"INFLATION, WEALTH AND THE REAL RATE OF INTEREST"

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Robert Mundell has argued that inflation, even when fully anticipated, alters the long-run equilibrium level of the real interest rate (1).

"... the money rate of interest rises by less than the rate of inflation and therefore the real rate of interest falls during inflation. This conclusion is based on the fact that inflation reduces real money balances and that the resulting decline in real wealth stimulates increased saving. Real conditions in the economy are altered by purely monetary phenomenon. The evils or benefits of inflation cannot be attributed solely to the failure of the community to anticipate it (2)."

Mundell obtains this result by postulating stable flow demands for assets, whereas -- as is argued here -- saving and investment arise from an inequality between actual and desired asset stocks. The model developed in this paper will include both the stock and flow demands for assets and demonstrates that inflation will permanently alter the real interest rate only if it can change the desired stock of capital. The desired stock of capital may be positively or negatively related to the rate of inflation, so that the real rate of return on capital may decrease or increase due to inflation. The model will then be used to obtain the time path of the real interest rate, and the effects of improperly anticipated inflation will be shown.

Mundell's analysis is based on the following diagram:

The curve labeled IS shows the combinations of real cash balances and real
Interest rates which maintain equilibrium in the goods market. The slope of the IS is positive since:

"... an increase in the real rate of interest lowers investment causing a deflationary gap, while an increase in real balances lowers saving, causing a compensating inflationary gap (3)."

The curve labeled LM shows the combinations of real interest rates and real cash balances which result in money market equilibrium. It has a negative slope for the usual reasons; the higher the opportunity cost of holding cash, the lower is the demand for cash balances. Mundell points out that the LM curve is defined in terms of the nominal interest rate. If, for some reason, the rate of inflation increases, the LM curve must shift downward by the rate of inflation, as is shown below.

Figure 2

The real rate of interest will decline from \( r_0 \) to \( r_1 \). Hence, argues Mundell, the Fisher Effect is incorrect.

Mathematically:

\[
I(r) = S(r, \frac{M}{P})
\]

\[
\frac{M}{P} = L(i, \frac{M}{P})
\]

\[
r = i - \pi
\]

\[
\Rightarrow \frac{dr}{d\pi} = \frac{\frac{\partial S}{\partial M} \frac{\partial L}{\partial i}}{\left(\frac{\partial L}{\partial r} - \frac{\partial S}{\partial r}\right) \left(1 - \frac{\partial L}{\partial M} \frac{\partial S}{\partial P}\right)} < 0
\]

Where: \( i = \) nominal interest rate

\( \pi = \) fully anticipated rate of inflation

and:

\( \frac{\partial S}{\partial M} < 0; \frac{\partial L}{\partial r} < 0; \frac{\partial S}{\partial r} > 0; \)

\( 0 < \frac{\partial L}{\partial M} < 1; \frac{\partial L}{\partial i} < 0 \)
As should be clear, Mundell's model ignores the effect of investment on the capital stock and, by necessity, the effect of a change in the capital stock on real output. Furthermore, saving -- asset accumulation -- is assumed to have no feedback effect upon wealth and next period's consumption and saving.

A Portfolio Balance Model

In this section, an alternative model, which captures the stock and flow relationships ignored in the Mundell model, will be developed. As in the Mundell model, only two distinct forms of wealth can be held, money and physical capital. At a point in time, in which wealth is fixed, asset holders can only allocate their assets between the different forms of wealth but cannot increase the size of their portfolios. The demand for an asset at any point in time can then be expressed as a proportion of wealth, where the constant of proportionality depends upon the rates of return on all assets. More generally:

1) \( m = 1(r_k, r_m, w) \)

2) \( k = k^D(r_k, r_m, w) \)

3) \( w = m + k \)

Where:\n
- \( m \) = real per capita cash balances
- \( k \) = real per capita capital
- \( r_k \) = real rate of return on capital
- \( r_m \) = expected real rate of return on cash balances, i.e., the negative expected rate of inflation
- \( w \) = real per capita wealth

The discussion of the point in time asset demand function implies the following sign restrictions (4):

\[
\frac{\partial l}{\partial r_k} + \frac{\partial k^D}{\partial r_k} = \frac{\partial l}{\partial r_m} + \frac{\partial k^D}{\partial r_m} = 0; \quad \frac{\partial l}{\partial w} + \frac{\partial k^D}{\partial w} = 1
\]

It is assumed that the demand for each asset is positively related to the level of wealth and its own rate of return and, hence, negatively related to the rate of return on the alternative asset.

Equations 1 through 3 relate to demands for assets at a point in time and
convey no information concerning asset accumulation. It seems reasonable to assume that saving is proportional to the difference between desired and actual wealth. Since the desired or target level of wealth depends solely upon income and the rates of return on assets, real saving can be represented by:

\[
dw \frac{dw}{dt} = \alpha \left[ w^*(y, r_k, r_m) - w \right]
\]

Where: 
- \( w^* \) = desired or target level of per capita wealth
- \( y \) = real per capita income

and:
- \( \frac{dw^*}{dr_k} \geq 0 \)
- \( \frac{dw^*}{dr_m} \geq 0 \)
- \( 0 < \frac{\partial w^*}{\partial y} < 1 \)

Asset holders do not know the rate of inflation with certainty and are assumed to increase their perception of the inflation rate in such a way that the change in the expected rate of inflation is proportional to the actual minus the expected inflation rate (5), i.e.,

\[
dr_m = a \left[ r_m - r_m^e \right]
\]

Where: 
- \( r_m \) = actual rate of return on money, i.e., negative rate of inflation.

