INTERGENERATIONAL CONSUMER WELFARE IN DYNAMIC COMMODITY MARKETS

Greg Hertzler

Staff Paper 131 May, 1983

INTERGENERATIONAL CONSUMER WELFARE IN DYNAMIC COMMODITY MARKETS

By Greg Hertzler[1]

In commodity markets, both prices and the cohorts of consumers change over time. Previous stabilization literature applied to changing generations incorporates a preference for inequity, does not require those generations who gain from lower prices to compensate those who lose, and assumes instability which preserves arithmetic mean prices. This paper examines preferences for both equity and specific forms of inequity, enforces actual income compensation to make the Pareto criterion applicable, and includes both arithmetic and geometric mean preserving price instability. Previous stabilization conclusions are recast into a dynamic context and new ordinal criteria based upon demand and expenditure functions are derived.

I. Introduction

The analysis of consumer welfare in the presence of supply induced price instability has a long and controversial history. Theoretically valid measures are difficult to obtain even while policies must be implemented which require quantification of benefits and costs. Consumer surplus as the area under a Marshallian demand curve has been used extensively to approximate the areas under compensated demand curves. Currie, et. al. [7], presents a survey of surplus measures and Willig [23] places bounds upon the measurement errors. Hausman [11] integrates Marshallian demands to obtain indirect utility and expenditure functions from which compensated demands are derived.

Hanoch [9] and Turnovsky, et. al. [20] discuss surplus measures as special cases of the more general convexity/concavity criterion of the indirect utility function. Vartia [21] presents algorithms for computing income compensations which avoid the path dependence problems of line integration in surplus computations.

Surplus measures were first proposed to analyze discrete policy alternatives. For example, a public works project must compare equilibrium without the project to equilibrium if the project were completed. It is not obvious, however, that criteria for static equilibria always apply to dynamic commodity markets as they evolve over time. Two critical assumptions in the static theory of consumer welfare will be relaxed in this paper.

First, prices are usually defined to be symmetrically distributed. Randomness in production such as the effects of weather on agricultural crops is a justification often cited. The dynamic analog to symmetrically distributed prices is a set of differential equations for price movements which preserve the arithmetic means. Other forms of price instability may be more appropriate in different circumstances. Preserving the geometric means may better model dynamic commodity markets with differential equations in terms of rates of change. Geometric mean preserving prices were proposed by Fleming, et. al. [8] and are discussed by Turnovsky, et. al. [20]. An empirically correct definition of price instability will prove to be crucial in drawing welfare conclusions.

A second assumption of previous welfare theory treats consumers as a single group. Actually, cohorts of consumers continually change as they are born, age, and die. Although there is overlapping, each instantaneous generation will differ from every other. Consumers who enjoy lower prices are not all the same as those who may later be penalized with higher prices. Difficult issues of intergenerational comparisons through preferences for equity or some form of inequity, and questions of income compensation between generations must be addressed.

In this paper the life-cycle components of individual consumer decisions are ignored. Instead utilities of different generations will be combined in the context of a social welfare function. Welfare by definition of Bergson [3] and Harsanyi [10] is a functional composite of individual utilities. If a cross section of all individuals at an instant of time meets the conditions of Muellbauer [16] and aggregates into a consistent market, the social welfare function becomes a weighted sum of each generation's aggregate utility.

Although the implications of various choices for the intergenerational welfare weights can be explored, the ultimate choice is an ethical decision for the researcher. One choice might be a strict preference for equity. Another choice might incorporate a time preference either for the present or the future. Traditionally, all welfare weights are implicitly assumed to be equal. Though time neutral, this is still a preference for a specific kind of inequity, and, unless income is actually transferred to those who must pay higher prices, instability is Pareto noncomparable. Some consumers will prefer stability while others prefer instability. A transfer mechanism, as proposed by Hicks [12] and Kaldor [14], is designed in this study to admit Pareto comparisons under preferences for inequity.

There is considerable debate whether social welfare is cardinal or ordinal. While demand theory is able to assume ordinal commodity preferences because demand functions are independent of any monotomic transformations of utility, it is not clear whether intensity of preferences should be excluded from welfare considerations. Arrow [1] has taken an ordinal approach and has been criticized by Hildreth [13]. In this paper, cardinality must be assumed when the Pareto criterion does not apply. Otherwise, ordinality is sufficiently strong.

An important point to emphasize is that welfare analysis in commodity markets may require different assumptions for different situations. The researcher must implicitly or explicitly assign welfare weights to generations and decide what kinds of income compensation may be appropriate. Finally it will be necessary to empirically determine the best definition of price instability and possibly estimate a demand system.

Several conclusions are derived in this paper which place previous welfare results in context. Welfare improvements from instability are possible only when society prefers inequity between generations. With inequity, the welfare measure of choice is the Pareto criterion. Pareto comparable and ordinal welfare comparisons are performed in this paper by requiring those who gain from instability to actually compensate those who would otherwise lose. When previous welfare results are applied to changing generations, cardinal interpersonal comparisons are required because some consumers gain at the expense of others.

To give an overview of the paper, Section 2 details the assumptions to be made and derives a dynamic measure of changes in welfare over time. Section 3 considers possible choices of welfare weights. Two forms of price instability are defined in Section 4. Section 5 specifies functional forms for income compensation between generations. An expanded representation of the linear expenditure system is presented in Section 6 to allow for risk aversion, risk neutrality, or risk affinity in determining the convexity/concavity properties of indirect utility. The main body of conclusions is contained in Section 7 where various combinations of preferences for equity/inequity, price changes, income compensations, and risk preferences are analyzed. Finally a brief summary follows in Section 8.

2. Welfare Changes over Time

Several assumptions will be made in specifying a social welfare function. These are: 1) Both prices and generations of consumers change continuously over time; 2) Each generation spans only an instant and aggregates consistently into a cross sectional market whose utility does not depend upon the utility levels of previous or future generations; 3) Social welfare is the weighted sum of utilities over generations; 4) Income is exogenous although transfers of income may be endogenously determined; 5) The commodity choice set is fixed and is the same for each generation.

From these assumptions it follows that each generation's utility is time independent and can be weighted and added to the utility of all other generations. Moreover, individual utilities are time additive because each person belongs to more than one instantaneous generation. Further, the analysis is only partial, with income determination and productive decisions given.

The policy analyst studying price instability would wish to

$$\text{(1)} \qquad \text{Maximize } W = \int_{t=0}^{\infty} e^{-\delta_t t} \Omega_t U(q_{1t}, \ldots, q_{nt}) dt,$$

subject to

$$\text{(2)} \qquad \Omega_t y_t = \Omega_t \sum_{j=1}^{n} p_{jt} q_{jt}, \qquad t \in [0, \infty],$$

where W is total social welfare, U is the utility function of each representative individual which is twice continuously differentiable, strictly increasing, and strictly quasi-concave, Ω_t is the population within each generation, y_t is per capita income, p_{it}, i=1,...,n are commodity prices, q_{it}, i=1,...,n are per capita demand functions, and $e^{-\delta_t t}$ is a weighting factor assigned to each individual within a generation. Population cancels from both sides of the aggregate budget constraint, equation 2, and may grow at an exogenous rate ω. It can be rewritten as $\Omega_t = \Omega_0 e^{\omega t}$, with the welfare weights for each generation becoming $\Omega_0 e^{(\omega-\delta_t)t}$.

Life cycle aspects of the consumer decision problem are ignored, combining time and equity preferences in the same discount factor. Problems can arise if $\omega-\delta_t$ is nonnegative, as discussed by Wan [22]. Briefly stated, convergence is assured only if $\omega-\delta_t$ is negative. Whenever $\omega-\delta_t > 0$, future weights will become infinitely large as t approaches infinity. If $\omega-\delta_t = 0$, convergence is difficult to check and it becomes impossible to distinguish between alternate time paths as t increases. In this study, $\omega-\delta_t$ will be either positive, zero, or negative and a criterion for measuring changes in welfare must be developed which is not constrained by convergence problems.

