



# The Economics of Public Beta Testing\*

Zhengrui Jiang<sup>†</sup>, Kevin P. Scheibe, Sree Nilakanta, and Xinxue (Shawn) Qu  
College of Business, Iowa State University, 2340 Gerding Business Building, Ames, IA  
50011-1350, e-mail: zjiang@iastate.edu, kscheibe@iastate.edu, nilakant@iastate.edu,  
quxinxue@iastate.edu

## ABSTRACT

A growing number of software firms now rely on public beta testing to improve the quality of their products before commercial release. While the benefits resulting from improved software reliability are well recognized, some important market-related benefits have not been studied. Through word-of-mouth, public beta testers can accelerate the diffusion of a software product after its release. Additionally, because of network effects, public beta testers can increase users' valuation of a product. In this study, we consider both reliability-related and market-related benefits, and develop models to determine the optimal number of public beta testers and the optimal duration of testing. Our analyses show that public beta testing can be profitable even if word-of-mouth and network effects are the only benefits. Furthermore, when both benefits are considered, there is significant "economies of scope"—the net profit increases at a faster rate when both word-of-mouth and network effects are significant than when only one benefit is present. Finally, our sensitivity analyses demonstrate that public beta testing remains highly valuable to software firms over a wide range of testing and market conditions. In particular, firms will realize greater profits when recruiting public beta testers who are interested in the software but unable to afford it. [Submitted: February 19, 2015. Revised: February 26, 2016. Accepted: February 29, 2016.]

**Subject Areas:** *Bass Model, Network Effects, Software Reliability, and Word-of-Mouth.*

## INTRODUCTION

The software market landscape has changed. Brick-and-mortar stores no longer play a major role in distributing software products to end-users. A growing proportion of software sales is delivered through the Internet, enabling end-users to preview, test, purchase, and review software products without leaving their homes or offices. While this new software delivery is faster and more cost-efficient, it also brings challenges to the software market.

---

\*The authors would like to thank the reviewers and editors of the *Decision Sciences* Journal for their helpful feedback and suggestions. Additionally, we would like to thank Dr. James McElroy for providing valuable comments. With the help of this community, our manuscript continued to improve to its present form, and we are very grateful.

<sup>†</sup>Corresponding author.

Analogous to physical products, the success or failure of a new software product depends on two critical factors: (i) high product quality and (ii) a sustainable market share. Even in the pre-Internet market landscape, these goals were difficult to achieve due to rapid technological changes. Now, the Internet has lowered the barrier of market entry, resulting in additional competition and a more volatile market. Consequently, it is more challenging to attract a sufficient number of users to reach a sustainable market share. Furthermore, software firms face increasing pressure to reduce the development lifecycle (Baskerville, Levine, Pries-Heje, Ramesh, & Slaughter, 2001). Due to the unique characteristics of software products, quality assurance is often more difficult than for physical products. Time pressure can further exacerbate the problem.

Fortunately, while the Internet is a major cause of the problem, it also offers a solution. The focus of this study is on using an Internet-enabled extension of traditional beta testing to address the aforementioned challenges.

### **Traditional Beta Testing Versus Public Beta Testing**

Beta testing begins after alpha testing is complete and all known major issues are resolved. Traditionally, a prerelease beta version is distributed to select members of the user community, who then test the product in real-world environments and report problems back to the development team. Typically, the development team and beta testers maintain close communication throughout the testing process.

More recently, numerous software firms are using the Internet to recruit larger numbers of beta testers. This new Internet-enabled beta testing practice is referred to as *public beta testing*. Public beta testing is applicable to both system software and application software. Some well-known examples include Microsoft's Windows Vista community technology previews, Google Wave and Cloud Print (Meyer, 2009; Riofrio, 2011), and mobile applications for smart phones and tablets. For example, some iPhone app developers publish apps on iBetaTest.com, thus allowing testers from around the world to test the apps and report issues back to developers. As of October 2015, iBetaTest.com listed over 16,400 beta testers from 36 countries (iBetaTest.com 2015).

In addition to the improvements in software reliability commonly associated with traditional beta testing, public beta testing brings two important market-related benefits. The first is improved speed of diffusion realized through the word-of-mouth of public beta testers, and the second is increased product valuation due to network effects.

### **Benefits Derived from Word-of-Mouth and Network Effects**

Word-of-mouth is traditionally viewed as verbal communication between individuals, but in the diffusion of innovation literature it is not limited to oral communications (Awad & Ragowsky, 2008). In the Internet age, product information spreads through various online communication channels such as product reviews, blogs, and social media, which are also forms of word-of-mouth. Based on their own experiences, public beta testers share information about a new product, which can speed up the diffusion of the product after its commercial release. Faster diffusion can lead to a number of benefits. First, the number of adoptions during a time

window will increase, leading to higher revenue. Second, some users may adopt the product earlier than they would otherwise, thus increasing the present value of sales. Third, the “critical mass” or sustainable market share can be reached sooner, thereby reducing the risk of market failure.

Researchers have documented the marketing effects of beta testing for general product categories, including software products (Dolan & Matthews, 1993). Businesses have realized and taken advantage of the word-of-mouth influences of (public) beta testers. For instance, Khan (2008) notes that beta testing can provide the first external exposure of a new software product, and develop early adopters to accelerate time to revenue. Google has placed some of their products in public beta tests for up to five years, not only to create a better product, but also to lay a foundation for a word-of-mouth marketing force (Gillin & Schwartzman, 2010, p. 53). In fact, public beta testing is being called the “secret formula” and most important tool for viral marketing (Gul, 2014). In a more specific example, as a company that provides public beta testing user management as a service for developers, Centercode (centercode.com) states that while public beta testing can be used for product improvement, it is more often used as a marketing tactic to increase visibility and “buzz” surrounding a prereleased product (“Centercode for Public Betas,” 2015).

Another market-related benefit of public beta testing is the increased valuation of the product, realized through network effects—the notion that a consumer values a product more if there are a greater number of users (Katz & Shapiro, 1985). Prior studies in the network effects literature have presented empirical evidences showing that more users can indeed lead to higher software prices (e.g., Brynjolfsson & Kemerer, 1996, Gallaughner & Wang, 1999). For instance, Brynjolfsson and Kemerer find that “a one percent increase in a product’s installed base was associated with a 0.75% increase in its price.” If public beta testers are allowed to continue using the software after its commercial release, the network of users increases from the outset. In addition, a higher speed of diffusion due to the word-of-mouth of public beta testing can increase the number of adopters throughout a product’s demand window, thus increasing the expected network size and hence the perceived value of the product.

## **Related Literature**

The present research is broadly related to the software development and maintenance literature. For instance, prior studies have examined the optimal release policy while considering the market opportunity cost (Jiang, Sarkar, & Jacob, 2012) and the optimal software enhancement policy during a software lifecycle (Ji, Kumar, Mookerjee, Sethi, & Yeh, 2011).

Because beta software versions may be viewed as a form of free software, it is appropriate to briefly review the related literature. A few studies have analyzed the benefits of offering free software from a network effects perspective (e.g., Haruvy & Prasad, 1998; Gallaughner & Wan 1999). Two recent articles suggest that the benefit resulting from the word-of-mouth effect warrants offering the software for free (Jiang, 2010; Jiang & Sarkar, 2010). Further, public beta testers may be considered seeds in a population of potential adopters, hence this study is also

related to the prior literature on network seeding to create social contagions and network effects (Aral, Muchnik, & Sundararajan, 2013; Dou, Niculescu, & Wu, 2013).

Surprisingly, formal research on beta testing and associated decision-making is rare. Wiper and Wilson (2006) use Bayesian statistical methods to estimate the failure rate during the beta testing phase of a project. The proposed Bayesian model is then applied to determine the duration of beta testing and the number of beta testers needed. While this research considers the reliability-related benefit of beta testing, it ignores the market-related benefits resulting from word-of-mouth and network effects associated with public beta testing. In a more recent study that examines the impact of software process maturity and customer error reporting on software release policy, August and Niculescu (2013) find that with a high process maturity and a high quality product, a software firm should delay the release of its product as the size of beta test increases. Their study does not attempt to derive an optimal number of beta testers or an optimal duration of beta testing. Further, the two market-related benefits are not considered. As a result of these differences, our findings diverge from theirs.

To date, we have not found any formal research that explicitly considers the reliability-related benefit as well as the market-related benefits resulting from word-of-mouth and network effects. This study fills this gap and addresses the following questions: How many public beta testers should be recruited and how long should public beta testing last under different market scenarios? How is the decision affected when not all of the three benefits are present? How is the optimal public beta testing policy affected by various environmental conditions?

We would like to emphasize that although the economic benefits derived from software testing, word-of-mouth, and network effects are *separately* well documented in different prior literature, we demonstrate the necessity to *simultaneously* consider all three benefits for public beta testing. We build on classic models from three distinctive bodies of literature, and develop an integrated modeling framework to help decide the optimal number of public beta testers and the optimal duration of testing. Such a solution would not be possible by considering only one benefit at a time. Finally, we study the characteristics of the optimal solutions under different conditions.

