

## Estimating Freeway Travel Time and Its Reliability Using Radar Sensor Data

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### ABSTRACT

Travel time and its reliability are intuitive system performance measures for freeway traffic operations. This paper proposes a method to estimate travel times based on data collected from roadside radar sensors, considering spatially correlated traffic conditions. Link-level and corridor-level travel time distributions are estimated using these travel time estimates and compared with the ones estimated based on probe vehicle data. The maximum likelihood estimation is used to estimate the parameters of Weibull, gamma, normal, and lognormal distributions. According to the log likelihood values, lognormal distribution is the best fit among all the tested distributions. Corridor-level travel time reliability measures are extracted from the travel time distributions. The proposed travel time estimation model can well capture the temporal pattern of travel time and its distribution.

Keywords: probe vehicles; queue spillback; radar sensor data; travel time reliability

### 1. INTRODUCTION

From drivers' perspective, travel time and its reliability are considered as more intuitive measures of service quality, compared to the levels of service defined in the highway capacity manual (Chen et al., 2003). Highly reliable travel times allow for arriving at work or other destinations on time in the context of personal travel and facilitate just-in-time logistics services in freight operations, while highly variable travel times indicate unpredictable trip times and low quality of transportation services (Turochy and Smith, 2002).

With the advances in sensing technology, a number of travel time estimation methods have been proposed based on data collected from various sources (e.g., Soriguera and Robusté, 2011a; Tam and Lam, 2008). Reviews of the research efforts on travel time estimation and prediction methods can be found in Mori et al. (2015) and Vlahogianni et al. (2014). In particular, loop detectors have been widely used to measure traffic conditions at specific locations. Link travel times can be estimated by simply extending the point speed measurements to the entire link (Van Lint and Van Der Zijpp, 2003; Soriguera and Robusté, 2011b; Bovy and Thijs, 2000). Moreover, to capture the traffic dynamics along the link, several methods have been proposed to estimate travel time based on kinematic wave theory and other traffic flow theories (e.g., Van Arem et al., 1997; Coifman, 2002; Zhang, 2006; Kesting and Treiber, 2008; Deniz et al., 2013; Aksoy and Celikoglu, 2012). Some travel time estimation methods are essentially based on flow conservation and propagation principles (e.g., Celikoglu 2007; Celikoglu 2013a,b). Castillo et al. (2014) proposed a method that considered both probabilistic and physical consistency of traffic random variables to estimate link- and route-level traffic flows and travel times. Moreover, a number of queuing-based travel time models have been developed in the literature (e.g., Daganzo, 1995; Nie and Zhang, 2005; Lei et al., 2013). These queuing-based models used a vertical queue or point-queue to describe traffic dynamics at bottlenecks. The point-queue models assume that the length of the queue is zero and the link has unlimited storage capacity. As a result, point-queue-based models usually ignore the spillback from a downstream bottleneck.

In addition, various approaches have been developed to estimate travel time reliability (e.g., Richardson, 2003; Oh and Chung, 2006; Kwon et al., 2011). One way to examine travel time variation is to look at the distribution of travel times. Based on travel time distributions, various reliability measures can be derived, including the standard deviation of travel times, buffer time, 90th or 95th percentile travel times, buffer index, planning time index, and the probability that a trip can be successfully completed within a specified time interval (Dong et al., 2006; Tu et al., 2007; Higatani et al., 2009). Different functional forms have been used to describe link travel time distributions. Van Lint and Van Zuylen (2005) and Susilawati et al. (2010) pointed out that travel time distributions were skewed and had a long upper tail. Based on travel time data collected using the automatic vehicle identification system, Li et al. (2006) suggested that a lognormal distribution best characterized the distribution of travel time when a large time window (e.g., in excess of 1 hour) was under consideration and in the presence of congestion, and a normal distribution was more appropriate for departure time windows on the order of minutes. Moreover, after using Weibull, exponential, lognormal, and normal distributions to fit the travel time data collected from dual-loop detectors, Emam and Al-Deek (2006) suggested that lognormal distribution was the best fit. Furthermore, to determine the probability of accomplishing a trip within a time window, a corridor-level reliability measure needs to capture the variability in the total travel times of multiple roadway segments along the corridor. Although travel times can be easily integrated across time (successive time frames constituting a trip) and space (adjacent links constituting a path), travel time distributions are generally non-additive because of the spatial and temporal correlations. Considering the correlation among multiple bottlenecks along a freeway corridor, the travel time along a stretch of freeway can be computed as the sum of a set of correlated link travel times. Accordingly, the corridor-level travel time distributions, as well as various travel time reliability measures, can be estimated.

