CONSUMER'S CHOICE IN INSURANCE

by

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1950
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I. INTRODUCTION

The treatment of life insurance as a problem in "rational" behavior in this paper focuses attention on a narrow sector of the whole problem of insurance. The suggested approach to the problem is not presented as the only proper one, but as one of the possible approaches.

In the material which has been written on life insurance there has been a large amount of thought devoted to the problems concerned with the selling of risk protection. Most of the literature primarily concerned with the problems of buying risk protection is devoted to presenting descriptive information on the many variations in types and kinds of insurance which have been developed to suit varying financial problems. Very little has been written on how to decide on how much insurance a person should buy.

One reason why there is a bewildering number of kinds of insurance is that risk protection has been sold in combination with a large number of different ways to save money. If the head of the household is interested mainly in protecting himself and his family against
financial loss, many authorities, such as Gilbert,\(^1\) Stewart,\(^2\) and the authors of the publications of the Temporary National Economic Committee,\(^3\) suggest separating this protection from the savings program.

The logic of this separation is based on the assumption that the individual may wish to buy protection even though his income may be reduced to the point where he no longer chooses to save. If the protection and savings programs are combined, he must continue to save in order to be protected; or, if he chooses not to save, he is left facing the possibility of financial losses without the protection he wants to have.

Having decided that, according to his preferences, it is better to buy life insurance not combined with a savings program, (such as renewable, term insurance), the consumer may have difficulty in finding it available in the market. He may have to buy a type with a relatively small amount of savings, such as straight life.

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If he does find renewable, term insurance available, he is still faced with the problem of whether to buy any insurance at all; and if any — how much. Advice on this problem of the amount to buy consists either of giving tables of recent figures on how much insurance has been purchased by families in various income levels\(^1\), or of giving tables showing the percentages of the income which the "average family" devotes to various budget classifications, including "savings and life insurance."

Life insurance is a means of providing a burial fund or protection for dependents against possible financial loss due to the death of the income producer. If the consumer has no one dependent on him for income, and if he has adequate assets for burial costs, then he does not need risk protection, since the possibility of his death involves no financial risk. If the consumer does have financial dependents he must then decide whether he can "afford" (i.e. whether he prefers) to spend for possible future needs in contrast to spending for present needs for himself and his dependents.

If a person wishes to maximize the satisfactions in goods and services obtainable from a limited purchasing power, he can not do it by consulting figures showing how other people prefer to spend their money. These may help him in his thinking, but actually he can find maximum satisfaction only by choosing the items which he finds most satisfying. When he knows, or thinks he knows, what his needs will be, an additional complication presents itself in buying insurance. The choice for many goods and services is not complicated by a lack of knowledge about the future. The choice in buying life insurance is not so simple. It is complicated by the individual's inability to say what his needs will be, since they depend on what events occur in the future.

The development of a general theory of rational behavior in decisions involving partial knowledge, or ignorance, concerning what will happen has been presented by Von Neumann and Morgenstern. The application of their theory to decisions on whether to purchase life insurance, and how much to buy, creates a number of

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theoretical and practical problems. These problems are presented and discussed in the following pages.

The basic problems which enter into the formulation of a rational decision on insurance are: the selection of a principle of decision-making which will satisfy the consumer's requirements concerning the results of the indicated choice; the extent to which the consumer is able to compare the utilities in each possible outcome from a choice, and the effect of the amount of knowledge concerning the occurrence of the events which determine the outcomes of the alternative choices.
II. CLASSIFICATION OF THE PROBLEM OF SECURING
RISK PROTECTION AS ONE OF "RATIONAL"
DECISION-MAKING.

A. The meaning of "rational"
Deciding on insurance is treated in this paper
as a behavior problem. A behavior problem may be
treated either as a descriptive problem (how people
behave) or a normative problem (how people "should"
behave, i.e., what is "rational").

While the buyer of life insurance generally con-
tracts for protection against possible financial loss
(term insurance) and for an investment program in one
policy, the decision is here separated into two parts.
The procedure for arriving at a "rational" decision
on whether to buy term insurance, and how much to
purchase is the subject to be given consideration. No
consideration of the investment program is presented
here.

It is not assumed in this discussion of choosing
insurance that all people who buy insurance do so in
a "rational" way, or that they wish to make the de-
cision "rationally". "Irrational" choice-making oc-
curs in all areas of selection. Whether or not con-
sumption occurs because of "rational" or "irrational"
decisions is a problem in determining how people behave. The primary considerations here are the factors involved if the consumer does want to choose "rationally".

"Rationality" will be used to mean that the choice has certain characteristics, especially transitivity and consistency, in relation to the individual's preference system. If the choice has transitivity, it means that the ordering of preference is such that if A is judged to have greater magnitude in terms of utility preference than B, and B has a greater magnitude than C, then A must have a greater magnitude than C.

The term "preference" is used in its wider meaning which includes the case of indifference. This preference may be symbolically written:

\[
A > B \\
B > C \\
A > B > C
\]

Consistency means that with the given factors unchanged in deciding on the ordering of A, B, and C, the repeated orderings of A, B, and C will remain unchanged.
Arrow\(^1\) has shown that while individual preferences systems can be consistent, when they are indicated by voting in a group choice they may show inconsistencies.

For any method of deriving social choices by aggregating individual preference patterns which satisfies certain natural conditions, it is possible to find individual preference patterns which give rise to a social choice pattern which is not a linear ordering.\(^2\)

B. Factors in decision-making

In making a decision there may be factors outside of the individual's control ("external events") as well as those under his control. If all the factors are controllable, the problem is simplified to that of deciding which outcomes are preferable.

By setting up the problem in a table the discussion of the possible solutions and outcomes is more easily presented.

\(^1\) Arrow, K. J. A difficulty in the concept of social welfare, Journal of Political Economy, 58:328-346, 1950

\(^2\) Ibid., p. 330
TABLE 1

<table>
<thead>
<tr>
<th></th>
<th>Event I</th>
<th>Event II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Choice 2</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

Table 1 shows two choices in the rows, and two external events which may occur in the columns, with the gains (or losses) in utility resulting from each of the four possible combinations indicated by A, B, C and D. The letters and their positions in the tables will remain constant throughout the paper.

C. Requirements for decision-making

In order to make a rational decision the individual must be able to state what the alternative choices and external events are, and what his knowledge is concerning the probability of the occurrence of these external events. He must then be able to assign a value to each outcome in terms of his estimation of its utility. The assignment of magnitude of the gain (or loss) for each outcome is not considered in terms of money, but in terms of utility or satisfactions. The relative utility is the pertinent information, not the money values, since
with a limited budget in choosing between two items which may be purchased for the same price, one item may have greater, less, or equal potential satisfactions compared to another item. When A has a greater magnitude than B, this means that the gain (or loss) of satisfaction is estimated to be greater in A than that in B.

D- Principles of decision-making

The first principle of decision-making suggested for use is the maximization of the minimum average utility or satisfaction. The long-run average is meant whenever the term "average" is used in this paper. This principle is not one designed to guaranty to the person making the choice that each time he makes the decision the utility will be the greatest. Rather than this, the principle is designed to guaranty that the minimum of the average utility will be the greatest over a period involving many repetitions of the choice. It will be shown how variations of this principle apply in each case discussed.

In those cases in which there is no knowledge of the occurrence of external events, the principle of minimizing the maximum average regret is suggested as
a possible alternative to maximizing the minimum average gain. The implications in its use will be discussed more fully in connection with the technique needed to determine a solution where there is no knowledge of the occurrence of external events.

This discussion is concerned with insurance so these principles of decision-making seem to be appropriate. If the person deciding wishes to gamble and feels that he cannot make any estimation of the probabilities (Case III in this thesis), other principles of choice would be more appropriate. These are described later. What principle of choice would be used by people in certain situations is an empirical question which can not be answered here.

Carnap\(^1\) presents a clear analysis of the problem of a "rule for determining decisions" in which he begins with the "rule of high probability" and progresses, step by step, to "Rule R\(_5\) Among the possible actions choose that one for which the estimate of the resulting utility is a maximum."\(^2\) In each step he makes a

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\(^2\) Ibid., p. 269
revision of the previous rule and shows by example how much more generally the revision of the previous rule may be applied. Then he points out the revised rule's limitations which indicate the need for further modification until he arrives at Rule R₅. This principle of decision-making is utilized for Case II of this thesis, and modified in Case III to meet the problem of deciding in those cases in which no estimate of the probabilities can be given.
III. CASES TO BE CONSIDERED, GROUPED
ACCORDING TO THE EXTENT OF KNOWLEDGE
OF THE OCCURRENCE OF EXTERNAL EVENTS
INFLUENCING THE OUTCOMES OF VARIOUS CHOICES.

The cases to be discussed may be grouped according
to the "type of knowledge" about the occurrence of
events determining the outcomes. (We have assumed that
the individual has been able to given an ordering to
the outcomes.) The "types of knowledge" are classified
into three cases according to the extent of the sub-
jective, a priori knowledge about the occurrence of
events. By using "subjective knowledge" the outcomes
are not defined by questions of fact, but by what the
individual believes to be true. The three cases are:

Case I- Decisions in which the individual knows
(or thinks he knows) which event will
occur.

Case II- Decisions in which the individual can
(or thinks he can) state the likelihood
of the occurrence (the probability) of
each of the "external events".

Case III- Decisions in which the individual be-
lieves he has no knowledge of the prob-
abilities of the occurrence of the
"external events".
A. Case I. Decisions in which the individual knows which event will occur.

1. Description of grouping.

In Case I the principle of choice is to maximize the utility. This means choosing the outcome with the maximum satisfaction, or the greatest utility. A person has a choice between two alternatives. If he knew that several different events might occur, but he got information that made him certain that Event I would occur, then he would make the choice in which the outcome was more satisfactory. Table 2 illustrates this decision.

2. Solution.

An ordering of the outcomes is necessary for a solution. That is, the individual must be able to say that outcome A is greater than C, or that outcome C is greater than A. The meaning of mathematical symbols as used here is the same as in a standard algebra textbook.\(^1\)

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TABLE 2

Gain

Ordering of outcomes
A > C

<table>
<thead>
<tr>
<th>Choice 1</th>
<th>Event I</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Choice 2</td>
<td>C</td>
</tr>
</tbody>
</table>

Solution: Select Choice 1

B. Case II. Decisions in which the individual knows the likelihood of the occurrence (the probability) of each of the "external events".