In the next section, we introduce the models used to quantify the different benefits of public beta testing, which will then be used to develop the integrated modeling framework.

## **ADOPTED MODELS**

We adopt three well-known models from three distinct bodies of literature to form the theoretical basis for this study.

### **The G-O Reliability Growth Model and an Extension**

In order to understand how public beta testing can improve the quality of a software product, we need a software reliability growth model (SRGM). In the related software reliability literature, numerous SRGMs have been proposed (Pham, 2006).

The G-O model (Goel & Okumoto, 1979) model is widely considered to be the most commonly used software reliability prediction model (Amin, Grunske, & Colman, 2013). Hence, we employ the G-O model to estimate the public beta testers' bug detection efficiency. The G-O model adopts the following key assumption:

**Assumption 1:** The lifetimes of all bugs are mutually independent, and the instantaneous bug detection rate at any given time is proportional to the number of undetected bugs at that time.

Assumption 1 is one of the most common assumptions in SRGMs and has ample empirical support (e.g., Ehrlich, Prasanna, Stampfel, & Wu, 1993; Wood, 1996). The assumption implies that the time it takes to detect each bug follows an independent and identical exponential distribution. Let the failure rate of each bug be  $h$ , the lifetime of a bug, i.e., the time it takes to detect a bug, follows the exponential distribution  $g(t) = be^{-ht}$ , and the probability that the bug will be found by time  $t$  is  $G(t) = 1 - e^{-ht}$ . We denote the number of undetected bugs at the start of testing by  $N$ . Based on the G-O model, the expected number of undetected bugs at time  $t$ , denoted by  $u(t)$ , equals

$$u(t) = Ne^{-ht}. \quad (1)$$

**Assumption 2:** Public beta testers evaluate the tested software independently.

This assumption is reasonable because most beta testers do not coordinate with others on their testing activities. This assumption is not included in the G-O model because the amount of testing resources is considered exogenous in that model. It is needed in the present study because the number of public beta testers is a decision variable.

Based on assumptions 1 and 2, we can calculate public beta testers' collective bug detection efficiency. Specifically, if we denote the average bug failure rate due to one public beta tester's testing by  $\lambda$ , and the number of public beta testers at time  $t$  by  $Z$ , then the bug failure rate as a result of the beta testers' collective testing effort at time  $t$  equals  $h = \lambda Z$ .

Suppose all public beta testing starts from time zero. If the number of bugs at the start of testing is  $N$ , the expected number of undetected bugs at the end of public beta testing ( $\tau$ ) is

$$u(\tau) = Ne^{-\lambda Z \tau}. \quad (2)$$

## The Bass Model

The product diffusion literature in marketing centers around the seminal Bass model (Bass, 1969). Although there are alternative diffusion models (e.g., normal distribution, logistic distribution) in the diffusion of innovation literature, our own research shows that the Bass model is still the most widely used model in explaining the diffusion of software products (e.g., Nascimento & Vanhonacker, 1988, Givon, Mahajan, & Muller, 1995, Teng, Grover, & Guttler, 2002, Jiang & Sarkar, 2010). We believe the reasons behind the wide adoption of Bass model are its interpretability, parsimony, and wide acceptance in business research. Consequently,

we adopt the Bass model to capture the impact of public beta testers' influences on the speed of software diffusion.

The Bass model assumes that the probability of a potential adopter adopting at time  $t$  is proportional to the number of existing adopters by time  $t$ . The influence of existing customers on potential adopters is also referred to as the *word-of-mouth* effect. The Bass model can be represented by the following equation:

$$dF(t)/dt = [p + qF(t)][1 - F(t)]. \tag{3}$$

In this model,  $p$  refers to the *coefficient of innovation*, and captures the potential adopters' intrinsic propensity to adopt or the amount of influence mass media has on the rate of adoption;  $q$  is the *coefficient of imitation*, and represents the amount of word-of-mouth influence each existing adopter exerts on potential adopters.  $F(t)$  and  $f(t)$  in Equation (3) represent the cumulative and instantaneous rate of adoption, and take the following forms:

$$F(t) = \frac{(1 - e^{-(p+q)t})}{(q/p)e^{-(p+q)t} + 1}, \tag{4}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{(p + q)^2}{p} \frac{e^{-(p+q)t}}{[(q/p)e^{-(p+q)t} + 1]^2}. \tag{5}$$

To keep track of the actual number of adoptions, we can rewrite Equation (3) as

$$dY(t)/dt = [p + (q/m)Y(t)][m - Y(t)], \tag{6}$$

where  $m$  is the market size (i.e., the total number of potential adopters who are willing to buy the product), and  $Y(t)$  represents the cumulative number of adoptions by time  $t$ . In addition, we use  $S(t)$  to denote the instantaneous adoption rate at time  $t$ . As expected,  $Y(t)$  and  $S(t)$  are proportional to  $m$  at any point in time  $t$ . Hence,  $Y(t) = m \cdot F(t)$ ,  $S(t) = m \cdot f(t)$ .

### Modeling the Benefit of Network Effects

Network effects commonly refers to the notion that the benefit a consumer derives from a good increases with the number of other users of that good (Katz & Shapiro, 1985). As explained earlier, evidences of network effects have been found in many product categories, including software products (e.g., Brynjolfsson & Kemerer, 1996).

In order to quantify the benefits of network effects, we need to know how the network size (i.e., the number of users of a product) affects the benefit each user receives from consuming the product. Unlike the software reliability and product diffusion literature, the literature on network effects contains some debate on how best to represent network effects. Swann (2002) argues that the functional form for network effects depends, in part, on the type of network. Briscoe, Odlyzko, and Tilly (2006) discuss Sarnoff's law, Metcalfe's law and Reed's law along with a fourth functional form. After some comparison, we adopted Metcalfe's law because it is the most well-known and is also the most "moderate"—the representing curve lies approximately in the middle of the four compared curves (Briscoe et al., 2006).

Metcalfe's law states that the *aggregate value* for a two-way communications network, such as telephone networks, is proportional to the square of the size of the network (Gilder, 1993). Metcalfe's law implies that the *average value* per user (i.e., the aggregate value divided by the number of users) is linearly related to the network size. Other studies suggest that the benefit a user receives from consuming a product comes from two sources, a network-independent benefit and an additional benefit resulting from network effects (Saloner & Shepard, 1995; Kauffman, McAndrews, & Wang, 2000). Most software products are useful as stand-alone applications and exhibit network effects. Therefore, based on prior research, we propose the following functional form to represent the benefit of consuming a software product:

$$v(Q) = \alpha + \beta \cdot Q, \quad (7)$$

where  $Q$  denotes the size of the network,  $\alpha$  is the "stand-alone" benefit, and  $\beta$  represents the amount of increase in benefit with each additional user joining the network. Here we assume that  $\alpha$  varies from user to user and follows a uniform distribution, while  $\beta$  is fixed for all users. We have tried other variations (e.g., allowing  $\beta$  to vary across users, a concave instead of linear function), and the findings are qualitatively similar.

In order to better understand the two market-related benefits of public beta testing, in the next three sections we first ignore the reliability-related benefit. The benefit resulting from word-of-mouth is examined in Section 3. In Section 4, we analyze the case where both network effects and word-of-mouth are significant. The most general case where both market-related and reliability-related benefits are significant is discussed in Section 5. For all cases, we assume that all public beta testers receive a complete product free of charge at the end of testing. Hence after commercial release, they automatically become adopters of the product they have tested.

## WORD-OF-MOUTH ONLY

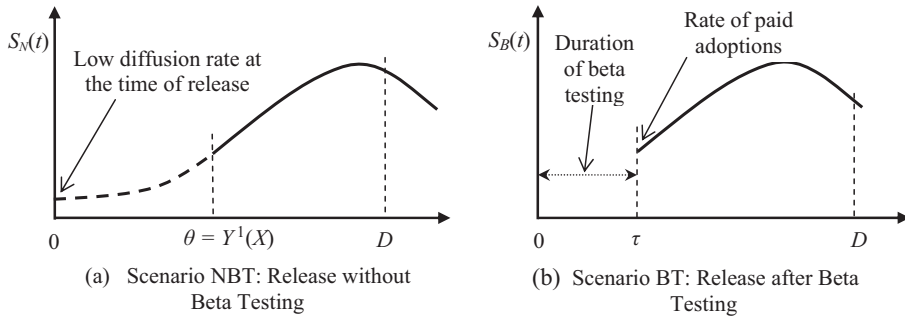
We first focus on public beta testers' word-of-mouth and assume that the network effects and their contributions to software reliability are negligible. This is a reasonable case to consider when the focal product is not used to support communication or coloration across users, and the quality of the beta version is sufficiently high or the cost of software failures is sufficiently low.