This paper presents methods to estimate corridor-level travel time reliability measures based on roadside radar sensor and probe vehicle data. As probe vehicles directly collect travel time data, link-level and corridor-level travel time distributions can be easily estimated. In the absence of direct measurement of travel times, point measurements of traffic conditions obtained from loop detectors or roadside sensors are used to estimate travel time and reliability measures along a stretch of urban freeway. In particular, the flow rates and speeds measured by roadside radar sensors on consecutive freeway segments are used to estimate link travel time distributions and correlation coefficients between links. Accordingly, the corridor-level travel time reliability measures are developed.

## **2. DATA DESCRIPTION**

Two independent data sources are used in this study to examine travel time and its reliability at the link and corridor levels—probe vehicle data and radar sensor data. The speed and volume data collected by radar sensors at fixed locations are used to estimate travel time and its distribution. The probe vehicle travel time data are used as the baseline to verify the accuracy of estimated travel times.

### **2.1 Probe Vehicle Data**

The probe vehicle travel time data used in this study is provided by INRIX, a commercial company that provides real-time traffic data collected from in-vehicle transponders on commercial vehicles and increasingly with cell phones in passenger cars. In the Des Moines metropolitan area, INRIX probe vehicle network covers all of the first, second, and third class roads, as well as the highway network. In this paper, the probe vehicle travel time data are queried from Regional Integrated Transportation Information System (RITIS), which archives INRIX probe vehicle data at 1-minute aggregation intervals. This dataset provides time-stamped segment-based speeds, travel times, historical average speed, free flow speed, and confidence score. As stated in the INRIX Interface Guide (2014), the data represent real-time data only when the

confidence score equals 30; otherwise the value is estimated from historical data. Consequently, the travel times used in this study are those with the confidence score of 30.

As the INRIX travel times are provided segment by segment, a temporally stitched algorithm (Chase et al., 2012) is adopted to generate probe vehicles at 1-minute time intervals. The temporally stitched algorithm is intended to simulate the experienced travel time of a probe vehicle traveling along the corridor. In this paper, the probe vehicle travel times are used as the ground truth.

## 2.2 Radar Sensor Data

In recent years, the Iowa Department of Transportation (DOT) has been placing Wavetronix radar sensors along interstates and major highways in the state. The majority of sensors are in the major metropolitan areas and provide valuable information for the DOT in terms of incident management, traffic operations, and planning. The existing Iowa DOT Wavetronix sensors cover the highway network in the Des Moines metropolitan area. These sensors count vehicles, by lane and classification, and register vehicle speeds. The aggregated data were obtained through an online data portal maintained by TransSuite. The data can be aggregated at different time intervals—20 seconds; 5, 15, 30, 60 minutes; and 24 hours. In order to be consistent with the travel time data generated from INRIX, the 20-second data are aggregated into 1-minute data and used to estimate travel times. The aggregated data obtained from TransSuite include volume, average speed, and average occupancy, by lane. The volume can be broken down by vehicle class as well.

On-ramps and off-ramps are potential bottlenecks on freeways (Bertini and Malik, 2004; Newell, 1999; Liu and Danczyk, 2009). As a result, roadway sensors are usually placed close to ramps, as illustrated in Figure 1. In such cases, both the ramp flow and the mainline flow can be monitored using a side-fired radar sensor, as well as the space mean speed.

The radar sensors sometimes report extreme values due to malfunction. Such abnormal data are identified and removed using the rules proposed by Vanajakshi (2005), as detailed in Table 1.

Since the proposed travel time estimation method needs to use volume and speed data during each time interval, the missing data are handled by the procedure shown in Figure 2. Basically, if a significant amount of data is missing on a certain day, that day is removed from the analysis. If data are missing only for a short time period, the data are imputed based on the data collected during the same time period on other days.