The time additivity assumption allows welfare at each instant of time to be expressed as the LaGrangian

$$(3) \qquad W_t = \Omega_0 e^{(\omega-\delta_t)t} U(q_{1t},\dots,q_{nt}) + \lambda_t(y_t - \sum_k p_{kt}q_{kt}), \qquad t \in [0, \infty),$$

with λ_t the marginal social welfare of income.

First order conditions of 3 can be solved for demands q_{it}, $i=1,\ldots,n$, at each instant of time t and substituted into direct utility, $U(q_t)$, to give the indirect utility function, $V(p_t, y_t)$, and its inverse, the expenditure function, $m(V_t, p_t)$. Welfare in turn is an unconstrained function of indirect utility,

$$(3') \qquad W_t = \Omega_0 e^{(\omega-\delta_t)t} V(p_{1t},\ldots,p_{nt},y_t).$$

The time derivative of 3' is

$$(4) \qquad \frac{dW_t}{dt} = \frac{\partial \Omega_0 e^{(\omega-\delta_t)t} V_t}{\partial t}$$

$$= \Omega_0 e^{(\omega-\delta_t)t} [(\omega-\delta_t - \dot{\delta}_t t)V_t + \sum_j \frac{\partial V_t}{\partial p_{jt}} \dot{p}_{jt} + \frac{\partial V_t}{\partial y_t} \dot{y}_t],$$

where $\dot{\delta}_t$, $\dot{p}_{it}$, $i=1,\ldots,n$ and $\dot{y}_t$ are derivatives with respect to time. Equation 4 gives the direction of change in welfare, dW_t/dt. The problem of "many surpluses" does not arise. Burns [5, 6] and Silberberg [19] discuss the path dependence of line integrals as it relates especially to compensating and equivalent variations. If dW_t were desired, path dependence in 4 could be avoided by completely defining the time paths of prices and income. In this study, prices and income vary simultaneously and continuously according to differential equations. Price instability is socially preferred or not depending upon whether welfare is convex or concave with respect to time. Convexity is determined by comparing the slopes of welfare at different points in time.

As it is written, Equation 4 can be applied in arbitrarily complex situations. Structural relationships for welfare weights between generations, price movements, income compensations, and even indirect utility can be imposed to draw specific conclusions. The more restrictions, the less general the conclusions will be.

For example, suppose income was constant, prices either increased or decreased over time in a way which preserved the arithmetic means, and all welfare weights were equal. Then $\dot{y}_t$, $\omega - \delta_t$, and $\dot{\delta}_t$ would be zero, and the direction of change in welfare, dW_t/dt, would depend only upon positive or negative price changes, $\dot{p}_{it}$, $i=1,\ldots,n$, multiplied by marginal indirect utilities, $\partial V_t/\partial p_{it}$, $i=1,\ldots,n$. These are the assumptions of previous price stabilization literature recast into the present dynamic context. Of the infinite variety of possible assumptions, the following sections will analyze a few of the most interesting.

3. Choice of Welfare Weights

A (Laspeyres type) cost of living index is denoted by the expenditure ratio $m(V_0, p_t)/m(V_0, p_0)$. Expenditures for fixed utility level V_0 are compared at initial prices, p_0, and later prices, p_t. By analogy, a true "util" of living index is the ratio $V(p_t, \bar{y})/V(p_0, \bar{y})$. For a fixed income, utility at initial prices, p_0, is compared to utility at later prices, p_t. A generation which gains (loses) from price changes will have a util index greater than (less than) unity.

The point of view with a strict preference for equity will require relative welfare weights to be inversely related to the util index,

$$(5) \qquad \frac{1}{e^{(\omega - \delta_t)t}} = \frac{V(p_t, y_t)}{V(p_0, y_0)} ,$$

where income, y_t, is allowed to vary over time. Equation 5 implies $V_0 = e^{(\omega-\delta_t)t} V_t$. As utility changes it is continuously discounted or compounded until every generation contributes equally to social welfare.

Other points of view embody a preference for inequity. Affinity for the present, time neutrality, and affinity for the future prevail whenever relative welfare weights are decreasing, constant, or increasing over time, respectively. Welfare weights can be assumed to grow at constant rate $\omega-\delta$ and

$$(6) \qquad e^{(\omega-\delta)t} <=> 1$$

when $\omega-\delta <=> 0$. With a preference for inequity, generations do not contribute equally to social welfare.

4. Price Changes over Time

Two classes of differential equations would appear to be the most useful in defining price instability, i.e., prices which preserve the arithmetic means, and prices which preserve the geometric means. Two further assumptions are made. 1) Prices change within the time interval, $t \varepsilon [0, 1]$. 2) Those prices which do change move in the same direction over this interval, either increasing or decreasing.

Of a wide class of differential equations which would preserve the arithmetic means, perhaps the simplest is

$$\dot{p}_{it} = c_i \qquad (i=1,\ldots,n), \tag{7}$$

where c_i is a constant, either positive or negative. Prices obeying 7 are uniformly distributed and will be termed arithmetic mean preserving, (AMP). With p_{i0} being the initial price of each commodity i, the differential equations (probability densities) in 7 have the solutions (distributions)

$$p_{it} = p_{i0} + c_i t, \qquad (i=1,\ldots,n), \tag{8}$$

where $t \varepsilon [0,1]$. At time $t=1/2$, price $p_{i1/2}$ equals the arithmetic mean of all prices, $p_{i0} + 1/2c_i$, $i=1,\ldots,n$.

A set of differential equations expressed as rates of change is

$$\dot{p}_{it} = c_i p_{it}, \qquad (i=1,\ldots,n). \tag{9}$$

This form of price instability has skewed distributions which will be termed geometric mean preserving, (GMP). The solutions (distributions) for differential equations (probability densities) in 9 are

$$(10) \qquad p_{it} = p_{i0}e^{c_i t}, \qquad (i=1,\ldots,n).$$

Price $p_{i1/2}$ again equals the geometric means of all prices, $p_{i0}e^{1/2c_i}$, $i=1,\ldots,n$.

The arithmetic mean for a given set of prices can be compared to the geometric mean of the same prices by the inequality

$$(11) \qquad \frac{1}{T}\int_{t=0}^{T} p_{it}dt > e^{\frac{1}{T}\int_{t=0}^{T} \ln p_{it}dt}, \qquad (i=1,\ldots,n),$$

where T=1. The arithmetic mean of a set of prices always exceeds the geometric mean of the same prices unless they are stable and $p_{it}=p_{i0}$ for all t.

Such a comparison is misleading, however, because the time paths of prices generated by 8 will differ from those generated by 10 and they cannot be directly compared. Given initial price p_{i0} and arbitrary constant c_i,

$$(12) \qquad p_{i0} + c_i t <=> p_{i0}e^{c_i t}, \qquad (i=1,\ldots,n).$$

Only if prices are stable with $c_i 0$, $i=1,\ldots,n$, are AMP and GMP time paths equal. Whether AMP prices exceed GMP prices or not depends upon the magnitudes of p_{i0} and t, and the magnitude and sign of c_i.

5. Income Changes over Time

A simplification in much of the price stabilization literature is the assumption of a stable income, i.e.,

(13) $y_t = \bar{y}$

and

(14) $\dot{y}_t = 0$

where $\bar{y}$ is a constant.