Based on whether a potential adopter's reservation price (i.e., the highest price she is willing to pay) is higher than the sale price of a software product or not, we classify all potential adopters of the product into two classes. A potential adopter belongs to the *high-valuation* class if her reservation price is equal to or higher than the sale price; otherwise the adopter belongs to the *low-valuation* class. For simplicity, we assume an adopter's reservation price of a product equals the benefit received from consuming the product, as defined in Equation (7).

### High-Valuation Testers Lead to Left-Shifting of Diffusion Curve

To accurately capture the effect of public beta testers' word-of-mouth influence, we first assume that all public beta testers are from the class of high-valuation

**Figure 1:** Diffusion curves with and without beta testing.



potential adopters. To differentiate from the total number of public beta testers (denoted by  $Z$ ), we denote the number of high-valuation testers (i.e., those from the high-valuation class) by  $X$ . For simplicity, we assume that the word-of-mouth influence from each beta tester is the same as that from a paying adopter.

When all public beta testers are high-valuation adopters, each one of them represents a loss of potential revenue to the software firm. The benefit is that their word-of-mouth can help speed up the diffusion of the product after commercial release. We next examine this tradeoff in detail.

For virtually all software products, there exists a finite demand window (Cohen, Eliashberg, & Ho, 1996). Assuming that the beta version is available at time zero, we denote the end of the demand window by  $D$ . The duration of such a demand window is often decided by various external factors such as advancements in hardware technology, operating systems, or competing software products. In most cases, if a firm delays the commercial release, the end of the demand window is unlikely to extend as well. Therefore, we assume that  $D$  is exogenous.

Public beta testers help increase the speed of diffusion after commercial release. However, when all public beta testers are high-valuation adopters, each tester represents a loss of revenue to the firm. This is because they might purchase the product if they were not recruited as public beta testers. We illustrate this tradeoff using the two release scenarios shown in Figure 1. Figure 1(a) represents the *hypothetical* scenario where a software product is released without public beta testing, which we label as *scenario NBT*. The diffusion curve in the figure shows that, without the testers' word-of-mouth, the diffusion of the new software would start from a low rate, and thus it may take some time for the diffusion to reach a desirable rate. In contrast, Figure 1(b) depicts the scenario where the software is released after a public beta testing period (i.e., from time 0 to  $\tau$ ), which is labeled as *scenario BT*. The solid curve captures the (paid) diffusion rate after commercial release. By comparing this curve with the one in Figure 1(a), we can see that by conducting public beta testing, although the commercial launch is delayed, the diffusion can start from a higher rate because of the word-of-mouth influence from the public beta testers.



Based on the basic premise of the Bass model, we arrive at the following conclusion (all proofs are relegated to the Online Supplement):

**Lemma 1:** When all public beta testers are from the high-valuation class of potential adopters, as a result of their word-of-mouth, the diffusion curve starting from time  $\tau$  under Scenario BT completely overlaps with the diffusion curve starting from time  $\theta$  under Scenario NBT, where  $\theta$  is a function of the number of public beta testers  $X$ :

$$\theta = Y^{-1}(X) = \frac{\ln[(pm + qX)/(pm - pX)]}{p + q}. \quad (8)$$

From Lemma 1, we conclude that the effect of public beta testers' word-of-mouth is equivalent to left-shifting the hypothetical diffusion curve without public beta testing by time  $\theta$ . By forgoing the potential revenue from time 0 to  $\theta$  under the hypothetical curve, the firm is able to jump-start the diffusion immediately after beta testing.

We now derive the present value of the revenue generated after beta testing. Based on Lemma 1, we can use the diffusion rate under Scenario NBT to obtain the diffusion rate under Scenario BT. For expositional convenience, we denote the diffusion rates in Figures 1(a) and (b) by  $S_N(t)$  and  $S_B(t)$ , respectively. Then,

$$S_N(t + \theta) = S_B(t + \tau). \quad (9)$$

Therefore, with a discount rate  $r$ , and a constant price  $\rho$  throughout the demand window, the time-discounted revenue after beta testing is

$$\begin{aligned} \text{Revenue}(X, \tau) &= \rho \int_{\tau}^D S_B(t) e^{-rt} dt = \rho \int_0^{D-\tau} S_B(t + \tau) e^{-r(t+\tau)} dt \\ &= \rho \int_0^{D-\tau} S_N(t + \theta) e^{-r(t+\tau)} dt = \rho \int_{\theta}^{\theta+D-\tau} S_N(t) e^{-r(t-\theta+\tau)} dt. \end{aligned} \quad (10)$$

In Equation (10),  $e^{-r(t-\theta+\tau)}$  is the continuous-time discount factor. For notational simplicity, the subscript “ $N$ ” is omitted in the rest of the analyses.

Based on Equation (10), we analyze the hypothetical case where word-of-mouth is the only benefit of public beta testing and all public beta testers are from the high-valuation group of potential adopters, and derive the optimal solution and other findings. The details are provided in Part II of the Online Supplement.

### Taking into Consideration Low-Valuation Beta Testers

The left-shifting of the diffusion curve is the result of the word-of-mouth influence of the high-valuation testers. We now examine the more realistic case where public beta testers are recruited from both high-valuation and low-valuation potential adopters. Here we assume that a low-valuation beta tester and a high-valuation beta tester have the same amount of word-of-mouth influence. As defined earlier, low-valuation potential adopters have a reservation price lower than the sale price, implying that they would not purchase the software if charged the regular sale price.

Therefore, by admitting low-valuation potential adopters as public beta testers, the firm still benefits from their word-of-mouth, but does not lose any potential revenue from them.

We denote the ratio of low-valuation to high-valuation public beta testers by  $\delta$ .<sup>i</sup> Given that  $X$  represents the number of high-valuation testers, the number of low-valuation testers equals  $\delta X$ . Because the Bass model considers only those potential adopters who can buy the product at the sale price, low-valuation potential adopters are not counted in the market size  $m$ . Therefore,  $Y(t)$  as a function of  $m$  also counts only high-valuation adopters. Taking into consideration the low-valuation public beta testers ( $\delta X$ ) not counted in  $Y(t)$ , Equation (6) can be modified to:

$$dY(t)/dt = \{p + (q/m)[Y(t) + \delta X]\} [m - Y(t)] = [\hat{p} + (q/m)Y(t)][m - Y(t)],$$

where  $\hat{p} = p + (q/m)\delta X$ . (11)

From (11), we conclude that accounting for the word-of-mouth of the low-valuation beta testers is equivalent to increasing the coefficient of innovation ( $p$ ) by  $(q/m)\delta X$ . With the higher coefficient of innovation, we have the revised diffusion rate function,

$$\hat{S}(t) = \frac{m(\hat{p} + q)^2}{\hat{p}} \frac{e^{-(\hat{p}+q)t}}{[(q/\hat{p})e^{-(\hat{p}+q)t} + 1]^2}. \tag{12}$$

From Equations (10) and (12), we can obtain the discounted revenue generated throughout the demand window when both low-valuation and high-valuation beta testers are considered:

$$Revenue(X, \tau) = \rho \int_{\hat{\theta}}^{\hat{\theta}+D-\tau} \hat{S}(t) e^{-r(t-\hat{\theta}+\tau)} dt, \tag{13}$$

where  $\hat{\theta} = \frac{\ln[(\hat{p}m+qX)/(\hat{p}m-\hat{p}X)]}{\hat{p}+q}$ .

As is common in the literature on economics of information goods, we assume that the marginal cost of producing an additional software copy is zero. Because we ignore the reliability-related cost and benefit in this section, the net profit derived from the software product during the demand window, denoted by  $V(X, \tau)$ , equals the time discounted revenue shown in Equation (13). Therefore, the public beta testing decision problem is formulated as

$$\begin{aligned} \text{Max}_{X, \tau} V(X, \tau) &= \rho \int_{\hat{\theta}}^{\hat{\theta}+D-\tau} \hat{S}(t) e^{-r(t-\hat{\theta}+\tau)} dt, \\ \text{s.t. } \hat{S}(t) &= \frac{m(\hat{p} + q)^2}{\hat{p}} \frac{e^{-(\hat{p}+q)t}}{[(q/\hat{p})e^{-(\hat{p}+q)t} + 1]^2}, \\ \hat{\theta} &= \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q}, \\ \hat{p} &= p + (q/m)\delta X. \end{aligned} \tag{14}$$

<sup>i</sup> We assume that the value of  $\delta$  depends on the demographics of the underlying potential adopter population, and can be estimated from previous data or using survey methods.

Here,  $\hat{S}(t)$  is the sales rate at time  $t$ ,  $e^{-r(t-\hat{\theta}+\tau)}$  is the discount factor,  $\hat{\theta}$  represents the amount of left-shifting illustrated in Figures 1 and discussed in Lemma 1, and  $\hat{p}$  captures the “extra” word-of-mouth influence of the low-valuation public beta testers.

