ELECTROMECHANICAL MODELING OF ULTRASONIC TRANSDUCERS

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INTRODUCTION

A general model of an ultrasonic flaw measurement system can be developed using the fundamental reciprocity formulation of Auld [1]. This general model can be reduced to a more explicit form by assuming that the waves incident on the flaw are quasi-plane waves, resulting in the Thompson-Gray measurement model [2]. In the Thompson-Gray model, the frequency components of received voltage, $V_0(\omega)$, in a pitch-catch immersion setup can be written in a product fashion as

$$V_0(\omega) = \beta(\omega)P(\omega)M(\omega)T_1(\omega)C_1(\omega)A(\omega)T_2(\omega)C_2(\omega) \tag{1}$$

where $P(\omega)$ accounts for the time delay in going from the transmitting transducer to the receiving transducer, $M(\omega)$ is due to the material attenuation, $T_1(\omega)$ and $T_2(\omega)$ are transmission terms that characterize the amplitude changes when going through the fluid-solid interfaces on transmission and reception, respectively, $C_1(\omega)$ and $C_2(\omega)$ are diffraction correction terms that account for the finite beam characteristics of the transducers on transmission and reception, and $A(\omega)$ is the far field scattering amplitude of the flaw. The term $\beta(\omega)$ is an "efficiency factor" that is a function of the electrical properties of the pulser/receiver, the associated cabling, and the ultrasonic transducers. Thus, $\beta(\omega)$ accounts for all the electrical to mechanical and mechanical to electrical conversion processes that contribute to the entire measurement process.

In the use of Eq. (1), it has been possible to obtain explicit models (either analytical or numerical) for all the terms that appear in that equation except for the attenuation term, $M(\omega)$, and the efficiency factor, $\beta(\omega)$, both of which can be determined experimentally in well characterized reference experiments [3]. Although this is a viable approach, obtaining $\beta(\omega)$ in particular in this fashion does not allow one to predict the effects of changes of parameters such as transducers, cables, or pulser/receiver characteristics and settings. Thus, it is desirable to also model explicitly the components that make up the $\beta(\omega)$ factor as well. Two important parts of $\beta(\omega)$ are the properties of the transmitting and receiving transducers.

Here, we will describe some explicit models of these components of $\beta(\omega)$ and demonstrate with an elementary example of how such transducer models can be used to predict the changes seen in a measured signal due to elements contained in $\beta(\omega)$.

LUMPED PARAMETERS IN THE MASON MODEL

As the basis for our transducer modeling, we will use the well-known Mason model [4], which treats the transducer as an equivalent three port electromechanical device containing an electrical port and two mechanical ports that correspond to the backing and radiation faces of a piezoelectric crystal [5]. If the backing conditions are specified by connecting the backing port terminals to a specific impedance then the three port Mason model can be reduced to a two port transmission line model instead, and the electrical and radiation ports are related through a "ABCD" matrix, i.e.

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \tag{2}$$

where (V_3, I_3) are the voltage and current at the electrical port and (F_1, v_1) are the force and velocity at the acoustic output port. Two port models such as given by Eq. (2) are entirely 1-D in nature, i.e. they deal with only "lumped parameters" such as (V_3, I_3) and (F_1, v_1). However, an ultrasonic transducer generates a spatially varying distribution of pressure and velocity over the face of the transducer, so in the use of 1-D models, the question arises as to how the force and velocity terms appearing in Eq. (2) are related to the underlying distributed parameters. To answer this question, consider, for example an immersion transducer. As done by Auld, one can use electromechanical reciprocity to relate the electrical and acoustic ports of the transducer, giving, for two different solutions *a* and *b*:

$$V_3^a I_3^b - V_3^b I_3^a = \int_{S_T} \left\{ p^a(\mathbf{x}_s,\omega) v_n^b(\mathbf{x}_s,\omega) - p^b(\mathbf{x}_s,\omega) v_n^a(\mathbf{x}_s,\omega) \right\} dS(\mathbf{x}_s) \tag{3}$$