The 1-minute interval data from 7:00 a.m. to 9:30 a.m. on weekdays from December 1, 2013, to December 1, 2014, are used in this study. After the outliers are removed and the selected missing data are replaced, the availability of radar sensor data and real-time INRIX data is shown in Figure 3. To validate the proposed model against INRIX travel times, data need to be available from both sources. The plot between the black lines indicates the data in April 2014, when both the INRIX and sensor data are available on most days for all links. The inconsistency in missing data of INRIX and sensor data can cause the difference between model-based travel time and INRIX travel time reliability indices.

## 3 METHODOLOGY

### 3.1 Spatial Correlation of Link Travel Times

In order to examine the spatial correlation of travel times, the correlation coefficient is computed to represent the relation between link travel times. Eq. 1 describes the cross-correlation between travel times of different links:

$$\rho_{x,y} = \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y} \quad (1)$$

where,

$x_i$  and  $y_i$  are the travel times of two different links;  
 $\mu_x$  and  $\mu_y$  are the mean of  $x_i$  and  $y_i$ , respectively; and  
 $\sigma_x$  and  $\sigma_y$  are the standard deviation of  $x_i$  and  $y_i$ , respectively.

Historic travel time data collected during work days in 2014 on I-235 in Des Moines are used to establish correlation between links. A heat map of correlations between the links of the I-235 corridor (consisting of 20 segments) is shown in Figure 4.

Significant correlations among link travel times indicate that the links along the corridor should not be considered as independent when examining corridor level travel time reliability. As expected, the correlations between a link and its adjacent upstream or downstream links are generally higher than the correlations between the link and other links. This finding is consistent with some previous studies, such as Park and Rilett (1999) and Zou et al. (2014). Moreover, Zou et al. (2014) also pointed out that a decreasing trend of cross-correlation value between two links can be observed as the distance between two links increases.

### 3.2 Travel Time Estimation

Consider a corridor with N potential bottlenecks. Assume that each bottleneck (i.e., sensor location) is a node and the road segments between these nodes are represented by links with homogeneous capacity. Denote node 1 as the start point and node N as the last node. The segment between node M and node M+1 is denoted as link M. Figure 5 illustrates the node-link representation for part of the corridor, from node M to node M+3. An on-ramp or off-ramp might be connected to a node. The on-ramp or off-ramp is denoted as “ramp of M.” For example, in Figure 5 the on-ramp that is connected to node M+1 is denoted as “ramp of M+1.”

In order to construct a numerically tractable model for computing corridor-level travel time, the first-in, first-out property is assumed to ensure that any vehicles that enter the link first would leave the link first (Lei et al., 2013). In addition, traffic breakdowns can be detected when speed drops significantly (say, 10 mph) and the low speed sustains for a long period (e.g., 15 minutes) (Dong and Mahmassani, 2009). Considering the spatial correlation between links, three possible conditions might occur when estimating travel time of link M. Under each condition, a travel time calculation method is proposed.

The first condition is when no breakdown occurs on link M and link M+1. The travel time of link M at time t can be estimated based on the length of link and the average of speeds measured at two ends of the link, as follows.

$$T_{c1}[M, t] = \frac{2 * D[M]}{S[M, t] + S[M+1, t]} \quad (2)$$

where,

D[M] is length of link M; and  
S[M] and S[M+1] are speeds measured at node M and M+1 at time t, respectively.

The second condition is when the breakdown occurs at bottleneck M+1, causing congestion on link M. The travel time of link M at time t is calculated as follows. Assuming vehicles in the platoon are traveling at the

same speed, the spacing between two vehicles in the platoon on link M can be calculated as

$$Space[M, t] = d_0 + S[M + 1, t] * \tau \quad (3)$$

where,

$d_0$  is the initial space between vehicles;

$\tau$  is the reaction time; and

$S[M+1]$  is the speed measured at node M+1 at time t.

The number of vehicles on link M at time t can be computed as

$$x[M, t] = x[M, t - 1] + (F[M, t - 1] - F[M + 1, t - 1] + R[M, t - 1] + R[M + 1, t - 1]) * t_{interval} \quad (4)$$

where,

$t_{interval}$  is the length of the time intervals;

$x[M, t - 1]$  is the number of vehicle on link M at time t-1;

$F[M, t-1]$  and  $F[M+1, t-1]$  are the flow rates measured at node M and M+1 at time t-1, respectively; and

$R[M, t-1]$  and  $R[M+1, t-1]$  are the ramp flow rates measured at node M and node M+1 at time t-1, respectively. The on-ramp flow rates are positive. The off-ramp flow rates are negative.