Theoretically, when the reliability-related benefit is absent, the optimal public beta testing duration is zero, i.e.,  $\tau^* = 0$ . In practice, this means that a firm should just try to recruit the optimal number of testing as quickly as possible. Therefore, in this and the next section, we focus on deriving the optimal number of public beta testers. Due to its complexity, a closed-form solution for problem (14) cannot be derived; however, we are able to obtain several analytical findings:

**Proposition 1:** If public beta testers’ word-of-mouth effect is the only benefit of public beta testing, the optimal number of high-valuation beta testers ( $X^*$ ) and the net profit under the optimal solution ( $V^*$ ) always increases proportionally with the target market size ( $m$ ), i.e.,  $X^*(m) = km$ ,  $V^*(m) = lm$ , where  $k$  and  $l$  are positive constants independent of  $m$ .

From Proposition 1, we conclude that when word-of-mouth effect is the only benefit, there is no “scale economy” — the optimal number of public beta testers and the net profit both increase at exactly the same rate as the market size.

Besides Proposition 1, we are able to identify some analytical properties regarding the impact of the key model parameters on the optimal solution. To reduce repetition, they will be discussed under the most general problem formulation presented in Section 5.

In the absence of a closed-form solution, we resort to a numerical method to obtain the optimal solution. In order to determine the optimal number of public beta testers and the optimal duration of public beta testing (i.e.,  $X^*$  and  $\tau^*$ ), we need to know (i) the diffusion curve after release, determined by the three parameters  $m$ ,  $p$ , and  $q$ , (ii) the number of undetected bugs at the start of beta testing ( $N$ ) and the average bug failure rate due to a beta tester’s testing ( $\lambda$ ), and (iii) the unit cost of an undetected bug ( $c$ ). Among these parameters, the diffusion path can be projected based on data for analogous products (Bayus, 1993; Bass, Gordon, Ferguson, & Githens, 2001). The parameters  $N$  and  $\lambda$  can be estimated from bug detection data for similar products or initial bug detection data for a new product. The expected cost of a software failure can be estimated by domain experts. This cost should include both the direct cost of identifying and fixing the bug and the indirect cost such as liability cost and the loss of goodwill resulting from a software failure.

As no prior study has taken into consideration both software reliability growth and software diffusion, we are unable to find any diffusion data and bug detection data for the same software project. Instead of using randomly generated values, we estimate the key parameter values based on data for multiple real-world software products. The coefficient of innovation and coefficient of imitation are first estimated from a spreadsheet sales data for United Kingdom (Givon et al., 1995) as  $p_1 = .002$ ,  $q_1 = 0.648$ . In addition, we estimate the two coefficients based on data for two apps from Apple’s App Store (*Words with Friends* and *Tiny Tower*). The average coefficient values for the two apps are  $p_2 = .133$ ,  $q_2 = 1.34$ . These parameter values imply a much faster diffusion rate. Numerical analyses based

on the “slow” and “fast” coefficient values lead to qualitatively similar findings under most cases. Instead of reporting two separate results, we choose to use the averages of the two sets of parameter values in our following numerical analyses (i.e.,  $p = .067$ ,  $q = 0.992$ ). The market potential (i.e., the number of high-valuation potential adopters) is assumed to be  $m = 1$  million. In addition, we assume that there are equal numbers of low-valuation and high-valuation public beta testers, i.e.,  $\delta = 1.0$ .

The expected number of undetected bugs at the start of beta testing equals  $N = 500$  and the bug detection rate per thousand public beta testers is  $\lambda = 0.1$ . These two parameters are estimated and adjusted based on bug detection data for a real-time control software (Pham, 2006, pp. 144–145). The expected cost of a software failure  $c$ , is assumed to be \$50,000. The price of the software ( $\rho$ ) is set to \$50 and the duration of the demand window ( $D$ ) is 10 years. The annual discount rate ( $r$ ) is assumed to be 6%. Unless otherwise noted, the aforementioned parameter values are used in all numerical analyses of this study.

Based on these parameter values, we obtained the following optimal solution: the optimal number of high-valuation public beta testers is  $X^* = 16.0$  K (thousand), and the resulting net profit is  $V^* = \$42.63$  million. As explained earlier, the optimal testing duration is zero when the reliability-related benefit is not considered. In this case, public beta testers are recruited solely to take advantage of their word-of-mouth effect. The solution confirms that public beta testing can be beneficial (otherwise we should have  $X^* = 0$ ), even if the word-of-mouth is its only benefit.

## **BOTH WORD-OF-MOUTH AND NETWORK EFFECTS**

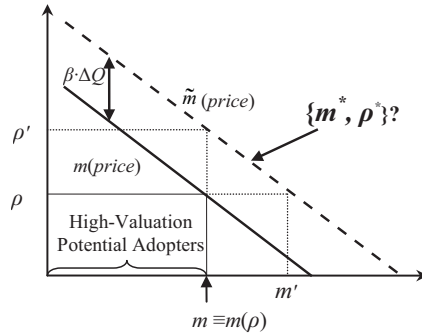
In this section, we examine the benefits of public beta testing resulting from network effects and word-of-mouth while still ignoring the benefit derived from improved software reliability. Given the benefit derived from word-of-mouth has already been analyzed in Section 3, we first show how the benefit of network effects can be modeled. For simplicity, we assume that each public beta tester and each future buyer contribute equally to network effects.

### **Quantifying the Benefit of Network Effects**

To isolate the benefit derived from network effects, we temporarily ignore public beta testers’ word-of-mouth, i.e., network effects represent the only benefit of public beta testing.

The number of adopters we use to calculate network effects is different from that used to calculate the amount of word-of-mouth effect at a given time. This is because future adopters can increase the utility of current adopters due to network effects whereas future adopters have no word-of-mouth influence on current adopters. According to Economides (1996), in the presence of network effects, a user’s valuation of a product depends on “the expected number of units of goods to be sold.” Therefore, we let the network size that determines a user’s valuation equal the total number of adopters by the end of the demand window  $D$ . Now, suppose that as a result of recruiting public beta testers, the network size

**Figure 2:** How to take advantage of high consumer valuation?



increases by  $\Delta Q$ . Because of network effects, every potential adopter’s reservation price also increases. Based on (7), the amount of increase, denoted by  $\Delta v$ , equals

$$\Delta v = \beta \cdot \Delta Q. \tag{15}$$

We next examine the financial implication of a higher reservation price. As defined in Section 3, high-valuation potential adopters are those with a reservation price higher than the sale price. Therefore, the number of high-valuation potential adopters depends on both the sale price and users’ reservation price. With everyone’s reservation price fixed, the number of high-valuation potential adopters is expected to decrease as the price increases. This relationship can be represented by the monotonically decreasing demand curve  $m(\text{price})$  shown in Figure 2. From this figure, we can see that if the sale price is set to the “base” price  $\rho$ , the number of high-valuation potential adoptions equals  $m(\rho)$ . For notational simplicity, we let  $m \equiv m(\rho)$ . Now, if every potential adopter’s reservation increases by  $\beta \cdot \Delta Q$ , the demand curve shifts upward to  $\tilde{m}(\text{price})$ . Facing this higher demand curve, a firm has a number of options. They can (i) raise the price by  $\beta \cdot \Delta Q$  to take full advantage of the increased consumer valuation (the market size remains the same at  $m$ ), (ii) keep the original price  $\rho$  to enjoy a large market size  $m'$ , or (iii) select a price, market size point  $\{m^*, \rho^*\}$  that maximizes the total profit. In this section, we focus on the first option. The third option is more complex and will be analyzed in Section 5.

We still assume that a firm recruits  $(1+\delta)X$  public beta testers, among which  $X$  are from the high-valuation class, and  $\delta X$  are from the low-valuation class. Because all  $X$  public beta testers are recruited from the  $m$  high-valuation potential adopters and they will receive the product for free as an incentive, there will be  $(m - X)$  potential adopters remaining after the commercial version is released.

Without the word-of-mouth effect of beta testers, the left-shifting illustrated in Figures 1(a) and (b) is not applicable. Because the number of remaining potential adopters decreases from  $m$  to  $(m - X)$ , the rate of paid adoptions after public beta

testing becomes  $(m - X)f(t)$ , hence the total number of paying adopters by the end of the demand window is

$$(m - X)F(D - \tau), \tag{16}$$

where  $f(t)$  and  $F(t)$  are defined in (4) and (5), respectively. By counting both the paying adopters and the public beta testers, the total network size by the end of demand window  $D$  becomes

$$Q(X) = (m - X)F(D - \tau) + (1 + \delta)X. \tag{17}$$

Without public beta testers, the network size by the end of demand window is  $Q(0) = mF(D)$ . Hence, increasing the number of testers from 0 to  $X$  grows the eventual network size by

$$\begin{aligned} \Delta Q &= Q(X) - Q(0) = [(m - X)F(D - \tau) + (1 + \delta)X] - m \cdot F(D) \\ &= X[1 - F(D - \tau)] - m[F(D) - F(D - \tau)] + \delta X. \end{aligned} \tag{18}$$

From Equation (18), it is clear that all else being equal, recruiting low-valuation public beta testers strictly increases the network size at the end of the demand window. We would like to point out, however, that  $\Delta Q$  can be negative under certain conditions (e.g., when the testing duration is long), in which case the curve for  $\tilde{m}(price)$  will be below that for  $m(price)$ .