1. Description of grouping.

Case II contains those decisions in which the individual can state the likelihood (probability of the occurrence of each of the "external events". (The decisions in Case I could be considered to have an "external event" with the probability of one). In Case II the person does not know what will occur, but on the basis of his knowledge or experience is able to say that one of several events will certainly occur, and that each event has a specified
probability of occurring. For example, he can say that Event I will occur one-half the time, and Event II will occur the other half of the time. This is the case generally considered in discussions of economic risk.

There has been some confusion in the literature on probability concerning its meaning. Carnap differentiates the two basic concepts of probability as follows:

The two concepts are (1) probability = degree of confirmation; (ii) probability = relative frequency in the long run. Strictly speaking, there are two groups of concepts, since both for (i) and (ii), there is a classificatory, a comparative, and a quantitative concept; however, at the present moment, we may leave aside these distinctions. 1

He distinguishes further the concept of relative frequency by saying that it is an empirical concept.

This is not to be understood as saying that its definition refers to nonlogical concepts, which is obviously not the case, but merely as saying that its ordinary application, that is, its application to factual properties as arguments, is to be formulated in factual, empirical statements; in other words, the determination of its values in ordinary cases is an empirical procedure. 2

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1 Carnap, Rudolph. op.cit. p. 25
2 Ibid., p. 34
Keynes¹ basic work on probability is concerned with its meaning as the degree of confirmation.

Some of the confusion from the use of the same word for two basically different ideas comes from the fact that they have much in common.

In spite of the fundamental difference between the concepts of probability¹ and probability², many theorems concerning these concepts show a striking analogy. Later discussions will throw some light from various angles on the basis of this analogy. We shall see that in certain cases probability¹ may be interpreted as an estimate of the relative frequency or probability². Later, on the basis of an analysis of sentences with the help of their ranges, it will be seen that probability¹ can likewise be regarded as a ratio of the measures of two classes; but there remains the important difference that in this case the ratio is determined in a purely logical way, while in the case of probability² it is determined empirically.²

Tintner³ in discussing the contributions of Carnap's theory says: "It has been possible to construct a 'pure' theory of prediction, estimation,

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² Carnap, Rudolph, op.cit. p. 34
testing hypotheses and choice between statistical hypotheses which is independent of pragmatic considerations." It is the concept of degree of confirmation which is used in this paper.

For the purpose of discussing the solution, Case II can be divided into two parts, A and B. Case IL-A contains decisions in which the outcomes, with respect to each external event, from one choice are all respectively more satisfactory than the outcomes from any other choice. Case II-B contains decisions in which there is no choice as specified in Case IL-A, but which can be placed in Case II according to the "type of knowledge" of the likelihood of the occurrence of events.

2. Solutions indicated by variations in preferences.

In Case II the principle of choice is the maximizing of the average utility from each choice. The solution for Case IL-A requires only an ordering of the outcomes in terms of the individual's preferences, since the choice which has a greater utility in each outcome, respectively, will give the maximum average utility. The solution in Case IL-B requires that the individual can do more than order the outcome; he must
be able to give them a numerical value. That he can do so in the case of risk is based on some assumptions which have been presented by Von Neumann and Morgenstern\(^1\) and developed by Marschak.\(^2\)

3. **Solutions are dependent upon the measurement of utility.**

These assumptions stated by Marschak have been listed in an easily understood manner by Friedman and Savage\(^3\) as follows:

(1) The system is complete and consistent; that is, an individual can tell which of two objects he prefers or whether he is indifferent between them, and if he does not prefer C to B and does not prefer B to A, then he does not prefer C to A. (In this context, the word "object" includes combinations of objects with stated probabilities; for example, if A and B are objects, a 40-60 chance of A or B is also an object).

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(2) Any object which is a combination of other objects with stated probabilities is never preferred to every one of these other objects, nor is every one of them ever preferred to the combination.

(3) If the object A is preferred to the object B, and B to the object C, there will be some probability combination of A and C such that the individual is indifferent between it and B.

With these assumptions and the discussion by Hurwicz\(^1\) and Stone\(^2\) of the main contributions to economic theory of Von Neumann and Morgenstern, a person with limited mathematical training can acquire a clear understanding of the basis for the theory of measurable utility.

The idea of measurable utility has been given considerable attention in the development of economic theory. Early writers on the subject tried to set up a common unit of measurement, which would measure the amount of utility of each useful object.\(^3\) However, the difficulties in defining

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the unit of measurement so that it would be possible
to make interpersonal comparisons of the utility val-
ue assigned to the same object caused many theorists
to conclude, until recently, that utility could not
be given a numerical evaluation.

In welfare economics such interpersonal compar-
isons of utility are important. However, in the devel-
opment of economic theory on consumer choice, the need
for exact interpersonal comparisons of utility is less
urgent than the need for an exact evaluation of the
utilities of alternative acquisitions by one person or
one economic unit. This numerical evaluation does not
indicate a fixed value assignable to objects as they
become available. Rather than this, the numerical
evaluation indicates changes in the level of "happiness"
or "state of satisfaction" in which an individual be-
lieves he would be, if he made one of two or more
choices. The unit of satisfaction does not require
exact definition except in the mind of the person de-
ciding. That is, he needs to be able to say that one
object has twice the utility of the other. Bishop¹
states:

¹ Bishop, Robert L. Consumer's surplus and cardinal
utility, Quarterly Journal of Economics, 57:421-449,
1943.
This manner of choosing a utility unit is no more arbitrary than the decision in choosing any unit of measurement. For example, the yard is the length of a standard bar of specified composition at a certain temperature. The utility unit, however, has the distinctive feature that it is purely subjective, it is not necessarily the marginal utility of B when anyone consumes that amount. If a different unit of utility had been chosen, the resulting utility curve would have differed from our curve only by a scalar constant; just as a yard differs from a foot.

Samuelson states that the removal of these "introspective, psychological elements" from the concept of utility may cause the remainder to be meaningless empirically.¹

There is a vital need for realizing the differences between utility considerations and monetary considerations.² Vickrey³ appears to mix these two factors in the decision problem when he states:

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Considerations other than the maximization of the mathematical expectation of utility enter the picture. Insurance may be taken out because it is the "sound" thing to do, without much thought as to the net cost. Or insurance may be purchased to avoid the responsibility for taking precaution against the casualty involved. Jewelry, for example, is often insured against theft or loss not so much because the loss would seriously affect the economic status of the owner, but rather to avoid the worry which the possibility of loss would otherwise cause. Indeed, it is probably true to say that if the jewelry is so valuable in relation to the income of the owner that insurance would be justified on the grounds of maximizing the expectation of utility alone, then the ownership of the jewelry is in itself an unwise extravagance.

If the jewelry is insured rationally, the utility of the money used for the insurance premium, rather than for any other purchase, results in a maximization of the expected utility. The monetary considerations Vickrey mentions enter into the utility estimations whether or not the money value of the jewelry is high in comparison with the income of the person owning it.

To make this case of risk and the need of numerical evaluation clearer, consider a case in which one of two possible events must occur. The person choosing
knows that Event I will occur (1/16) of the time. He knows that he can assign the following ordering to each outcome of the four possible combinations of events and choices. In this illustration, and in the following ones, the outcomes will be indications of dissatisfactions since the insurance problem is one mainly concerned with losses. In insurance the application of the principle of maximizing the minimum average satisfaction would mean minimizing the maximum average losses.

### TABLE 3

**Loss**

Ordering of A, B, C, D:

\[ A > C > D > B \]

<table>
<thead>
<tr>
<th>Event I (1/16)</th>
<th>Event II (15/16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>A</td>
</tr>
<tr>
<td>Choice 2</td>
<td>C</td>
</tr>
</tbody>
</table>

Solution (non-numerical utilities):

Choice 2.

The person choosing can say that in terms of dissatisfaction that A is greater than B, that C is greater than D, that D is greater than B, and that A is greater than C, and therefore that \( A > C > D > B \). Unless he
can say how much greater numerically, he can not arrive at a method of deciding which to choose, in the light of his knowledge of probabilities, which will give him the least possible average loss. Since Choice 1 contains the greatest loss (A), he might always choose Choice 2 to try to minimize his average loss. However, if he knows that Event I will occur during only one of the sixteen periods, he would certainly not want to select Choice 2 every time in order to avoid a maximum loss (A) unless it is relatively very large. It will be shown that the point where (A) is large enough to change his decision to always choosing 2 can be determined only through the assignment of numerical values to the outcomes.

Without a numerical evaluation of the outcomes the person deciding cannot tell which choice will give him the maximum average satisfaction. According to Friedman this idea of a numerical evaluation with attempts to maximize the average utility has been considered by writers on economic problems for a long time.

The idea that choices among alternatives involving risk can be explained by the maximization of expected utility is ancient, dating back at least to D. Bernoulli's celebrated analysis of the
St. Petersburg paradox. It has been repeatedly referred to since then but almost invariably rejected as the correct explanation - commonly because the prevailing belief in diminishing marginal utility made it appear that the existence of gambling could not be so explained.

Bernoulli's analysis is especially interesting since it is the only early use of calculus in economics which is generally known.

Friedman and Savage have shown that gambling and willingness to pay for insurance, which may appear to be irrational if engaged in by the same person, can be explained by an extended analysis of the total utility curve.

Houthakker is another authority who has considered the theory of choice and attempted to integrate the indifference analysis and the revealed preference


approach to utility analysis. He states:

The existence of indifference surfaces implies the existence of a utility function, for to each class of equivalent batches we can assign a number (the "utility" of that class), giving higher numbers to preferred classes. As is well known, this can be done indefinitely many ways, since only ordinal properties are involved.

Houthakker then analyzes Samuelson's consumption theory based on "revealed preferences" and finds that the two are the same.

We have shown that a theory based on semi-transitive revealed preference entails the existence of ordinal utility, while the property of semi-transitivity itself was derived from utility considerations. The "revealed preference" and "utility function" (or "indifference surface") approaches to the theory of consumers' behavior are therefore formally the same.


3 Houthakker, H. S., op. cit., p. 173
The concept of measurable utility has an important bearing on such economic problems as the structure of an income tax,\(^1\) social welfare budgets,\(^2\) and on many decisions involving general welfare.\(^3\) Whether or not the utility analysis for an individual as described here, can have validity in its application to groups is not a problem for discussion here. The following example is designed to show that a rational decision in the case of risk is dependent on a numerical analysis of utility by the individual making the decision.