Assuming that the increment of vehicles during the period adds to the queue, the number of vehicles in the queue (or queue size) can be computed as

$$Q[M, t] = (F[M, t] - F[M + 1, t] + R[M, t] + R[M + 1, t]) * t_1 + x[M, t] \quad (5)$$

where,

$t_1$  is the free flow travel time on link M.

The queue length is

$$L_Q = Q[M, t] * (L_V + Space[M, t]) \quad (6)$$

where,

$L_V$  is the average vehicle length.

The deceleration distance can be calculated, for vehicles entering link M at the speed of  $S[M]$  and needing to decelerate before joining the slow moving traffic traveling at the speed of  $S[M+1]$ .

$$D_s = \frac{S^2[M, t] - S^2[M+1, t]}{2a} \quad (7)$$

where,

$a$  is the deceleration rate.

The sum of free flow travel distance, deceleration distance, and queue length equals the length of link M; that is,

$$D[M] = D_s + S[M, t] * t_1 + L_Q \quad (8)$$

The free flow travel time  $t_1$  can be solved for as follows:

$$t_1 = \frac{D[M]-D_S-x[M,t]*(L_V+Space[M,t])}{(F[M,t]-F[M+1,t]+R[M,t]+R[M+1,t])*(L_V+Space[M,t])+S[M,t]} \quad (9)$$

As a result, the travel time of link M at time t can be calculated:

$$T_{c2}[M, t] = t_1 + \frac{L_Q}{S[M+1,t]} + \frac{S[M,t]-S[M+1,t]}{a} \quad (10)$$

The third condition is when the breakdown occurs at bottleneck M+2 at time t. Under this condition, if the queue spills back onto link M, the travel time of link M would be impacted by the breakdown; otherwise, the travel time of link M can be estimated in the same fashion as when no breakdown occurs.

Similar to the second condition, the average spacing between two vehicles in the platoon, number of vehicles, queue size, and deceleration distance on link M+1 can be derived by changing M and M+1 in Eq. 3 to Eq. 7 to M+1 and M+2, respectively. Therefore, the following situations are taken into consideration.

When the queue length is longer than the length of link M+1, the travel time is calculated as follows:

$$T_{c3}[M, t] = \frac{D[M]+D[M+1]-L_Q-D_S}{S[M,t]} + \frac{L_Q-D[M+1]}{S[M+2,t]} + \frac{S[M,t]-S[M+2,t]}{a} \quad (11)$$

When the queue length is shorter than the distance of link M+1, but the queue length plus deceleration distance is longer than the distance of link M+1, the travel time can be calculated as follows:

$$T_{c3}[M, t] = \frac{D[M]+D[M+1]-L_Q-D_S}{S[M,t]} + \frac{S[M,t]-S[M+2,t]}{a} * \frac{D_S+L_Q-D[M+1]}{D_S} \quad (12)$$

If the sum of the queue length and deceleration distance are shorter than the distance of link M+1 (i.e., the breakdown at bottleneck M+2 has no impact on travel time on link M), the travel time estimation method for link M is same as the method described under the first condition.

Furthermore, empirical studies have documented that flow breakdown does not necessarily occur at the same prevailing flow level, and thus pre-breakdown flow rate (i.e., the flow rate observed immediately before traffic breaks down) has been treated as a random variable in order to model the probabilistic nature of traffic breakdown (Brilon et al., 2005; Dong and Mahmassani, 2009). This results in a probability of breakdown occurring at a given flow (demand) level. The probability distribution function of the pre-breakdown flow rates has been calibrated to follow the Weibull distribution based on data samples from freeway sections in California, USA (Dong and Mahmassani, 2009b; Kim et al., 2010) and Germany (Brilon et al., 2005). The pre-breakdown flow distribution function expresses the probability that traffic breaks down in the next time interval (for a given time discretization).

$$P[M, t] = 1 - e^{-\left(\frac{F[M,t]}{\sigma}\right)^s} \quad (13)$$

where,

$P[M,t]$  is the pre-breakdown probability at node M at time t;  
 $s$  is the shape parameter,  $\sigma$  is the scale parameter; and  
 $F[M,t]$  is the flow rate measured at node M at time t.