Due to network effects, a larger (smaller) network size leads to a higher (lower) product valuation. In order to maintain the same number of high-valuation potential adopters, the sale price needs to be changed.

**Lemma 2:** If network effects are present while the public beta testers' word-of-mouth effect is absent, a firm can change its product price while maintaining the same number of high-valuation adopters. Specifically,

$$\begin{aligned} \tilde{m}(\rho') &= m(\rho), \text{ if } \rho' = \rho + \beta X[1 - F(D - \tau)] \\ &\quad - \beta m[F(D) - F(D - \tau)] + \beta \delta X. \end{aligned} \tag{19}$$

Based on Lemma 2, we analyze the hypothetical case with network effects as the only benefit of public beta testing, and derive the optimal solution. We find that the market size is above a threshold, the optimal number of high-valuation beta testers ( $X^*$ ) is a linear function and the net profit under the optimal solution ( $V^*$ ) is a quadratic function of the target market size ( $m$ ). The details are provided in Part II of the Online Supplement.

### Optimal Policy with Both Word-of-Mouth and Network Effects Considered

With public beta testers' word-of-mouth and their contribution to network effects both taken into account, we next show how the optimal public beta testing solution can be obtained.

Without their word-of-mouth effect, public beta testers' impact on the network size is given by (18). Now, with their word-of-mouth considered, and diffusion accelerating, the expected network size by the end of the demand window should further increase. Similar to Lemma 2, we are able to arrive at the following conclusion:

**Lemma 3:** If public beta testers' word-of-mouth and network effects are both present, then

$$\tilde{m}(\rho') = m(\rho'), \text{ if } \rho' = \rho + \beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X], \quad (20)$$

where  $\hat{p} = p + (q/m)\delta X$ ,  $\hat{F}(t) = \frac{(1 - e^{-(\hat{p}+q)t})}{(q/\hat{p})e^{-(\hat{p}+q)t} + 1}$ , and  $\hat{\theta} = \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q}$ .

This lemma reflects the joint benefits of word-of-mouth and network effects. The word-of-mouth effect can lead to a larger network by the end of the demand window, thus allowing a firm to charge a higher price while still maintaining the same number of high-valuation potential adopters. Note that in Equation (20), the coefficient of innovation is increased from  $p$  to  $\hat{p}$  to reflect the word-of-mouth of low-valuation testers (see Equation (11)). As a result, the amount of left-shifting and the cumulative adoption function are revised as well.

Based on Lemma 3, we have the formulation for the scenario where both word-of-mouth and network effects are significant:

$$\begin{aligned} \text{Max}_{X, \tau} V(X, \tau) &= \rho' \cdot \int_{\hat{\theta}}^{\hat{\theta} + D - \tau} \hat{S}(t) e^{-r(t - \hat{\theta} + \tau)} dt, \\ \text{s.t. } \hat{S}(t) &= \frac{m(\hat{p} + q)^2}{\hat{p}} \frac{e^{-(\hat{p}+q)t}}{[(q/\hat{p})e^{-(\hat{p}+q)t} + 1]^2}, \\ \rho' &= \rho + \beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X], \\ \hat{F}(t) &= \frac{(1 - e^{-(\hat{p}+q)t})}{(q/\hat{p})e^{-(\hat{p}+q)t} + 1}, \\ \hat{\theta} &= \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q}, \\ \hat{p} &= p + (q/m)\delta X. \end{aligned} \quad (21)$$

Problem (21) is a generalization of problem (14); when  $\beta = 0$ , the former reduces to the later.

Again, despite lack of a closed-form solution, we are able to obtain several analytical findings. First, in contrast to Propositions 1, we have the following conclusion:

**Proposition 2:** If word-of-mouth and network effects are the only benefits of public beta testing and public beta testing is beneficial, the optimal number of high-valuation beta testers ( $X^*$ ) and the net profit ( $V^*$ ) both increases at an increasing speed with the target market size ( $m$ ), i.e.,

$$\frac{dX^*(m)}{dm} > 0, \quad \frac{d^2X^*(m)}{dm^2} > 0, \quad \frac{dV^*(m)}{dm} > 0, \quad \text{and} \quad \frac{d^2V^*(m)}{dm^2} > 0. \quad (22)$$

We have found that when only word-of-mouth or network effects (see Part II of Online Supplement) are present, the optimal number of high-valuation beta testers increases linearly with the potential market size. When both word-of-mouth and network effects are considered, Proposition 2 shows that it is optimal to increase the number of high-valuation public beta testers at a faster pace than the rate of increase in market size. Although recruiting more high-valuation testers leads to a larger loss of potential revenue, the net profit still grows at a higher rate than under the cases when only one benefit is present. This is similar to the notion of “economies of scope” in economics. The reason behind this compounding effect is as follows. The word-of-mouth effect increases the speed of diffusion, which not only can increase the number of product sales, but also necessarily leads to a larger network size and subsequently a higher valuation for the product. The network effects, in turn, make the increased speed of diffusion even more valuable because each additional buyer can bring in a higher profit to the firm.

We adopt the same parameter values used in the previous section, and find that the optimal number of high-valuation public beta testers is  $X^* = 283.3\text{K}$  and the resulting net profit is  $V^* = \$52.89$  million. Owing to network effects, the sale price can be increased from \$50 to \$78.3. By comparing this solution with the previous solution ( $X^* = 16.0\text{K}$ ,  $V^* = \$42.63$  million when word-of-mouth is the only benefit), we can see that the optimal number of public beta testers and the net profit are both significantly higher when both market-related benefits are present. This confirms the existence of *economies of scope* associated with the benefits of public beta testing.

## ALL BENEFITS CONSIDERED

In this section, in addition to word-of-mouth and network effects, we also take into account the benefit resulting from public beta testers’ contribution to software reliability.

### Impact of Public Beta Testing on Software Reliability

The benefit resulting from improved software reliability is measured by the reduction in the cost of software failures in the field, which includes both the direct cost of fixing the bug and indirect costs such as potential liability and loss of goodwill. Consistent with the existing literature (e.g., Dalal & Mallows, 1988; Ehrlich et al., 1993), we assume that the total cost of software failures in the field is a linear function of the number of undetected bugs at the time of release, and we denote the expected cost of software failure caused by each undetected bug by  $a$ .

We assume that each of the  $(1+\delta)X$  public beta testers has the same bug detection efficiency, regardless of whether the tester belongs to the low-valuation or high-valuation class. Based on Equation (2), the total expected cost of software failures equals

$$C_1(X, \tau) = aNe^{-\lambda(1+\delta)X\tau}. \quad (23)$$



Note that even if a bug is detected during beta testing, fixing it still incurs a cost. We denote the average cost of fixing a bug detected during beta testing by  $b$ . Then, the total cost of fixing bugs detected during public beta testing is

$$C_2(X, \tau) = bN(1 - e^{-\lambda(1+\delta)X\tau}). \tag{24}$$

As the cost is expected to be higher if a bug is detected after the product is released, we denote the difference between the two costs by  $c$  (i.e.,  $c = a - b$ ). For expositional convenience, we refer to  $c$  as the *unit cost of an undetected bug*. Then, the sum of the cost of fixing bugs detected during beta testing and the cost of software failures equals

$$\begin{aligned} C(X, \tau) &= bN(1 - e^{-\lambda(1+\delta)X\tau}) + aNe^{-\lambda(1+\delta)X\tau} \\ &= bN(1 - e^{-\lambda(1+\delta)X\tau}) + (b + c)Ne^{-\lambda(1+\delta)X\tau} = bN + cNe^{-\lambda(1+\delta)X\tau}. \end{aligned} \tag{25}$$

In the RHS of Equation (25), the term  $bN$  is a constant with respect to  $X$  and  $\tau$ , hence it does not affect the optimal value of the decision variables. Furthermore, the cost of fixing a bug detected during public beta testing is significantly lower than the cost of software failure caused by the same bug after the product is released (i.e.,  $b < c$ ). Therefore, the first term  $bN$  is insignificant compared to the second term  $cNe^{-\lambda(1+\delta)X\tau}$ . For simplicity and clarity, we consider only the second term in our subsequent analyses. This term essentially represents the consequence of not detecting all bugs before release. We denote this cost by *BugCost*( $X, \tau$ ) and refer to it as the *total cost of undetected bugs*:

$$\text{BugCost}(X, \tau) = cNe^{-\lambda(1+\delta)X\tau}. \tag{26}$$

**Price as a Decision Variable**

In the previous two sections, we have focused on the first option illustrated in Figure 2, i.e., assuming that price is set at a level such that the number of high-valuation potential adopters remains the same even as consumer valuation of the product increases. In this section, we consider the third option shown in Figure 2, where we assume that the price itself is a decision variable to be optimized.

