INTERNATIONAL TRADE, FACTOR MARKET DISTORTIONS AND THE OPTIMAL DYNAMIC SUBSIDY

by Harvey E. Lapan

No. 1

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International trade theorists have long favored free trade, arguing that the only proper basis for a tariff is in response to some monopoly power in international trade. Pragmatic politicians, on the other hand, wary of the wrath of their constituents, have often resorted to the use of tariffs, arguing that the protection afforded by tariffs was necessary to prevent unemployment. Recent papers on the theory of domestic distortions and optimal policy interventions can be interpreted as an attempt to reconcile these two divergent views. For example, if factors are immobile and if distortions exist in the factor markets (due to factor price rigidities), then it is argued that the optimal policy intervention is not a tariff, which destroys the equality between the MRS and FRT, but rather a wage subsidy. This policy prescription, which recognizes the pragmatic difficulties politicians must face, offers a feasible remedy that is proved superior to the use of tariffs.

However, one can argue that the theory of optimal policy interventions goes a bit too far. While it may well be true that factors are not instantaneously mobile, they certainly do become mobile through time; capital depreciates, new workers enter the labor force, and factories can be moved intertemporally. The policies that eliminate unemployment at the same time destroy the incentives for resource reallocation (particularly if the institutional constraint stipulates that factor prices must be the same in all sectors). Thus, while the standard

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wage subsidy that would promote full employment may be efficient in the short run, it may prove inefficient in a dynamic setting.

Nevertheless, this does not imply that the efficient long-run policy entails no interventions; rather, it is likely that it will call for some wage subsidy which is smaller than the static optimum subsidy. It is the purpose of this paper to discuss how this dynamic optimum subsidy is determined and to specify its implications for the economic system.

I. THE PROBLEM

For our analysis we shall use a standard two sector, one factor trade model.[1] Let the two commodities be M and C, and let N_m and N_c represent the labor employed in each sector. Then:

$$(1)\quad M = F_m(N_m);\ C = F_c(N_c);\ F_i' > 0,\ F_i'' < 0;\ i = c, m.$$

Furthermore, choose C as the numeraire and let P represent the relative price of M.

In addition, assume that the number of potential workers in each sector (L_i) is fixed at any moment, and that the total supply of available workers is fixed for the period in question:

$$(2)\quad N_i \le L_i,\ i = c, m;\ L_c + L_m = \bar{L},\ \text{fixed.}$$

Institutionally, it is assumed that firms behave competitively in factor (and product) markets, so that workers are paid their marginal value product (and there cannot be excess demand for factors in either sector). However, it is also assumed that, because of distortions in the factor market, the wage rate must be the same in each sector.[2] Thus, in the absence of any

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[1] If desired, it can be assumed that other factors are used in each sector; but that these are fixed for the period under study. Harris and Todaro [4] use this model.

[2] The alternative assumption frequently used in the literature is that factor prices are downward rigid. Either assumption gives qualitatively similar results, and our assumption seems more consistent with the long-run behavior of the economy.

wage subsidies, we have:

$$(3)\quad F_c'(N_c) = PF_m'(N_m);\ N_c = L_c \text{ or } N_m = L_m$$

The above three equations constitute a long-run equilibrium if commodity markets clear and if full employment occurs in each sector.

To simplify the analysis, let us assume that the economy in question is open and "small," so that it can trade at unchanging terms of trade.[3] Assuming the economy is in long-run equilibrium it will be producing at some point, such as B in Figure 1, on the long-run production possibility frontier and will consume somewhere on the price line tangent to the production possibility frontier at B.[4]

Now suppose that, due to external conditions, the terms of trade shift; for example, assume the country was exporting C and let its terms of trade improve (P falls). If factors are immobile, the short-run production possibility frontier is given by ABD; and if no distortions exist in the factor markets, the country will continue to produce at B and will benefit from the improved terms of trade. However, if factors are immobile and if factor prices must be the same in each sector, an excess supply of labor will develop in M and output will occur somewhere along the open segment AB. Thus, it is conceivable that the improved terms of trade might leave the country worse off.

As has been clearly demonstrated in the literature [1, 5, and 6], the optimal policy under these circumstances is a wage subsidy to M that restores production at point B. Thus, if we let $\bar{P}$ be the old price at which resources

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[3] This simplifying assumption, which is common in the literature [for example, Bhagwati and Srinivasan (2)] allows us to ignore the utility function (or demand functions) for commodities. Also, it implies that the optimum tariff is zero. The analysis would not be substantially altered if we dropped this assumption, or assumed a closed economy.

[4] The long-run production possibility frontier represents the efficient production locus when labor is assumed perfectly mobile.

were fully employed, and P^* the new world price ($P^* < \bar{P}$), and if S^* represents the optimum wage subsidy to sector M, then:

(4) $F_c'(L_c) = [\bar{P}F_m'(L_m)] = [P^*F_m'(L_m)]/(1-S^*)$; $S^* = [(\bar{P} - P^*)/\bar{P}] > 0$

This subsidy restores full employment and enables the country to benefit from its improved terms of trade.

However, this equilibrium is still sub-optimum in a long-run context since the change in the terms of trade moves the long-run optimum production point from B to E (in Figure 1). The obvious question is what determines intertemporal factor migration and what is the best policy to be pursued in a dynamic context. If, for example, factor migration between sectors is determined by economic factors, such as different wages or unemployment rates in each sector, then the static optimum subsidy (S^*) destroys any economic incentives for migration since, by assumption, wages are equalized between sectors while, by choice, unemployment is reduced to zero in each sector.[5] Thus, while S^* may be efficient in a short-run context, it may not be optimal when considered in a long-run context.[6]

Clearly, the government may have other tools at its disposal that will enable it to maintain full employment in the short-run while encouraging

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[5] Wage rates are equalized across sectors by assumption; an alternative assumption frequently employed in the literature is that real wages are inflexible downward. The implications of this latter assumption depends upon how "real" wages are defined. For example, if wages in each sector are assumed "inflexible downward" in terms of purchasing power of C, this coincides with our model (assuming we start from equilibrium); if they are rigid in terms of M, no unemployment will occur due to a fall in P (though it will if P rises), but wage differentials can occur between the sectors. Finally, if real wages are defined in terms of some weighted price index, some subsidy may still be necessary to maintain full employment and wage differentials will occur. However, nothing guarantees that the wage differentials implied by the optimum static subsidy are optimal in terms of encouraging migration.

[6] Even if no distortions exist in factor markets, so that full employment (at B) occurs without subsidization, it does not follow that this solution is optimal in a long-run context when we are interested in encouraging migration between sectors.