Price changes over time may be Pareto noncomparable if the generations paying higher prices prefer stability while the generations paying lower prices prefer instability. Pareto comparability is assured if an income transfer mechanism requires those who gain to actually compensate those who lose. If each generation is endowed with a fixed income, $\bar{y}$, the utility that would prevail at stabilized prices is $V(\bar{p}, \bar{y})$ where $\bar{p}$ is a vector of either arithmetic or geometric mean prices. For the two kinds of price instability discussed in the previous section, $\bar{p}=p_{1/2}$ and $V(\bar{p}, \bar{y})=V(p_{1/2}, \bar{y})$.

First assume $k \leq n$ prices are rising in the interval $t \in [0,1]$ making $c_i > 0$, $i=1,...,k$ in 7, 8, 9, and 10, and $p_{i0} < p_{i1}$. Between times $t \in [1/2,1]$ generations will tend to prefer mean prices since indirect utility is nonincreasing in prices, i.e., $V_{1/2} \geq V_t$. The amount of additional income required to make them indifferent between stable and higher prices is $y_t^c = m(V_{1/2}, p_t) - m(V_{1/2}, p_{1/2})$ where y_t^c is the negative of compensating variation. Similarly, when prices are falling, $c_i < 0$ and $p_{i0} > p_{i1}$. Generations of consumers in the interval $t \in [0, 1/2]$ will require compensation $y_t^c = m(V_{1/2}, p_t) - m(V_{1/2}, p_{1/2})$. Of course this compensation must be paid by the generations benefiting from lower prices. When $c_i > 0$, these consumers are in the time interval $t \in [0, 1/2]$. When $c < 0$, they reside in $t \in [1/2, 1]$.

For rising prices, the income transfer mechanism is

$$\begin{aligned}(15a)\qquad y_t &= \bar{y}-y^c_{1-t} = \bar{y}-m(V_{1/2},p_{1-t})+m(V_{1/2},p_{1/2}) \\ &= 2m(V_{1/2},p_{1/2})-m(V_{1/2},p_{1-t}), \qquad t \in [0,1/2],\ c_i>0, \quad (i=1,\dots,k),\end{aligned}$$

$$\begin{aligned}(15b)\qquad y_t &= \bar{y}+y^c_t = \bar{y}+m(V_{1/2},p_t)-m(V_{1/2},p_{1/2}) \\ &= m(V_{1/2},p_t), \qquad t \in [1/2,1],\ c_i>0, \qquad (i=1,\dots,k).\end{aligned}$$

The transfer mechanism decreases income when prices are lower in the interval $t \in [0,1/2]$, while income in the interval $t \in [1/2,1]$ is increased. Notice the time subscripts in 15a require consumers paying the lowest prices, p_{i0}, $i=1,\dots,k$ to compensate those paying the highest prices, p_{i1}. When $t=1/2$ no compensation is paid.

The changes over time of 15a and 15b are

$$(16a)\qquad \dot{y}_t = -\dot{y}^c_{1-t} = -\sum_j \frac{\partial m(V_{1/2},p_{1-t})}{\partial p_{j(1-t)}}\dot{p}_{j(1-t)}, \quad t \in [0,1/2],\ c_i>0,\ (i=1,\dots,k),$$

$$(16b)\qquad \dot{y}_t = \dot{y}^c_t = \sum_j \frac{\partial m(V_{1/2},\ p_t)}{\partial p_{jt}}\dot{p}_{jt}, \qquad t \in [1/2,1],\ c_i>0, \qquad (i=1,\dots,k).$$

Conversely, when prices are high initially and falling, the transfer mechanism is

$$(17a)\quad y_t = \bar{y}+y_t^c = \bar{y}+m(V_{1/2},p_t)-m(V_{1/2},p_{1/2})$$

$$= m(V_{1/2},p_t), \qquad t \in [0,1/2],\ c_i<0, \qquad (i=1,\ldots,k),$$

$$(17b)\quad y_t = \bar{y}-y_{1-t}^c = \bar{y}-m(V_{1/2},p_{1-t})+m(V_{1/2},p_{1/2})$$

$$= 2m(V_{1/2},p_{1/2})-m(V_{1/2},p_{1-t}), \qquad t \in [1/2,1],\ c_i<0, \quad (i=1,\ldots,k).$$

Taking derivatives,

$$(18a)\quad \dot{y}_t = \dot{y}_t^c = \sum_j \frac{\partial m(V_{1/2},p_t)}{\partial p_{jt}}\dot{p}_t, \qquad t \in [0,1/2],\ c_i<0, \qquad (i=1,\ldots,k),$$

$$(18b)\quad \dot{y}_t = \dot{y}_{1-t}^c = -\sum_j \frac{\partial m(V_{1/2},p_{1-t})}{\partial p_{j(1-t)}}\dot{p}_{j(1-t)}, \ t \in [1/2,1],\ c_i<0, \quad (i=1,\ldots,k).$$

By definition, $y_t=m(V_t,p_t)$, but for rising prices in 15b, $y_t=m(V_{1/2},p_t)$. This implies $V_{1/2}=V_t$ when $t \in [1/2,1]$ and $c_i>0$, $i=1,\ldots,k$. Conversely, falling prices, $c_i<0$, $i=1,\ldots,k$, and $t \in [0,1/2]$ in 17a imply $V_t=V_{1/2}$. Those generations who would otherwise lose from instability are compensated to the point of indifference between stability and instability.

6. Functional Forms for Demand Systems

In most cases, it will be necessary to specify a demand system before applied welfare conclusions can be reached. The welfare problem as defined in this study assumes consistent cross sectional aggregations within each generation of consumers. Consistency requires a price independent generalized linear expenditure function (PIGL) as formulated by Muellbauer [16]. Members of the PIGL class must satisfy

$$(19a) \quad m(V_t,p_t) = y_t = [a(p)^{-\varepsilon}+h(V_t)b(p_t)^{-\varepsilon}]^{-1/\varepsilon}, \; \varepsilon \neq 0,$$

$$(19b) \quad \ln m(V_t,p_t) = \ln y_t = \ln a(p)+h(V_t)\ln b(p_t), \; \varepsilon = 0,$$

where $a(p_t)$ and $b(p_t)$ are functions of prices, $h(V_t)$ is a monotonic transformation of utility, and ε is a parameter. A common member of the PIGL class is the linear expenditure system (LES). The LES is derivable from the additive Stone-Geary direct utility function as discussed by Phlips [17]. Direct additivity implies restrictions on the substitutability between commodities which will be exploited in the examples of Section 7 to draw a priori conclusions. The welfare measures to be derived could just as easily apply to other demand systems. However, any flexible functional forms will almost surely need to be estimated before conclusions are possible.

When ε =-1 in 19a, expenditures are linear. Further assumptions of LES are

$$(20) \quad a(p_t) = \sum_j \alpha_j p_{jt}, \qquad b(p_t) = \prod_j p_{jt}^{\beta_j},$$

$$(21) \quad 0 \leq \alpha_i < q_i, \; 0 < \beta_i < 1, \; \sum_j \beta_j = 1, \qquad (i=1,\ldots,n),$$

$$(22) \quad h(V_t) = V_t^{1/\upsilon}, \; \upsilon > 0,$$

where υ is a strictly positive parameter.

Equation 20 gives specific functional forms for $a(p_t)$ and $b(p_t)$. Restrictions on parameters necessary to meet the condition of negativity of the direct substitution effect, Slutsky symmetry, and adding up are shown in 21. Equation 22 contains the assumptions about transformation h. It is usual to let $h(V_t)=e^{V_t}$, but the modified transformation in 22 is more flexible and allows the measurement of cardinal risk preferences.