If the demand function without network effects is

$$m(\text{price}) = M - k \cdot \text{price}, \tag{27}$$

as shown in the proof of Lemma 3, the demand function with network effects considered is

$$\begin{aligned} \tilde{m}(\text{price}) &= M - k \cdot \text{price} + k\beta \cdot \Delta Q \\ &= M - k \cdot \text{price} + k\beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X], \end{aligned} \tag{28}$$

where  $\hat{p} = p + (q/m)\delta X$ ,  $\hat{F}(t) = \frac{(1 - e^{-(\hat{p}+q)t})}{(q/\hat{p})e^{-(\hat{p}+q)t} + 1}$ , and  $\hat{\theta} = \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q}$ .

**Problem Formulation and Solution**

By subtracting the cost of undetected bugs as shown in Equation (26) from the objective function of problem (21), and incorporating the new demand function

(28), we obtain the formulation for the general case where all three benefits of public beta testing are present:

$$\begin{aligned}
 \text{Max}_{X, \tau, \text{price}} V(X, \tau, \text{price}) &= \text{price} \cdot \int_{\hat{\theta}}^{\hat{\theta}+D-\tau} \hat{S}(t) e^{-r(t-\hat{\theta}+\tau)} dt - cN e^{-\lambda(1+\delta)X\tau}, \\
 \text{s.t. } \hat{S}(t) &= \frac{\tilde{m}(\hat{p} + q)^2}{\hat{p}} \frac{e^{-(\hat{p}+q)t}}{[(q/\hat{p})e^{-(\hat{p}+q)t} + 1]^2}, \\
 \tilde{m} &= M - k \cdot \text{price} + k\beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X], \\
 \hat{F}(t) &= \frac{(1 - e^{-(\hat{p}+q)t})}{(q/\hat{p})e^{-(\hat{p}+q)t} + 1}, \\
 \hat{\theta} &= \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q}, \\
 \hat{p} &= p + (q/m)\delta X.
 \end{aligned} \tag{29}$$

It can be shown that Proposition 2 remains valid for (29). Furthermore, given  $X$  and  $\tau$ , we can derive closed-form expressions for the profit maximizing price and market size.

**Proposition 3:** With all three benefits of public beta testing considered, given  $X$  high-valuation potential adopters and a testing duration  $\tau$ , the optimal sale price and the corresponding market size are, respectively,

$$\hat{\rho} = M/(2k) + \beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X]/2, \tag{30}$$

$$\hat{m} = M/2 + k\beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X]/2. \tag{31}$$

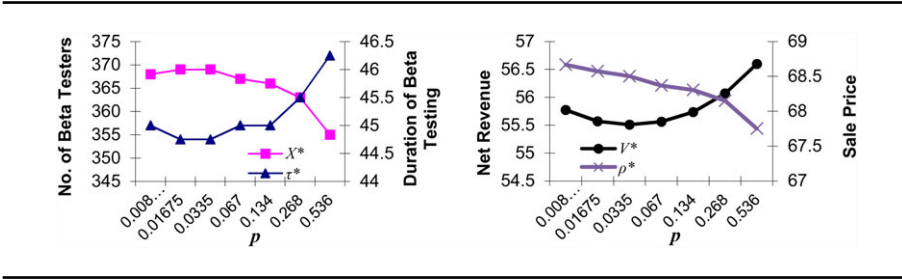
**Corollary 1:** If  $\rho$  is the optimal price without network effects, and  $\rho'$  is the price that keeps the number of high-valuation potential adopters unchanged in the presence of network effects (see Lemma 3), then the optimal sale price with network effects considered is  $(\rho + \rho')/2$ .

Corollary 1 essentially provides an answer to the optimal {price, market size} point we are seeking in Figure 2, i.e., if  $\rho$  was the optimal price without network effects, the best price with network considered lies precisely in the middle of  $\rho$  and  $\rho'$  shown in the figure.

Based on Proposition 3, we can simplify (29) to a problem with two decision variables:

$$\begin{aligned}
 \text{Max}_{X, \tau} V(X, \tau) &= \hat{\rho} \cdot \int_{\hat{\theta}}^{\hat{\theta}+D-\tau} \hat{S}(t) e^{-r(t-\hat{\theta}+\tau)} dt - cN e^{-\lambda(1+\delta)X\tau}, \\
 \text{s.t. } \hat{S}(t) &= \frac{\tilde{m}(\hat{p} + q)^2}{\hat{p}} \frac{e^{-(\hat{p}+q)t}}{[(q/\hat{p})e^{-(\hat{p}+q)t} + 1]^2}, \\
 \hat{m} &= M/2 + k\beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X]/2, \\
 \hat{\rho} &= M/(2k) + \beta[m\hat{F}(\hat{\theta} + D - \tau) - mF(D) + \delta X]/2,
 \end{aligned}$$

**Figure 3:** Impact of the coefficient of innovation ( $p$ ).



$$\hat{F}(t) = \frac{(1 - e^{-(\hat{p}+q)t})}{(q/\hat{p})e^{-(\hat{p}+q)t} + 1},$$

$$\hat{\theta} = \frac{\ln[(m\hat{p} + Xq)/(m\hat{p} - X\hat{p})]}{\hat{p} + q},$$

$$\hat{p} = p + (q/m)\delta X. \tag{32}$$

Using the previous parameter values, and letting  $M = 2$  million and  $k = 20$ , we obtain the following solution:  $X^* = 367.2\text{K}$ ,  $\tau^* = 0.18$  years, and  $V^*$  is \$55.56 million. Due to the increased network size, the price is raised from \$50 to \$68.38, and the number of high-valuation potential adopters increases from 1 million to 1.37 million. As expected, when the reliability-related benefit is considered, a positive public beta testing duration is warranted.

**Sensitivity Analysis**

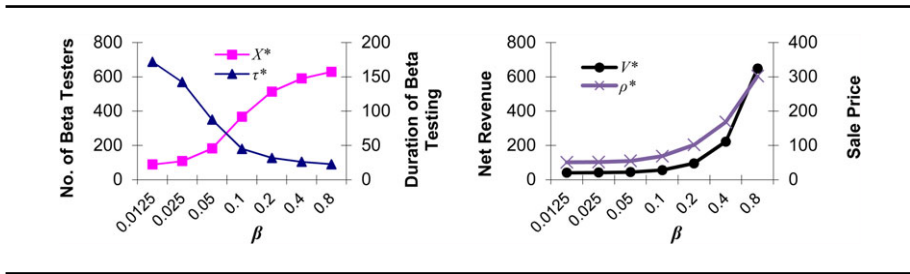
Even with both market-related and reliability-related benefits considered, public beta testing is not guaranteed to be beneficial. When it is indeed beneficial (i.e.,  $X^* > 0$ ), the impacts of several key model parameters on the net profit are summarized in the following proposition:

**Proposition 4:** With all benefits considered, the net profit under the optimal solution ( $V^*$ ) increases monotonically with the coefficient of innovation ( $p$ ), the coefficient of imitation ( $q$ ), the ratio of the low-valuation to high-valuation public beta testers ( $\delta$ ), and the bug failure rate ( $\lambda$ ), whereas it decreases with the unit cost of undetected bug ( $c$ ).

We conduct sensitivity analyses for all key model parameters in problem (32). Due to space limitation, we only present results for five of them.

In the Bass model, the coefficient of innovation ( $p$ ) and the coefficient of imitation ( $q$ ) jointly determine the speed of diffusion. As the impacts of the two parameters are similar, we only present the sensitivity analysis result for  $p$ , as summarized in Figure 3. The left chart shows the number of beta testers (in thousands) and the duration of public beta testing (in days). The right chart displays the net profit (in millions of dollars) and the sale price. From these results, we

**Figure 4:** Impact of sensitivity of reservation price to network size ( $\beta$ ).

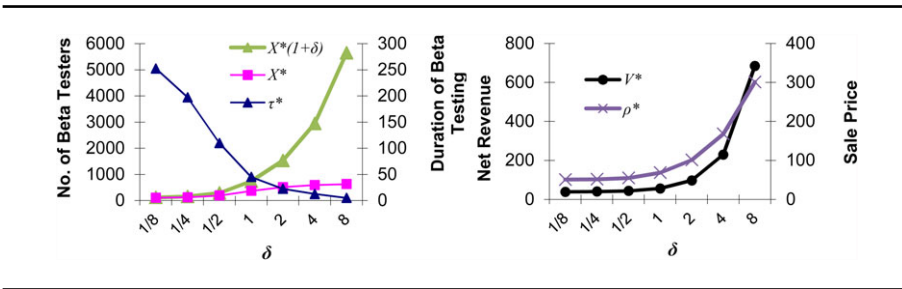


can see that as the value of  $p$  increases, the optimal number of high-valuation public beta testers tends to decrease while the duration of public beta testing tends to increase. This is because with a higher  $p$  value, each tester’s word-of-mouth influence is larger, hence a smaller number of beta testers are sufficient to speed up the diffusion. Therefore, the firm can recruit fewer testers to reduce the loss of potential revenue. To ensure that the desired reliability is attainable, though, the testing duration should be extended. Figure 3 also shows that the pricing curve closely follows the curve representing  $X^*$ . This is because when the ratio of low-valuation to high-valuation customers is fixed, a smaller  $X^*$  implies fewer low-valuation public beta testers are recruited, thereby decreasing the eventual network size and subsequently the sale price. With all factors considered, the net profit turns higher as the value of  $p$  increases.

