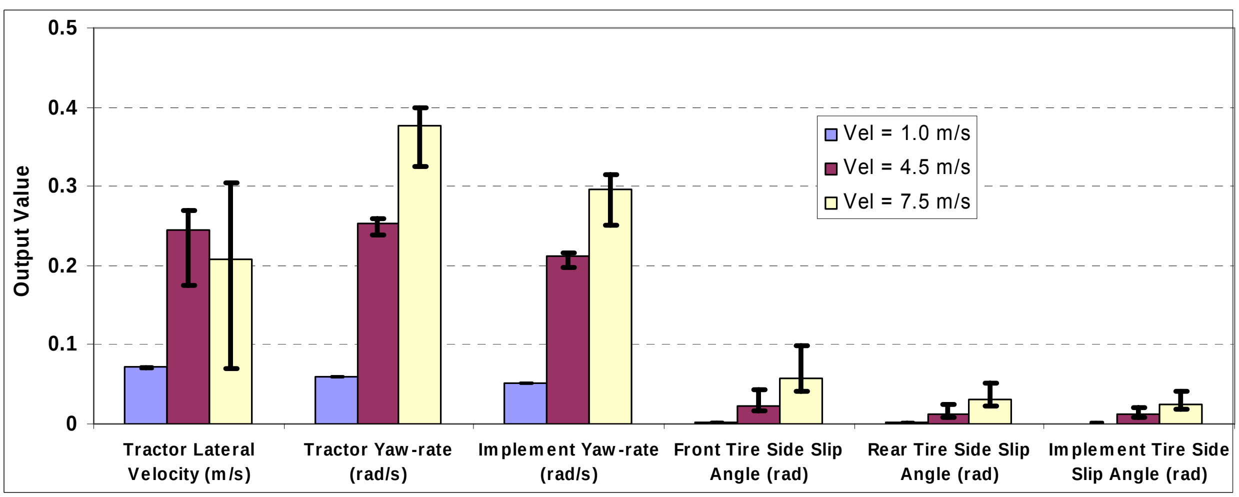
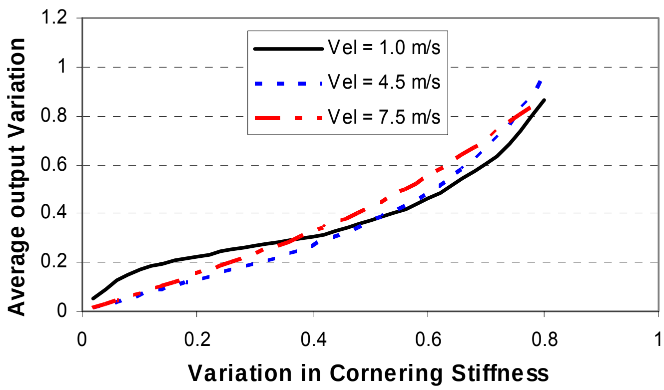
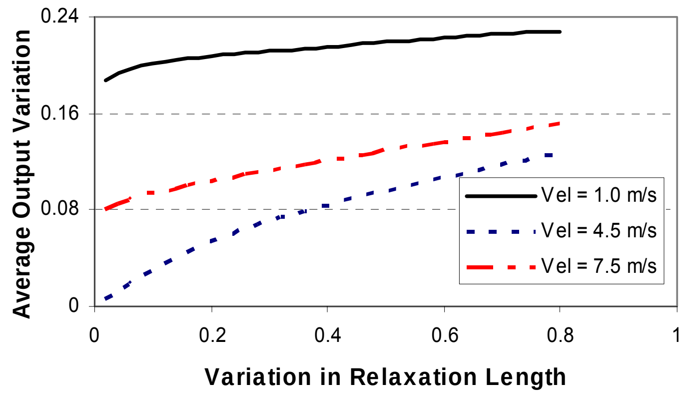



**Fig. 7: Changes in state/output variables for a 50% change in cornering stiffness parameters at 1.0 m/s, 4.5 m/s and 7.5 m/s. The solid bars represented the values of the output variables for the nominal parameters and the vertical lines represented the values of the output variables when the cornering stiffness parameters were increased and decreased by 50% of the nominal values.**