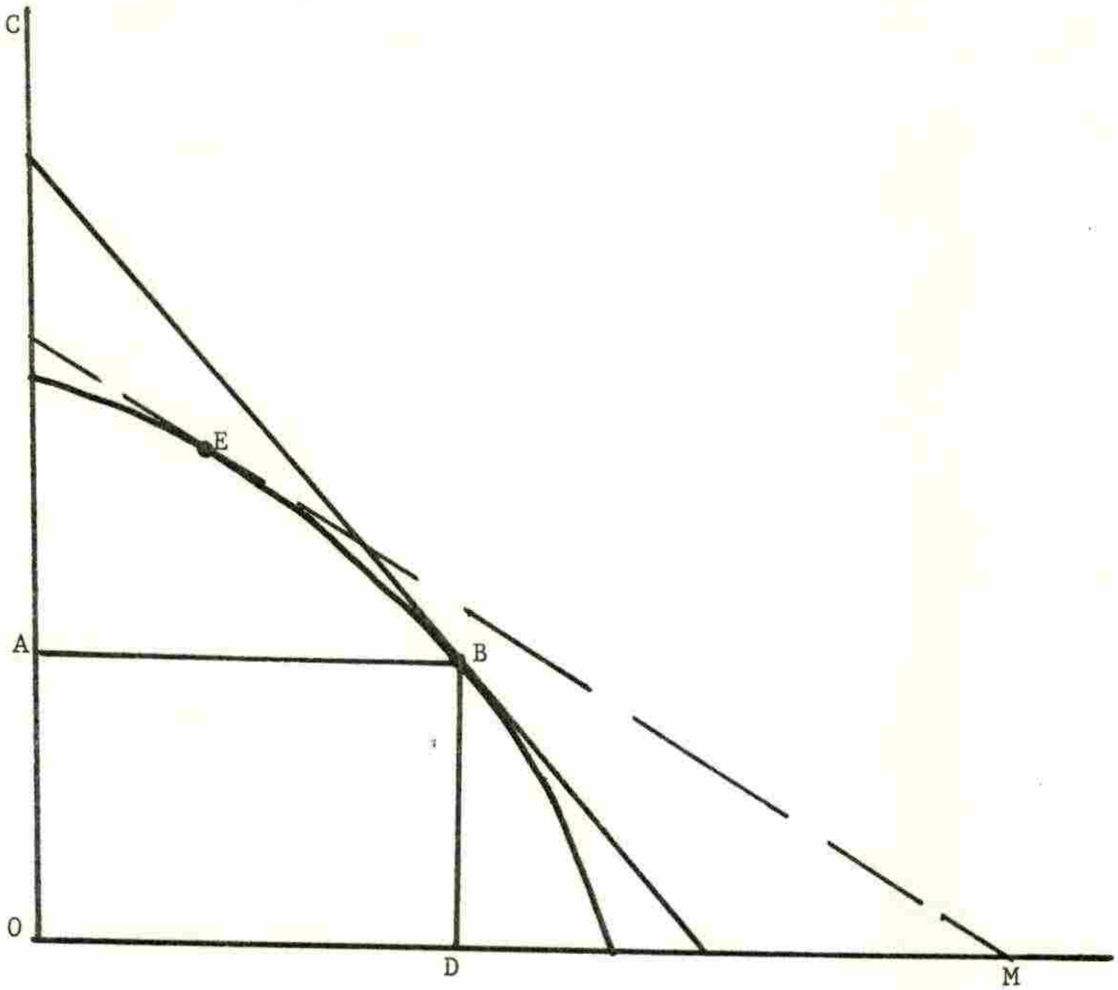

Figure 1

migration in the direction of long-run equilibrium. One obvious policy tool, suggested in Harris and Todaro [4] would be simply to force (or restrict) migration in a particular direction while maintaining the optimal subsidy (S^*) needed to guarantee full employment of resources if there were factor market distortions (naturally, S^* changes through time as labor migrates). However, this particular policy is most likely to be opposed because it infringes on personal liberty (and it is not likely to be very efficient if all workers are not identical).

The economic tools the government should use to maintain full employment and encourage migration depend upon the causes of migration and the institutional constraints.[7] If migration rates between sectors depend upon differences in real incomes between the sectors, the government should attempt to enlarge these differences, while at the same time pursuing a policy that maintains full employment in all sectors. For example, if we do <u>not</u> assume that wages must be equalized across sectors, then the government could apply large wage subsidies to sector C (where labor is more productive) to encourage migration toward C, while at the same time use the minimum wage subsidy necessary to promote full employment in M. Even if wages <u>must</u> be equalized across sectors, the government could encourage migration to C by imposing a lump sum tax on workers in M (or a lump sum transfer to workers in C), while at the same time subsidizing wages in M in order to promote full employment.

However, the above analysis is subject to criticism on several grounds. It assumes that somehow labor (effectively) demands a minimum real wage but apparently cannot effectively demand a minimum real standard of living. Specifically, the above policy can work because it assumes that a wage subsidy

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[7] In the following analysis we assume that labor's marginal value product is higher in C, so that the government wishes to encourage migration toward C while maintaining full employment in F (by means of an optimum subsidy).

can be applied to sector M to meet minimum wage demands and promote full employment, while (some of) the income from this wage subsidy can be removed by lump sum taxation (to promote real income differences between the sectors) without altering the wage demands of laborers in this sector.

Moreover, it can be argued that the economic efficiency of the wage subsidy to C (to promote migration) depends crucially upon the assumption that the return to labor is a pure economic rent. If each individual's labor supply decision were responsive to the wage rate, then the subsidy in C would create inefficiencies by causing a discrepancy between the marginal value product of labor and that worker's marginal rate of substitution between consumption and leisure. Thus, while the wage subsidy (to the more efficient sector) attempts to promote migration, and hence dynamic efficiency, it produces static inefficiency.

Given the preceding discussion, we are skeptical about the ability of the government to costlessly reallocate resources in the economy. Thus, for the remainder of the paper we explicitly assume this reallocation <u>cannot</u> be achieved costlessly, and that it is the unemployment rate (in M) that determines the rate of out-migration. One might rationalize this assumption by arguing that workers are risk averse and hence are unwilling to move to an unfamiliar situation (a new industry) if there is a reasonable chance of finding a job in their own sector. In addition, if it is <u>assumed</u> that wages must be the same in all sectors, the assumption that unemployment rates determine migration is equivalent to the assertion that migration is determined by the differences in expected wages between the sectors.[8]

In the analysis that follows, no assumption will be made about wage rigidity in either sector; rather, the key relation will be the effect of

[8] For further details on this point, see Harris and Todaro [4].

unemployment rates on migration. Nevertheless, when we determine the optimal time path of the unemployment rate (that balances the gains from migration against the costs of unemployment), we can infer the wage subsidy (or tax) that would be needed to support this unemployment rate, and we can then compare our results to that obtained for the optimum static subsidy.