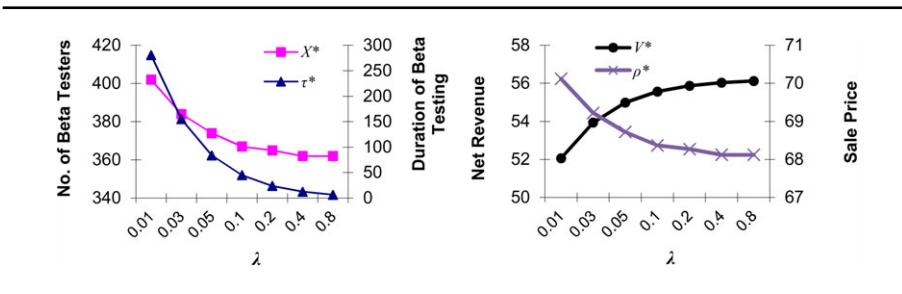
In our model, the parameter  $\beta$  reflects the sensitivity of users’ reservation price in response to changes in the network size. A higher  $\beta$  implies that the reservation price increases more rapidly as the network size grows. To evaluate the impact of this parameter on the optimal solution, we vary the value of  $\beta$  from 0.00625 (low increase in reservation price) to 0.4 (high increase in reservation price), and repeat the analysis. Figure 4 shows the results. We observe that as the value of  $\beta$  increases, it becomes beneficial to recruit more public beta testers. This is because each additional public beta tester becomes more valuable as the value of  $\beta$  increases. The optimal testing duration decreases because with more testers, the desired level of reliability is reached sooner. A larger number of beta testers also imply a bigger word-of-mouth effect, which leads to a faster speed of diffusion. Similarly, more testers lead to a larger network size, thus allowing the firm to charge a higher price. Although more high-valuation beta testers cause a higher loss of potential revenue, the loss is more than offset by a faster speed of diffusion and a higher sale price. The net profit increases monotonically as a result.

The ratio of the low-valuation to high-valuation public beta testers ( $\delta$ ) depends on the percentage of low-valuation potential adopters in the population. It may also be controlled if the firm is able to identify certain low-valuation potential adopters. In order to understand the impact of this ratio, we repeat the numerical analysis while varying the value of  $\delta$  from 1/8 to 8. The results are summarized in Figure 5. We can see that the optimal number of high-valuation public beta testers increases with  $\delta$ . The total number of public beta testers, which include both

**Figure 5:** Impact of ratio of low-valuation to high-valuation beta testers ( $\delta$ ).



**Figure 6:** Impact of bug detection efficiency ( $\lambda$ ).

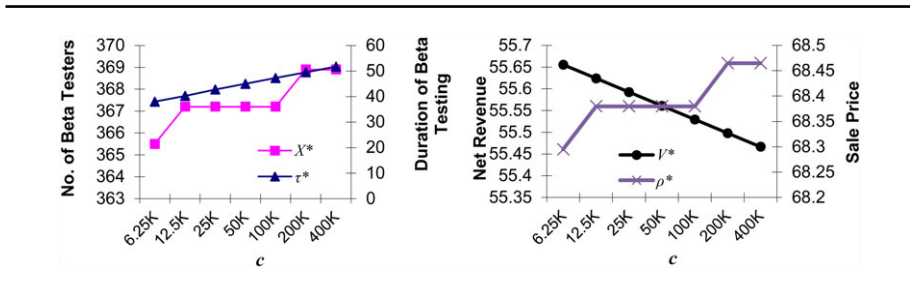


high-valuation and low-valuation testers, increases at an even higher speed. Due to the large number of testers, the optimal testing duration decreases significantly, and the sale price increases due to network effects. As a result of a shorter testing duration, quicker speed of diffusion, and higher sale price, the net profit increases monotonically with a larger  $\delta$  value. This is expected because all else being equal, more low-valuation public beta testers is always more beneficial. Therefore, unless it is very costly to target them, a firm should try to increase the proportion of low-valuation testers when recruiting public beta testers.

The parameter  $\lambda$  represents the public beta testers' bug detection efficiency. The sensitivity analysis results for  $\lambda$  are shown in Figure 6. We conclude from the figure that if bugs are easier to detect or if the beta testers are more efficient at bug detection, both implying a higher  $\lambda$ , the firm can recruit fewer high-valuation public beta testers and reduce the duration of public beta testing. The sale price decreases because fewer beta testers imply a smaller network size. However, because the firm can start selling the product sooner, and the loss of potential revenue due to free adoption by high-valuation public beta testers is smaller, the net profit increases.

Depending on how mission-critical a software product is, the cost of an undetected bug can vary greatly. In order to understand how sensitive the solution is to the unit cost of an undetected bug, we vary the value of  $c$  from \$6.25K to \$400K. The results are summarized in Figure 7. As expected, as the cost of undetected bugs increases, the optimal number of public beta testers also increase and testing lasts longer. With more testers admitted, the network size increases, and the firm can charge a higher price. However, because more testers lead to a

**Figure 7:** Impact of the unit cost of an undetected bug ( $c$ ).



larger loss of potential revenue, and software failures are costlier, the expected profit decreases.

**CONCLUDING REMARKS**

With the accessibility of the Internet and the expanding mobile application environment, public beta testing has gained tremendous popularity in the software industry. This study provides an in-depth analysis of both the reliability-related and the market-related benefits of public beta testing. We show that in addition to improved software quality, market-related benefits accrue because public beta testing can speed up product diffusion and increase potential users’ valuation of a software product.

Both the methodologies and findings of this study are of practical significance. The new software landscape with online access to software products requires a new way of thinking—specifically, closing the gap between marketing and development. The traditional model of develop-test-market will handicap firms in this new software era. Firms can also take advantage of public beta testing to improve their product quality and secure a better market position in an increasingly volatile and competitive marketplace. In addition to demonstrating the importance of public beta testing, the models developed in this study can help firms make informed public beta testing decisions under different scenarios.

Another novel aspect of this study is that it draws on theories and models from three distinct bodies of literature: the software reliability literature from software engineering, the diffusion literature from marketing, and the network effects literature from economics. Taken together, these widely accepted theories/models allow us to illustrate the benefits of public beta testing. They also form the theoretical foundation for future studies on public beta testing.

There exist several interesting future research directions. For instance, we currently assume that all public beta testers will receive an unrestricted version of the tested software product. It would be interesting to analyze how the optimal solution is affected if the beta version can only be used for a limited period of time. Similarly, in this study we do not model consumers’ strategic behaviors in response to public beta testing. In a future study, it may be possible to take into consideration consumers’ intentions to maximize their own utility and examine the

subsequent impact on firms' optimal public beta testing policy. In yet another direction, one could incorporate software versioning and price discrimination into our model, examine how the different segments of the market are affected by network effects, and then derive the optimal public testing policy for this more complex scenario.