Salient properties of the modified LES are

$$(23)\qquad m(V_t,p_t) = y_t = \sum_j \alpha_j p_{jt} + V_t^{1/\upsilon} \prod_j p_{jt}^{\beta_j},$$

$$(24)\qquad V(p_t,y_t) = \left(\frac{y_t - \sum_j \alpha_j p_{jt}}{\prod_j p_{jt}^{\beta_j}}\right)^{\upsilon},$$

$$(25)\qquad \frac{\partial V_t}{\partial p_{it}} = -\upsilon V_t\left(\frac{\alpha_i}{y_t - \sum_j \alpha_j p_{jt}} + \frac{\beta_i}{p_{it}}\right), \qquad (i=1,\ldots,n),$$

$$(26)\qquad \frac{\partial V_t}{\partial y_t} = \upsilon V_t\left(\frac{1}{y_t - \sum_j \alpha_j p_{jt}}\right),$$

$$(27)\qquad q_i(V_t,p_{it}) = \frac{\partial m_t}{\partial p_{it}} = \alpha_i + \left(V_t^{1/\upsilon} \prod_j p_{jt}^{\beta_j}\right) \frac{\beta_i}{p_{it}}, \qquad (i=1,\ldots,n),$$

$$(28)\qquad q_i(p_{it},y_t) = \frac{-\partial V_t/\partial p_{it}}{\partial V_t/\partial y_t} = \alpha_i + \left(y_t - \sum_j \alpha_j p_{jt}\right) \frac{\beta_i}{p_{it}}, \qquad (i=1,\ldots,n),$$

$$(29)\qquad \rho_t = \frac{-\partial \ln(\partial V_t/\partial y_t)}{\partial \ln y_t} = (1-\upsilon)\frac{y_t}{y_t - \sum_j \alpha_j p_{jt}},$$

$$\eta_{it} = \frac{\partial \ln q_{it}}{\partial \ln y_t} = \frac{\beta_i y_t}{p_{it} q_{it}}, \qquad (i=1,\ldots,n), \tag{30}$$

where q_{it}, i=1,...,n are demand functions, ρ_t is the income elasticity of the marginal utility of income, and η_{it}, i=1,...,n are income elasticities of demand.

All of the parameters are known once ordinal demand functions are estimated, except υ which must be determined separately, as discussed by Hanoch [9]. The elasticity of marginal utility, ρ_t, was first proposed by Arrow [2] and Pratt [18] as the coefficient of relative risk aversion. Measurements of ρ_t are discussed in Luce and Suppes [15] and experimentally obtained by Binswanger [4]. Once values for ρ_t are known, υ can be estimated.

When $0<\upsilon<1$, $\upsilon=1$, and $\upsilon>1$, $\rho>0$, $\rho=0$, and $\rho<0$ respectively, and consumers have risk aversion, risk neutrality, or risk affinity. For fixed υ, $0<\upsilon<1$, ρ_t recedes from infinity as income y_t, increases beyond subsistence level, $\Sigma_j \alpha_j p_{jt}$, and in the limit equals $1-\upsilon$. In other words, there is decreasing relative risk aversion, or, intuitively, generations of consumer with less income will be less inclined to prefer price instability.

7. Changes in Consumer Welfare under Selected Restrictions

A very general statement of changes in consumer welfare is contained in Equation 4. This section will obtain more specific results by imposing selected restrictions.

Time Preference for Equity

A widely applicable conclusion follows when a preference for equity in

Equation 5 prevails. Inverting 5, and taking logarithms, $(\omega-\delta_t)t=\ln V_0-\ln V_t$ which, when differentiated, becomes

$$(31) \qquad \omega-\delta_t-\dot{\delta}_t t = -[\sum_j \frac{\partial V_t}{\partial p_{jt}}\dot{p}_{jt}+\frac{\partial V_t}{\partial y_t}\dot{y}_t]/V_t.$$

Combining 5, 31, and 4,

$$(32) \qquad \frac{dW_t}{dt} = \frac{\partial \Omega_0 V_0}{\partial t} = \Omega_0 e^{(\omega-\delta_t)t}[0] = 0.$$

Proposition 1: When there is a preference for equity, neither price instability nor income changes will change social welfare, regardless of the form of indirect utility. Conversely, increases or decreases in welfare require a preference for some form of inequity.

Interpretation of Proposition 1 must be done carefully. It simply states that, if all generations count equally, social welfare is constant. It does not preclude price instability from which some generations may gain and some may lose. Presumably, those who prefer equity would also stabilize prices and/or implement income transfer mechanisms to equalize utilities over all generations. Proposition 1 says nothing about the level of total welfare given different price and income changes.

Whenever utility is constant over time, the preference for equity degenerates into time neutrality with $\omega-\delta_t$ and δ_t zero. Constant utility requires $\Sigma_j(\partial V_t/\partial p_{jt})\dot{p}_{jt}+(\partial V_t/\partial y_t)\dot{y}_t=0$ which becomes $\dot{y}_t=\Sigma_j\dot{p}_{jt}q_{jt}$ using Roy's Theorem. By definition, $\dot{y}_t = \Sigma_j(\dot{p}_{jt}\; q_{jt} + p_{jt}\; \dot{q}_{jt})$, implying $\dot{q}_{it}=0$, i-1,...,n, and making demands, q_{it}, constant. Vartia [21] presents algorithms for approximating constant demand differential equations in income to an arbitrary degree of accuracy. Drawing upon Vartia's results, the following proposition is presented for completeness with very little elaboration.

Proposition 2: When there is a preference either for equity or time neutrality and generations are compensated to maintain constant utility, consumers prefer price instability if constant income compensated demands exceed constant demands at stable prices. Otherwise, consumers are indifferent or prefer stability. This is an ordinal and Pareto comparable result.

The income compensation mechanisms of this study are easy to solve in closed form because they are anchored to the reference level of utility, $V_{1/2}$, which also happens to be the level of utility at stable prices. If some generations gain while others are indifferent, it would be possible to increase compensation so that all gain equally. However, once demands are compensated to be constant over all generations, welfare is constant over time for both stable and unstable prices, and there is no common reference level of utility. An approximation algorithm such as Vartia's becomes necessary to quantify benefits.

Pareto Noncomparability, Time Neutrality, and AMP Prices

When there is a preference for inequity and no compensation between generations is enforced, the Pareto criterion does not apply. All welfare weights will be set equal in Equation 6. Price changes are defined in 7, with constant income as in 14. The convexity/concavity of W_t can be determined by evaluating differences in the slope, dW_t/dt, at two arbitrary points, t=1 and t=0.

$$(33) \qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0\Big[\sum_j \frac{\partial V_t}{\partial p_{jt}}\dot{p}_{jt}\Big]\Big|_0^1$$

$$= \Omega_0 [\sum_j (\frac{\partial V_1}{\partial p_j} - \frac{\partial V_0}{\partial p_{j0}}) c_j].$$

Equation 33 is a second order condition which is positive for W_t convex over time. When there are n prices, any subset, k, $0<k<n$, of those prices can be unstable. For the important case of $k=1$, $c_1 \neq 0$, and $c_2=c_3=,\ldots,=c_n=0$. Convexity/concavity of social welfare depends upon the convexity/concavity in p_{1t} of indirect utility, V_t. If V_t is linear in price p_{1t}, $\partial V_1/p_{11} = \partial V_0/p_{10}$, and society is indifferent between stable and unstable p_1. Because V_t is nonincreasing in prices with $\partial V_t/\partial p_{1t} \leq 0$, whenever V_t is convex in p_{1t} marginal utilities will differ and at least one of them will be negative. An increase in price p_{1t} requires $c_1>0$, $p_{10}<p_{11}$ and $|\partial V_0/\partial p_{10}|>|\partial V_1/\partial p_{11}|$ or $\partial V_0/\partial p_{10}<\partial V_1/\partial p_{11} \leq 0$. Equation 33 is positive because $\partial V_0/\partial p_{10}$ dominates and is negative. Similarly, when $c_1<0$, $p_{i0}>p_{i1}$, $0 \geq \partial V_0/\partial p_{10} > \partial V_1/\partial p_{11}$ and again 33 is positive.