II. OPTIMAL LABOR TRANSFER

Suppose the economy is out of long-run equilibrium, in the sense that labor's marginal value product differs between sectors. In particular, let us assume that the marginal value product of labor is larger in sector C.[9] Furthermore, assume that labor is not instantaneously mobile, but that it moves through time in response to unemployment. We seek to determine the optimum time path of unemployment, assuming that the central planner wishes to maximize the present discounted value of the stream of income over the interval (0, T).[10]

Let production and factor endowments be as described in equations (1) and (2):

$$(1)\quad C = F_c(N_c);\ M = F_m(N_m);\ F_i' > 0,\ F_i'' < 0$$

$$(2)\quad N_i \leq L_i;\ L_c(t) + L_m(t) = \bar{L};\ \bar{L}\text{ fixed}$$

Since we wish to encourage migration towards C, it will always be optimal

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[9] The difference in marginal value products may have arisen because of some exogenous change in the terms of trade. Also, the analysis could readily be generalized to handle the case in which labor's marginal value product is larger in M.

[10] For simplicity, we maintain the assumption that the economy is open and "small." Then, the maximization of the PDV of the income stream is equivalent to utility maximization over the period if either: (i) the marginal utility of income is constant and r is the rate of time preference; or if (ii) the country can borrow or lend on world capital markets at the rate r. In the latter case, the planner still must determine the optimum borrowing and lending pattern in order to obtain the proper consumption stream. Our results would not be qualitatively altered by assuming utility, instead of income, is maximized.

to maintain full employment there. Thus, labor migration towards C can be postulated as a function of the unemployment rate (u) in M:

$$(5)\quad \dot{L}_c = -\dot{L}_m = \phi(u)L_m;\ u \in [0, 1];\ \phi(0) = 0;\ \phi' > 0, \phi'' \leq 0$$

From the definitions and the initial conditions:[11]

$$(6)\quad N_c = L_c;\ N_m = L_m(1-u);$$

$$(7)\quad F_c'(L_c(0)) > PF_m'\ (L_m(0))$$

Given (1), (2), (5), (6) and (7), we seek to maximize:

$$(8)\quad V = \int_0^T [F_c(L_c) + PF_m(N_m)]e^{-rt}dt$$

The problem as stated is an optimal control problem in one state variable ($L_c(t)$) and one control variable (u). It is interesting to note that this problem is very similar to the optimal growth problem and that the unemployment rate plays the role of "saving." The Hamiltonian is:

$$(9)\quad H = [F_c(L_c) + PF_m(N_m)]e^{-rt} + \lambda[L_m\ \ \phi(u)]$$

In (9), $\lambda(t)$ represents the value, discounted to time 0, of transferring a worker at time t to sector C. It is convenient to define q(t) so that q(t) represents the value at time t of such a transfer:

$$(10)\quad q(t) = \lambda(t)e^{rt}$$

Optimizing the Hamiltonian over u yields:[12]

$$(11)\quad H_u = L_m e^{-rt}[q\phi'(u) - PF_m'(N_m)] \leq 0;\ u \in [0, 1]$$

In addition, for an optimum path we derive the canonical equations and the transversality condition:

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[11] To generalize the analysis, we could make net migration between sectors some function of the unemployment rates in each sector. However, it will always be optimal to maintain full employment in one sector and thus, given our assumption that labor is initially more productive in C, equation (5) follows.

[12] We assume $F_m'(0) = \infty$; thus, it will never be optimal to have the labor force in M completely unemployed. In addition, we assume $\phi'(0)$ is finite; if it is not, some unemployment will always be optimal (if marginal value products differ) and the static optimum subsidy will never be dynamically efficient.

$$(12)\quad \dot{q} = [r + \phi(u)]q - [F_c' - P(1-u)F_m']$$

$$(13)\quad \dot{L}_c = L_m \phi(u)$$

$$(14)\quad q(T) = 0;\ \text{and } L_c(0) \text{ given}$$

By the assumptions on F_i'' and ϕ'', any path that satisfies (11), (12), (13) and (14) constitutes an optimum solution to the problem. In discussing this solution, it is convenient to consider separately the two cases: $r = 0$ and $r > 0$.

Case i): $r = 0$

For this case, (12) reduces to:[13]

$$(12')\quad \dot{q} = [\phi(u)q] - [F_c' - P(1-u)F_m']$$

A stationary solution to this problem is obtained by solving (11), (12') and (13) for $\dot{L}_c = \dot{q} = 0$:

$$(15)\quad (L_c^*, q^*) \text{ s.t.: } F_c'(L_c^*) = PF_m'(L_m^*);\ L_m^* = \bar{L} - L_c^* \quad q^* = [PF_m'/\phi'(0)]$$

Note that this stationary solution, which would be optimal for $L_c(0) = L_c^*$, implies the equalization of marginal value products across sectors.

Also, it can be shown that the solution to the differential equations (for $L_c < L_c^*$) represents a saddle-point, and that the unique path which converges to (L_c^*, q^*) represents the turnpike--the optimum solution if T is unbounded.[14] In order to study the properties of an optimal solution, it is useful to plot the phase diagrams of the system in (L_c, q) space.

First, consider (11); if $H_u < 0$ for all u, then we should choose $u = 0$.

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[13] If $r = 0$, the integral defined in (8) will not converge as $T \to \infty$. However, an equivalent problem would be to minimize the deviation of actual output from maximum output: $[F_c(\bar{L}_c) + PF_m(\bar{L}_m) - F_c(L_c) - PF_m(N_m)]$, where $F_c'(\bar{L}_c) = PF_m'(\bar{L}_m)$. Since this integral converges, and since the two problems give equivalent conditions, we can proceed with the problem as stated.

[14] Naturally, if T is unbounded, the transversality condition given by (14) must be suitably modified. Also, note that $L_c(0) < L_c^*$ by assumption, so we are only interested in the solution for $L_c < L_c^*$. If $L_c(0) > L_c^*$, we should want to allow migration out of C, and hence unemployment in C. We shall say more on this point in footnote 20.

