ACOUSTIC WAVES IN A PIEZOELECTRIC PLATE IMMERSED IN A CONDUCTIVE FLUID

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INTRODUCTION

Leaky Lamb Wave (LLW) has drawn interest from the research society for its application in the nondestructive evaluation of plate-like structures. Starting from isotropic plate[1] to plates with anisotropic materials such as fiber-reinforced composites[2], the LLW problem increase complexity in a very fast manner. With the dielectric effect included in the LLW problem, a piezoelectric plate immersed in a dielectric fluid can be extended from the framework setup by the NDE LLW study. Among these studies, Nayfeh et. al.[3] investigated the influence of piezoelectricity on Lamb wave propagation. However only the mechanical effect of the fluid is treated in their study, leaving the dielectric effect with two extreme assumptions: either short-circuit or open-circuit condition. Later, Yang and Chimenti[4] modeled the piezoelectric LLW problem with fluid dielectric effect included and conducted extensive experiments[5]. The influence of the fluid conductivity on the wave propagation in a piezoelectric plate is the major interest in the current research. Theoretical modeling as well as experimental measurements are presented in this report.

THEORY

Using the method of partial wave analysis [4] we modeled the reflection coefficient of a plane wave incident on a piezoelectric plate loaded by a dielectric fluid. The reflection coefficient can be written as:

$$R = \frac{AS - YY'}{(A + iY)(S - iY')} \qquad (1)$$

$$A = A_{OP} - \varepsilon_f A_{SH} \qquad (2a)$$

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$$S = S_{OP} + \varepsilon_f S_{SH} \qquad\qquad (2b)$$

$$Y = Y_{OP} - \varepsilon_f Y_{SH} \qquad\qquad (2c)$$

$$Y' = Y'_{OP} - \varepsilon_f Y'_{SH} . \qquad\qquad (2d)$$

Here A, S, Y, and Y' are functions of mechanical and electrical properties of the piezoelectric plate and mechanical properties of the fluid. A_{OP}, S_{OP}, Y_{OP}, and Y'_{OP} are the open-circuit solutions; while A_{SH}, S_{SH}, Y_{SH}, and Y'_{SH} are the short-circuit solutions. As for the case of dielectric fluid loading, the dielectric constant of the fluid, ε_f, plays as a multiplier combining the effect of short-circuit and open-circuit effect total as the dielectric effect.

This work for the investigation of acoustic waves propagating in a piezoelectric plate immersed in dielectric fluid[4] can be extended to the case of dielectric/conductive fluid case. We treat the dielectric/conductive fluid as a lossy dielectric fluid by modeling the conductivity, σ, as the imaginary part of the dielectric constant of the fluid. i.e.

$$\varepsilon_f = \varepsilon_L + i\frac{\sigma}{\omega} . \qquad\qquad (3)$$

Here ε_f is the complex dielectric constant of the fluid, σ is the conductivity, and ω is the angular velocity of the oscillation field.

EXPERIMENT

Figure 1 is the block diagram showing the measurement configuration for LLW of a piezoelectric plate·immersed in a conductive fluid. Shown in the bottom of Figure 1, the piezoelectric plate is placed in a reservoir holding about 4 liter of conductive fluid. The reservoir is fastened at a rotation table, with which the azimuthal angle is measured. During the experiments, the temperature is monitored with a thermometer, while the conductivity is simultaneously monitored by a conductivity meter.

The piezoelectric sample is a piece of X-cut Lithium Niobate (LiNbO$_3$) plate. The thickness is 1.043 mm. The X-axis of the crystal is positioned upward, with θ measuring the incident (as well as the reflected) angle of the ultrasonic beam. The azimutal angle is described by Φ, with 0 degree parallel to -Z and 90 degree parallel to -Y crystal directions. Conductive fluid with various levels of conductivity is prepared by adding NaCl into deironized water. In this research, we have fluid samples of conductivity of 8 μS/cm (pure deironized water), 180 μS/cm, 420 μS/cm, 610 μS/cm, 810 μS/cm, 1010 μS/cm, and 1200 μS/cm.

The transducers are positioned by a stepper motor-driven X-Y-Z-θ position system. The step size of θ is 0.225 degree. The transducers locate about 50 mm above the piezoelectric plate. According to [6], the horizontal spacing between the two transducers is adjusted such that the center line of the transmitting and receiving transducers intercept the top surface of the piezoelectric plate at a spacing equal to the radius transducer.