More than one price can be unstable, $1<k \leq n$, in which case V_t may be convex in all k prices. Hanoch [9] and Turnovsky, et. al. [20] give sufficient but not necessary conditions for indirect utility to be convex, and cite conventional surplus measures as a special case. The static result translated into the present dynamic context is summarized as:

Proposition 3: When there is a preference for time neutrality, there is no compensation between generations, and both stable and unstable prices preserve the same arithmetic means, price instability is socially preferred if indirect utility is strictly convex in the unstable prices. This is a cardinal and Pareto noncomparable result because some generations of consumers are harmed.

The traditional wisdom holding consumers to prefer supply induced price instability embodies the assumptions of Proposition 3, especially cardinality and AMP prices, and the empirical statement that indirect utility is usually convex. Marginal indirect utility for the LES in 25 can be substituted into 33 to give

$$(34)\qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[\sum_j(V_0(\frac{\alpha_j}{\bar{y}-\Sigma_0} + \frac{\beta_j}{p_{j0}}) - V_1(\frac{\alpha_j}{\bar{y}-\Sigma_1} + \frac{\beta_j}{p_{j1}}))c_j],$$

$$= \Omega_0\upsilon[\sum_j(\frac{\alpha_j(\bar{y}-\Sigma_0)^{\upsilon-1}}{\Pi_0^\upsilon} + V_0\frac{\beta_j}{p_{j0}} - \frac{\alpha_j(\bar{y}-\Sigma_1)^{\upsilon-1}}{\Pi_1^\upsilon} - V_1\frac{\beta_j}{p_{j1}})c_j],$$

where, to conserve notation, Σ_0 is $\Sigma_{j=1}^k\alpha_j p_{j0}+\Sigma_{j=k+1}^n\alpha_j p_{jt}$, Σ_1 is $\Sigma_{j=1}^k a_j p_{j1} + \Sigma_{j=k+1}^n\alpha_j p_{jt}$, Π_0 is $\Pi_{j=1}^k p_{j0}^{\beta_j}\Pi_{j=k+1}^n p_{jt}^{\beta_j}$, and Π_1 is $\Pi_{j=1}^k p_{j1}^{\beta_j}\Pi_{j=k+1}^n p_{jt}^{\beta_j}$, $0\leqslant k\leqslant n$.

If all prices are stable, $k=0$ and $c_i=0$, $i=1,\ldots,n$. Welfare will not change over time. If some prices are unstable, $c_i>0$, $i=1,\ldots,k$, and welfare is unconditionally convex with respect to time if consumers have either risk neutrality or risk affinity. This is shown by examining two cases.

First, let $c_i>0$, $p_{i0}<p_{i1}$, $i=1,\ldots,k$, and $V_0\geqslant V_1$. The second term in the summation, $V_0\beta_i/p_{i0}$, will dominate the fourth term, $V_1\beta_i/p_{i1}$. As long as $\upsilon\geqslant 1$, the first term will dominate the third, because $\Sigma_0<\Sigma_1$ and $\Pi_0<\Pi_1$. Equation 34 is positive. Conversely, if $c_i<0$, $p_{i0}>p_{i1}$, $i=1,\ldots,k$, $V_0\leqslant V_1$, and $\upsilon\geqslant 1$, the fourth term will dominate the second, and the third will dominate the first. The result multiplied by negative c_i again makes 34 positive.

Corollary 3a: Consumers with risk neutrality or risk affinity always gain under the conditions of Proposition 3, given the linear expenditure system.

If some prices increased while others decreased, Σ_t, Π_t, and V_t in 34 could either increase, decrease, or remain unchanged. For this reason, all prices are assumed to move in the same direction. When consumers are risk averse $0<\upsilon<1$, and the first and third terms cannot be compared without estimates of the demand parameters α_i and β_i, and risk coefficient ρ_t.

The sufficient conditions for convexity of Turnovsky, et. al. [20] require all income elasticities of demand, η_i, to be equal for the k commodities with unstable prices and equal to unity if k=n. Thus $\eta_i = \eta_k$, $i=1,\ldots,k\leq n$, and $\eta_k=1$ if k=n. Consumers will gain from price instability if $2\eta_k \geq \rho_t$. This condition is more likely to be met for income inelastic commodities and low degrees of risk aversion. For $\rho_t \leq 0$ it is always met, independently verifying Corollary 3a, although Corollary 3a does not require common income elasticities of demand.

Pareto Noncomparability, Time Neutrality, and GMP Prices

Equal welfare weights in 6, price changes preserving the geometric means as in 9, and constant income according to 14 give

$$(35) \qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[\sum_j \frac{\partial V_t}{\partial p_{jt}} \dot{p}_{jt}]\Big|_0^1$$

$$= \Omega_0[\sum_j(\frac{\partial V_1}{\partial p_{j1}} p_{j1} - \frac{\partial V_0}{\partial p_{j0}} p_{j0})c_j].$$

Whereas Equation 33 is positive whenever indirect utility is convex in prices, Equation 35 is positive whenever indirect utility is convex in the logarithm of prices. This is noted by the fact $\partial V_t/\partial \ln p_{it} = (\partial V_t/\partial p_{it})p_{it}$, $i=1,\ldots,n$.

Proposition 4: When there is a preference for time neutrality, there is no compensation between generations, and both stable and unstable prices preserve the same geometric means, price instability is socially preferred if indirect utility is strictly convex in the logarithms of unstable prices. This is a cardinal and Pareto noncomparable result because some generations of consumers are harmed.

In 33, it was shown consumers could gain if some prices are rising, $c_i>0$, $p_{i0}<p_{i1}$, $i=1,\ldots,k$, because $\partial V_0/\partial p_{i0}<\partial V_1/\partial p_{i1}<0$. However, for positive and rising prices, $(\partial V_0/\partial p_{i0})p_{i0}$ will decrease in absolute value relative to $(\partial V_1/\partial p_{i1})p_{i1}$. Similarly, for falling prices, $c_i<0$, $p_{i0}>p_{i1}$, $i=1,\ldots,k$, $(\partial V_1/\partial p_{i1})p_{i1}$ will decrease relative to $(\partial p_0/\partial p_{i0})p_{i0}$. With the same marginal utilities, it is possible for 33 to be positive and 35 negative.

Corollary 4a: Consumers will be less likely to prefer geometric mean preserving price instability than arithmetic mean preserving instability.

One unambiguous case can be observed as υ approaches zero in the LES. Indirect utility, V_t, will approach unity in 24, and marginal indirect utility in 25 will approach $-\upsilon(\alpha_i/(y_t-\Sigma_j\alpha_j p_{jt}) + \beta_i/p_{it})$. Equation 35 approaches

$$(36)\qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0\upsilon[\sum_j(\frac{p_{j0}\alpha_j}{\bar{y}-\Sigma_0} + \beta_j - \frac{p_{j1}\alpha_j}{\bar{y}-\Sigma_1} - \beta_j)c_j].$$

When prices are increasing, $c_i>0$, $p_{i0}<p_{i1}$, $i=1,\ldots,k$, and the summation in 36 is negative. When prices are decreasing, $c_i<0$, $p_{i0}>p_{i1}$, $i=1,\ldots,k$, and the summation is again negative. Multiplied by a very small positive υ change in welfare is very near zero, but negative.

Corollary 4b: Very risk averse consumers can never gain from geometric mean preserving price instability, given the linear expenditure system.