Thus, the border between full employment and unemployment is defined by the values of (L_c, q) such that $H_u = 0$ at $u = 0$:

$$(16) \quad g(q, L_c) = [q\phi'(0) - PF_m'(L_m)] = 0$$

It is clear this locus is positively sloped and asymptotic to the line $L_c = \bar{L}$; the curve AB in Figure 2 is the graph of $g(q, L_c) = 0$. Moreover, above the locus $u > 0$ and $\dot{L}_c > 0$, whereas below it $u = 0$, $\dot{L}_c = 0$. Furthermore, how u responds to changes in q and L_c can readily be derived from (11) (assuming an interior solution):

$$(17) \quad \partial u/\partial q = -\phi'/[q\phi'' + PL_mF_m''] > 0$$

$$(18) \quad \partial u/\partial L_c = -[(1-u)PF_m'']/[q\phi'' + PL_mF_m''] < 0$$

Thus, loci of constant unemployment rates are positively sloped in (L_c, q) space.

Next, consider the locus of points such that $\dot{q} = 0$. From (11), u can be written as an implicit function of (L_c, q); therefore:

$$(19) \quad \dot{q} = 0 \text{ defines } h(q, L_c) = \phi q - [F_c' - P(1-u)F_m'] = 0$$

For $u = 0$, it is clear there is a unique value of L_c (equal to L_c^* as determined in (15)) such that $\dot{q} = 0$; for $u > 0$, we calculate, using (17), (18) and (19):

$$(20) \quad \partial h/\partial q = [\phi(u)] - PN_mF_m''(\partial u/\partial q) > 0$$

$$(21) \quad \partial h/\partial L_c = -F_c'' - [P(1-u)^2 q\phi' F_m'']/[q\phi' + PL_mF_m''] > 0$$

Thus, the locus $\dot{q} = 0$ is negatively sloped above the curve AB, and points above this locus correspond to $\dot{q} > 0$. The curve DE in Figure 2 depicts the $h(q, L_c) = 0$ locus.

The choice of an optimal solution thus entails choosing the value of $q(0)$; in turn, this will depend on $L_c(0)$ and T, the length of the planning horizon. If the horizon is unbounded, the optimal choice is the turnpike and the corresponding value of $q(0)$. Note that everywhere on the turnpike solution corresponds to some unemployment, but the economy asymptotically approaches the allocation where labor's marginal value product is equal across sectors.

For finite horizons, the initial value of $q(0)$ chosen will lie below the turnpike; in particular, the shorter the horizon, the smaller the value of $q(0)$ (and hence the smaller the initial unemployment rate).[15] Further, for sufficiently small planning horizons, it will not be optimal to incur any unemployment.[16] To see this, suppose we have followed an optimal path for the interval $(0, T - \tau)$, and let $L_c(T - \tau)$ be the corresponding value of labor. The cost of incurring any further unemployment over the interval (dt) is A:

$$(22) \quad A \simeq P \cdot L_m \cdot u \cdot F_m'(N_m) \cdot dt$$

The benefit, in terms of more future output, if resources will be fully employed over the remaining interval $(\tau - dt)$ is B:

$$(23) \quad B \simeq (F_c' - PF_m'(L_m))\, \phi(u) \cdot L_m \cdot \tau \cdot dt, \text{ where:}$$

$$(24) \quad dL_c = -dL_m = \phi(u) \cdot L_m \cdot dt$$

If $(B - A)$ is nonpositive at $u = 0$, it will not pay to incur any further unemployment; thus, if:

$$(25) \quad \tau \leq [PF_m'(L_m)]/[\phi'(0)(F_c' - PF_m')],$$

it is optimum to choose $u = 0$.

Several conclusions follow from (25). First, it is clear that for any finite horizon it will not be optimal to transfer labor so that $L_c(T) = L_c^*$. Moreover, for any finite horizon there should always be a terminal period with full employment of resources. Further, for small T or $L_c(0)$ near L_c^*, it may not pay to incur any unemployment.

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[15] Paths above the turnpike correspond to maximization of (8) subject to a terminal constraint on $L_c(T)$.

[16] This case corresponds to the optimum static subsidy referred to earlier (if distortions exist in factor markets).

