Chemically Mediated Quantum Criticality in NbFe2

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Laves-phase Nb1+c,Fe2−c is a rare itinerant intermetallic compound exhibiting magnetic quantum criticality at c_{cr} \sim 1.5\%Nb excess; its origin, and how alloying mediates it, remains an enigma. For NbFe2, we show that an unconventional band critical point above the Fermi level E_F explains most observations and that chemical alloying mediates access to this unconventional band critical point by an increase in E_F with decreasing electrons (increasing %Nb), counter to rigid-band concepts. We calculate that E_F enters the unconventional band critical point region for c_{cr} > 1.5\%Nb and by 1.74\%Nb there is no Nb site-occupation preference between symmetry-distinct Fe sites, i.e., no electron-hopping disorder, making resistivity near constant as observed. At larger Nb (Fe) excess, the ferromagnetic Stoner criterion is satisfied.

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Quantum criticality emerges from collective low-energy excitations leading to a second-order phase transition at zero temperature (T) [1]. In the study of correlated-electron materials, e.g., high-T superconductors and heavy-fermion compounds, understanding such a critical phenomenon remains a principal challenge, and locating any existing quantum critical points (QCPs) is difficult. Near these QCPs, quantum fluctuations (rather than thermal fluctuations) are observed to give rise to exotic effects, e.g., non-Fermi-liquid behavior [2]. In intermetallic compounds, the situation appears simpler, where, by varying nonthermal order parameters, such as applied pressure, external magnetic field, or chemical doping, various phases near a QCP can be accessed, i.e., ferromagnetic (FM), antiferromagnetic (AFM), and paramagnetic states. Importantly, strong electron-electron interactions are not required for an alloy to show correlated behavior, as now confirmed in Fe-As compounds [3]. The Laves phase of Nb_{1+c},Fe_{2−c} is one important example and featured in recent reviews of quantum criticality in weak magnets [4]. At stoichiometry the susceptibility exhibits Curie-Weiss behavior down to a spin-density wave (SDW) transition at T_{sdw} \approx 10 K, whereas at \sim 1.5\%Nb excess a QCP is observed where the SDW collapses and non-Fermi-liquid behavior occurs [5,6]. For larger Nb excess (c > 0), hole doping or Fe excess (c < 0, electron doping), a FM transition is always observed [5,6]. Indeed, the sensitivity of the magnetic state to Nb deficiency has long been known [7,8].

Although well characterized experimentally, our understanding of the properties of Nb_{1+c},Fe_{2−c} is lacking, especially how chemical disorder affects the magnetic transitions and QCP (and the FM onset at larger dopings). Recent theoretical work studied the electronic properties of NbFe2 [9] but did not address the critical chemical effects. Of course, including properly the effects of disorder in this class of systems, specifically at such a small doping (c), is a considerable challenge. Attempting to address doping, recent studies use the virtual crystal approximation [10] or supercells (ordered array of dopants) [11], having severe shortcomings, as we discuss.

In metals low-energy excitations lie at or near the Fermi surface (FS), and, thus, some unique spectral feature at or near $E_F$ is typically required to drive a transition, as with FS nesting [12,13] or van Hove band critical points [14] types of ordering [15,16]. Such FS features have been rarely identified as the origin of quantum criticality. We detail how an unconventional band critical point (UBCP), i.e., an accidental saddle-point dispersion, above $E_F$ is responsible for the QC behavior in metallic Nb_{1+c},Fe_{2−c}. We show that chemical disorder mediates, via d-state hybridization, access to this UBCP even with fewer electrons for increasing %Nb, which the rigid-band or virtual crystal approximation cannot describe, effects well known in alloy theory.

We use an all-electron, Korringa-Kohn-Rostoker (Green’s function) electronic-structure method and the coherent-potential approximation [17] to calculate the electronic dispersion, density of states, total energies, and doping site preferences in Nb_{1+c},Fe_{2−c}. See Ref. [18] for details and examples for (dis)ordered alloys. In agreement with experiment, we calculated $c_{cr}$ “onset” at 1.5\%Nb, and, at 1.74\%Nb, when $E_F$ lies exactly at the UBCP, we find that Nb has no site-occupation preference between the two symmetry-distinct Fe sites, favoring a homogeneous solute distribution and no electron-hopping disorder, suggesting a near constant resistivity. We find competing FM and AFM states, as observed, that are associated with the competing wave vectors from the UBCP. We show that the FM Stoner criterion in Nb_{1+c},Fe_{2−c} is obeyed for larger Nb (or Fe) excess. If the UBCPs are removed from consideration, none of these results hold. We conclude that the QCP...
occurs via chemical-disorder-mediated access to the UBCP above $E_F$.