The average loss \((E)\) from Choice 1 would be \((1/16) (A) + (15/16) (B)\), and from Choice 2 would be \((1/16) (C) + (15/16) (D)\). Unless some numerical

---


value can be assigned to A, B, C and D, it is not possible to know whether the average loss from Choice 2 is greater than, equal to, or less than that from Choice 1. If the numerical evaluations, using this same ordering of A, B, C and D are given as in Table 4, the average losses from choosing 1 or 2 every time are:

Choice 1, Average loss = \( \frac{1}{16} (12) + \frac{15}{16} (0) = \frac{3}{4} \)

Choice 2, Average loss = \( \frac{1}{16} (6) + \frac{15}{16} (5) = \frac{5}{16} \)

**TABLE 4**

*Loss*

Ordering of A, B, C, D

\[ A > C > D > B \]

\( (12) (6) (5) (0) \)

<table>
<thead>
<tr>
<th></th>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/16</td>
<td>15/16</td>
</tr>
<tr>
<td>Choice 1</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Choice 2</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Solution: Select Choice 1

The least average loss (3/4) results from Choice 1. This is a reversal of the solution from non-numerical
outcomes which indicated Choice 2 to avoid the maximum loss (A).

With these numerical values, A is not sufficiently large to make its rare occurrence give an average loss greater than that from selecting Choice 2 every time. It is easy to see that by changing the numerical values of A, B, C and D, and keeping the same ordering and probability of occurrence, that the average loss from always choosing Choice 1 could be greater. For example:

**TABLE 5**

**Loss**

Ordering of A, B, C, D

\[ A > C > D > B \]

(32) (8) (1) (0)

<table>
<thead>
<tr>
<th></th>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/16</td>
<td>15/16</td>
</tr>
</tbody>
</table>

Choice 1  A = 32  B = 0
Choice 2  C = 8  D = 1

Solution: Select Choice 2

The average losses (E) from always choosing 1 or 2 would be:

Choice 1,  \( E = \frac{1}{16} (32) = 2 \)
Choice 2,  \( E = \frac{1}{16} (8) + \frac{15}{16} (1) = 1 \frac{7}{16} \)
In this solution with the given numerical set of outcomes, Choice II gives the minimum average loss. This is a reversal of the previous solution using outcomes with the same ordering, but different numerical values.

The solution for Case II is such that it guarantees the best average result with the limited knowledge of the occurrence of events. If a person knew (as in Case I) what would happen, the loss would be less (or the gain greater).

Usually the forecasts will have proved more or less wrong; the total utility of each individual will have proved smaller than it would have been if he had known the future; these will be dynamic losses — or (mal-investment losses) — the larger, the less the man's skill (or luck) in forecasting.¹

An algebraic proof of the need for numerical ordering and the determination of the critical points of value at which changes in value are effective in determining the solution follows.

Algebraic proof of the need for numerical ordering of the outcomes in the case of risk in which the outcomes from one choice are not all respectively less

than the outcomes from any other choice:

**TABLE 6**

**Loss**

(1) Assume $A > 1$

(2) Let $q = \text{known probability of Event I, then } (1-q)$

is the probability of Event II.

(3) Let $(p) = \text{the percentage of the total number of}$

choices when Choice 1 is to be made, then $(1-p)$

is the percentage of the total for Choice 2.

(4) Let $E = \text{the average total loss}$.

<table>
<thead>
<tr>
<th>Choice 1 $(p)$</th>
<th>Event I $(q)$</th>
<th>Event II $(1-q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1 $(p)$</td>
<td>$A$</td>
<td>$0$</td>
</tr>
<tr>
<td>Choice 2 $(1-p)$</td>
<td>$0$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

The average total loss is the quantity to be mini-

mized by the selection of a value of $(p)$, which will

make this quantity the least.

\[
E = (p)(q)(A) + (1-p)(q)(C) + (1-p)(1-q)
\]

\[
= p(qA) + Cq - p(qC) + 1-p-q+pq
\]

\[
= p(qA) - qC - 1+q + (Cq+1-q)
\]

\[
= p \left[ q(A-C) - (1-q) \right] + (Cq+1-q)
\]
E is composed of two terms:
\[ p \left[ q \left( A-C \right) - (1-q) \right] \text{ and } (Cq+1-q) \]
Let \( M = \left[ q \left( A-C \right) - (1-q) \right] \)
\[ K = (Cq+1-q) \]
Then \( E = pM+K \), where \( M \) and \( K \) are independent of \( p \).
(1) \( M = 0 \) when \( (A-C) \) is equal to \( \left( \frac{1-q}{q} \right) \)
When \( M = 0 \), \( E = K \) (which \( p \) is chosen
does not affect \( E \))
(2) \( M \) is positive when \( (A-C) \) is greater than \( \left( \frac{1-q}{q} \right) \)
When \( M \) is positive, \( E \) increases with increases
in \( p \); hence, the lowest value of \( p \) minimizes
\( E \). The lowest value of \( p \) is zero. (When \( p 
\) is zero, the decision is to select Choice II
always).
(3) \( M \) is negative when \( (A-C) \) is less than \( \left( \frac{1-q}{q} \right) \)
When \( M \) is negative, \( E \) decreases with increases
in \( p \), hence the greatest value of \( p \) minimizes
\( E \). The greatest value of \( p \) is one. (When \( p 
\) is one, the decision is to select Choice I
always).

In order to determine the value of \( (p) \) which min-
imizes the average total loss, \( (E) \), it is necessary to
know whether \( (A-C) \) is greater than, or less than, \( \left( \frac{1-q}{q} \right) \).
This requirement means that a numerical ordering of the
outcomes is necessary.
4. **Uncertainty - a special case of risk.**

Another concept of risk is that case discussed by Knight\(^1\) as uncertainty. He uses risk to indicate a "dispersion of the frequency distribution of alternative events and uncertainty as the degree of ignorance about this frequency distribution.\(^2\)

Makower and Marschak proposed to use "uncertain" to indicate the existence of risk, i.e., "that no event is assigned a probability of one.\(^3\)

An exact differentiation between risk, uncertainty and lack of any knowledge of the future has not been accepted generally. However, such a differentiation is proposed by Marschak, and later by Weston.

Uncertainty involves future events about which there is incomplete knowledge or whose probability of occurrence is not one. Quantitative measures associated with uncertain future events are not single valued. If the dispersion of the probability distribution of the likelihood

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3 Ibid.
of the future event is not zero, uncertainty exists. Individuals may hold single-valued expectations of future events for which several alternative actual values may be experienced. Uncertainty exists, therefore, even through expectations are single valued...... Thus risk and uncertainty are not categories which differ in kind or degree. Risk is a subset of uncertainty.

Tintner has described the concept as follows:

Subjective risk deals with the case in which there exists a probability distribution of anticipations which, however, is itself known with a certainty (probability one). Subjective uncertainty assumes that there is an a priori probability of the probability distributions themselves, i.e., a distribution of probability distributions.


......... The pure theory of production under technological risk and uncertainty, Econometrica, 9:308-312, 1941.


An example of this "a priori probability of the probability distributions themselves" is as follows: A person may believe either of two events may occur with the following probabilities: Event I (1/4), Event II (3/4).

However, there is another set of two events, x and y, one of which must occur, but which are mutually exclusive. The occurrence of both x and y depends on the occurrence of Event I. If Event I occurs they have the following probabilities: Event x (2/3), Event y (1/3). Uncertainty, or the probability distribution of the probabilities, (Events x and y dependent on Events I and II) can be reduced to the case of risk previously discussed by the following process.

The person deciding wishes to know the probability of the occurrence of Event x or Event y. A chart indicating the dependence of Events x and y on Event I could be set up as follows:

<table>
<thead>
<tr>
<th>Event I (1/4)</th>
<th>Event II (3/4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event x (2/3)</td>
<td>Event y (1/3)</td>
</tr>
</tbody>
</table>

Event x has a probability of occurrence of (2/3) (1/4) or (1/6), and Event y has the probability of
All the data he is able to use indicate that the total distribution contains which are unusual in the total distribution for a specified period of time. Such an insurance for a specified period of time, probability may be important. However, in determination, economic planners, other parameters than the total Hart, has pointed out that for certain types of natural, he has not been any empirical value. The extent of the dependence of events is the amount of if it will not occur (1 - q) would be (g/6), that is, the occurrence of event X would be (1/6). The value of q in an event depends on the probability of the occurrence of a certain event. However, if the occurrence of event X is (4/10) or (2/10).
G. L. S. Shackle\textsuperscript{1} has treated this problem of decision-making by what he calls a focus-value solution to be used in decisions involving a "non-additive measure of uncertainty", or ignorance. In defense of the applicability of this principle to everyday problems he states:

The notion that the power, effectiveness or significance of some set of elements can depend, not in any sense upon their sum, but on the power and effect of the most powerful amongst them, is not after all so recondite. The difficulty of climbing a particular orag may consist in the cumulative exhaustion involved in overcoming a succession of difficulties, but it often will depend, instead, on the question whether the greatest of the "individual" difficulties can be overcome.\textsuperscript{2}


\textsuperscript{2} Ibid., p. 74
C. *Case III. Decisions in which there is no knowledge of the probabilities of the "external events".*

1. *Description of grouping.*

The third and last group for consideration (and the one in which insurance problems may fall) is the group of situations in which the probabilities of the events are unknown. The problem of determining a course of action from several possible choices when there is a complete lack of information with regard to the exact situation which will prevail is one which occurs frequently, but one which has not been given very decisive consideration by economists until recently. Under static conditions the "trial and error" method might be used to approach the optimum result, but under dynamic conditions the "trial and error" method does not yield comparable results.¹

Von Neumann and Morgenstern in their book² have presented their solution to the problem in certain simplified situations, with the hope that further work will lead to solutions in less simplified situations. The application of their method of solution

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² Von Neumann, J. and Morgenstern, O. *op.cit.* pp. 143-186.
is being made in many fields, such as military science, where decisions must be made without any knowledge of the probabilities of occurrence of various events.\(^1\)

In the case of decisions on insurance, to simplify the discussion, the problem may be reduced to a choice of one of two courses of action, (to insure or not to insure) the outcome of which depends on the occurrence or non-occurrence of a given situation (destruction of the insurable item). The person deciding believes there is no information available concerning the likelihood of its occurrence. This paper is an attempt to show the applicability of the Von Neumann solution to decisions regarding the amount of insurance to be carried by a family with a desire to make a rational choice. The element of complete ignorance about whether or not destruction of the insurable item will occur is the factor which makes the use of their method necessary, since if one knew when destruction would occur, he would be able to decide when to insure, and when not to. If he could

---

state his knowledge of the probabilities of destruction, the decision would fall into Case II for solution. The insurance company is not in a position of ignorance. It can compute the expected loss it will take in insuring a large number of items on the basis of selection of the policy-holders, and from many years of experience. "Insurance reduces risks through the operating of the 'law of large numbers'. Risk is not transferred to the insurance producer; it is substantially eliminated by the insurance process."¹

The individual has no similar basis for determining the probability that his insurable item will be destroyed. He may be aware of the number of items destroyed per thousand as tabulated by the insurance company, but this general data may not be particularly applicable to the specific characteristics of his insurable item. If he feels or thinks that it is applicable, then he can state the probabilities for destruction and the solution is determined as indicated in Case II. If, on the other hand, the individual believes that circumstances are such that he cannot make any consistent evaluation of the probabilities of

¹ Weston, J. F., op.cit., p. 44
destruction or non-destruction, then he can proceed on the basis of ignorance concerning what may happen during the period under consideration.