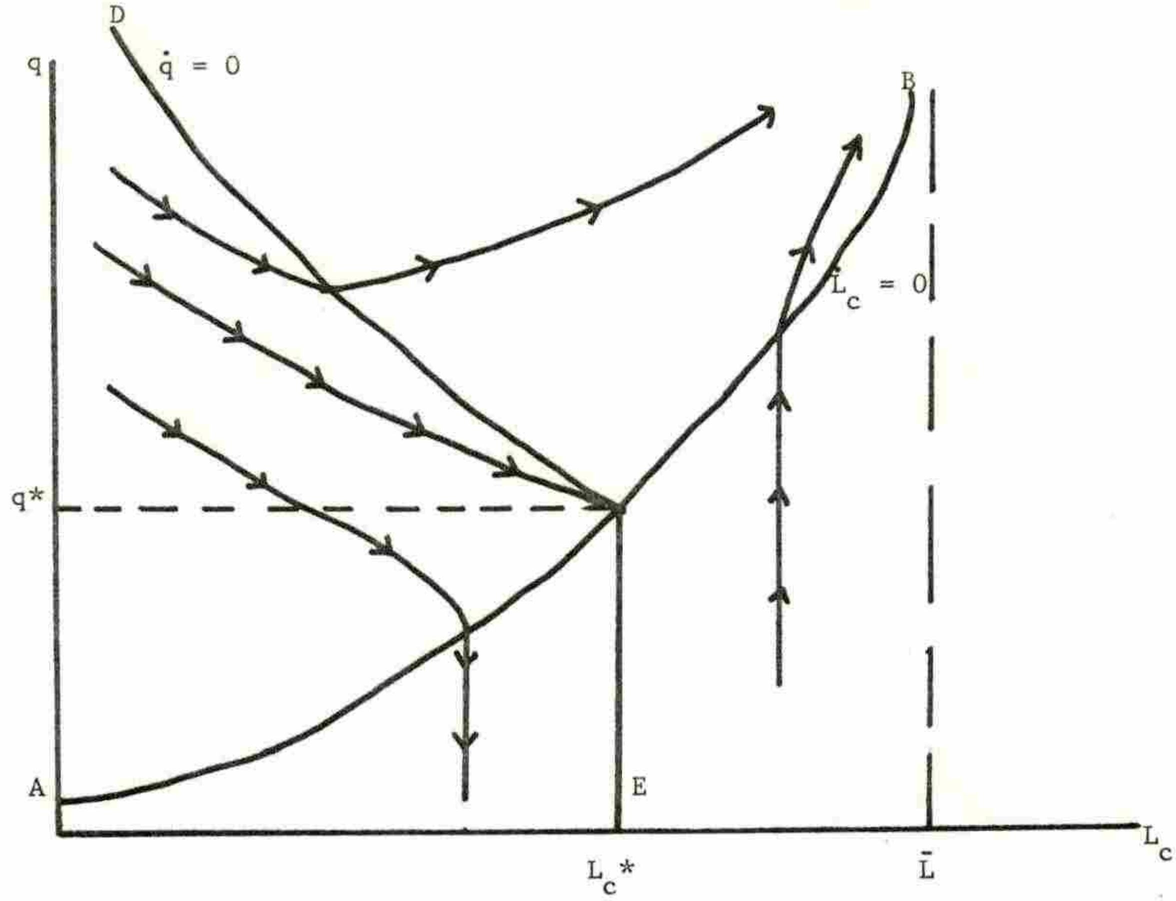

Figure 2; r = 0

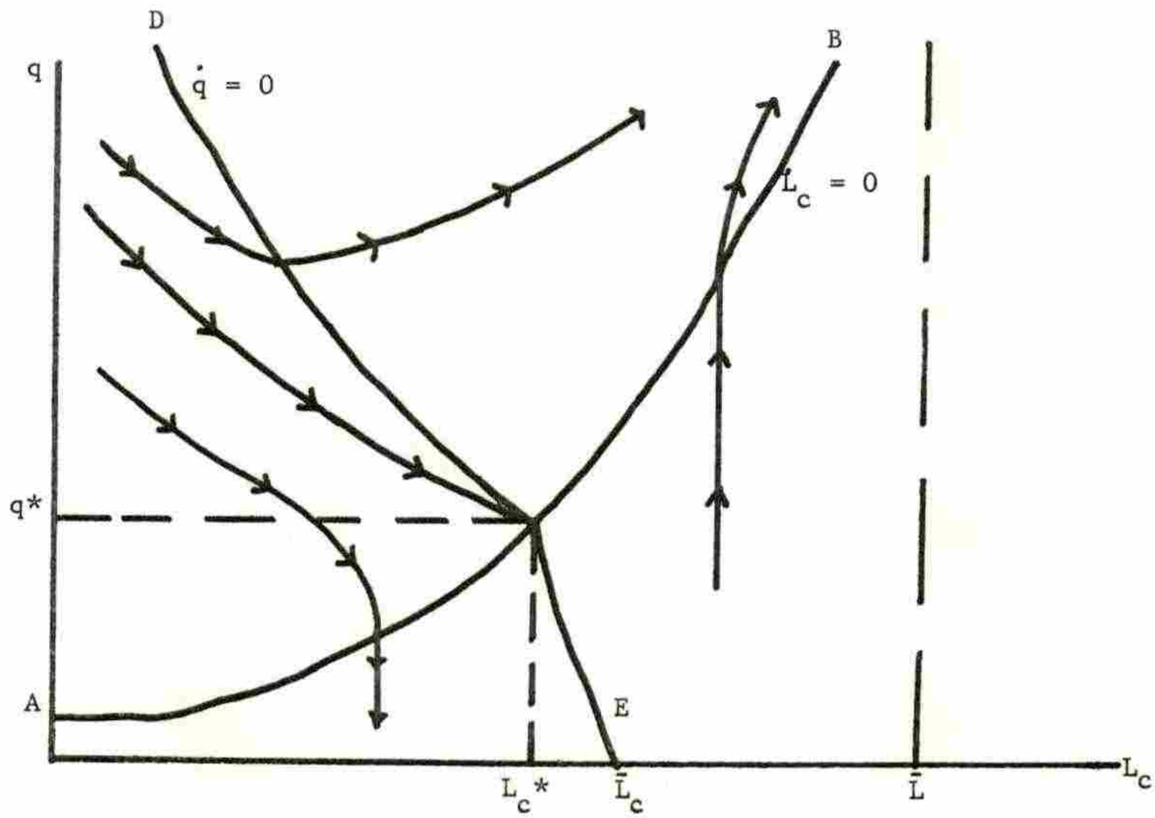

Figure 3; r > 0

It is readily seen that the terminal value of L_c (on the optimal path) increases with T;[17] thus, from (25), it follows that longer planning periods correspond to longer periods of unemployment and longer periods of full employment of resources. In conclusion, the optimal path for any finite horizon can be characterized as follows:

$$(26) \quad t \in (0, T - \tau),\ u > 0,\ \dot{L}_c > 0$$
$$t \in (T - \tau, T),\ u = 0;$$

where τ is determined by (25). Also, we may have $\tau = T$ in some cases, whereas τ always increases with T. We shall see that similar results hold for the case $r > 0$.

Case ii): $r > 0$

The principal difference in this case is that future benefits are discounted, so that even for an infinite horizon, it will never pay to fully reallocate labor so that marginal value products are equated across sectors. The AB locus, as defined by equation (16) is unaltered by the positive discount rate; however, the $\dot{q} = 0$ locus is altered, as represented by (12). The intersection of these two curves, as shown in Figure 3, is (L_c^*, q^*):

$$(27) \quad F_c'(L_c^*) - PF_m'(L_m^*) = [PF_m'(L_m^*) \cdot r]/\varphi'(0) > 0$$
$$q^* = [F_c'(L_c^*) - PF_m'(L_m^*)]/r$$

Thus, this solution, which corresponds to a stationary point, does not imply the equalization of marginal value products.

Though the $\dot{q} = 0$ locus is altered by the assumption $r > 0$, qualitatively it looks much the same as for $r = 0$. For $u > 0$, the locus is negatively sloped and points above it correspond to $\dot{q} > 0$; for $u = 0$, the locus remains negatively sloped (rather than becoming vertical) and crosses the L_c axis

[17] This can be demonstrated by comparing the optimal paths for the horizons T and (T + dT) respectively.

at the labor allocation that equalizes labor's marginal value product in the two sectors.[18] For $L_c(0) < L_c^*$, the solution to the differential equations, (L_c^*, q^*) represents a saddle-point, and the unique path which converges to this point represents the turnpike--the optimum solution if T is unbounded.

In discussing the optimum solution, it is necessary to consider two separate cases. If $L_c(0) < L_c^*$, the solution path is qualitatively the same as that discussed for the case $r = 0$. The infinite time horizon solution corresponds to the turnpike; any finite time horizon corresponds to a choice of $q(0)$ that lies below the turnpike.[19] Moreover, the finite time horizon solution is characterized by an initial period of unemployment and a final period of full employment of resources (unless T is small, in which case the initial period is degenerate). Further, as the horizon increases, both of these periods increase in length.