Thus, the expected travel time of link M is

$$T_E[M, t] = [(1 - P[M, t])(1 - P[M + 1, t]) + (1 - P[M, t])P[M + 1, t]] * T_{c1}[M, t] + P[M, t](1 - P[M + 1, t]) * T_{c2}[M, t] + P[M, t]P[M + 1, t] * T_{c3}[M, t] \quad (14)$$

where,

$P[M, t]$  and  $P[M+1, t]$  are the pre-breakdown probabilities at nodes M and M+1 at time t, respectively.

Consequently, the vehicle that departs from node M at time t would arrive at node M+1 at time  $t+T_E[M]$ . The travel time estimation procedure presented above is repeated to estimate travel time on link M+1 using measurements collected at time  $t+T_E[M]$ . The corridor-level travel time from bottleneck 1 to bottleneck N can be calculated as the sum of the time-dependent link travel times:

$$T_{corridor} = \sum_{i=1}^N T_E[i] \quad (15)$$

The proposed model detects different spillback conditions and uses the queue length and deceleration distance to calculate the delay at the bottleneck with queue spillback. However, there is a limitation of the proposed model. If the breakdown occurs between two sensors and the queue does not propagate to a sensor located upstream of the bottleneck, the model would not be able to detect the breakdown.

In order to evaluate the performance of the proposed model, the travel time estimation method proposed by Vanajakshi et al. (2009) is compared with the proposed method. In Vanajakshi et al. (2009) the travel time is calculated as follows:

$$T_E[M, t] = \begin{cases} \frac{D[M] [K[M, t-1] + K[M, t]]}{2} \frac{F[M+1, t]}{2 * D[M]} & F[M + 1, t] > 500 \text{ veh/hr/ln} \\ \frac{2 * D[M]}{S[M, t] + S[M+1, t]} & \text{otherwise} \end{cases} \quad (16)$$

where,

$K[M, t-1]$  and  $K[M, t]$  are the density measured at node M at time t-1 and t, respectively.

In addition, a naïve approach is also tested to estimate link travel time solely based on the point measurement of speeds, that is, using Eq. 2 to calculate link travel time. The corridor travel time is simply the summation of the link travel times.

### 3.3 Travel Time Distribution

Four statistical distributions are considered to fit the data, as shown in Table 2.

Based on the travel time distribution, various reliability measures can be derived, including the standard deviation of travel times, 95th percentile travel times, buffer time index, and planning time index. The planning time index is defined as the ratio of the 95th percentile travel time to the free flow travel time. The buffer time index is the ratio of buffer time (i.e., the difference between 95th percentile travel time and the average travel time) to average travel time.

## 4. RESULTS

In this section, the proposed methodology is applied to estimate the travel time of part of I-235, as shown in Figure 6. This 6-lane freeway section (3 lanes in each direction) is one of the busiest freeways in West Des Moines, Iowa, USA. The locations of roadway sensors are shown in Figure 6. All the sensors are

located in the merging/diverging areas, where the sensors can collect data from the ramps and the main road.

#### 4.1 Travel Time Calculation

The model-based travel time (MTT), Vanajakshi et al. (2009) travel time, naïve-approach based travel time, and INRIX travel time (INRIX-TT), represented as travel time index, are plotted in Figure 7. Since congestion generally occurred during the morning peak on weekdays at the study site, the travel times are estimated for each 1-minute interval from 7:00 a.m. to 9:30 a.m. with one month of data from April 2014. Figure 5 compares the time-dependent travel times estimated by different methods on an example day. It shows that the model-based travel time index estimation followed the pattern of the INRIX travel time index well, at both the link and corridor levels. The naïve-approach and Vanajakshi et al. (2009) model, however, underestimate the delay in terms of congestion duration and severity. Similar patterns are observed for other days as well.

To show the spread of the breakdown, the speed contour during the congested period, from 7:20 a.m. to 8:30 a.m., is plotted in Figure 8. It can be seen that the speed drops started at sensor 3 and propagated to sensor 1. At sensor 4 the traffic is free flowing.