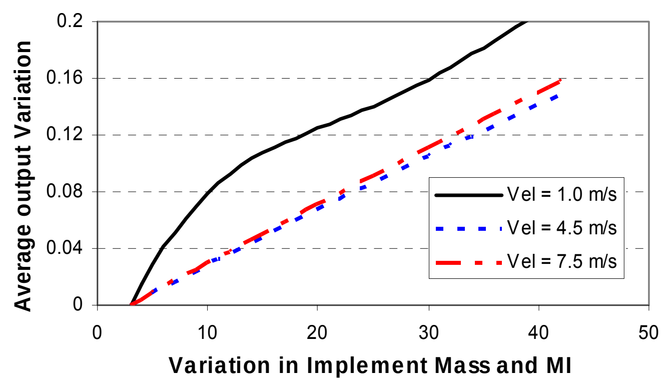
The six state/output variables (Fig. 1) were used to calculate the average output variation of the system with respect to the variation in system input parameters (Fig. 8). At all velocities, the average output variation increased exponentially with the variation in cornering stiffness parameters (Fig. 8a). The result showed that if a 10% average variation in system responses is acceptable, the cornering stiffness parameters must be estimated within 7%, 18% and 18% of actual/nominal values respectively at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities. The relationship between average output variation and variation in relaxation length parameters was fairly linear at all velocities (Fig. 8b). The output variation was always more than 10% at 1.0 m/s forward velocity. At other velocities, a 75% variation in the relaxation length parameters caused about 10% average output variation. Similarly, the relationship between average output variation w.r.t. the variation in the implement inertial parameters was also linear (Fig 8c) at 4.5 m/s and 7.5m/s forward velocities. Up to 30% variation in the implement mass and MI was acceptable considering a 10% output variation to be acceptable at these velocities. At 1.0 m/s, however, the average output variation increased rapidly in the beginning. Only about 12% variation in the implement inertial parameters was acceptable for a 10% average output variation.




(a)




(b)




(c)

**Fig. 8: The relationship between average variation over the six output variables of the system and the variation in a) cornering stiffness parameters, b) tire relaxation length parameters, and c) implement inertial parameters (mass and MI). The variations were calculated at 1.0 m/s, 4.5 m/s and 7.5 m/s forward velocities.**

## 4.0 Conclusions:

Local sensitivities of a tractor and single axle grain cart system were calculated using a derivative-based method. Due to the complexity in deriving analytical sensitivity indices, a dynamic system numerical simulation technique was used. The work was important in understanding the effect of parameter variations on the tractor and towed implement system responses. The system parameters were ranked based on the relative importance of the parameters to the system responses. Specifically, the following are the conclusions from this work.

- In general, variation in system parameters and system outputs were related in a non-linear fashion. At the nominal values and 4.5 m/s forward velocity, a 10% variation in the cornering stiffness parameters resulted to a 5% average output variation where as an 80% variation in the cornering stiffness resulted to a 80% variation in the output.
- Overall, cornering stiffness was the most influential parameter of the tractor and a single axle towed implement system dynamics. At the nominal point, the system was equally sensitive to both of the tractor front and rear tire cornering stiffness parameters. The system was least sensitive to implement inertial parameters.
- Tire relaxation length parameters became less influencing to the system dynamics as the forward velocity was increased where as the cornering stiffness parameters, in general, became more influential with the increasing forward velocity.

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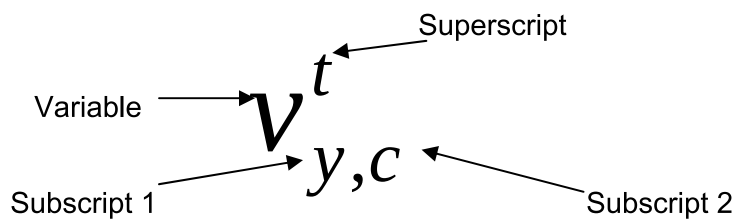
## Notation and List of Variables:




*Variable:* The variable itself.

Big or bold letter – vector or matrix, small letter – scalar

*Superscript:* Denotes whether the variable is related to tractor or implement.

t – tractor, i – implement

*Subscript 1:* Specifies the co-ordinate axis the variable corresponds to.

x – x axis, y – y axis, z – z axis

*Subscript 2:* Specifies the location the variable corresponds to.

f – front tire axle, r – rear tire axle, c – center of gravity, p – toe pin (hitch point)

List of variables

| | |
|---|---|
| $\alpha$ | side slip angle or the angle between the direction the tire is going and the direction it is facing. The velocity vector to the right of the tire is positive and reverse is negative. |
| $\alpha_0$ | steady state side slip angle |
| $\gamma$ | yaw rate |
| $\delta$ | steering angle |
| $\lambda$ | angle between tractor heading and implement heading |
| $\sigma$ | relaxation length |
| $\varphi$ | heading angle |
| a | distance between front axle and CG of tractor |
| b | distance between rear axle and CG of tractor |
| c | distance between hitch point and CG of tractor |
| $C_\alpha$ | cornering stiffness |
| d | distance between hitch point and CG of implement |
| e | distance between rear axle and CG of implement |
| F | force |
| $I_z$ | yaw moment of inertia |
| m | mass |
| p | parameter vector |

| | |
|---|---|
| S | local sensitivity measure |
| u | longitudinal velocity |
| v | lateral velocity |
| X-Y | world coordinates |
| x'-y' | vehicle coordinates |
| y | position of CG in y- axis of the world co-ordinate system |

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