Von Mises\(^1\) considers that there is no empirical basis for assigning any probability to the likelihood of a person dying during a given period of time.

We have nothing to say about the chances of life and death of an individual even if we know his conditions of life and health in detail. The phrase, "probability of death", when it refers to a single person has no meaning at all for us.

He states that as a member of one "collective" he would have a probability of death of a certain amount, if this were the proper basis of determining it. "He is, furthermore, a member of a great number of other collectives which can be easily defined, and for which the calculation of the probability of death may give as many different values."\(^2\)

In order to eliminate personal elements and to clarify the essential factors, the object to be insured is selected to be some perishable object, such as a car or a barn, which can be replaced by the payment for loss from the insurance company. Insurance


\(^2\) Ibid., p. 23
on a person's life is actually only insurance on the loss of his earning power in the event of death, and not on the social losses to his family. The social losses are not insurable. Thus the application to life insurance of the theory arrived at on the basis of considering a replaceable, perishable object does not require any changed assumptions.

The principle used in the solution of Case III is to minimize the maximum average loss of utility. To clarify the requirements for solution, Case III is divided into Case III-A and Case III-B. This grouping is indicated by whether or not the solution points to always making the same choice each time the decision must be made.

2. Solutions indicated by variations in preferences

   a. Solution is independent of whether or not item is insured at the time of decision-making.

The factors to be considered in the estimations of the losses are those which cause changes in the state of satisfaction at the time of estimating the effects of the possible results. The factors in the present status from which changes in satisfaction are to be estimated are:
(1) $L =$ the loss of the satisfactions obtainable from the use of the insurable item which can be replaced by a loss payment from the accepted policy.

(2) $N =$ the loss of the satisfactions obtainable from the use of the insurable item not replaceable by the loss payment.

(3) $W =$ worry, or dissatisfaction over the possible destruction of the item.

(4) $P =$ the loss of the enjoyment of the satisfactions obtainable by using the premium money for the purchase of other goods and services.

The estimates of increased or decreased dissatisfaction concerning the four possible situations if the item has not been insured, are as follows:

Status - (Item has not been insured previously)

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Dissatisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy; Destruction</td>
<td>$L + N$</td>
</tr>
<tr>
<td>Refuse policy; No destruction</td>
<td>No change in status</td>
</tr>
<tr>
<td>Accept policy; Destruction</td>
<td>$N + P - W$</td>
</tr>
<tr>
<td>Accept policy; No destruction</td>
<td>$P - W$</td>
</tr>
</tbody>
</table>
The factors in the present status which the decision concerning the choice may change are arbitrarily selected, but the conclusions drawn are not affected. Another relationship of the factors to be considered would be evident if the status, at the time of the decision, were taken to be that of having the item already insured. The decision concerning the policy then would be whether to renew the policy by accepting, or to drop the policy by refusing. The estimates of dissatisfaction would then give the following relationship of the factors:

Status - (Item has not been insured previously)

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Dissatisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy; Destruction</td>
<td>[(L + N + W) - P]</td>
</tr>
<tr>
<td>Refuse policy; No destruction</td>
<td>[(W - P)]</td>
</tr>
<tr>
<td>Accept policy; Destruction</td>
<td>[N]</td>
</tr>
<tr>
<td>Accept policy; No destruction</td>
<td>No change in status</td>
</tr>
</tbody>
</table>

Assignment of numerical values to the factors used in evaluating the outcomes is not necessary to the development of theory, but it may clarify the point concerning the status used. Tables assigning the numerical values to these factors, with the two starting points for change in status, indicate that the conclusions are
unaffected by the arbitrary selection of the present status (a negative value for an outcome indicates a gain in utility). Using either status and the same evaluations, the solution is to accept the policy. Status I will be used for this discussion. The method of solution will be explained later. These factors determined the estimations of the outcomes in the previous tables, but their influence has not been essential to the development of the theory prior to this point.

TABLE 7

Loss
Status I - Item has not been insured previously

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Accept policy</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Column minima</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

Solution: Accept policy.
Status II - Item has been insured previously

\[ A = (L + N + W) - P = 9 \]

\[ B = W - P = -3 \]

\[ C = N = 2 \]

\[ D = \text{No change in satisfaction} = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>9</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>Accept policy</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Column minima</td>
<td>2</td>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Accept policy.

b. Explanation of individual variations in estimations of the amount of dissatisfaction in each outcome. In Table 7, and in the following tables, numbers have been used to indicate a quantitative state of dissatisfaction for L, N, P and W. These numbers which an individual might give, depend upon the individual's reaction to the situation in which he finds himself. Such factors as whether or not his father favored insurance, his educational background, his occupation, his previous financial losses on insured and uninsured items, the amount of assets available to
withstand possible financial loss may enter into his reaction indicating how much he worries over possible financial loss.¹

A person's previous experience with insurance is a particularly important factor. If he comes from a family which has experienced the loss of an uninsured value which resulted in dire need, that person may resolve to buy a great deal of protection for his family, and would be very worried about it if there was no risk protection. Another individual may have had a father who was "insurance poor" and he might determine that such concern over the possibility of future need was of less importance than satisfying the present needs of his family. He, therefore, would give a high rating to F.

The estimation of L, N, P and W, for determining the total dissatisfaction in a given outcome, must be a subjective process of comparing and weighing the factors individually, and in relation to ratings assignable to each of the others. If there is a great deal of worry, then foregoing alternative

satisfactions will not be deemed such a hardship. If a family has secured investments providing an income sufficient to satisfy the "basic needs" of the dependents, then the worry over possible loss would be diminished, and the dissatisfaction from foregoing alternative uses of the money would be increased. If a member of the family is able to step into the earning role without great hardship in family adjustment, the dissatisfaction in the loss of the earning power of the income producer, L, will not be as great as it would be if such an arrangement could not be made. In some rural areas, family relationships are such that the resources of any and all of the sons and daughters of a family are considered to be available to meet the emergency needs of any one member. This would influence the ratings given to L, N, P and W. Depending on the social and psychological reactions of an individual to his environment, these ratings will indicate the variations in dissatisfaction assignable to each outcome.

The importance of past experience in economic decisions has been pointed out by Georgescu-Roegen.
The classical theory of choice depicted the homo economicus as having an invariant in quantitative terms, independent of his past experiences. The intermediate positions adopted before the ultimate equilibrium was reached had, in the classical theory, no influence upon the process by which the equilibrium was obtained. Such a point of view led to a theory that was not only a poor approximation to the economic reality but also essentially different from it. Demand curves were considered as depending solely upon income and other prices, and when attempts to obtain statistical demand curves valid for the next period failed, economists turned to challenge the constancy of economic laws and implicitly the foundations of any economic theory. The above analysis shows that those attempts were vitiated not by a false stand regarding the constancy of economic laws, but by the ignorance of other possible shifts of the demand curves, distinct from those caused directly by a variation of prices or of income.¹

Carnap² discusses the problem of deciding on an insurance policy in connection with the suggested rule for decision-making of maximizing the expected utility. His analysis is designed to show that in the process of maximizing utility the decision-maker can rationally pay more than the cost of the insurance determined by the empirically observed ratio of losses. His discussion of the problem considers the relationship of


² Carnap, Rudolph. op.cit. pp. 269-279.
the two factors - the money value of the insurable item and the money value of the total assets of the decision-maker considered in terms of his utility maximization.

This analysis is basic, since it shows that the decision to insure based on maximization of utility can be rational even if the money value of the premium is much higher than the rate indicated by the number of losses per thousand items insured. In the previous discussion of the factors which determine the estimates of losses in utility in each outcome, these two basic considerations are supplemented by the addition of utility estimates of the possible alternative uses of the premium money and the amount of worry over the possible loss of the insurable item.

c. Case III-A.

(1) Solutions indicating always accepting the policy.

Case III-A under Case III (choices which have unknown probabilities of the external events) is one in which a "pure strategy" is indicated, i.e., the decision always falls on one choice, rather than on one choice a certain percentage of the time.
TABLE 8

Loss

Ordering of outcomes A, B, C, D.

\[ A > C > D > B \]

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Accept policy</td>
<td>C</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>Column minima</td>
<td>C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution: Accept policy

Maximum expectation of loss = C

If the values of losses can be arranged according to Table 8, then the decision is determined, always accept the policy — since the average loss in satisfactions from refusing (A or B) would always be greater than the average loss from accepting. Whether destruction occurs or not does not affect the decision. Perhaps an explanation of the evaluations given in this ordering would make the determination of the decision clearer. A gets the greatest value because destruction of the uninsured item would be the greatest loss. B gets zero value because there is no change in
status. C and D are net losses or gains in satisfaction which arise because of the difference between P and W. C is a greater loss than D because destruction of the item brings non-replaceable losses (N).

The average loss from always refusing the proposed policy is always greater, no matter what happens, than that from always accepting the policy. Therefore, the choice of accepting is strictly determined, and indicates a pure strategy (accepting 100% of the time). The assignment of numerical values, as in Table 9, is not necessary for determinacy, but may be done to illustrate the point. For this solution L is always greater than (P - W). This means that the income is high enough so that the loss (P) from foregoing alternative purchases, such as food or shelter, is not as high as the loss (L) of the item if it is destroyed. A negative value in the outcome indicates net satisfaction.
TABLE 9

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>A = 13</td>
<td>B = 0</td>
<td>13</td>
</tr>
<tr>
<td>Accept policy</td>
<td>C = -2</td>
<td>D = -3</td>
<td>-2</td>
</tr>
<tr>
<td>Column minima</td>
<td>-2</td>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Accept the policy

Maximum expectation of loss = -2
(2) Solution indicating always refusing the policy.