Performance measures, including mean square error (MSE) and mean absolute percentage error (MAPE), are calculated based on the one-month data, as follows.

$$MSE = \frac{\sum(\text{estimated}-\text{actual})^2}{\text{number of observations}} \quad (17)$$

$$MAPE = \frac{\sum \frac{|\text{estimated}-\text{actual}|}{\text{actual}}}{\text{number of observations}} * 100\% \quad (18)$$

Table 3 compares the values of the performance measures of all the methods at both the link and corridor levels. As it can be seen, the proposed method outperforms other methods.

Additionally, Table 4 shows the impact of data aggregation on the performance of the proposed model. When the aggregation level increases, the error of the proposed model increases. The differences in errors of three methods become less noticeable at larger aggregation levels. For example, with 1-minute aggregation level data, the proposed model is significantly better than the other two; with 5-minute aggregation level data, the proposed model performs similarly to the Vanajakshi et al. (2009) model.

#### 4.2 Travel Time Distribution

The maximum likelihood estimation is used to fit the distributions. To evaluate the goodness of fit, the log-likelihood value of each distribution is summarized in Table 5. Since the lognormal distribution has the largest log-likelihood value, it was selected as the best distribution to fit the travel time data.

The weekday data for the peak 15-minute travel times (7:45 a.m. to 8:00 a.m.) from December 1, 2013, to December 1, 2014, were used to estimate the travel time distribution. After removing the outliers, the correlation between link 1 and link 2 of the INRIX data and model-based travel time is 0.83 and 0.97, respectively. The proposed MTT method slightly overestimated the correlation. The travel time distributions are shown in Figure 9. The MTT distribution captured the tendency of the INRIX travel time distribution well.

Figure 10 plots the cumulative distribution functions of the lognormal distributions estimated based on the

INRIX travel time and model-based travel time.

Table 6 compares the travel time reliability indices of model-based travel time estimates and INRIX travel times. At corridor level, all MTT reliability indices are within 10% error range, compared to the ones calculated based on INRIX travel times. At the link level, although the means and standard deviations of MTTs are close to those of the INRIX travel times, the 95th percentile travel time, planning time index, buffer time, and buffer time index show fairly significant discrepancies, up to 70%. The MTT method overestimates reliability indices in most of the cases.

The errors in the proposed travel time estimation model could be attributed to several factors. First, the first-in, first-out assumption does not take lane change behavior into consideration in the calculation. As a result, the number of vehicles approaching the bottleneck might be underestimated or overestimated by the model. Second, missing values from the radar sensor data might also cause errors in estimating travel times, thus causing errors in computing travel time reliability indices.

## 5. CONCLUSIONS

This paper proposed a travel time estimation model that considers the spatially correlated traffic conditions. Link- and corridor-level travel time distributions were estimated using probe vehicle data and roadside radar sensor data. Corridor-level travel time reliability measures were extracted from the travel time distributions. Compared to the probe vehicle data from INRIX, the proposed travel time estimation model captured the patterns of travel time and its distribution well. Moreover, the inconsistency of the missing data of INRIX and sensor data can cause the travel time reliability be overestimated/underestimated by the MTT method.

The proposed model provides a method to assess corridor-level travel time and its distribution using the point measurements collected from the side-fired radar sensors. In future research, the impacts of lane-changing behavior and the temporal correlation on the travel time can be incorporated into the model. Moreover, it is desirable to examine and consider the distinct car-following behavior of passenger cars and heavy vehicles in the travel time estimation model.

## REFERENCES

- Aksoy, G., & Celikoglu, H. B. (2012). Modeling and distribution of link travel times: an analysis on Istanbul freeways. *Procedia-Social and Behavioral Sciences*, 54, 117–128.
- Bertini, R. L., and Malik, S. (2004). Observed dynamic traffic features on freeway section with merges and diverges. *Transportation Research Record: Journal of the Transportation Research Board*, 1867(1), 25–35.
- Bovy, P. H., and Thijs, R. (Eds.). (2000). Estimators of travel time for road networks: New developments, evaluation results, and applications. Delft, Netherland: Delft University Press.
- Brilon, W., Geistefeldt, J., & Regler, M. (2005). Reliability of freeway traffic flow: a stochastic concept of capacity. *In Proceedings of the 16th International symposium on transportation and traffic theory*. Maryland: College Park. 19-21 July 2005