where we have used lumped parameters on the electrical port but left the acoustical port as a general two-dimensional distribution of pressure and velocity over some active area, S_T, of the transducer face. For immersion probes, it is customary to treat the transducer as a velocity source having a specific (frequency independent) spatial distribution. For example, given a distribution function, $f(\mathbf{x}_s)$, we could write

$$v_n(\mathbf{x}_s,\omega) = v_1(\omega) f(\mathbf{x}_s) \tag{4}$$

Once the velocity is specified in this manner, the pressure distribution, $p(\mathbf{x}_s,\omega)$, must be found by solving for the fields in the fluid for the specific problem under consideration. As Eq. (3) shows, however, once $p(\mathbf{x}_s,\omega)$ is found, we can define a "force" as a weighted integral of this pressure given by

$$F_1 = \int_{S_T} f(\mathbf{x}_s) p(\mathbf{x}_s,\omega) dS \tag{5}$$

so that Eq. (3) then reduces to an equivalent 1-D model where

$$V_3^a I_3^b - V_3^b I_3^a = F_1^a v_1^b - F_1^b v_1^a \tag{6}$$

For such a reciprocal system, it can be shown that (V_3, I_3) are related in a linear fashion to (F_1, v_1) through a ABCD transfer matrix of the form given in Eq. (2).

This discussion shows that when a transducer is treated mechanically as a velocity source of the type given by Eq. (4), the "force" appearing in a 1-D transducer model must be defined consistently in the form of Eq. (5). For example, for a circular planar transducer of radius a, where the weighting function, $f(\mathbf{x}_s)$, is taken as only a function of the radial distance, r, from the transducer center [6] as

$$f(\mathbf{x}_s) = \frac{(n+1)}{\pi a^2}\left(1 - \frac{r^2}{a^2}\right)^n \quad (n = 0,1,2) \tag{7}$$

the velocity, v_1, is then just the average velocity on the face of the transducer but the force as computed from Eqs. (5) and (7) is not just the average pressure multiplied by the transducer area except when $n = 0$ (piston transducer model).

RADIATION IMPEDANCE

The relationships described in the previous section are important when one wishes to use a 1-D transducer model such as Eq. (2) in a specific problem. Consider, for example, the case when we let the transducer radiate directly into an infinite fluid medium. Then at the acoustic output port

$$F_1(\omega) = Z_r(\omega) v_1(\omega) \tag{8}$$

where we will call the proportionality constant, $Z_r(\omega)$, the radiation impedance, where from Eqs. (5) and (8):

$$Z_r(\omega) = \frac{\int_{S_T} f(\mathbf{x}_s) p(\mathbf{x}_s, \omega) dS}{v_1(\omega)} \tag{9}$$

In many discussions of the use of such 1-D models, it is assumed that $Z_r(\omega)$ is given simply by the acoustical impedance of a 1-D traveling wave in the fluid, i.e.

$$Z_r = \rho c\, v_1\, S_T \tag{10}$$

where ρ and c are the density and wave speed, respectively, of the fluid and S_T is the active area of the transducer. However, a transducer does not put out purely a 1-D plane wave and rigorously $Z_r(\omega)$ must be computed from Eq. (9) instead. Thus, the question arises as to the validity of Eq. (10). Fortunately, Greenspan [6] has calculated such integrals of the weighted pressure for the three cases described by Eq. (7). For $n = 0$, we have, as mentioned previously, a piston model, whereas $n = 1$ corresponds to a "simply-supported" model, and $n = 2$ a "clamped" model [6]. Figure 1 shows a result of calculating the normalized radiation impedance versus non dimensional frequency, ka, in these three cases, where $Z_r(\omega)$ has been normalized by a factor $\rho c V S_T$, where the velocity, V, is given in terms of the average velocity, v_1, as $V = v_1$ $(n = 0)$, $V = 4v_1/3$ $(n = 1)$, $V = 9v_1/5$ $(n = 2)$, so that all the curves asymptote to the same value of one at high frequencies. As Fig. 1 shows, for ka larger than approximately 10, all three cases reduce to a constant equivalent plane wave value,

$$Z_r = \rho c V S_T. \tag{11}$$

Since typical NDE transducers radiating into water often have ka values of 100 or greater, for such cases it could be expected that using the equivalent plane wave value will be an excellent approximation. Simulations performed by connecting the transducer model to a