TABLE 10

Ordering of outcomes:
C > D > A > B

<table>
<thead>
<tr>
<th>Loss</th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Accept policy</td>
<td>C</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>Column minima</td>
<td>A</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse the policy

Maximum expectation of loss = A

If the values of losses can be arranged according to the above ordering in Table 10, then the decision is determined—always decide against being insured under the policy. An example of a situation in which this assignment of values might be given would be the decision on an insurance policy on an item already insured for its full money value, assuming that this were possible. The losses (P) from foregoing alternative satisfactions by using the money for insurance may be
high, and there is no worry \((W)\). The possible loss from refusing is small, since the item's replaceable value \((L)\) is already covered by other policies. The assignment of numerical values might be given as in Table II.

**Table II**

<table>
<thead>
<tr>
<th>Loss</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(L = 0)</td>
<td>(A = L + N = 1)</td>
</tr>
<tr>
<td>(P = 6)</td>
<td>(B = \text{No change} = 0)</td>
</tr>
<tr>
<td>(W = 0)</td>
<td>(C = N + P - W = 7)</td>
</tr>
<tr>
<td>(N = 1)</td>
<td>(D = P - W = 6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Accept policy</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Column minima</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution: Refuse the policy.

Maximum expectation of loss = 1

For a second example in which the decision is strictly determined to refuse the policy, consider the evaluations which might be given to the losses in deciding whether to insure an item already insured for
part of its full value, say 90%. There are some losses if the item is not insured under the contemplated policy for the remaining 10%, but the losses from foregoing alternative satisfactions are judged to be much greater. The numerical values in this example might be as in Table 12.

**TABLE 12**

<table>
<thead>
<tr>
<th>Loss</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 3</td>
<td>A = L + N = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P = 6</td>
<td>B = No change in status = 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W = 1</td>
<td>C = N + P - W = 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N = 1</td>
<td>D = P - W = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>4 0</td>
<td>4</td>
</tr>
<tr>
<td>Accept policy</td>
<td>6 5</td>
<td>6</td>
</tr>
<tr>
<td>Column minima</td>
<td>4 0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse the policy.

Maximum expectation of loss = 4

A third example of Case III-A in which the decision is strictly determined to refuse the proposed policy is
a simple explanation of the situation in which no insurance is taken out on a valuable item, the destruction of which would mean a definite loss in satisfactions (L), and on which there is considerable worry over its possible destruction. At the same time, the budget is so restricted that paying the insurance premium (P) may mean a greater loss in foregoing alternative satisfactions (such as food or shelter). Numerical values for this example might be assigned as in Table 13.

### TABLE 13

**Loss**

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 5</td>
<td>A = L + N = 6</td>
<td></td>
</tr>
<tr>
<td>P = 11</td>
<td>B = No change in status = 0</td>
<td></td>
</tr>
<tr>
<td>W = 4</td>
<td>C = N + P - W = 8</td>
<td></td>
</tr>
<tr>
<td>N = 1</td>
<td>D = P - W = 7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse policy</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Accept policy</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Column minima</td>
<td>6</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse the policy.

Maximum expectation of loss = 6
(3) **Algebraic proof that, based on the estimation of losses, the solutions for decisions on insurance are always "strictly determined".** From the previous examples it has been indicated that in the case of insurance using the principle of minimizing the maximum average loss, that the solution is always strictly determined. This means that with a certain set of evaluations on the possible outcomes from refusing to take the policy or accepting it, the least average loss will result if the choice is always refusing, rather than refusing part of the time and accepting part of the time. Another set of valuations might indicate constant acceptance of the policy. Von Neumann and Morgenstern\(^1\) have proved that the choice is strictly determined when the minimum of the row maxima is in the same cell in the table as the maximum of the column minima. In order to use their proof for the statement that all choices, regarding whether or not to insure are strictly determined for a pure strategy it is necessary to prove that in all cases in decisions on insurance the minimum of the row

\(^1\) Von Neumann and Morgenstern, op.cit. pp. 98-110.
maxima will always be the same as the maximum of the column minima. This can be done by assuming that the dissatisfactions resulting from the destruction of the insurable item, no matter which choice is made, are greater than the dissatisfactions from the possible choices if no destruction occurs. An algebraic proof follows:

**TABLE 14**

**Loss**

1. Let \( b \) = the amount of dissatisfaction under refusal by which the outcome from destruction is greater than that from no destruction.

2. Let \( d \) = the amount of dissatisfaction under acceptance by which the outcome from destruction is greater than that from no destruction.

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>( a + b )</td>
<td>( a )</td>
<td>( a + b )</td>
</tr>
<tr>
<td>Accept</td>
<td>( c + d )</td>
<td>( c )</td>
<td>( c + d )</td>
</tr>
</tbody>
</table>
Possible combinations of the two column minima are:

I. \((a + b)\) and \((a)\)
II. \((c + d)\) and \((c)\)
III. \((a + b)\) and \((c)\)
IV. \((c + d)\) and \((a)\)

I. If \((a + b)\) and \((a)\) are the column minima, then \((a + b)\) is the minimum of the row maxima and, therefore, the "minimax" (minimum of the row maxima and maximum of the column minima).

II. If \((c + d)\) and \((c)\) are the column minima, then \((c + d)\) is the minimum of the row maxima and, therefore, the minimax.

III. If \((a + b)\) and \((c)\) are the column minima, then

\((1)\) \((a + b) > c\)

or \((2)\) \((a + b) < c\)

Since \(c\) is the column minimum, \(a > c\) therefore \((a + b) > c\), hence \((a + b)\) is the minimax.

IV. If \((c + d)\) and \((a)\) are the column minima, then

\((1)\) \((c + d) > a\)

or \((2)\) \((c + d) < a\)

Since \(a\) is the column minimum, \(c > a\), therefore, \((c + d) > a\), hence \((c + d)\) is the minimax.
d. **Case III-B**

(1) **Solution from the application of the principle of minimizing the maximum average regret.** Thus in insurance the decision which will minimize the maximum average loss is always strictly determined indicating a pure strategy - either for constant acceptance (when $P - W$ is less than $L$), or for constant refusal (when $P - W$ is greater than $L$). However, there is a factor which enters into a decision with regard to whether to accept a policy which has not been considered. This is the satisfaction from having made the right decision; or conversely the dissatisfaction, or regret, from having made the wrong decision. This regret at accepting or refusing the policy depends on the difference in the estimated losses between the two decisions. Those decisions in which the person deciding wishes to use the regret principle fall into Case III-B.

(2) **Solution indicating accepting and refusing the policy in a fixed ratio as indicated by the use of a random device.**

In the case in which the decision was determined always to be insured (Case III-A), a regret table could be set up for it as indicated in Table 15, *Regret.*
TABLE 15

Loss

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Accept</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Column minima</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Always accept policy.
Maximum expectation of loss = 3

Regret

Let \( p \) = % of refusals  \( A' = A - C = 3 \)

\( 1 - p \) = % of acceptances  \( B' = \) No regret

\( C' = \) No regret

\( D' = D - B = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse ((p))</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Accept ((1-p))</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Column minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
\[ p = \frac{2}{5} \]
\[ 1 - p = \frac{3}{5} \]

Solution from regret principle: Refuse policy \( \frac{2}{5} \) of the time; accept policy \( \frac{3}{5} \) of the time.

Maximum expectation of regret using the solution = \( \frac{1}{5} \)

Maximum expectation of regret from always accepting = 2

The assignment of values in the Table 15 Regret results from determining what the net loss is, knowing what the occurrence of the destruction is. A is the regret over refusing after deducting the loss which would have occurred if the decision had been to accept the policy. C and B are zero because the decision is right as to the occurrence of destruction, and there is no regret. D is the regret in the event of no destruction due to loss of alternative satisfactions from the unnecessary use of the premium money for insurance.

When the loss of alternative satisfactions caused by using the money to pay for risk protection (P) is equal to the worry or dissatisfaction over the possibility of loss (W), i.e., P = W, there is no regret. If it is not possible to set up a regret table with
a numerical value other than zero for D (Accept policy; item not destroyed) then the application of the principles of minimizing the maximum average loss and regret indicates the same solution of selecting the same choice every time the decision is made.

However, if W is less than P, it is possible to set up a regret table, and the solution indicated is not a pure strategy. Looking at Table 15 Regret and following the principle of minimizing the maximum average regret, one would say that the decision arrived at from the loss table is unaltered - always accept, since the maximum loss in regret from refusing is 3 and from accepting is 2. It is here, however, that the Von Neumann and Morgenstern theory of obtaining the minimum of the maximum average loss by the use of a "mixed strategy" becomes applicable. A "mixed strategy" means that the same choice is not made every time, but a specified per cent of the time. They have shown that by the use of a random device, used with a ratio between refusal and acceptance determined by their method, it is possible to guaranty a smaller average loss than
with any other method. Wald\(^2\) has discussed this problem in his work.

(a) **Determination of the ratio.** The determination of the ratio \((p)\) for refusal and \((1-p)\) for acceptance can be made by equating the values of the losses in terms of regret which occur if there is destruction, and if there is no destruction.

Let \(p = \) the \(\%\) of times to refuse; then \(1-p = \) the \(\%\) of times to accept.

---

1 VON NEUMANN and MORGENSTERN, ibid., pp. 143-186.


TABLE 16

Regret

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse (p)</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Accept (1-p)</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

Total loss if destruction occurs

\[ p(A) + (1-p) (C) = p(B) + (1-p)D \]
\[ p(A) + C - p(C) = p^B + D - p^D \]
\[ p(A - C - B + D) = D - C \]
\[ p = \frac{D - C}{A + D - C - B} \]

With the values given in Table 16 Regret the value of p, as determined by the formula is

1. \[ p = \frac{(2) - (0)}{(3) + (2) - (0) - (0)} = 2/5 \]
2. \[ 1 - p = 3/5 \]

The process of equating the losses from destruction or non-destruction frees the decision from the influence of the occurrence of events. This means that no matter what happens the decision-maker
is guaranteed that the average loss in regret will not be greater than a stated amount - and it may be much less.

Algebraic proof that this value of \( p \) gives the minimum value of the maximum expectation of loss:
Let the values be given as in Table 17.

**TABLE 17**

<table>
<thead>
<tr>
<th>Regret</th>
<th>( q )</th>
<th>( 1-q )</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destroyed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refuse ( p )</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Accept ( 1-p )</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Column minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse 1/2 the time; accept 1/2 the time.