However, if $L_c(0) > L_c^*$ (but $L_c(0) < \bar{L}_c$, where $\bar{L}_c$ is the labor allocation that equalizes marginal value products), the optimal solution is to do nothing <u>regardless</u> of the length of the planning period.[20] Intuitively, this occurs because future benefits are discounted. The cost of incurring positive unemployment during the period (dt) is as described in (22); however, the benefits that accrue, assuming full employment is to be

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[18] Equation (21) is unaltered by $r > 0$; in (20), there is an extra term, equal to r, on the RHS. From (12), $\dot{q} = 0$, $u = 0$, and $F_c' = PF_m'$ implies $q = 0$.

[19] As for $r = 0$, the smaller T, the smaller $q(0)$.

[20] Throughout, we have assumed $L_c(0) < \bar{L}_c$, so that migration towards M is never desirable. However, by permitting positive unemployment in C, the solution could be rendered symmetric. In this case, the $\dot{q} = 0$ curve would be extended to values of $q < 0$ (since moving labor to C is undesirable) and there would be an A'B' locus that defines the region where unemployment in C is undesirable. Then, there would be two saddle point solutions, one for $L_c^* < \bar{L}_c$ and one for $\hat{L}_c > \bar{L}_c$. If $L_c(0) \in [L_c^*, \hat{L}_c]$ full employment is always optimal. Finally, for $r = 0$, $\hat{L}_c = L_c^* = \bar{L}_c$.

maintained over the remaining time (τ -dt), must be discounted. Thus, (23) becomes:

$$(23') \quad B' = (F_c' - PF_m')\phi(u) \cdot L_m \cdot [(1-e^{-r\tau})/r] \cdot dt$$

Therefore, it is not worthwhile to incur any (more) unemployment if:

$$(25') \quad e^{-r\tau} \geq [F_c' - PF_m'(1 + (r/\phi'(0)))]/[F_c' - PF_m']; \; r \neq 0$$

In particular, if the numerator is negative, the optimal solution, regardless of T, is u = 0 throughout.

Thus, the presence of a positive discount rate increases the likelihood that it will not pay to incur any unemployment. Consequently, the optimum static subsidy can be dynamically optimum if the planning period is short, the discount rate is large, or marginal value products do not differ widely across sectors.[21] Given the optimum path, as described above, let us now attempt to describe how particular economic variables change along this path.

III. THE ECONOMIC PROPERTIES OF THE OPTIMAL PATH

The previous section has described the qualitative properties of an optimal solution; in this section we plan to describe some of the quantitative properties of this solution. As we have seen, it is possible that the optimal solution coincides with doing nothing--in this case, the economic variables remain constant over the planning horizon, and the only appropriate policy is the one that maintains full employment in each sector. If factor price rigidities exist, it may be necessary to use wage subsidies to maintain full employment, and these wage subsidies would correspond to the optimum static subsidy discussed in the literature.

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[21] For the infinite horizon case, only the presence of discounting can make the static solution efficient.

However, suppose the optimum path entails incurring some unemployment during the initial period. What, then, can we deduce about the properties of this path? From the solution, it is evident that along an optimum path (for $u > 0$) $L_c(t)$ is increasing and q is decreasing; thus, it follows from (17) and (18) that the unemployment rate falls through time along this path.[22] Furthermore, it is readily seen that the value of output increases along this path through time:

$$(28) \quad Y \equiv F_c(L_c) + PF_m(N_m);$$

$$(29) \quad \dot{Y} = [F_c' - P(1-u)F_m'] \, \dot{L}_c - (PL_m F_m') \, \dot{u}$$

But, $\dot{u} < 0$; and, from (12) it can be seen that $[F_c' - P(1-u)F_m'] > 0$ since $\dot{q} < 0$. Thus, regardless of the discount rate, the optimum path will be characterized by increasing GNP.

Also, it can be seen that the marginal value product of the last <u>employed</u> worker will be larger in sector C.[23] From (11) and (12):

$$(30) \quad \dot{q} = [r + \phi - u\phi']q - [F_c' - PF_m'(N_m)] < 0$$

But $[\phi - u\phi'] \geq 0$ for all u since $\Phi'' \leq 0$; therefore, $[r + \Phi - u\phi']q \geq 0$, and $\dot{q} < 0$ implies $[F_c' > PF_m'(N_m)]$. Consequently, everywhere along the optimal path the marginal value product of labor is larger in C.

This latter result implies that if factor price rigidities exist that entail factor price equalization across sectors, the optimal policy will entail subsidizing wages in M.[24] Naturally, at any moment of time, the optimal subsidy is less than that prescribed by the static conditions if

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[22] However, it is also apparent that the initial unemployment rate is an increasing function of T and a decreasing function of $L_c(0)$.

[23] By assumption, if all workers are employed, $F_c'(L_c) > PF_m'(L_m)$. However, the unemployment in M implies that the marginal value product of the last worker hired exceeds $PF_m'(L_m)$.

[24] If there are no factor price rigidities and if labor moves only in response to unemployment, then some non-market policy would be required to encourage migration. However, if factor prices are flexible, it is reasonable to assume labor migrates in response to wage differentials; this case has been discussed earlier in the paper.

$u > 0$. How this dynamically optimum subsidy changes through time is a point we shall return to shortly.

In order to deduce further properties of the optimum path, it is necessary to make particular assumptions on the form of the functions. A very useful assumption, and one that is reasonable for small u is: $\Phi'' = 0$.[25] Since any finite horizon path consists of a final portion for which $u = 0$, it follows that this approximation is appropriate near the end of the period of unemployment.

What happens to total employment in M on the optimal path? Clearly, two opposing forces are at work--the total labor force is shrinking (in M), but unemployment rates are falling. The net impact depends on the relative sizes of these two effects. Without specific assumptions on Φ'', or F_i'', it does not appear possible to describe how N_m changes through time. However, if $\Phi'' = 0$, the answer is apparent. From (11), for $u > 0$:

$$(11')\quad \Phi'(u) \cdot q = \Phi'(0) \cdot q = PF_m'(N_m);\ \dot{q} < 0$$

Since q decreases, if $\Phi'' = 0$, then $PF_m'(N_m)$ must also decrease through time. Thus, for $\Phi'' = 0$, total employment in M rises along the optimum path ($u > 0$).

Furthermore, this same result holds during the final portion (of the period of unemployment) for any finite horizon path. During this portion u must be small, and since $\dot{q}$ does not tend to zero as u tends to zero (for finite T), it follows that $[\Phi'(u) \cdot q]$, and hence $[PF_m'(N_m)]$, must be falling. Therefore, towards the end of the period of unemployment, N_m must rise.