Pareto Comparability, Time Neutrality, and AMP Prices

Enforcing compensation between generations will allow welfare rankings by the Pareto criterion. Roy's Theorem and Shepard's Lemma in the form $-\partial V_t/\partial p_{it} = (\partial V_t/\partial y_t)q_{it} = (\partial V_t/\partial y_t)\partial m_t/\partial p_{it}$, $i=1,\ldots,n$, are applied in the following derivations. It is also necessary to find $\dot{p}_{i(1-t)}$, $i=1,\ldots,k$ in 16a and 18b. Substituting $1-t$ for t in Equation 8, $\dot{p}_{i(1-t)} = -c$, $i=1,\ldots,k$ for arithmetic mean preserving prices. With equal welfare weights in 6, AMP price changes from 7, changes in compensated income for increasing prices from 16a and 16b, the difference in the changes in welfare becomes

$$\text{(37a)}\quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[\sum_j \frac{\partial V_t}{\partial p_{jt}} c_j + \frac{\partial V_t}{\partial y_t} \sum_j \frac{\partial m(V_{1/2}, p_{1-t})}{\partial p_{j(1-t)}} c_j]\Big|_0^{1/2}$$

$$= \Omega_0[\sum_j \frac{\partial V_t}{\partial p_{jt}} c_j + \frac{\partial V_t}{\partial y_t} \sum_j \frac{\partial m(V_{1/2}, p_t)}{\partial p_{jt}} c_j]\Big|_{1/2}^1$$

$$= \Omega_0[\sum_j (\frac{\partial V_1}{\partial p_{j1}} + \frac{\partial V_1}{\partial y_1} \frac{\partial m(V_1, p_1)}{\partial p_{j1}}) c_j]$$

$$- \Omega_0[\sum_j (\frac{\partial V_0}{\partial p_{j0}} + \frac{\partial V_0}{\partial y_0} \frac{\partial m(V_1, p_1)}{\partial p_{j1}}) c_j]$$

$$= \Omega_0 \frac{\partial V_0}{\partial y_0} [\sum_j (q_j(p_0, y_0) - q_j(V_1, p_1)) c_j], \qquad c_i > 0, \qquad (i=1,\ldots,k),$$

where $V_{1/2}=V_1$ for increasing prices. A completely analogous derivation for falling prices and the compensation mechanism of 18a and 18b gives

$$(37b) \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \frac{\partial V_1}{\partial y_1}[\sum_j (q_j(V_0,p_0)-q_j(p_1,y_1))c_j], \qquad c_i<0, \qquad (i=1,\ldots,k).$$

For falling prices, $V_0 = V_{1/2}$. In both 37a and 37b, the generations paying higher prices have been compensated to make them indifferent between stable and unstable prices. By the Pareto criterion, welfare is convex, with 37a and 37b positive, if the generations benefitting from low prices can pay compensation and still gain. Notice the magnitudes of 37a and 37b depend upon cardinal marginal utilities of income, but the signs of 37a and 37b depend only upon a relative price change weighted sum of differences in ordinal demand functions, q_{i0} and q_{i1}, $i=1,\ldots,k$.

Propositon 5: When there is a preference for time neutrality, generations who would otherwise lose are compensated to the point of indifference and both stable and unstable prices preserve the same arithmetic means, price instability is socially preferred if the relative price change weighted sum of quantities demanded by generations paying both lower prices and compensation exceeds the relative price change weighted sum of quantities demanded by generations paying higher prices but receiving compensation. Otherwise consumers are either indifferent or prefer stability. This is an ordinal and Pareto comparable result because no generations of consumers are harmed.

To achieve further insight into the effects of compensation, the uncompensated cardinal measure in 33 can be rewritten as

$$(33') \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[\sum_j(\frac{\partial V_0}{\partial \bar{y}}q_j(p_0,\bar{y}) - \frac{\partial V_1}{\partial \bar{y}}q_j(p_1,\bar{y}))c_j],$$

the sign of which depends upon differences in the marginal utilities of income. Compensation in 37a and 37b eliminates the effects of different marginal utilities of income by varying income itself.

Equations 37a and 37b are easy to apply. Given the expenditure and indirect utility functions, compensated demands are known once ordinary demands are estimated. Further, the expenditure function at fixed levels of indirect utility gives compensated incomes as functions of mean income, mean and extreme prices. In particular, for LES expenditures in 23, increasing AMP prices in 7, and compensation as in 15a and 15b,

$$(38a) \quad y_0 - \Sigma_0 = 2(\Sigma_{1/2}+V_{1/2}^{1/\upsilon}\Pi_{1/2})-\Sigma_1-V_{1/2}^{1/\upsilon}\Pi_1-\Sigma_0$$

$$= V_{1/2}^{1/\upsilon}(2\Pi_{1/2}-\Pi_1)$$

$$(38b) \quad y_1 - \Sigma_1 = \Sigma_1+V_{1/2}^{1/\upsilon}\Pi_1-\Sigma_1$$

$$= V_{1/2}^{1/\upsilon}\Pi_1$$

where $2\Sigma_{1/2} = \Sigma_1+\Sigma_0$ for AMP prices. Substituting marginal utility of income and demand functions from 26, 27, and 28, incorporating 38a and 38b, and noting $\beta_i/p_{it}=(\partial\Pi_t/\partial p_{it})/\Pi_t$, $i=1,\ldots,n$, 37a becomes

$$(39a) \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0\upsilon V_0[\sum_j(\frac{\beta_j}{p_{j0}} - \frac{y_1-\Sigma_1}{y_0-\Sigma_0}\frac{\beta_j}{p_{j1}})c_j]$$

$$= \Omega_0 \upsilon V_0 [\sum_j (\frac{\beta_j}{p_{j0}} - \frac{\Pi_1}{2\Pi_{1/2}-\Pi_1}\frac{\beta_j}{p_{j1}})c_j]$$

$$= \Omega_0 \upsilon V_0 [\sum_j (\frac{\partial\Pi_0/\partial p_{j0}}{\Pi_0} - \frac{\partial\Pi_1/\partial p_{j1}}{2\Pi_{1/2}-\Pi_1})c_j], \qquad c_i>0, \qquad (i=1,\ldots,k).$$

The Cobb-Douglas type function, Π_t, is negative semidefinite in its linearly homogenous form dictated by 21. Thus, $\partial\Pi_0/\partial p_{i0} \geq \partial\Pi_1/\partial p_{i1}$ for increasing prices, and $2\Pi_{1/2}-\Pi_1 \geq \Pi_0$ since $\Pi_{1/2} \geq 1/2(\Pi_0+\Pi_1)$. Equation 39a is always nonnegative. Parallel derivations for falling prices transform 37b into

$$(39b) \qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \upsilon V_1 [\sum_j (\frac{\partial\Pi_0/\partial p_{j0}}{2\Pi_{1/2}-\Pi_0} - \frac{\partial\Pi_1/\partial p_{j1}}{\Pi_1})c_j], \qquad c_i<0, \qquad (i=11,\ldots,k),$$

where $\partial\Pi_0/\partial p_{i0} \leq \partial\Pi_1/\partial p_{i1}$ for falling prices and $2\Pi_{1/2}-\Pi_0 \geq \Pi_1$. Thus, for $c_i<0$, $i=1,\ldots,k$, 39b is also nonnegative.

Corollary 5a: Consumers never lose under the conditions of Proposition 5, given the linear expenditure system.

The ordinal welfare criteria in 37a and 37b can be applied with any demand system, though flexible functional forms impose very few restrictions. A priori conclusions may be difficult, and demand parameters may first need to be estimated.