Maximum expectation of loss = \( 1/2 \)

The average loss (\( E \)) is determined according to the following equation:
$E = (p) (q) (1) + p(1-q)(0) + (1-p) (q) (0) + (1-p)(1-q) (1) = pq + 1-p-q + pq = 2pq - q + 1 - p = q(2p-1) + 1-p$

Finding a value of $p$ to make the loss due to $q$ equal to zero means making $q$'s coefficient equal to zero.

$$(2p-1) = 0$$

$$p = 1/2$$

When $p = 1/2$

$E = q (2 \left(\frac{1}{2}\right) - 1) + \frac{1}{2} = \frac{1}{2}$

Let $Z$ = an increment in $p$ to change the average loss ($E$).

$Z$ must have a value not equal to zero and equal to less than $\frac{1}{2}$ since $p + Z$ cannot be greater than 1.

$E$ (Expectation of loss) = $q \left[ 2(p + Z) - 1 \right] + 1 - (p+Z)$

$= q \left[ 2(\frac{1}{2} + Z) - 1 \right] + 1 - (\frac{1}{2}+Z)$

$= q \left( 1 + 2Z - 1 \right) + 1 - \frac{1}{2} - Z$

$= \frac{1}{2} + 2qZ - Z$

$= \frac{1}{2} + Z \left( 2q - 1 \right)$

If $q$ becomes equal to one, then the greatest loss which would be expected would be $\frac{1}{2} + Z$. In order to make the maximum average loss the minimum, $(\frac{1}{2} + Z)$ must be made a minimum. It is a minimum where $Z = 0$. 

(b) **Method of applying the ratio to obtain the solution.** When the percentage of times to accept or refuse the policy is determined, by finding the value of \( p \) as indicated, its application depends on the use of a random device for deciding each time the decision must be made. If \( p = \frac{1}{2} \), such a random device might be a box in which there are three white marbles and one black one, all exactly alike except for their color. The box is shaken, and then tipped so that one marble can fall out. If the black one falls out, the policy is rejected; if a white one falls out, the policy is accepted. A table of random numbers might be used as indicated by a textbook on statistics.¹

Using the numerical values of estimated losses for \( A \) and \( C \) and the calculated value of \( p \), the maximum average loss which the person can expect is

\[
p(A) + (1-p)(C) \; ; \text{or an equal amount} - p(B) + (1-p)D.
\]

Any variation in the rate of destruction \( q \) would not change the determination of \( p \) since the value as determined gives the minimum of the maximum average regret independent of the rate of destruction. For example with Table 18, where \( p = 1/3; \)

E (regret) = 1 1/3

If q = 1/3, E (regret) = (1/3) (1/2) (4) +
2/3(1/2) (2) = 4/6 + 2/3 = 4/3 = 1 1/3

If q = 3/4, E (regret) = (1/3) (3/4) (4) +
2/3(1/4) (2) = 1 + 1/3 = 4/3 = 1 1/3

TABLE 18

<table>
<thead>
<tr>
<th>Regret</th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>p = 1/3 Refuse</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1-p = 2/3 Accept</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Column minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse 1/3 of the time; accept 2/3 of time.

Maximum expectation of regret = 1 1/3

(c) Limitations of the solution. It must be remembered, however, that the value of (p) was determined on the assumption that there was no information concerning the value of (q). A different value of (p) can be found which will give a smaller guaranty of maximum average regret if there is a subjective a priori probability given for the occurrence of destruction as in Case II. Using Table 18 for an example,
if the occurrence of destruction \( q \) is known to have a probability of one, then a value of \( p = 0 \) will give a guaranty of a maximum average regret of zero. This is less than the \( 1 \ 1/3 \) average regret previously found to be the minimum guaranty of the maximum average regret, assuming no subjective \textit{a priori} probability for the occurrence of destruction.

As previously stated there is no guaranty that each time the person makes the decision he will lose the least (have the least regret). It is possible that the greatest regret might occur the first time the mixed strategy was used, or that it would occur the first two times it was used. The guaranty is that the average, or the mean of the regret, will be the least.

It has been common among economists to insist that the means of the amounts of commodities are not the only parameters of their joint distribution that are relevant to the man's decisions, and possibly not the most important ones, and attempts were made to specify which additional, or alternative parameters - e.g., the higher moments - should be considered. These writers, eg., Marschak, Tintner, often failed to make clear that the statement "the average amounts of goods are not alone relevant to the man's decision" does not contradict the statement that the average utility is maximized by him.
The latter proposition permits, in fact, to relate "risk-aversion", "advantage of diversification", and similar concepts of the older asset theory to the properties of the utility function of sure prospects.¹

The special case of the maximization of utility by a gambler is discussed by Friedman and Savage.²

Without the use of the random device with the calculated value of p, the decision might be to use a pure strategy. In the case illustrated by Table 12, the person deciding might always choose to accept, since the greatest possible loss in regret is that resulting from refusal. If destruction never occurred, which is a possibility if there is no information with regard to the occurrence of destruction, then the choice of always accepting gives a regret expectancy which has a maximum average value of 2. This is greater than the maximum average regret expectancy (1 1/3) using mixed strategies.


(d) **Solution indicated by variations in the amount of worry.** Solution indicating always accepting.

The use of mixed strategies makes the decision one of minimizing the maximum dissatisfaction (regret) at having chosen incorrectly. If there is a great deal of worry (i.e., \( W \) is almost equal to \( P \)) concerning the possible loss of the use of the insurable item, then there will be less regret at having chosen to accept the policy, and the regret table will approach closer and closer to the pure strategy indicated by Table 19, which was set up indicating losses in satisfaction.

If the dissatisfaction from worry equals or is greater than the dissatisfaction from losing alternative goods and services, then there will be no regret at having chosen to accept when no destruction occurs. The situation when \( P = W \) is illustrated by Regret Table 19 which shows a minimax and indicates the same pure strategy as the Loss Table 19 from which it is derived - always accept.
TABLE 19

**Loss**

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 5</td>
<td>A = L + N = 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P = 4</td>
<td>B = No change in status = 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W = 4</td>
<td>C = N + P - W = 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N = 1</td>
<td>D = P - W = 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>6 A</td>
<td>0 B</td>
<td>6</td>
</tr>
<tr>
<td>Accept</td>
<td>1 C</td>
<td>0 D</td>
<td>1</td>
</tr>
<tr>
<td>Column minima</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Always accept policy
Maximum expectation of loss = 1

**Regret**

\[ A' = A - C = 5 \]
\[ B' = No regret \]
\[ C' = No regret \]
\[ D' = D - B = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Accept</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Column minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Always accept policy
Maximum expectation of regret = 0
If the worry over possible loss is greater than the dissatisfaction from losing alternative satisfactions by paying the premium \((W > P)\), there is no regret at having paid the premium. A regret table, derived in the usual way, indicates either no dissatisfaction, or a net satisfaction, from accepting the policy, whether destruction occurs or not.

**TABLE 20**

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>11</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Accept</td>
<td>-3</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>Column Minima</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Accept policy

Maximum expectation of loss = \((-3)\)
### Regret

\[
A' = A - C = 14 \quad C' = \text{No regret} \\
B' = \text{No regret} \quad D' = D - B = -4
\]

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>14</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Accept</td>
<td>0</td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

**Solution:** Accept policy

**Maximum expectation of regret = 0**

Solution indicating accepting most of the time.

If \( W \) is only slightly less than \( p \), then the mixed strategy value of \( p \) is very small and leads, practically, to a very rare choice of refusing the policy. In the following example the random device used would indicate refusal less than \( 3\% \) of the time.
TABLE 21

Loss

\[ L = 35 \quad A = L + N = 36 \]
\[ P = 25 \quad B = \text{No change in satisfaction} = 0 \]
\[ W = 24 \quad C = N + P - W = 2 \]
\[ N = 1 \quad D = P - W = 1 \]

<table>
<thead>
<tr>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>36</td>
<td>0</td>
</tr>
<tr>
<td>Accept</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Column Minima</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution: Accept policy

Maximum expectation of loss = 2

Regret

\[ A' = A - C = 34 \quad C' = \text{No regret} \]
\[ B' = \text{No regret} \quad D' = D - B = 1 \]

<table>
<thead>
<tr>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>34</td>
<td>0</td>
</tr>
<tr>
<td>Accept</td>
<td>(1-p) = 97.2%</td>
<td>0</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution: Refuse 2.8% of the time; accept 97.2% of the time.

Maximum expectation of regret = .972
The greater the difference between $P$ and $W$, when $W$ is less than $P$, the greater the regret, and the greater the value of $(p)$ in using mixed strategies.

Solution indicating refusing most of the time.

If the dissatisfaction from having to pay the premium is large and the worry over the possibility of loss is small, then the random device used would have a ratio between indicators for refusal and acceptance heavily weighted for refusal. In the following example the random device used would indicate refusal 78% of the time.

**TABLE 22**

<table>
<thead>
<tr>
<th>Loss</th>
<th></th>
<th></th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 10$</td>
<td>$A = L + N = 11$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P = 10$</td>
<td>$B = \text{No change in satisfaction} = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W = 1$</td>
<td>$C = N + P - W = 10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N = 1$</td>
<td>$D = P - W = 9$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>11</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Accept</td>
<td>10</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Column Minima: 10 0

Solution: Accept the policy

Maximum expectation of loss = 10
Regret

\[
\begin{align*}
A' &= A - C = 1 \\
B' &= \text{No regret} \\
C' &= \text{No regret} \\
D' &= D - B = 9
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse p = .9</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Accept 1-p = .1</td>
<td>0</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Column Minima</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse 90% of the time; accept 10% of the time.

Maximum expectation of regret = .9

This case, in which the dissatisfaction from having to forego alternative uses of the money is as great as the worry over possible loss, is the situation in which many people find themselves. It occurs when they are on such a limited budget that transferring any funds to risk protection creates a hardship, or when they have secured enough protection so that the occurrence of destruction would not create a financial crisis. Without using the solution indicated by Table 22 - Regret, but being aware of the regret at choosing wrong, the person might choose to refuse the policy every time. The solution offers a method of minimizing the maximum amount of dissatisfaction in terms of regret, by refusing the greater part of the time.
Solution indicating always refusing. If the dissatisfaction from paying the premium (P) is large; the worry over the possible loss small, and the estimate of the dissatisfaction in the actual loss of the insurable item (L) is comparatively small both the loss and regret principles indicate always refusing. This occurs when \((P - W) > L\). Table 23 shows this situation which might occur where there is a low income or where, with a higher income, the decision is with regard to insuring for the last small part of the value of the item.