- Castillo, E., Calviño, A., Nogal, M., & Lo, H. K. (2014). On the probabilistic and physical consistency of traffic random variables and models. *Computer-Aided Civil and Infrastructure Engineering*, 29(7), 496–517.
- Celikoglu, H. B. (2007). A dynamic network loading process with explicit delay modelling. *Transportation Research Part C: Emerging Technologies*, 15(5), 279–299.
- Celikoglu, H. B. (2013a). Reconstructing freeway travel times with a simplified network flow model alternating the adopted fundamental diagram. *European Journal of Operational Research*, 228(2), 457–466.
- Celikoglu, H. B. (2013b). Flow-based freeway travel-time estimation: A comparative evaluation within dynamic path loading. *Intelligent Transportation Systems, IEEE Transactions on*, 14(2), 772–781.
- Chase Jr., R. T., Williams, B. M., Roupail, N. M., and Kim, S. (2012). Comparing reported travel times between temporally-stitched and simultaneous methods at various aggregation intervals. In *Transportation Research Board 91st Annual Meeting* (No. 12-0194). Washington D.C., January 22-26, 2012
- Chen, C., Skabardonis, A., and Varaiya, P. (2003). Travel-time reliability as a measure of service. *Transportation Research Record: Journal of the Transportation Research Board*, 1855(1), 74–79.
- Coifman, B. (2002). Estimating travel times and vehicle trajectories on freeways using dual loop detectors. *Transportation Research Part A: Policy and Practice*, 36(4), 351–364.
- Daganzo, C. F. (1995). Properties of link travel times under dynamic load. *Transportation Research Part B: Methodological*, 29(2), 95–98.
- Deniz, O., Aksoy, G., and Celikoglu, H. B. (2013). Analyzing freeway travel times within a case study: Reliability of route traversal times. *Intelligent Transportation Systems (ITSC), 2013 16th International IEEE Conference on* (pp. 195–202). IEEE. October 6-9, 2013
- Dong, J., and Mahmassani, H. S. (2009). Flow breakdown and travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board*, 2124(1), 203–212.
- Dong, J., Lu, C., and Mahmassani, H. S. (2006). How reliable is this route? Predictive travel time and reliability, for anticipatory traveler information systems. *Transportation Research Record: Journal of the Transportation Research Board*, 1980, 117–125.
- Emam, E. B., and Al-Deek, H. (2006). Using real-life dual-loop detector data to develop new methodology for estimating freeway travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board*, 1959(1), 140–150.
- Higatani, A., Kitazawa, T., Tanabe, J., Suga, Y., Sekhar, R. and Asakura, Y. (2009) Empirical analysis of travel time reliability measures in Hanshin expressway network. *Journal of Intelligent Transportation Systems: Technology, Planning, and Operations*, 13(1), 28–38.
- INRIX, “INRIX Interface Guide,” 2014. [Online]. Available: [http://trafficjam.to/downloads/INRIX\\_Dev\\_Guide.pdf](http://trafficjam.to/downloads/INRIX_Dev_Guide.pdf) [Accessed 2015]