Pareto Comparability, Time Neutrality, and GMP Prices

The welfare effects of geometric mean preserving price instability can also be derived. In this case, prices change non-linearly over time. The form of $\dot{p}_{i(1-t)}$ in compensation mechanisms 16a and 18b is found by substituting 1-t for t in 10, or $\dot{p}_{i(1-t)} = -c_i p_{i(1-t)}$, $i=1,\ldots,k$. Combining this result with equal welfare weights in 6 and compensation for rising prices from 16a and 16b, the difference in the direction of change in welfare is

$$\text{(40a)} \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0\Big[\sum_j \frac{\partial V_t}{\partial p_{jt}} c_j p_{jt} + \frac{\partial V_t}{\partial y_t} \sum_j \frac{\partial m(V_{1/2}, p_{1-t})}{\partial p_{j(1-t)}} c_j p_{j(1-t)}\Big]\Big|_0^{1/2}$$

$$+ \Omega_0\Big[\sum_j \frac{\partial V_t}{\partial p_{jt}} c_j p_{jt} + \frac{\partial V_t}{\partial y_t} \sum_j \frac{\partial m(V_{1/2}, p_t)}{\partial p_{jt}} c_j p_j\Big]\Big|_{1/2}^1$$

$$= \Omega_0\Big[\sum_j \Big(\frac{\partial V_1}{\partial p_{j1}} p_{j1} + \frac{\partial V_1}{\partial y_1} \frac{\partial m(V_1, p_1)}{\partial p_{j1}} p_{j1}\Big) c_j\Big]$$

$$- \Omega_0\Big[\sum_j \Big(\frac{\partial V_0}{\partial p_{j0}} p_{j0} + \frac{\partial V_0}{\partial y_0} \frac{m(V_1, p_1)}{p_{j1}} p_{j1}\Big) c_j\Big]$$

$$= \Omega_0 \frac{\partial V_0}{\partial y_0}\Big[\sum_j (p_{j0} q_j(p_0, y_0) - p_{j1} q_j(V_1, p_1)) c_j\Big], \quad c_i > 0, \quad (i=1,\ldots,k),$$

with $V_{1/2}=V_1$ for increasing prices. The analogous derivation for falling prices gives

$$\text{(40b)} \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \frac{\partial V_1}{\partial y_1}\Big[\sum_j (p_{j0} q_j(V_0, p_0) - p_{j1} q_j(p_1, y_1)) c_j\Big], \quad c_i < 0, \ (i=1,\ldots,k),$$

where $V_0=V_{1/2}$ for falling prices.

<u>Proposition 6: When there is preference for time neutrality, generations who would otherwise lose are compensated to the point of indifference, and both stable and unstable prices preserve the same geometric means, price instability is socially preferred if the relative price growth weighted sum</u>

<u>of expenditures by generations paying both lower prices and compensation exceeds the relative price growth weighted sum of expenditures by generations paying higher prices but receiving compensation. Otherwise, consumers are indifferent or prefer stability. This is an ordinal and Pareto comparable result because no generations of consumers are harmed</u>.

When there are nonlinear and increasing GMP prices in the LES, 15a and 15b give

$$(41a) \quad y_0 - \Sigma_0 = 2(\Sigma_{1/2}+V_{1/2}^{1/\upsilon}\Pi_{1/2})-\Sigma_1-V_{1/2}^{1/\upsilon}\Pi_1-\Sigma_0$$

$$= V_{1/2}^{1/\upsilon}(2\Pi_{1/2}-\Pi_1)+2\Sigma_{1/2}-(\Sigma_1+\Sigma_0)$$

$$(41b) \quad y_1 - \Sigma_1 = E_1+V_{1/2}^{1/\upsilon}\Pi_1-\Sigma_1$$

$$= V_{1/2}^{1/\upsilon}\Pi_1$$

It follows from 41a and 41b that $y_0-\Sigma_0<y_1-\Sigma_1$. When prices are increasing, $2\Pi_{1/2}<2\Pi_1$, $2\Pi_{1/2}-\Pi_1<\Pi_1$, while $2\Sigma_{1/2}<(\Sigma_1+\Sigma_0)$ because GMP prices are convex over time. Neither $y_0-\Sigma_0$ or $y_1-\Sigma_1$ can become negative, and $0<y_0-\Sigma_0<y_1-\Sigma_1$.

Substituting for marginal utility of income and demands from 26, 27, and 28, 40a becomes

$$(42a) \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0\upsilon V_0[\sum_j(\frac{p_{j0}\alpha_j}{y_0-\Sigma_0} + \beta_j - \frac{p_{j1}\alpha_j}{y_0-\Sigma_0} - \frac{y_1-\Sigma_1}{y_0-\Sigma_0}\beta_j)c_j]$$

$$= \Omega_0 \upsilon V_0 [\sum_j (\frac{\alpha_j}{y_0-\Sigma_0}(p_{j0}-p_{j1})+\beta_j(1-\frac{y_1-\Sigma_1}{y_0-\Sigma_0}))c_j], \quad c_i>0, \quad (i=1,\ldots,k).$$

With rising prices, $p_{i0}<p_{i1}$, $i=1,\ldots,k$ and $0<y_0-\Sigma_0<y_1-\Sigma_1$, making the difference in the direction of change in welfare negative in 42a. Similar derivations for falling GMP prices transform 40b into

$$(42b) \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \upsilon V_1 [\sum_j (\frac{\alpha_j}{y_1-\Sigma_1}(p_{j0}-p_{j1})+\beta_j(\frac{y_0-\Sigma_0}{y_1-\Sigma_1}))c_j], \quad c_i<0, \quad (i=1,\ldots,k).$$

In this case, $p_{i0}>p_{i1}$, $i=1,\ldots,k$ and $y_0-\Sigma_0>y_1-\Sigma_1\geq 0$. Multiplier c_i is negative and 42b is always negative.

<u>Corollary 6a: Consumers always lose from instability under the conditions of Proposition 6 and prefer stability, given the linear expenditure system.</u>

A comparison of Corollaries 5a and 6a emphasizes the importance of an empirically correct definition price instability. If there were a single composite commodity, n=1 and 40a and 40b become

$$(40a') \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \frac{\partial V_0}{\partial y_0} [y_0-y_1]c_i, \qquad c_i>0, \qquad (i=1),$$

$$(40b') \quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0 \frac{\partial V_1}{\partial y_1} [y_0-y_1]c_i, \qquad c_i<0, \qquad (i=1).$$

By the compensation mechanism of 15a and 15b, $y_0<y_1$ for a rising price, $c_i>0$, making 40a' negative. By 17a and 17b, $y_0>y_1$ for a falling price, $c_i<0$, with 40b' also negative.

Corollary 6b: When there is a single composite commodity, consumers always lose under the conditions of Proposition 6 and prefer stability.

Time Preferences and Pareto Comparability

If there is a social preference either for the present of the future, the relative welfare weights of generations will decline or rise over time, respectively in Equation 6. It will be assumed $\omega-\delta_t=\omega-\delta$, $\dot{\delta}_t=0$, causing welfare weights to grow or decline at a constant rate. For increasing AMP prices and compensation between generations, 4 becomes

$$(43a)\quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[(\omega-\delta)(e^{(\omega-\delta)}V_1-V_0)]$$

$$+ \Omega_0\frac{\partial V_0}{\partial y_0}[\sum_j(q_j(p_0,y_0)-q_j(V_1,p_1))c_j], \qquad c_i>0, \qquad (i=1,\ldots,k).$$

Equation 43a is equivalent to 37a with the addition of the first term involving nonzero $\omega-\delta$. When AMP prices are falling with $\omega-\delta$ nonzero, the analog of 37b is

$$(43b)\quad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[(\omega-\delta)(e^{(\omega-\delta)}V_1-V_0)]$$

$$+ \Omega_0 e^{(\delta-\omega)}\frac{\partial V_1}{\partial y_1}[\sum_j(q_j(V_0,p_0)-q_j(p_1,y_1))c_j], \quad c_i<0, \ (i=1,\ldots,k).$$

The point of indifference for stable prices is no longer $\frac{dW_t}{dt} - \frac{dW_0}{dt} = 0$, but rather

$$(44) \qquad \frac{dW_t}{dt}\Big|_0^1 = \Omega_0[(\omega-\delta)(e^{(\omega-\delta)}\bar{V}-\bar{V})]$$

which happens to be zero only when $\omega-\delta=0$, and is positive otherwise. If, in either 43a or 43b, $q_{i0}=q_{i1}$, $i=1,\ldots,k$, then $V_0=V_1=\bar{V}$, and consumers are indifferent between instability in whichever of 43a or 43b applies and stability in 44.