**TABLE 23**

<table>
<thead>
<tr>
<th>Loss</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(L = 5)</td>
<td>(A = L + N = 6)</td>
<td>(B = \text{No change in satisfaction} = 0)</td>
<td>(C = N + P - W = 10)</td>
<td>(D = P - W = 9)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Accept</td>
<td>10</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Column Minima</td>
<td>6</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse policy

Maximum expectation of loss = 6
### Regret

<table>
<thead>
<tr>
<th></th>
<th>Destroyed</th>
<th>Not Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>A' = No regret</td>
<td>C' = C - A = 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B' = No regret</td>
<td>D' = D - B = 9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Accept</td>
<td>4</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Refuse policy

Maximum expectation of regret = 0

(e) **Characteristics of solutions to be considered in the application of the regret principle of decision-making.** Application of the regret principle to more than two alternatives.

When the regret principle is used in decisions involving more than two choices, the solution indicated may not be consistently indicated by using pairwise groupings of the sets of choices. That is, if Choice A is preferred to three choices, A, B and C, it may not be preferred if only Choices A and B are considered. This is a different problem than the requirement for rational behavior previously discussed, that the ordering of outcomes in terms of preferences be transitive. This characteristic is inherent in the concept of regret since
regret depends on what alternative actions could have been taken. An example of the circularity of decisions which may result from the pairwise application of the regret principle to those alternatives is given in Tables 24 to 26.

**TABLE 24**

<table>
<thead>
<tr>
<th></th>
<th>Event I</th>
<th>Event II</th>
<th>Event III</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>10</td>
<td>0</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>8</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Choice 3</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Choice 3

**Regret**

<table>
<thead>
<tr>
<th></th>
<th>Event I</th>
<th>Event II</th>
<th>Event III</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>10</td>
<td>0</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Choice 3</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution (without mixed strategies): Choice 3
In Table 24 the three choices are considered as a group. The principle of minimizing the maximum average regret is used. To illustrate the point more clearly, mixed strategies will not be considered. Choice 3 is preferred to Choice 1 or 2. If Choice 3 is not a possibility, the regret is different, as indicated in Table 25, and the solution is that Choice 2 is preferred to Choice 1.

**TABLE 25**

<table>
<thead>
<tr>
<th>Loss</th>
<th>Event I</th>
<th>Event II</th>
<th>Event III</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>10</td>
<td>0</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>8</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Choice 1

<table>
<thead>
<tr>
<th>Regret</th>
<th>Event I</th>
<th>Event II</th>
<th>Event III</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution: Choice 2

If only Choices 1 and 3 are considered, Table 26 shows that Choice 1 is preferred to Choice 3. This is a
reversal of the decision if three alternatives are considered.

**TABLE 26**

<table>
<thead>
<tr>
<th></th>
<th>Loss</th>
<th></th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Event I</td>
<td>Event II</td>
<td>Event III</td>
</tr>
<tr>
<td>Choice 1</td>
<td>10</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Choice 3</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Column Minima</td>
<td>5</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Solution: Choice 3

<table>
<thead>
<tr>
<th></th>
<th>Regret</th>
<th></th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Event I</td>
<td>Event II</td>
<td>Event III</td>
</tr>
<tr>
<td>Choice 1</td>
<td>5</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Choice 3</td>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution: Choice 1

To summarize, if three choices are considered, Choice 3 > Choice 2 > Choice 1; but if considered by pairs, Choice 1 > Choice 3. Because regret is based on all possible alternatives, a consistent decision depends on a consideration of the same alternatives.
Regret as a characteristic reaction of an advisor.

Whether or not regret over a rational decision is the result of a rational process is not a question for consideration here. The fact that this reaction (of regret) is a factor in decision-making for many is not generally questioned. The application of the regret principle is proposed as a method of incorporating this factor in the decision process in a rational way.

An advisor is in a position where he is particularly aware of the regret factor. Where there is no knowledge of the occurrence of events, if he indicates a certain choice based on the loss principle and the events occur so that another choice would have been much better, his position as an advisor may be jeopardized. This may be true regardless of the fact that he gave the decision as the result of a rational process, utilizing the principle of minimizing the maximum average loss. An example of an extreme case in Table 27 may serve to illustrate this point. In this case there is only a small difference in the maximum expectation of loss between the two choices, but a large difference in the expectation of regret.
TABLE 27

<table>
<thead>
<tr>
<th></th>
<th>Event I</th>
<th>Event II</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>99</td>
<td></td>
</tr>
</tbody>
</table>

Solution (using loss principle) = Choice 1
Minimum average loss = 99

<table>
<thead>
<tr>
<th></th>
<th>Event I</th>
<th>Event II</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1</td>
<td>99</td>
<td>0</td>
<td>99</td>
</tr>
<tr>
<td>Choice 2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Column Minima</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Solution (using loss principle) = Choice 1
Maximum expectation of loss = 99
Maximum expectation of regret = 99

Regret principle solution: Choice 2 99% of the time; Choice 1 1% of the time.
Maximum expectation of loss = 99.99
Maximum expectation of regret = 0.99
With these values given as in Table 27 it is obvious that while Choice 1 guarantees a smaller maximum average loss than Choice 2, the difference is slight (1), but the regret over choosing incorrectly if Event I occurs would be great (99). The solution using the regret principle with a pure strategy would be to take Choice 2 every time and incur a maximum average regret of (1) while increasing the maximum expectation of loss from (99) to (100). The Von Neumann and Morgenstern solution using a mixed strategy will guaranty a smaller average regret of (.99). By using the regret principle and a mixed strategy, the average loss is increased (.99), but the expectation of regret is reduced from (99) to (.99).

An advisor, therefore, would be more apt to find the solution using the regret principle since he, himself, does not suffer the small increase in the expectation of loss which it gives. The great decrease in the expectation of regret decreases the likelihood that people following his advice will be sorry that they consulted him.

Whether or not the decision is made by an advisor or by an individual alone in the case indicated by Table 27, it is the writer's opinion that most people would use the regret principle. This is only an opinion which might be
the subject of empirical testing. Such an investigation might attempt to determine how great an importance is attached to regret in relation to the increase in the expectation of loss which the use of the regret principle incurs. Such an empirical investigation encounters the difficulty of correlating what a person says he would do with what he would actually do.

The writers of the copy contained in the many advertisements for insurance stress the regret factor. A vivid presentation is shown of the difficulties of the bereaved family of an uninsured man, and their regret that he did not heed the words of the life insurance salesmen.

The regret principle versus alternative principles of decision-making.

The use of the loss principle ignores the existence of the psychological factor of regret. If the regret principle is used exclusively it may result in a great increase in the expectation of loss. In order to reduce the expectation of regret the decision-maker must be willing to increase the expectation of loss.

Each principle of decision-making requires assumptions which limit the range of its application. The loss principle adopts the pessimistic attitude that
the worst is going to happen. The solution given guarantees that the individual will suffer the least if the most unfavorable event occurs.

In certain cases in which the outcomes from the decision do not involve critical changes in the individual's status, the principle of selecting the choice which may result in the minimum of the minimum average loss or the maximum of the maximum average utility of the alternative choices might be used. This implies the optimistic assumption that the best outcome will happen more frequently than the worst outcome. The principle used in deciding depends not only on the nature of the decision, but also on the psychological attitudes of the decision-maker.

A description of these principles of decision-making has been concisely presented by Modigliani.¹

If one examines the class of rules or decisions which yield a definite solution even when the relevant probability distributions are unknown, there are two rules which appear to be particularly interesting because they represent limiting types:

(1) Maximining of income: \[ \text{Max. } x \text{ Min. } u \]
\[ Z(x,u) \] where \( Z \) denotes income, \( x \) the commitment and \( u \) the event unknown at the time of decision making. This is the procedure typical of the "cautious pessimist". It implies the "pessimistic" expectation that nature will always tend to produce the situation most adverse to the operator and the "cautious" unwillingness to take a chance about it.

(2) Maximaxing of income: \[ \text{Max. } x \text{ Max. } u \]
\[ Z(x,u) \]. This is the procedure typical of the "reckless optimist". It implies the "optimistic" expectation that nature will tend to produce the situation most favorable to the operator and the "reckless" disregard of the desirability of hedging against the possibility of a worse occurrence.

These two rules of decision thus appear to correspond to limiting cases in a psychological sense. Moreover, in a class of situations which, it is conjectured, is empirically important they represent also limiting cases in a quantitative economic sense. In such cases commitments outside the range marked off by the maximin and maximax choice are absolutely irrational in the sense that increasing (or decreasing) the commitment will produce a larger income no matter which set of admissible events will materialize. Every commitment within the range is rational in the sense that there is no other possible
commitment which will produce a higher income under every one of the events considered possible. The choice within the range will depend primarily on psychological dispositions. The rule of minimaxing regret, which in such cases yields a solution within the above-mentioned range and hence belongs to the class of rational decisions, appears to be, also psychologically, a very reasonable rule, striking a balance between the two extremes.

Extent of knowledge limits the ability to select the most favorable result. (Maximization of utility).

The solutions given by the use of principles of decision-making are based on the assumption that the individual is able to give an ordering to the outcomes as described earlier in this paper. If the individual obtains new information concerning the effect of the loss of the insured item, or concerning the probability of its destruction, he might be able to give a different set of evaluations which would guaranty a greater minimum utility. If he fails to utilize the information he has in his evaluations of the outcomes it would have the same effect as ignorance concerning that information.
Extension of the method of solution to
decisions on life insurance involving more than two
choices.

The decision in life insurance is particularly
suited to consideration in terms of a two-by-two table.

The outcomes from the choices can depend on only two
events during the specified time period – the life or
the death of the individual. If he become ill and is
able to provide only half the normal income for the
family the outcome from the decision is the same as
though he were well. There is no possible gradation
of the effect of the factors outside of his control –
destruction or non-destruction of the life of the
insured.

Just as the number of events outside of the in-
dividual's control is limited to two, the number of
choices concerning a single policy is limited to two –
acceptance or refusal. Insurance is not marketed in
terms of the purchase of a part of a policy. However,
if more than one policy is considered it may have a
face value which is a fractional part of the face value
of another. The following discussion shows that there
would be no advantage gained if the number of choices considered at one time were enlarged to include several different policies.