## SUPPORTING INFORMATION

Additional supporting information may be found in the online version of this article at the publisher's website:

### Online Supplements

## REFERENCES

- Amin, A., Grunske, L., & Colman, A. (2013). An approach to software reliability prediction based on time series modeling. *Journal of Systems and Software*, 86(7), 1923–1932.
- Aral, S., Muchnik, L., & Sundararajan, A. (2013). Engineering social contagions: Optimal network seeding in the presence of homophily. *Network Science*, 1(2), 125–153.
- August, T., & Niculescu, M. F. (2013). The influence of software process maturity and customer error reporting on software release and pricing. *Management Science*, 59(12), 2702–2726.
- Awad, N. F., & Ragowsky, A. (2008). Establishing trust in electronic commerce through online word of mouth: An examination across genders. *Journal of Management Information Systems*, 24(4), 101–121.
- Baskerville, R., Levine, L., Pries-Heje, J., Ramesh, B., & Slaughter, S. (2001). How Internet software companies negotiate quality. *Computer*, 34(5), 51–57.
- Bass, F. M. (1969). New product growth for model consumer durables. *Management Science*, 15(5), 215–227.
- Bass, F. M., Gordon, K., Ferguson, T. L., & Githens, M. L. (2001). DIRECTV: Forecasting diffusion of a new technology prior to product launch. *Interfaces*, 31(3) S82–S93.
- Bayus, B. L. (1993). High-definition television—Assessing demand forecasts for a next-generation consumer durable. *Management Science*, 39(11), 1319–1333.
- Briscoe, B., Odlyzko, A., & Tilly, B. (2006). Metcalfe's law is wrong. *IEEE Spectrum*, 43(7), 34–39.
- Brynjolfsson, E., & Kemerer, C. F. (1996). Network externalities in microcomputer software: An econometric analysis of the spreadsheet market. *Management Science*, 42(12), 1627–1647.
- Centercode for Public Betas. (2015). Retrieved October 24, 2015, from <http://www.centercode.com/beta/tests/public/>.

- Cohen, M. A., Eliashberg, J., & Ho, T. H. (1996). New product development: The performance and time-to-market tradeoff. *Management Science*, 42(2), 173–186.
- Dalal, S. R., & Mallows, C. L. (1988). When should one stop testing software. *Journal of the American Statistical Association*, 83(403), 872–879.
- Dolan, R. J., & Matthews, J. M. (1993). Maximizing the utility of customer product testing—Beta-test design and management. *Journal of Product Innovation Management*, 10(4), 318–330.
- Dou, Y., Niculescu, M. F., & Wu, D. J. (2013). Engineering optimal network effects via social media features and seeding in markets for digital goods and services. *Information Systems Research*, 24(1), 164–185.
- Economides, N. (1996). The economics of networks. *International Journal of Industrial Organization*, 14(6), 673–699.
- Ehrlich, W., Prasanna, B., Stampfel, J., & Wu, J. (1993). Determining the cost of a stop-test decision. *IEEE Software*, 10(2), 33–42.
- Gallaugh, J. M., & Wang, Y.-M. (1999). Network effects and the impact of free goods: An analysis of the Web server market. *International Journal of Electronic Commerce*, 3(4), 67–88.
- Gilder, G. (1993). Metcalfe's law and legacy, *Forbes*, v152, no.n6, 1993 Sept 13, pS158(9).
- Gillin, P., & Schwartzman, E. (2010). *Social marketing to the business customer: Listen to your B2B Market, generate major account leads, and build client relationships*. Hoboken, NJ: John Wiley & Sons, Inc.
- Givon, M., Mahajan, V., & Muller, E. (1995). Software piracy: Estimation of lost sales and the impact on software diffusion. *The Journal of Marketing*, 59(1), 29–37.
- Goel, A. L., & Okumoto, K. (1979). Time-dependent error-detection rate model for software reliability and other performance-measures. *IEEE Transactions on Reliability*, 28(3), 206–211.
- Gul, W. (2014). Secret Formula to Turn Beta Testing into a Viral Success for Startups. Retrieved October 25, 2015, from <http://www.cloudways.com/blog/beta-testing-for-startups-viral-marketing-success/>.
- Haruvy, E., & Prasad, A. (1998). Optimal product strategies in the presence of network externalities. *Information Economics and Policy*, 10(4), 489–499.
- iBetaTest.com (2015). iBetaTest.com, (available at [ibetatest.com](http://ibetatest.com)).
- Ji, Y., Kumar, S., Mookerjee, V. S., Sethi, S. P., & Yeh, D. (2011). Optimal enhancement and lifetime of software systems: A control theoretic analysis. *Production and Operations Management*, 20(6), 889–904.
- Jiang, Z. (2010). How to give away software with successive versions. *Decision Support Systems*, 49(4), 430–441.
- Jiang, Z., & Sarkar, S. (2010). Speed matters: The role of free software offer in software diffusion. *Journal of Management Information Systems*, 26(3), 207–239.



- Jiang, Z., Sarkar, S., & Jacob, V. S. (2012). Post-release testing and software release policy for enterprise-level systems. *Information Systems Research*, 23(3), 635–657.
- Katz, M. L., & Shapiro, C. (1985). Network externalities, competition, and compatibility. *American Economic Review*, 75(3), 424–440.
- Kauffman, R. J., McAndrews, J., & Wang, Y. M. (2000). Opening the "black box" of network externalities in network adoption. *Information Systems Research*, 11(1), 61–82.
- Khan, S. (2008). Building a Better Beta. Retrieved October 25, 2015, from <http://pragmaticmarketing.com/resources/building-a-better-beta?p=0>.
- Meyer, D. (2009). Google to open Wave for public beta, ZDNet, accessed July 23, 2011, available at <http://www.zdnet.com/news/google-to-open-wave-for-public-beta/323894>.
- Nascimento, F., & Vanhonacker, W. R. (1988). Optimal strategic pricing of reproducible consumer products. *Management Science*, 34(8), 921–937.
- Pham, H. (2006). *System software reliability*. Berlin; London: Springer.
- Riofrio, M. (2011). Google cloud print beta is limited but evolving, *PC World*. Feb 2, 2011, available at [http://www.pcworld.com/article/218573/google\\_cloud\\_print\\_review.html](http://www.pcworld.com/article/218573/google_cloud_print_review.html).
- Saloner, G., & Shepard, A. (1995). Adoption of technologies with network effects—An empirical-examination of the adoption of automated teller machines. *RAND Journal of Economics*, 26(3), 479–501.
- Sultan, F., Farley, J. U., & Lehmann, D. R. (1990). A meta-analysis of applications of diffusion models. *Journal of Marketing Research*, 27(1), 70–77.
- Swann, G. M. P. (2002). The functional form of network effects. *Information Economics and Policy*, 14(3), 417–429.
- Teng, J. T., Grover, V., & Guttler, W. (2002). Information technology innovations: General diffusion patterns and its relationships to innovation characteristics. *IEEE Transactions on Engineering Management*, 49(1), 13–27.
- Wiper, M. P., & Wilson, S. P. (2006). A Bayesian analysis of beta testing. *Test*, 15(1), 227–255.
- Wood, A. (1996). Predicting software reliability. *Computer*, 29(11), 69–77.

**Zhengruì Jiang** is an associate professor of information systems and the inaugural holder of the endowed Thome Professorship in Business at the College of Business, Iowa State University. He received his PhD in Management Science from the University of Texas at Dallas. His research projects cover broad areas including technical (design science) IS, economics of IT, and quantitative marketing. His primary research interests lie in business intelligence/analytics, diffusion of innovations, economics of information goods, software engineering, and project management. His research has been funded by the U.S. Agency for International Development (USAID) and National Natural Science Foundation of China (NFSC). He has published in leading academic journals including *Management Science*, *Information Systems Research*, *IEEE Transactions on Knowledge and*

*Data Engineering*, and *Journal of Management Information Systems*. He currently serves as an associate editor for *MIS Quarterly*, and is serving or has served as special issue editors for *Production and Operations Management* and *IEEE Transactions on Management Information Systems*. In addition, he served as program co-chairs for the 2014 Midwest Association of Information Systems Conference and the 2015 Big XII+ MIS Research Symposium.

**Kevin Scheibe** is an associate professor of management information systems. He has been at Iowa State University since 2003. His research interests include IT privacy and security, supply chain risk, supply chain analytics, decision support systems, and wireless telecommunications. He is a member of the Association for Information Systems and the Decision Sciences Institute. Dr. Scheibe has published in journals such as *Decision Sciences Journal*, *European Journal of Operations Research*, *Decision Support Systems*, *Journal of Information Privacy and Security*, and *Computers in Human Behavior*. He serves as an Associate Editor at *Decision Sciences Journal*. He received a PhD from Virginia Polytechnic Institute and State University.

**Sree Nilakanta** is an associate professor of management information systems at Iowa State University. He received his MBA and PhD in Information Systems from University of Houston. Dr. Nilakanta also holds a BS in Mechanical Engineering from Madras University. Dr. Nilakanta's research straddles both behavioral and technical domains of information systems. His current research interests are in privacy and security, organizational memory and knowledge management, and technology innovation. His research has appeared in *Management Science*, *Journal of Management Information Systems*, *Decision Support Systems*, *Omega*, *Information & Management*, *Journal Software and Information Technology*, *Journal of Strategic Information Systems*, and others. His current research focuses on applying analytics to areas of business processes and metadata. Dr. Nilakanta is the Director of the recently launched Master of Business Analytics program. Prior to that he was the Chair of the Supply Chain and Information Systems Department.

**Xinxue Qu** is a PhD student of management information system at the College of Business, Iowa State University. His primary research interest is business intelligence/analytics. He has presented his research at conferences including the Big XII+ MIS Research Symposium and the Workshop on Information Technology and Systems. He is a member of Association of Information Systems.