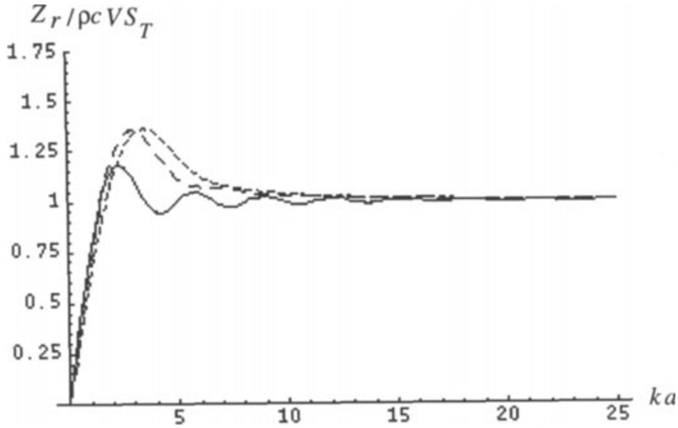

Figure 1. Normalized radiation impedance vs. non-dimensional wave number *ka* for $n = 0$ (solid line), $n = 1$ (long dashes), $n = 2$ (short dashes).

voltage source (taken as a decreasing exponential to represent the discharge from a pulser) have demonstrated that this indeed is the case, and that differences in the output wave form due to the use of Eq. (9) as opposed to Eq. (11) for the radiation impedance were minor except for simulations of very low frequency (< 1 MHz) transducers radiating into solids such as steel or aluminum. Ristic [5] has noted that the equivalent plane wave values are good approximations for piston transducers, so that our results demonstrate that this is also true for the non-uniform velocity profiles of the simply supported and clamped cases as well. However, note that except in the piston case, the equivalent plane wave velocity, V, that appears in Eq. (11) is not the average velocity on the face of the transducer.

USE OF TRANSDUCER MODELS IN A MEASUREMENT MODEL

Consider the problem shown in Fig. 2 where a sending transducer is driven by a voltage source, $V_s(\omega)$, and radiates into a fluid to a coaxial receiving transducer. We will use this simple case to demonstrate how having an explicit model of the efficiency factor, $\beta(\omega)$, can allow one to quantitatively examine the effects of the electrical and electromechanical parts of a measurement model in the same fashion as can be done with the purely mechanical terms.

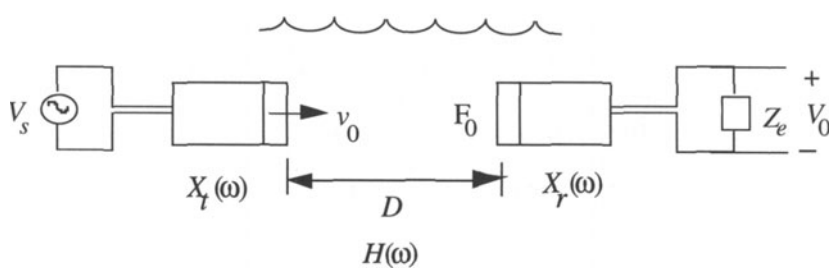

Figure 2. A simple ultrasonic setup where all the elements are modeled explicitly.