To understand chemical disorder effects, we first need to appreciate the structure of NbFe$_2$, which crystallizes in a C14 hexagonal Laves phase with space group $P6_3/mmc$ (no. 194). In terms of crystallography, NbFe$_2$ = Nb$_4$Fe$_2$ (2$a$)Fe$_6$ with a 12-atom unit cell: Fe$_{6b}$ sites form two kagome networks (⊥ to the $c$ axis) separated by Fe$_{2a}$ sites in a hexagonal sublattice, and Nb atoms occupy the interstices; see Figs. 1(a) and 1(b). The Wykoff positions (without inversion) are Nb at $4f(\frac{1}{3}, \frac{2}{3}, x)$, Fe$_{2a}$ at $2a(0, 0, 0)$, and Fe$_{6b}$ at $6h(y, 2y, \frac{3}{2})$. Calculations are performed with measured [5] structural parameters $a = 4.8401$ Å and $c = 7.8963$ Å and internal coordinates $x = 0.0629$ and $y = 0.1697$. Using the Korringa-Kohn-Rostoker coherent-potential approximation, we may study the effect of (in)homogeneous solute distributions. Disorder with antisite Nb on one or both Fe sublattices is written as Nb$_4$(Fe$_{2a}$)$_c$(Fe$_{6b}$)$_{1-c}$, or Nb$_4$(Fe$_{1-c}$)$_c$Nb$_6$ for the homogeneous case.

To identify critical electronic features and chemical disorder effects, we detail the dispersion $\epsilon(\mathbf{k}; E)$ and density of states (DOS). Korringa-Kohn-Rostoker uses constant-$E$ matrix inversion to get $\epsilon(\mathbf{k}; E)$, rather than constant-$\mathbf{k}$ diagonalization to get eigenvalues $\epsilon(\mathbf{k}; E)$. To handle (dis)ordered cases, we calculate the Bloch spectral function [13] $A_B(\mathbf{k}; E)$ on a grid of $32 \times 32 \times 24$ $\mathbf{k}$ points to project the dispersion. For ordered cases, $A_B(\mathbf{k}; E)$ yields $\delta(E - \epsilon(\mathbf{k}; E))$, i.e., bands; otherwise, it exhibits disorder-induced spectral broadening in $E$ and $\mathbf{k}$, related to the finite electron scattering length.

We now show that the $c$ dependence tied to the observed quantum criticality arises from the NbFe$_2$ dispersion above $E_F$. Our bands in Fig. 1(c) agree with those from full-potential methods [9]. The bands crossing $E_F$ along $\Gamma$-$M$ arise mainly from Fe$_{6b}$ $t_{2g}$ orbitals and lead to saddle-point dispersion slightly (6.6 meV) above $E_F$ with an unusual flat dispersive region (UBCP) [Figs. 1(c) and 2(b)]. These UBCPs near $E_F$ are not a result of symmetry but arise from accidental band crossings. Experimentally, Crook and Cywinski [19] inferred that the Fe$_{6b}$ $t_{2g}$ orbitals in the kagome nets play a critical role in the competing magnetic order associated with the quantum phase transition. The Nb$_{1+c}$Fe$_{2-c}$ phase diagram [5] shows the QCP onset at ambient pressure at $c_{cr} \sim 1.5\%$Nb (hole-doped) and extends to $c \approx 2.0\%$, with FM at larger doping. For a metal, only low-energy excitations near $E_F$ can be relevant for such low-$T$ transitions. By small tuning of a nonthermal order parameter, i.e., $c$, the unusual quantum phase transition behavior is observed and can be explained if these UBCPs are accessed.