In estimating the loss of utility in each outcome the evaluation is based on a change in the level of satisfaction measured from the level at which the person is when deciding. This means that in each table a choice should be included for no change in status (refusal of all policies under consideration). The evaluation of the change in utility by a policy with half the face value of another would not mean that the ratios of the evaluations for P and W would also be as one is to two. The loss in utility for P might be four times as great in paying twice as large a premium.

Therefore, the enlarged table is really a combination of several two-by-two tables. If only one choice is to be made the table will indicate the most advantageous.
TABLE 28

<table>
<thead>
<tr>
<th>Choice</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>L =</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>P =</td>
<td>0</td>
<td>6</td>
<td>9</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>W =</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>N =</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ A = L + N = 10 \]
\[ B = \text{No change} = 0 \]
\[ C, E, G, I = N + P - W \]
\[ D, F, H, J = P - W \]

<table>
<thead>
<tr>
<th></th>
<th>Not Destroyed</th>
<th>Destroyed</th>
<th>Row Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1 (Refuse all policies)</td>
<td>A = 10</td>
<td>B = 0</td>
<td>10</td>
</tr>
<tr>
<td>Choice 2 (Accept $1,000 policy)</td>
<td>C = 2</td>
<td>D = 1</td>
<td>2</td>
</tr>
<tr>
<td>Choice 3 (Accept $2,000 policy)</td>
<td>E = 6</td>
<td>F = 5</td>
<td>6</td>
</tr>
<tr>
<td>Choice 4 (Accept $3,000 policy)</td>
<td>G = 11</td>
<td>H = 10</td>
<td>11</td>
</tr>
<tr>
<td>Choice 5 (Accept $4,000 policy)</td>
<td>I = 14</td>
<td>J = 13</td>
<td>14</td>
</tr>
</tbody>
</table>

Column Minima            2  0

Solution: Choice 2

Table 28 shows that Choice 2 contains an outcome which is the minimax for the table. Having decided to accept Choice 2, the individual might remove it from the
table since it had no influence on utility estimates of the other outcomes. Choice 3 would then contain the minimax of the table. If Choice 3 were removed, Choice 1 would contain the minimax, which would indicate no more policies should be purchased.

A previous illustration in Tables, 24, 25 and 26 has demonstrated the effect of enlarging a table and using the regret principle. Because of this effect and the greater simplicity in evaluating the outcomes in terms of a small number of choices, the two-by-two table seems to satisfy the needs of a person seeking to make a rational decision on whether to buy risk protection and, if any - how much.
IV. SUMMARY

Insurance as a problem in "rational choice"

Behavior problems may be treated in descriptive terms (what people do) or in normative terms (what is "rational" or advisable to do). Deciding on insurance is treated here as a normative behavior problem. That is, whether to accept or reject a policy covering either a fraction, or the total monetary value, of the insurable item is regarded in this paper as a problem in "rational" decision-making.

Requirements of "rational" choice

"Rational" choice is assumed to require, among other characteristics, that the individual be able to state what mutually exclusive outcomes may result from each choice, and to order them transitively in terms of his preferences. (Transitivity means that if A is preferred to B, and B is preferred to C, then A must be preferred to C). The term "preference" is used in its wider meaning which includes the case of indifference. These orderings may be accomplished by associating with each outcome an entity called "utility" which is orderable, but not necessarily measurable.
Cases of choices grouped according to the "type of knowledge" about events.

The outcomes of decisions made depend on two types of factors, those under the individual's control (which choice is made) and those outside his control (the occurrence of various "external" events). The principles of choice-making are selected according to the "type of knowledge" about the occurrence of the external events. The "types of knowledge" are classified into three groups according to the extent of the subjective, a priori knowledge about the occurrence of the external events. By using subjective knowledge the outcomes are not defined by questions of fact, but by what the individual believes to be true. The three groups are:

Case I - Decisions in which the individual knows which event will occur.

Case II - Decisions in which the individual can state the likelihood (probability) of the occurrence of each of the external events.

Case III - Decisions in which the individual has no knowledge of the probabilities of the occurrence of the external events.
Principles of decision-making and requirements for solution

Case I. In Case I only one possible outcome corresponds to each choice, since it is known (subjectively) which external event will take place. The principle used to make the rational decision is that of utility maximization. Since insurance problems are commonly considered in terms of losses in the main part of this paper, the suggested variations of the principle of maximizing utility are used as principles of minimizing loss in utility. The rational choice is that followed by choosing the outcome with the highest utility attached to it. It is not necessary that utility be measurable here; it is sufficient that it be orderable.

Case II. In Case II there are several possible outcomes corresponding to each choice. The outcome depends on which external event takes place, and the probability of each external event is known.

The principle used to make the rational decision is that of maximizing the "average" utility. (In what follows, whenever the term "average" is used, it should be interpreted as meaning a long-run average, i.e., the
"mathematical expectation").

To clarify the requirements for a solution, Case II is divided into two parts, Case II-A and Case II-B. Case II-A contains decisions in which there is one choice for which the outcomes, with respect to each external event considered, are all respectively greater than the outcomes from other choices. It has been shown in this paper that in Case II-A an ordering of the outcomes is sufficient to determine the rational decision.

Case II-B contains decisions in which there is no choice as specified in Case II-A. In Case II-B it has been shown that measurable utility is needed in order to determine which choice gives the maximum average utility. It has been shown by J. Marshak that measurable utility is implied by certain axioms on "rational" behavior of individuals under risk. However, it would also be possible to construct a theory of behavior under risk which does not accept Marshak's postulate, and therefore, may imply lack of utility measurability. In the latter case, however, one could not use the concept of average utility. The present paper does not go in this direction.
Case III. In Case III, which typically includes decisions on buying insurance, there are two, alternative approaches to decision-making, the "utility" approach and the "regret" approach. The "utility" approach will be considered first.

The "utility" approach

One possible criterion for choosing is described as follows: the individual indicates for each choice the "worst" outcome, or the one with the lowest (minimum) utility; he then decides on the choice in which the "worst" outcome is better than the "worst" outcome from any other choice (the maximum of the minimum outcomes). This is called maximizing the minimum utility.

Another possible criterion is this: assume some arbitrary probabilities for each of the external events, and compute the "average utility" as in Case II. Then behave in such a way that this average utility becomes the highest possible, even for the most "pessimistic" assumption with regard to the probabilities of the external events. This is called maximizing the minimum average utility.

It has been shown by Von Neumann and Morgenstern that in an important class of cases, these two criteria
Since we have a minmax, hence they do not require measure to need it has been shown that all detentions on

(Case III-a). Furthermore, when the utitity approach

is not measurable (Case III-b), utitity is needed when a minmax exists

It has been shown in this paper that only observable

points in each of which there is no minmax.

there is a minmax, and Case III-b, containing situations in each of which

Case III-a, containing situations in each of which
guarantemente for a solution, Case III is divided into

utility (lead to the same results, to attain the re-

the minmax utility and maxmin utility does exist that the two criteria agreed) (maxmin utility
does not exist. It is only when a minmax

to called the "minmax." In a given situation the min-

"worst outcome (which is also the lowest of the best"

come for the different outcome), then highest,

happens to equal the highest of all the "worst out-

best outcome (for the different external outcome)

the highest utility outcome. If the lowest of all the

the "best" outcome for a given external event, i.e.,

describe the three cases of cases we need the concept of

lead to the same solution for rational behavior. To
In Case III-B, in which there is no minimax, Von Neumann and Morgenstern have shown that the two criteria described give different outcomes. The behavior which gives the maximum of the average utility minima is "randomized". That is, a device, such as a die, or a table of random numbers, is used for choosing each time a decision is made so that the average (the mathematical expectation) of each choice corresponds to its determined value. In order to determine the "optimum" values of the ratios between choices used on the randomizing device, the utilities of the outcomes must be given a numerical value. Since all decisions on insurance using the "utility" approach have been shown to fall into Case III-A, "randomized" behavior is not needed in these decisions.

\* The "regret" approach

If the principle of maximizing the minimum average utility is used, it neglects the dissatisfaction arising out of having made the "wrong" choice in regard to the actual occurrence of events. If regret is considered to be important, the principle used is the minimizing of the maximum average regret.
The decisions using the regret principle can be classified into Case III-A and Case III-B, with the same requirements concerning measurability of utility. It has been shown in this paper that when the regret principle is used, decisions on insurance may fall into either Case III-A or Case III-B; hence, insurance decisions determined on regret may or may not call for randomized behavior.

When the regret principle is used in decisions involving more than two choices, the solution indicated may not be consistently indicated by using pairwise grouping. That is, if Choice A is the "best" of three choices, A, B, and C, it may not be better than Choice B if only Choice A and B are considered, as the set of choices. This is a different problem than the requirement for rational behavior that the ordering of outcomes in terms of preferences be transitive. This characteristic of the comparison of more than two choices in terms of regret requires that the solution be considered only on the basis of the total set of alternative choices. Decisions on insurance problems can be solved on the basis of choice between two alternatives, so that this characteristic of the regret solution is not involved in decisions on insurance.
Extent of knowledge limits the minimization of the maximum average loss or regret

The solutions arrived at by using the principles of decision-making give the maximum of the minimum average utility, or the minimum of the maximum average regret according to the given ordering of the given outcomes. If the number of possible choices or external events is changed, or their ordering is revised, then the new solutions will correspond to the new set of outcomes, and their new ordering. Thus, the extent of the maximization, or the minimization, depends on the extent of the knowledge of the individual as to the outcomes from various choices and events.

Extension of the method of solution to decisions on life insurance involving more than two choices.

Decisions on life insurance using the maximization of utility principle, seem to be especially suitable to solution by the use of a two-by-two table. The outcomes are primarily the result of two possible occurrences during a specified time period - the life or death of the insured. The choices which affect the change in the level of satisfaction (indicated by the utility evaluations of the outcomes) are whether or not to accept a contract for
insurance. If more than one policy is considered in setting up a table in determining a solution, the results are the same as though each were considered separately.

A restatement of the important concepts formulated in this paper

1. Rational decisions, determined by maximizing the minimum average utility, which involve risk require measurable utility.

2. Rational decisions on insurance, determined by maximizing the minimum average utility, do not require measurable utility.

3. The value of the ratio indicating the percentage of times each choice should be made, as determined by Von Neumann and Morgenstern, is the only value of the ratio which gives the maximum of the minimum average utility.

4. Rational decisions on insurance, determined by maximizing the minimum average utility do not have solutions requiring randomized behavior.

5. Rational decisions involving insurance, determined by minimizing the maximum average regret, may require randomized behavior.
V. LITERATURE CITED


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