Finally, assuming factor prices must be equalized across sectors, let us inquire into the properties of the optimal subsidy. Two separate questions arise: (i) how does the relation between the optimum dynamic subsidy and

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[25] For any u, $\Phi(u) = \Phi(0) + \Phi'(0) \cdot u + \Phi''(\hat{u}) \cdot u^2/2$, for some $\hat{u} \in [0, u]$. Thus, if u is small, we can approximate: $\Phi(u) = \Phi'(0) \cdot u$, since $\Phi(0) = 0$.

the static subsidy change through time? and (ii) how does the optimum dynamic subsidy change along the optimum path?

Consider the first problem; since $L_c(t)$ increases through time, the static subsidy decreases. Also, since we start with $u > 0$, the dynamic subsidy must initially be less than the static subsidy. However, since $u = 0$ over the final portion of the path, the two are equal; thus, eventually the gap between them must be eliminated. If we define $S^*(t)$ as the optimum dynamic subsidy and $S(t)$ as the corresponding static subsidy, we have:

$$(31) \quad [(1 - S^*)/(1 - S)] = [F_m'(N_m)/F_m'(L_m)] > 1$$

Unless we know the properties of F_m'', it does not appear possible to say how this ratio changes through time (though $S^*(0) < S(0)$, and $S^*(\tau) = S(\tau)$ where $u(\tau) = 0$). If we assume a constant output-labor elasticity for M:

$$(32) \quad F_m(N_m) = N_m^{\alpha};\ \alpha \in (0, 1);\ \text{then:}$$

$$(31') \quad [(1 - S^*)/(1 - S)] = (1 - u)^{\alpha - 1}$$

Since $\dot{u} < 0$, the fraction $[(1 - S^*)/(1 - S)]$ decreases through time, approaching one. Thus, the ratio of the percent of wages paid by employers in the optimum dynamic case to the static case decreases through time. However, even with (32) it is not possible to say $(S - S^*)$ monotonically decreases through time; without (32) it does not seem possible to reach any specific conclusions.

Next, consider how the optimal dynamic subsidy (S^*) changes through time. By definition:

$$(33) \quad S^* = 1 - [PF_m'(N_m)/F_c'(L_c)] > 0$$

Clearly, L_c increases through time, so that F_c' falls; what happens to F_m' depends on $\dot{N}_m$. If $\dot{N}_m < 0$, F_m' increases and S^* falls through time. However, if $\dot{N}_m > 0$, as for $\Phi'' = 0$ and latter portions of the optimal path, we cannot determine <u>a priori</u> how S^* changes through time. In order to get more specific results, we need some assumptions on F_c'.

Consider the locus in (L_c, q) space for which S, a subsidy rate, is constant. Assuming $\Phi'' = 0$, we find from (11) and (33):

$$(34) \quad [\Phi'(0) \cdot q/F_c'] = (1 - S) \text{ for } u > 0; \text{ thus:}$$

$$(35) \quad [(dq/dL_c)(L_c/q)]_S = [L_c F_c''/F_c'] < 0$$

Equation (35) gives us the elasticity of an iso-subsidy line; note that it is independent of q and S, and depends only on L_c.

For the optimal path, we have from (11), (12) and (13):

$$(36) \quad [dq/dL_c] = [\dot{q}/\dot{L}_c] = [(r + \Phi'(0))q - F_c']/[\Phi'(0) \cdot u \cdot L_m] < 0$$

Letting $S^*(t)$ represent the optimal subsidy at t, and using (11), we have, for any point on the optimal path:

$$(37) \quad [(dq/dL_c)(L_c/q)] = [L_c/uL_m][(r(1 - S^*)-S^* \cdot \Phi'(0))/((1 - S^*)\Phi'(0))]$$

As $u \to 0$, the RHS in (37) tends to minus infinity; thus, subsidies must be rising along the final portion of the optimum path.[26] Note that this must hold even if $\Phi'' \neq 0$, since, for small u, equation (37) can be used as a suitable approximation.

Suppose $S^*(t)$ is not monotonic; then the optimum path must cross some iso-subsidy line at least twice. If $[L_c F_c''/F_c']$ is constant (as for a constant output-labor elasticity), the optimum path can cross a given iso-subsidy line at most two times, since $[L_c/uL_m]$ increases monotonically along the optimum path. Figure 4 depicts the relation between the iso-subsidy lines and the optimum path for a given time horizon. Note that, given L_c, smaller values of q correspond to larger subsidies.

If $\Phi'' = 0$ and $[L_c F_c''/F_c']$ is constant, then the time path of $S^*(t)$ can be characterized as follows: (i) for small T, S^* is constant and equal to the static subsidy; (ii) for intermediate values of T, $S^*(t)$ rises for $u > 0$, and then remains constant for $u = 0$; and (iii) for larger values of T, $S^*(t)$ decreases initially, reaches a minimum, and then increases until $u = 0$;

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[26]This assumes the planning horizon is finite.