For homogeneous doping, we find a chemical-disorder-mediated increase of $E_F(c)$ versus %Nb excess, or decreasing electron-per-atom ($e/a$) ratio [Fig. 2(a)]. Counter to rigid-band concepts, $E_F$ rises to UBCP and due to Fe-Nb (bond or antibond) alloying hybridization in kagome nets Fe bands shift lower, but bands from the pure Nb layer remain unaffected [Fig. 2(a), inset]. (An ordered array of impurities exaggerates the effect—see below—showing that disorder plays a key role.) At 1.74%Nb excess, UBCP lay at $E_F$ a 6.6 meV shift due to alloying and disorder (finite lifetime) effects; see Fig. 2(b). The 6.6 meV (or 77 K) sets the maximum temperature, as observed, for these effects to occur on stoichiometry. The dispersion and disorder-induced widths along $\Gamma$-$M$ for $0 \leq c \leq c_{cr}$ [Fig. 2(b)] estimate the QC range. By 1.65%, $E_F$ enters the spectral tails, giving zero-energy excitations into the anomalous dispersion; $E_F$ is maximally aligned with the UBCP by 1.74% (giving a Lifshitz-type transition; see below) and exits by 2%, where FM is observed. We conclude that the QCP occurs from alloying or disorder-mediated access to the UBCP inherent in NbFe$_2$ dispersion above $E_F$, detailed more below.

![FIG. 1 (color online).](image)

(a) Laves unit cell. (b) Fe$_{6b}$ sites form two kagome nets, with Nb (gray), Fe$_{2a}$ (red), and Fe$_{6b}$ (blue), with higher (lower) planes shaded. (c) NbFe$_2$ bands ($E_F$ at 0 eV). The highlighted box shows an UBCP above $E_F$.

![FIG. 2 (color online).](image)

(a) $\Delta E_F(c)$ shift due to %Nb excess vs $e/a$ for Nb$_4$(Fe$_{1-c}$)$_c$Nb$_6$. $E_F$ lies at UBCPs at $c_{cr} = 1.74\%$ but enters spectral tails at 1.65%. Inset (a) shows (und)oped $A_B^{\mathrm{inh}}(\mathbf{k}; E)$ along $\Gamma$-$M$, and (b) expands around UBCPs (bars are spectral widths due to disorder broadening).
We have performed supercell calculations to illustrate how sensitive $\Delta E_p(c)$ is to approximations used to address chemical effects, which ignore disorder. Supercells are numerically costly due to large cells needed with decreasing %Nb. We performed calculations at two concentrations with cells constructed by substituting a Nb atom for one Fe atom in a $2 \times 2 \times 1$ (6.25%Nb excess) and a $2 \times 2 \times 2$ (3.125%Nb excess) supercell. The supercells also yield a $\Delta E_p(c)$ increase, relative to $c = 0\%$, of 117.1 and 281.3 meV for 3.125% and 6.25%Nb excess, respectively. The coherent-potential approximation shifts are 65.4 and 158.1 meV, respectively, which lie on the curve in Fig. 2(a) at smaller $e/a$. So, supercells do not provide the correct occupancy probability nor hybridization across the kagome net, missing the key disorder effects. Nonetheless, supercell results do reinforce the fact that rigid-band or virtual crystal approximation concepts are invalid, missing the critical alloying and disorder effects.

Figure 3 compares the DOS for Nb$_{1+}$Fe$_{2-c}$ at $c_{cr}$ to 0%, which is similar to full-potential results [9,11]. Disorder broadening for $c > 0\%$ is evident. $E_F$ lies near a precipice of a DOS depression, which plays a role in forming a SDW state at $c = 0\%$. $n(E_F)$ is 3.59 states [eV × formula unit (f.u.)]$^{-1}$ for NbFe$_2$. Notably, for NbFe$_2$, unlike conventional band critical points [14], the saddle-point dispersion at $k$’s associated with the UBCP is (beyond) cubic ($e^k \propto k^3$) in one direction and quadratic in the orthogonal ($k_x$, $k_z$) plane, yielding a chemically mediated peak in $n(E_F)$ when $E_F$ and UBCP are aligned (Fig. 3, inset). From Stoner theory with interaction parameter $I$ [20], a FM instability occurs if $n(E_F)I > 1$. For pure Fe d electrons, $I$ was reported [21] between 0.7 and 0.9 eV. From susceptibility data for NbFe$_2$, the Stoner factor [1 $\nabla n(E_F)/n(E_F)$]$^{-1}$ was estimated [5] at $\sim$100. Our calculated $I$ for Nb$_{1+}$Fe$_{2-c}$ is almost constant versus $c$ (0.88 $\leq I \leq 0.9$), but $n(E_F)$ increases with doping, increasing $n(E_F)I$; e.g., $n(E_F)$ for 1.74%Nb increases to 3.90 states(eV f.u.)$^{-1}$. Stoner’s criterion is satisfied beyond 2%Nb, as observed, and discussed more below.