If the radiation impedance of a transmitting transducer is given as Z_r and the transducer is connected directly to the specified voltage source, V_s, as shown in Fig. 2, then $V_3 = V_s(\omega)$ and the two port model for the transmitter reduces to an explicit relationship between the output velocity of the transducer, $v_1 = v_0(\omega)$, and the voltage source. For the Mason model, this relationship is, explicitly,

$$v_0(\omega) = X_t(\omega)V_s(\omega) \tag{12}$$

where

$$X_t(\omega) = \frac{n\left(Z_b + jZ_0\tan\frac{kl}{2}\right)}{Z_rZ_b + Z_0^2\left(1 - \frac{2n^2}{\omega C_0 Z_0}\tan\frac{kl}{2}\right) - j(Z_b + Z_r)\left(Z_0\cot kl - \frac{n^2}{\omega C_0}\right)} \tag{13}$$

with Z_b being the acoustic impedance of the backing, l is the thickness of the piezoelectric crystal, Z_0 its acoustic impedance, C_0 is the clamped capacitance of the transducer, and $n = h_{33}C_0$, where h_{33} is the piezoelectric stiffness of the crystal. Similarly, for the receiving transducer, one can connect an electrical impedance, Z_e, to the output electrical port and obtain an expression for the output voltage, $V_0(\omega)$, as a function of the total force, $F_0(\omega)$, on the face of the receiving acoustic port:

$$V_0(\omega) = X_r(\omega)F_0(\omega) \tag{14}$$

where, analogous to Eq. (13), X_r is a function of the properties of the fluid and the electromechanical properties of the receiving transducer. X_r is also a function of the impedance, Z_e. Since X_r is a rather complex expression, because of space limitations we will not give it explicitly here.

If we neglect the attenuation of the fluid, for this problem, it is also possible to obtain an explicit analytical expression for the relationship between F_0 and v_0 in terms of a "diffraction correction integral", D_p, i.e. [7]

$$\frac{F_0}{v_0} = H(\omega) = Z_rD_p\left(ka^2 / D\right) \tag{15}$$

where

$$D_p(x) = 1 - \exp(-jx)\left[J_0(x) + jJ_1(x)\right] \tag{16}$$

in terms of Bessel functions J_0 and J_1. Combining all these results, it is possible to obtain a complete measurement model for this problem as

$$V_0(\omega) = \beta(\omega)H(\omega) \tag{17}$$

which is analogous to the Thompson-Gray measurement model form (Eq. (1)), but where now the efficiency factor, $\beta(\omega)$, is given explicitly as

$$\beta(\omega) = X_t(\omega)X_r(\omega)V_s(\omega) \tag{18}$$

When a mechanical parameter such as the distance, D, between the two transducers is changed, this has a profound effect on the form of the measured response. In fact, by examining the behavior of D_p, it can be easily found that

$$\begin{aligned} D_p(x) &\cong 1 \qquad (D \ll a) \\ D_p(x) &\cong jx/2 \quad (D \gg a) \end{aligned} \tag{19}$$

Equation (19) shows that the effect of increasing the distance D significantly is equivalent to a differentiation process on the received time domain signal. This can be seen in Fig. 3 where a received voltage is simulated for the two cases where the transducers are close together (D = 1 mm) and far apart (D = 1 m), for an input voltage again taken as a