When there are rising prices $c_i>0$, $p_{i0}<p_{i1}$, $i=1,\ldots,k$, a social preference for the present, $\omega-\delta<0$, and if $q_{i0}>q_{i1}$, $i=1,\ldots,k$ in 43a, then $V_0>V_1$ and a preference for the present with currently low but rising prices will amplify the benefits of instability. Conversely, if there are falling prices, $c_i<0$, $p_{i0}>p_{i1}$, $i=1,\ldots,k$, a preference for the future, $\omega-\delta>0$, and if $q_{i0}<q_{i1}$, $i=1,\ldots,k$ in 43b, then $V_0<V_1$ and a preference for the future with currently high and falling prices will amplify the benefits of instability.

Proposition 7: When generations who would otherwise lose are compensated to the point of indifference, and both stable and unstable prices preserve the same arithmetic means, either i) a preference for the present with rising prices, or ii) a preference for the future with falling prices will tend to make instability socially more desirable than if the time preference was neutral. This is an ordinal and Pareto comparable result.

The two cases of increasing prices with a preference for the future and decreasing prices with a preference for the present give ambiguous results.

Summary

In commodity markets, prices actually change over time. Cohorts of consumers are born, age, and die, and although there is overlapping, the consumers in the instantaneous generations enjoying lower prices are not all the same as those who are penalized by higher prices. This paper first presents a completely general method for theoretically and empirically measuring social welfare changes from dynamic price instability in commodity markets. Then four categories of restrictions are applied to draw specific conclusions. These categories consider 1) the social preference relating generations of consumers through time, 2) the empirical form of price changes, 3) actual income compensations between generations who gain and generations who lose, and 4) the form of the indirect utility function.

The welfare measures discussed are classified according to whether they embody a preference for equity or a preference for inequity. Preferences for inequity are further classified by their affinity for the present, time neutrality, or affinity for the future. Each of these inequity preferences can be subdivided according to whether the Pareto welfare criterion applies or not. Finally, different forms of price instability may apply to either Pareto comparable or Pareto noncomparable situations.

Several conclusions are reached. When a preference for equity prevails, each generation contributes equally and social welfare is constant over time.

Although price and income movements may actually alter each generation's utility, generations with higher utility levels are not allowed to influence social welfare more than generations with lower utility. Although not considered in detail, income compensation mechanisms can be constructed to equalize utilities. In this case, instability is socially preferred when constant income compensated demands exceed constant demands from stable prices.

A change in social welfare over time requires a preference for some form of inequity. When inequity is preferred, the time preference is neutral, and there is no compensation between generations, welfare measurement is inherently Pareto noncomparable. Those who pay lower prices will prefer instability, while those who pay higher prices will prefer stability. Unless marginal utility of income happens to be constant, measurement is also cardinal. In this category, traditional welfare results assuming arithmetic mean preserving prices, are confirmed in a dynamic context, and the analysis expanded to include geometric mean preserving prices. Consumers are found to prefer AMP over GMP prices.

Pareto comparable and ordinal criteria are derived by requiring consumers who gain from lower prices to actually transfer income until those who would otherwise lose from higher prices are indifferent between stability and instability. Enforcing compensation transforms the welfare measures into ordinal comparisons of compensated demand or expenditure functions. The derived criteria are powerful and simple to use. Path dependence properties of surplus measures are avoided by completely defining the time paths of prices and income, and while compensations are usually exogenously granted income variations, compensations in the new criteria are endogenously determined. Thus, once functional forms for price changes and a demand system are empirically estimated, compensation is known and compensated demands can be computed.

Finally, nonneutral time preferences are introduced. Consumers with an affinity for the present will tend to prefer currently low but rising prices. Consumers with an affinity for the future will tend to prefer currently high and falling prices.

Iowa State University

REFERENCES

[1] ARROW, K. J.: Social Choice and Individual Values. New York: John Wiley and Sons, Inc., 1951.

[2] _____: Essays in the Theory of Risk Bearing. Chicago: Markham Publishing Co., 1971.

[3] BERGSON, A.: "A Reformulation of Certain Aspects of Welfare Economics," Quarterly Journal of Economics, 52 (1938), 310-334.

[4] BINSWANGER, H. P.: "Attitudes toward Risk: Experimental Measurement in Rural India," Economic Growth Center, Discussion Paper 285, New Haven Connecticut: Yale University, June 1978.

[5] BURNS, M. E.: "A Note on the Concept and Measure of Consumer's Surplus," American Economic Review, 63 (1973), 335-343.

[6] _____: "On the Uniqueness of Consumer's Surplus and the Invariance of Economic Index Numbers," Manchester School, 45 (1977), 41-61.

[7] CURRIE, J. M., J. A. MURPHY, AND A. SCHMITZ: "The Concept of Economic Surplus and its Use in Economic Analysis," Economic Journal, 81 (1971), 741-799.

[8] FLEMMING, J. M., S. J. TURNOVSKY, AND M. C. KEMP: "On the Choice of Numeraire and Certainty Price in General Equilibrium Models of Price Uncertainty," Review of Economic Studies, 44 (1977), 573-583.

[9] HANOCH, G.: "Risk Aversion and Consumer Preferences," Econometrica, 45 (1977), 413-426.

[10] HARSANYI, J. C.: "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," Journal of Political Economy, 64 (1955), 309-321.

[11] HAUSMAN, J. A.: "Exact Consumer's Surplus and Deadweight Loss," American Economic Review, 71 (1981) 662-676.

[12] HICKS, J. R.: "Foundations of Welfare Economics," Economic Journal, 49 (1939), 696-712.

[13] HILDRETH, C.: "Alternative Conditions for Social Orderings," Econometrica, 21 (1953), 89-90.

[14] KALDOR, N.: "Welfare Propositions in Economics," Economic Journal, 49 (1939), 549-552.

[15] LUCE, R. D. and P. SUPPES, "Preference, Utility, and Subjective Probability," in Handbook of Mathematical Psychology, ed. by R. D. Luce, et. al. New York: John Wiley and Sons, 1965.

[16] MUELLBAUER, J.: "Aggregation, Income Distribution, and Consumer Demand," Review of Economic Studies, 62 (1975), 525-543.

[17] PHLIPS, L.: Applied Consumptions Analysis, Amsterdam: North Holland Publishing Company, 1974.

[18] PRATT, J. W.: "Risk Aversion in the Small and in the Large," Econometrica, 32 (1964), 122-136.

[19] SILBERBERG, E.: "Duality and the Many Consumer's Surpluses," American Economic Review, 62 (1972), 942-952.

[20] TURNOVSKY, S. J., J. SHALIT, AND A. SCHMITZ: "Consumer's Surplus, Price Instability, and Consumer Welfare," Econometrica, 48 (1980), 135-152.

[21] VARTIA, Y. O.: "Efficient Methods of Measuring Welfare Change and Compensated Income Terms of Ordinary Demand Functions," Econometrica, 51 (1983), 79-98.

[22] WAN, H. Y. Jr.: Economic Growth. New York: Harcourt Brace Jovanovich, 1971.

[23] WILLIG, R. D.: "Consumer's Surplus without Apology," American Economic Review, 66 (1976), 589-597.

1. The author is indebted to Gordon A. King and Arthur M. Havenner for their comments on earlier drafts.