Lifshitz transitions are mediated by FS topology changes (e.g., collapse of the FS neck or loss of pockets). An unconventional Lifshitz transition emerging near a marginal QCP was proposed in ZrZn$_2$ [22]. Such a zero-temperature, pressure-induced transition is also associated with access to an UBCP by an increase in $E_F$. The FS topology changes are reflected in a maximum in $n(E_F)$ at the Lifshitz point, as we found for hole doping in NbFe$_2$ at $c_{cr} = 1.74\%$ (Fig. 3, inset). Not only the topology but the FS volume is strongly dependent on doping, in NbFe$_2$ enhanced with hole doping and also observed in a hole-doped Ba$_{0.3}$K$_{0.7}$Fe$_2$As$_2$ [23].

To explore beyond the QCP, we studied doping in the Fe- and Nb-rich parts of the phase diagram. The DOS (atom- and impurity-projected) for 3.125% Nb- and Fe-rich Nb$_{1+}$Fe$_{2-c}$ is shown in Fig. 4. The Nb-rich (hole-doped) $n(E_F)$ increases to 4.66 states(eV f.u.)$^{-1}$. Fe-rich (electron-doped) alloys behave opposite to what is expected from rigid-band theory; i.e., with electron doping, $n(E_F)$ rises to 4.75 states(eV f.u.)$^{-1}$, which is mainly due to Fe-impurity DOS originating from Nb layers—see Fig. 4 (lower panel). These values of Nb-rich and Fe-rich $n(E_F)$ satisfy the FM Stoner criterion, as observed [5].

We have also studied how the nature of the UBCP (or QCP) is affected by inhomogeneous Nb doping on the two Fe sublattices. We find that, if only Fe$_{(h)}$ sites are doped, the UBCP lies at $E_F$ when $c_{cr}^{(h)} = 2.34\%$, for an average $c_{cr}^{(h)} = 2.34\% \times (6/8) = 1.75\%$. For the Fe$_{(h)}$-only case, dispersion is similar to the homogeneous case as in Fig. 1(c), because mainly Fe$_{(h)}$ $t_{2g}$ orbitals within the kagome sheets are involved. In contrast, if only Fe$_{(a)}$ sites are doped, the UBCP lies at $E_F$ when $c_{cr}^{(a)} = 8.7\%$, i.e., $c_{cr}^{(a)} = 8.7\% \times (2/8) = 2.02\%$. The shift of $E_F$ relative to dispersion in this case arises due to hybridization of Fe$_{(a)}$ states indirectly with Fe$_{(h)}$ states. Dispersion with Fe$_{(a)}$-only doping shows an increased slope of the UBCP compared to the other cases, changing the character and $c$ dependence of the response.

In Fig. 5, we report Nb site-preference energy differences ($E_F^{(h)} - E_F^{(a)}$) versus %Nb excess in

![FIG. 3 (color online). Nonmagnetic DOS per formula unit relative to $E_F$ for Nb$_{1+}$Fe$_{2-c}$ at $c_{cr}$ = 0 and 1.74%. Inset: DOS at $c_{cr}$ around $E_F$ (the vertical dashed line is $E_F$ at $c = 0\%$).](image)

![FIG. 4 (color online). Nonmagnetic DOS relative to $E_F$ for 3.125% Nb-rich (top) and 3.125% Fe-rich (bottom).](image)
Albeit weaker, it can occur from saddle-point topology, too. From the small caliper of the flat part of the UBCP, i.e., 0.055(2π/a), we estimate a |q| of 0.07 Å⁻¹ close to the 0.05 Å⁻¹ estimated from the experiment [6].