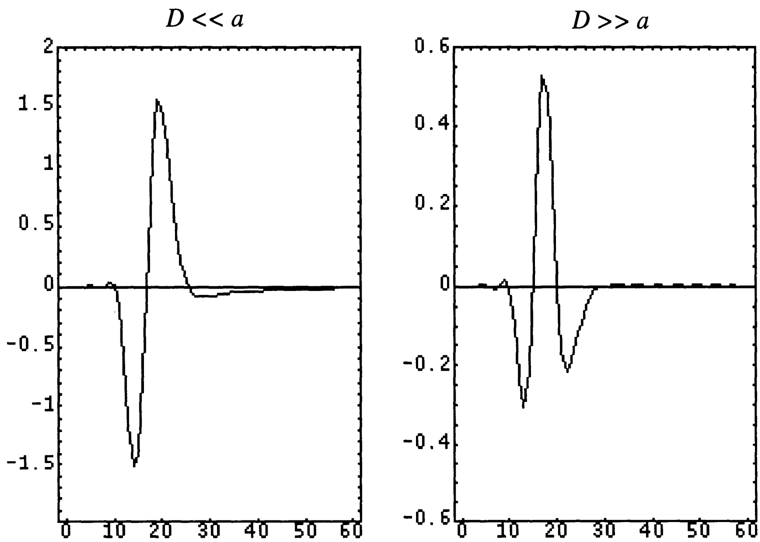

Figure 3. Received waveforms for the cases when 1) the distance $D \ll a$ and 2) $D \gg a$.

decreasing exponential pulse to simulate a typical pulser output. However, a similar behavior also can be observed by keeping D fixed and varying the electrical impedance at the receiver. By examining Eq. (17) in detail, one can show that

$$\begin{aligned} V_0(\omega) &\cong E(\omega) \qquad (Z_e \to \infty) \\ V_0(\omega) &\cong j\omega C_0 Z_e E(\omega) \quad (Z_e \to 0) \end{aligned} \tag{20}$$

so that the effect of reducing the output impedance at the receiver is also equivalent to a differentiation process. This can be seen in Fig. 4 where the output pulse changes from a bipolar form to a (differentiated) tripolar form as Z_e goes from $Z_e = 100\,k\Omega$ to $Z_e = 1\,\Omega$.

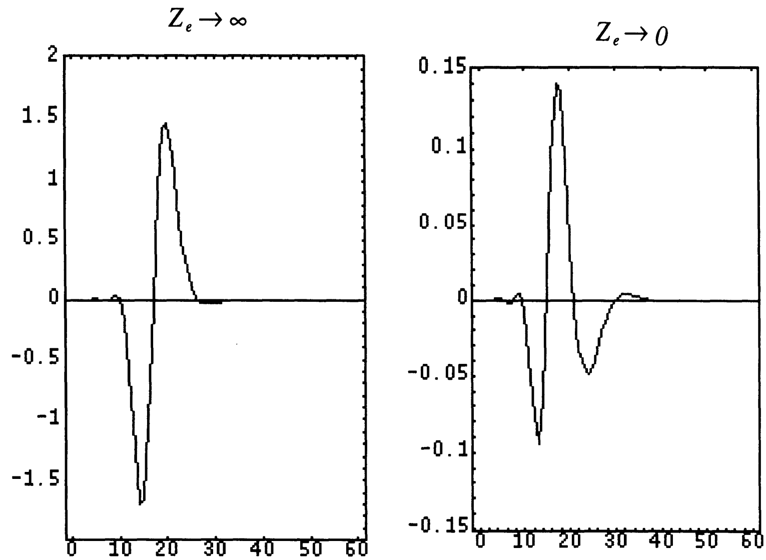

Figure 4. Received waveforms for the cases when 1) $Z_e \to \infty$, and 2) $Z_e \to 0$.

CONCLUSIONS

We have incorporated an electromechanical (Mason) model of an ultrasonic transducer into an ultrasonic measurement model so as to obtain an explicit expression for some of the important components that make up the efficiency factor. It has been shown that such 1-D electromechanical models can be rationally joined with the spatially varying fields generated at the acoustic output (or input) port of the transducer provided that 1-D lumped parameters of force and velocity are defined appropriately in terms of those spatially varying fields. At the transmitting transducer, it has been shown that except for very low frequency transducers radiating into a solid, the radiation impedance can be taken as simply an equivalent plane wave value.

Finally, we have shown through a simple example that by defining the transducers and other electrical parts of the measurement process explicitly, one can form up a complete measurement model where changes in the electrical components of the measurement setup can be examined in the same manner as done for the mechanical components. As shown, electrical characteristics, such as the output impedance of the receiver, can have a significant effect on the form of the measured output, and hence must be accounted for in any quantitative simulation of the entire measurement process.

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