- Kesting, A., and Treiber, M. (2008). Calculating travel times from reconstructed spatiotemporal traffic data. In *Proceedings of the 4th International Symposium Networks for Mobility, Stuttgart*. September 25 - 26, 2008
- Kim, J., Mahmassani, H. S., and Dong, J. (2010). Likelihood and duration of flow breakdown. *Transportation Research Record: Journal of the Transportation Research Board*, 2188(1), 19–28.
- Kwon, J., Barkley, T., Hranac, R., Petty, K., and Compin, N. (2011). Decomposition of travel time reliability into various sources. *Transportation Research Record: Journal of the Transportation Research Board*, 2229, 2011, 28–33.
- Lei, H., Zhou, X., List, G., and Taylor, J. (2013). Predicting corridor-level travel time distributions based on stochastic flow and capacity variations. Draft report. Washington, D.C.: Strategic Highway Research Program 2.
- Li, R., Rose, G., and Sarvi, M. (2006). Using automatic vehicle identification data to gain insight into travel time variability and its causes. *Transportation Research Record: Journal of the Transportation Research Board*, 1945(1), 24–32.
- Liu, H. X., and Danczyk, A. (2009). Optimal sensor locations for freeway bottleneck identification. *Computer-Aided Civil and Infrastructure Engineering*, 24(8), 535–550.
- Mori, U., Mendiburu, A., Álvarez, M., and Lozano, J. A. (2015). A review of travel time estimation and forecasting for Advanced Traveller Information Systems. *Transportmetrica A: Transport Science*, 11(2), 119–157.
- Newell, G. F. (1999). Delays caused by a queue at a freeway exit ramp. *Transportation Research Part B: Methodological*, 33(5), 337–350.
- Nie, X., and Zhang, H. M. (2005). A comparative study of some macroscopic link models used in dynamic traffic assignment. *Networks and Spatial Economics*, 5(1), 89–115.
- Oh, J.S., and Chung, Y. (2006). Calculation of freeway travel time variability from loop detector data. *Transportation Research Record: Journal of Transportation Research*, 1945, 12–23.
- Park, D., and Rilett, L. R. (1999). Forecasting freeway link travel times with a multilayer feedforward neural network. *Computer-Aided Civil and Infrastructure Engineering*, 14(5), 357–367.
- Richardson, A. J. (2003). Travel time variability on an urban freeway. TUTI Report 25-2003. Presented at the 25th Conference of Australian Institutes of Transport Research (CAITR), University of Adelaide. Adelaide, Australia. 3-5 December 2003
- Soriguera, F., and Robusté, F. (2011a). Highway travel time accurate measurement and short-term prediction using multiple data sources. *Transportmetrica*, 7(1), 85–109.
- Soriguera, F., and Robusté, F. (2011b). Requiem for freeway travel time estimation methods based on blind speed interpolations between point measurements. *Intelligent Transportation Systems, IEEE Transactions on*, 12(1), 291–297.

- Susilawati, S., Taylor, M. A., and Somenahalli, S. V. (2010). Travel time reliability measurement for selected corridors in the Adelaide Metropolitan area. *Journal of the Eastern Asia Society for Transportation Studies*, 8, 86–102.
- Tam, M. L., and Lam, W. H. (2008). Using automatic vehicle identification data for travel time estimation in Hong-Kong. *Transportmetrica A: Transport Science*. 4 (3): 179–194.
- Tu, H., Van Lint, J. W., and Van Zuylen, H. J. (2007). Impact of traffic flow on travel time variability of freeway corridors. *Transportation Research Record: Journal of the Transportation Research Board*, 1993(1), 59–66.
- Turochy, R. E., and Smith, B. L. (2002). Measuring variability in traffic conditions by using archived traffic data. *Transportation Research Record: Journal of the Transportation Research Board*, 1804(1), 168–172.
- Van Arem, B., Van Der Vlist, M. J., Muste, M. R., and Smulders, S. A. (1997). Travel time estimation in the GERDIEN project. *International Journal of Forecasting*, 13(1), 73–85.
- Van Lint, J. W. C., and Van Der Zijpp, N. J. (2003). An Improved Travel-time Estimation Algorithm using Dual Loop Detectors. *Transportation Research Record: Journal of the Transportation Research Board*, 1855, 41–48.
- Van Lint, J. W., and Van Zuylen, H. J. (2005). Monitoring and predicting freeway travel time reliability: using width and skew of day-to-day travel time distribution. *Transportation Research Record: Journal of the Transportation Research Board*, 1917(1), 54–62.
- Vanajakshi, L. D. (2005). Estimation and prediction of travel time from loop detector data for intelligent transportation systems applications (Doctoral dissertation, Texas A&M University).
- Vanajakshi, L. D., Williams, B. M., and Rilett, L. R. (2009). Improved flow-based travel time estimation method from point detector data for freeways. *Journal of Transportation Engineering*, 135(1), 26–36.
- Vlahogianni, E. I., Karlaftis, M. G., and Golias, J. C. (2014). Short-term traffic forecasting: Where we are and where we're going. *Transportation Research Part C: Emerging Technologies*, 43, 3–19.
- Zhang, W. (2006). Freeway travel time estimation based on spot speed measurements (Doctoral dissertation, Virginia Polytechnic Institute and State University).
- Zou, Y., Zhu, X., Zhang, Y., and Zeng, X. (2014). A space–time diurnal method for short-term freeway travel time prediction. *Transportation Research Part C: Emerging Technologies*, 43, 33–49.