Thus, from our direct calculations or estimated χ(q) features, paramagnetic, FM, and SDW (AFM) states compete near the QCP, until overwhelmed by a Stoner instability at larger |c|, shown above. Such competing magnetic behavior is observed [6] near the QCP, where the low-T resistivity [ρ(T) ∼ T²] exponent ν varies between 3/2 and 5/3, giving unusual non-Fermi-liquid behavior.

Quantifying the anomalous response further requires χ(q, c, ω), which is beyond the present scope. But the change in ω = 0 susceptibility, Δχ(Q, ω = 0), yields the change in the DOS at E_F, i.e., Δn(E_F), and, if large, a FM instability. Neal, Ylvisaker, and Pickett have extended the Moriya χ(q, c = 0, ω) theory to account for UBCPs, which yields an anomalous frequency response and competing FM and SDW states [25], agreeing with our direct calculations.

In summary, we have identified an accidental Fermi-surface (nonideal saddle-point) dispersion as the origin for observed behavior associated with the quantum criticality in Nb_{1+c},Fe_{2-c}. We find that Nb (hole) doping accesses unconventional band critical points in NbFe_2 that provide the necessary low-energy excitations for a (Lifshitz-type) transition. This origin explains most of the observed doping behavior in this QC intermetallic compound, specifically, (i) onset of c_{cr} %Nb excess for the QCP; (ii) almost constant resistivity versus c, (iii) competing magnetic states and temperature scale, (iv) observed SDW wave vectors, and, finally, (v) the stable FM states at large hole or electron doping (>2%). To explore the nature of the QC transition further, these electronic and chemical features can be incorporated into a model Hamiltonian, but they would not have been discovered without the full dispersion and chemical alloying effects detailed here.

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FIG. 5. Nb site-preference energy difference (E^{(6h)} - E^{(2a)}) versus %Nb excess in Nb_{1-c}Fe_2. E^{(c)} is the total energy when only the Fe^{(c)} sites are doped. At c_{cr} = 1.74%, no Fe site is favored, i.e., homogeneous solute distribution.

\[ Nb_{1-c}Fe_{2a}^{(2a)}Nb_{2c}^{(2a)}(Fe_{1-c}^{(6h)}Nb_{c}^{(6h)})_{2}, \text{ where } E^{(c)} \text{ is the total energy when only the Fe}^{(c)} \text{ sites are doped. At c}_{\text{cr}} = 1.74\%, \text{the UBCP lies at } E_F, \text{there is no energetically favored site occupancy and, therefore, no electron-hopping disorder effects, explaining the observed extremely slow variation of } \rho \text{ versus doping [5]. With no site preference, a homogeneous solute distribution is favored, further supporting our focus on this case. These results shed light on the microscopic phenomenon occurring at or near the QCP: The alignment of } E_F \text{ with the UBCP at 1.74\% provides the maximum response concomitant with no electron-hopping disorder and instability to both AFM and FM fluctuations.} \]

We now explore the stability of competing magnetic phases. Out of several small-cell, magnetic configurations in NbFe_2, we found an AFM state energetically favored by −10.45 m eV/atom over the nonmagnetic state, where local moments on the Fe^{(2a)} and Fe^{(6h)} sites are aligned antiparallel with values of 0.69 μ_B and −0.98 μ_B, respectively, and the Nb has an induced moment of 0.14 μ_B aligned with the Fe^{(2a)} site. The same ground state was found in other calculations [9,11]. The phase diagram indicates a SDW state for NbFe_2 with a highly itinerant nature (as for iron arsenides [3]), which competes with nearby FM states. The itinerancy of magnetism is clear from the spin density on Fe sites (see [24]), while carrier density peaks at c_{cr} (Fig. 3, inset).

To connect to scattering experiments, the UBCPs ε(k) exhibit zero-velocity quasiparticles near E_F at symmetry-equivalent, nonspecial k points Q_0(±1, 0) and Q_0(±1/2, ±(√3/2)], where Q_0 = 0.25 in units of the Γ-M caliper in Fig. 1(c). These Q’s remain relevant for off-stoichiometric alloys. The susceptibility, i.e.,

\[ χ(q) = \sum_k [f(ε_k) - f(ε_{k+q})][ε_{k+q} - ε_k]^{-1}, \]

can exhibit an enhanced response due to a convolution of (un)occupied states near E_F, as for FS nesting [12,13].

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