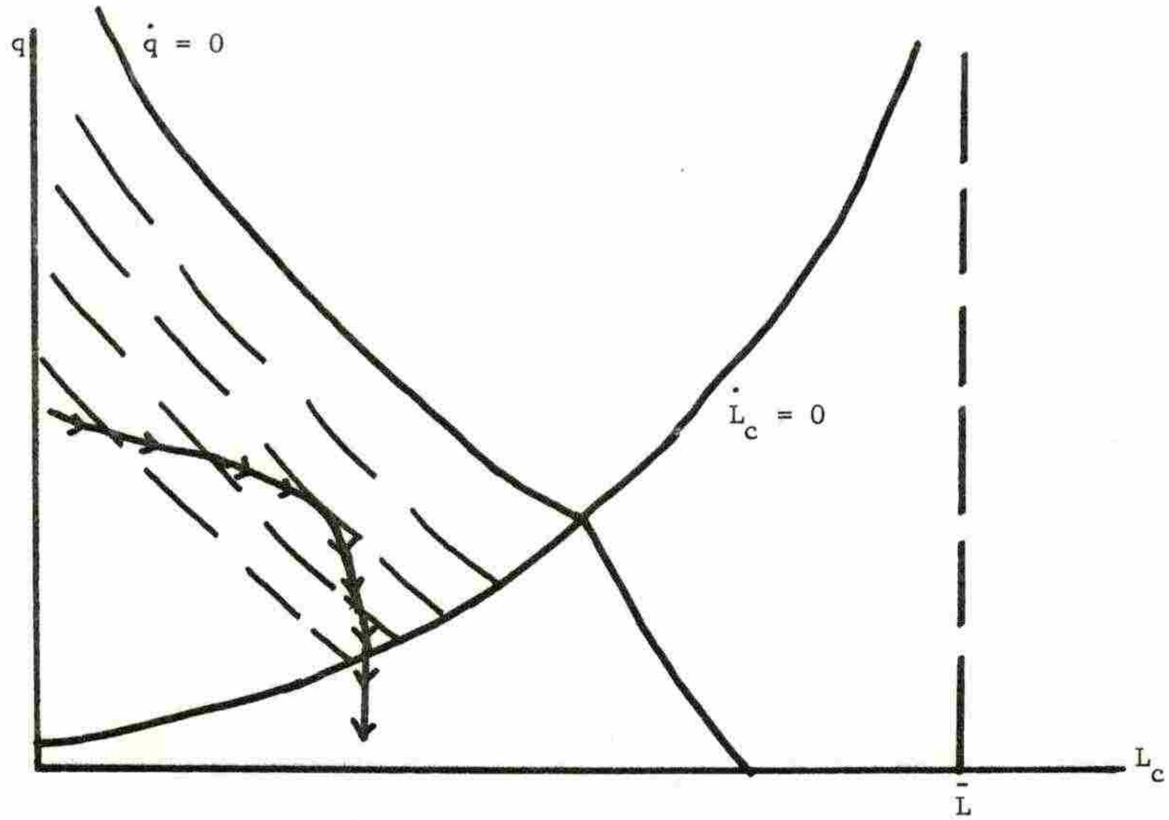

Figure 4

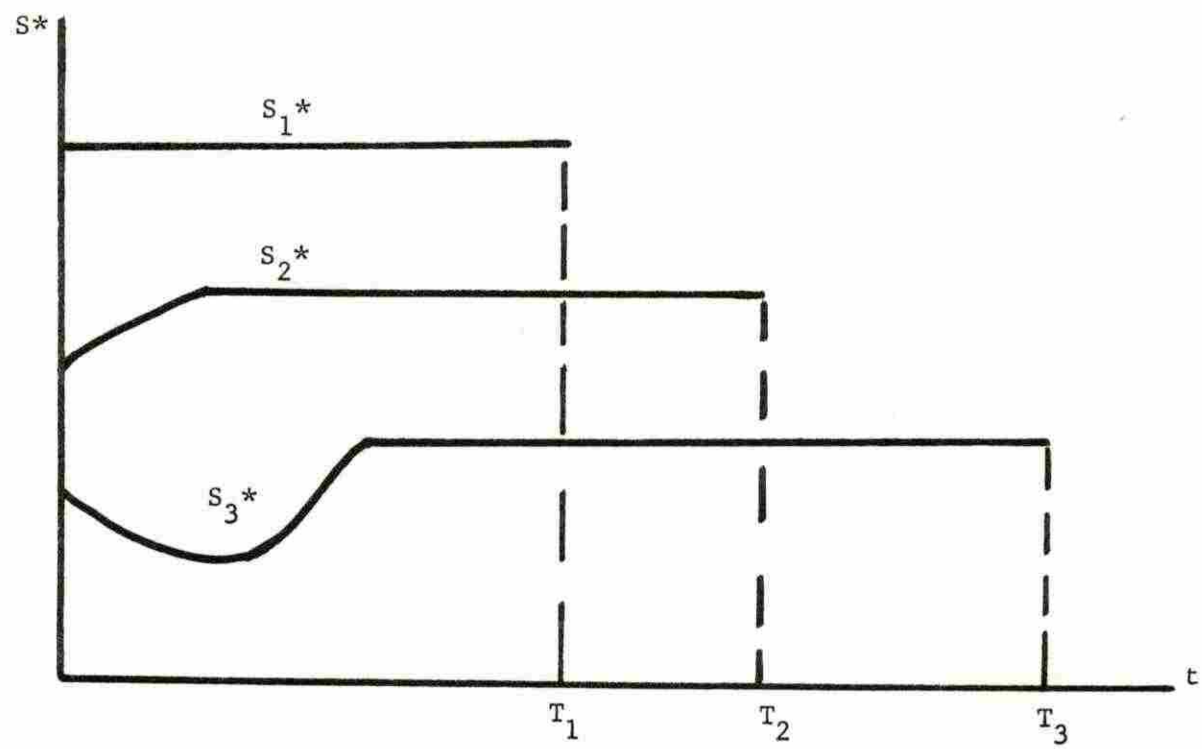

Figure 5

again, the latter portion of the path is characterized by constant S^*. Figure 5 depicts this result, assuming $L_c(0)$ is the same in all cases. Finally, note that, regardless of the assumptions on Φ'' and $[L_c F_c''/F_c']$, $S^*(t)$ must increase during the final stages of unemployment for any finite horizon path.

IV. CONCLUSION

We have seen that, if resources cannot be transferred costlessly, the static subsidy is inefficient in a dynamic context unless the planning horizon is short, or discount rates are large. Thus, in some sense the static subsidy represents myopic behavior. Moreover, we have shown how the optimum path can be determined and have characterized the properties of the path. In particular, we have seen that for long time horizons and low discount rates, it will always be optimal to have some unemployment initially; and the initial level of unemployment increases with the time horizon. Nevertheless, if wages must be equalized across sectors, some subsidy will always be needed; and we have discussed how this subsidy changes through time. Thus, a realistic policy must recognize that resources are not instantaneously mobile, but it must also recognize that too large a subsidy removes the incentives for intertemporal reallocation of resources.

REFERENCES

1) J. Bhagwati, V. K. Ramaswami and T. N. Srinivasan, "Domestic Distortions, Tariffs, and the Theory of Optimum Subsidy: Some Further Results," Journal of Political Economy, November 1969, 1005-1010.

2) ____________________ and T. N. Srinivasan, "On Reanalyzing the Harris-Todaro Model: Policy Rankings in the Case of Sector-Specific Sticky Wages," American Economic Review, June, 1974, 502-08.

3) G. Hadley and M. C. Kemp, Variational Methods in Economics, North-Holland, Amsterdam, 1971.

4) J. Harris and M. Todaro, "Migration, Unemployment and Development: A Two-Sector Analysis," American Economic Review, March, 1970, 126-42.

5) H. G. Johnson, "Optimal Trade Intervention in the Presence of Domestic Distortions," in R. E. Caves, et. al., eds., Trade, Growth, and the Balance of Payments, North-Holland, Amsterdam, 1965, 3-34.

6) S. Magee, "Factor Market Distortions, Production and Trade: A Survey," Oxford Economic Papers, March 1973, 1-43.