The economy in question is such that real per capita output is solely a function of the capital/labor ratio and the production function is well behaved, i.e.,

\[
y = y(k)
\]

Where: \( 0 < y' < 1 \); \( y'' < 0 \)

7) \( r_k = y'(k) \)

Long-run equilibrium (the particular solution to Equations 1 through 7) requires that desired and actual wealth be equal, and that the expected rate of return on money equal the actual rate of return on money. These conditions plus Equations 3 through 6 can be substituted into Equations 1 and 2 in order to yield the steady state solution for money and capital, i.e.,

1') \( m^* = 1^*(y(k^*), r_k(k^*), r_m) \)

Where: * designates a steady state solution of the associated variable.

2') \( k^* = k^{\text{eq}}(y(k^*), r_k(k^*), r_m) \)

It should be clear that \( \frac{\partial k^{\text{eq}}}{\partial r_m} \) may be positive, zero or negative, as an
increase in the rate of return on money will increase the desired stock of asset holdings but reduces the desired proportion of capital to wealth. Thus, an increase in the rate of inflation (decrease in $r_m$) may increase, decrease or leave unchanged the capital stock and, as a result, the real rate of interest. In contrast to Mundell's conclusion, it has been shown that inflation may lead to a reduction in real cash balances without producing a permanent change in the real interest rate.

Equations 2 through 7 (since Equation 1 is not independent of Equations 2 and 3) can be used to obtain the time path for $k$, $r^e_m$, $w$, $r_k$, $m$ and $y$. Allowing $D$ to represent the differential operator, and retaining only the linear terms of a Taylor Expansion, the coefficient matrix post multiplied by the column vector of unknowns can be represented by:

\[
\begin{bmatrix}
1 - \frac{\partial k}{\partial r_k} & \frac{\partial r_k}{\partial r_m} & 0 & -\frac{\partial k}{\partial w} & -\frac{\partial k}{\partial r^e_m} \\
-1 & -1 & 1 & 0 & 0 \\
\left(\frac{\partial w^*}{\partial y} + \frac{\partial w^*}{\partial r_k} \frac{\partial r_k}{\partial k}\right) & 0 & -(1+\frac{D}{\alpha}) & 0 & 0 \\
0 & 0 & 0 & -(1+\frac{D}{\beta}) & 0
\end{bmatrix}
\]

Setting the determinant of the coefficient matrix equal to zero, the solution set for $D$ will yield the characteristic roots of the system. The determinant of the coefficient matrix ($\Delta$) is:

\[
\Delta = (1 + \frac{D}{\beta}) \left[ -\frac{D}{\alpha} \left( 1 - \frac{\partial k}{\partial r_k} \frac{\partial r_k}{\partial r_m} + \frac{\partial k}{\partial w} \frac{\partial w^*}{\partial y} + \frac{\partial w^*}{\partial r_k} \frac{\partial r_k}{\partial k} \right) \right]
\]

Thus: $r_1 = -\beta$ ; $r_2 = -\alpha \left[ 1 - \frac{\partial k}{\partial w} \frac{\partial w^*}{\partial y} + \frac{\partial r}{\partial k} \frac{\partial k}{\partial r_k} + \frac{\partial w^*}{\partial r_k} \frac{\partial w^*}{\partial k} \right]$

The two characteristic roots are both negative, thus the system is stable. Regardless of whether the long-run value of the real interest rate is invariant
with respect to the rate of inflation, the characteristic roots show that the real interest rate changes during the transition period. Further, the smaller is $\beta$ -- the slower is the adjustment of the expected to the actual rate of inflation -- the longer it will take for the system to return to long-run equilibrium from any given displacement. Thus, incorrectly anticipated inflation acts to delay the return to full stock equilibrium.

The view taken in this paper is that individuals demand a stock of wealth and that saving and investment are attempts to equilibrate desired and actual asset holdings. In this approach, equilibrium--in each period of time--requires that the desired accumulation of wealth equal the actual accumulation of wealth. Since individuals, however, have a desired or target level of asset holdings, saving and investment in a period will increase actual wealth and thus change desired saving and investment in successive periods. Thus, it is clear that "real conditions in the economy are altered by purely monetary phenomenon", but this has been shown to be unambiguously true only in the short run. In the long run, it has been shown that it is possible for real cash balances to fall during inflation while the real interest rate may rise or fall depending upon whether cash balances and capital are complementary or substitute assets. This is in direct contradiction to Mundell's explanation of why the long run rate of interest is not invariant with respect to the rate of inflation.
FOOTNOTES


5. The target level of wealth can be allowed to depend upon expected income and the expected rate of return on capital without changing the major conclusions of this paper.