Frequency scanned tone-burst ultrasonic signals simulating quasi CW plane waves are launched to the sample to obtain the LLW spectra of the piezoelectric plate. The frequency of the tone-burst signal start from 1 MHz, at a step size of 20 Khz, to 8 MHz. The output voltage of HP3314A of 2 volt peak-to-peak is directly used to drive the PANAMETRIC V302 wide-band transducer of 10 mm diameter. Another transducer of the same type as the transmitter is used to detect the reflected signal, which combines specular and leaky components. The signal is then detected by a PANAMETRIC 5800PR receiver. The peak-to-peak amplitude of the detected signal is directly measured with the LeCroy 9310A digital oscilloscope, which is connected to personal computer through a IEEE 488 interface.
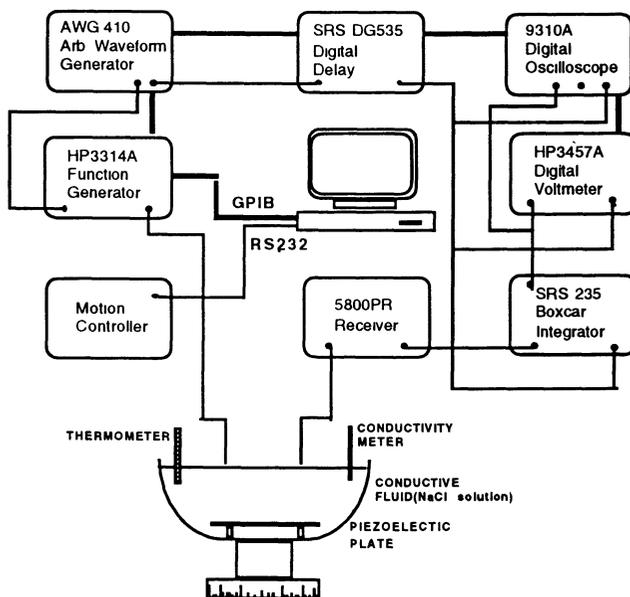

Figure 1: Schematic of the experimental setup

RESULTS AND DISCUSSIONS

Theoretical calculations on reflection coefficients based on equation (1) to (3) are presented in Figure 2 for X-cut LiNbO$_3$, Φ=45°, immersed in NaCl solutions of various conductivity. In a very natural way, the image format of the plane wave reflection coefficient displays the dispersion relations including the fluid effect. The horizontal axis represents fd in a unit of MHz-mm from 0.0 to 9.0; while the vertical axis is phase velocity in a unit of Km/sec from 2.0 to 10.0. The four images correspond to different conductivity of 8, 420, 610, and 1200 μS/cm.

First of all, the image format representation of dispersion relation has other feature beyond the capability of the traditional line-representation. For example, immediately from the images in Figure 2, we observe that some modes, like S$_0$ and A$_1$, are also "losing focus" in the
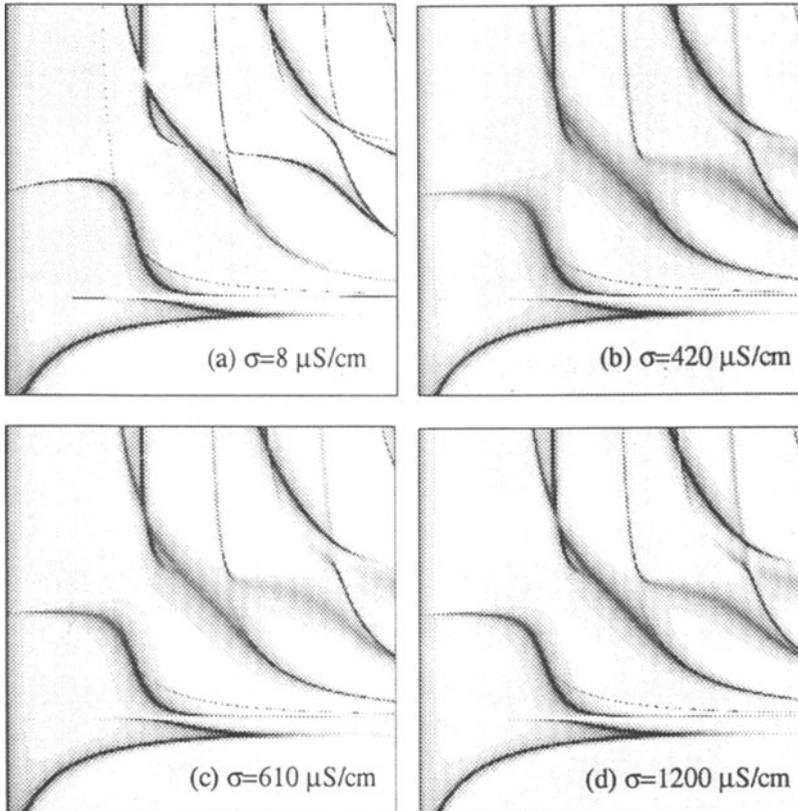

Figure 2: Results of theory showing reflection coefficients of X-cut LiNbO$_3$, Φ=45°, immersed in NaCl solutions.

dispersion space. This is because of that the conductivity of the fluid acting as lossy media is pulling these modes away from the real fd axis towards the complex fd space with increasing imaginary component.

Our major interest lies on the shift of the reflection minima influenced by the fluid conductivity. This can be roughly observed from Figure 2 for the low frequency part of the S_0 mode, but not very clear because the frequency shift is small. The influence of fluid conductivity on reflection minima can be better demonstrated if we subtract image 2(a) with image 2(b) to construct a differential image. Shown in Figure 3, the white lines in correspond to the

modes(reflection minima) with σ=8 μS/cm, while the black

lines correspond to σ=420 μS/cm.

The background gray scale represents equal reflection coefficient of 1.0. As shown in Figure 3, we do see significant shift of reflection minima for S_0, A_1, and some other higher order modes. The common rule for the shift of any mode is that the conductivity always tend to move the mode toward lower frequency with regard to all the shifted modes. But for the A_0 mode, there is virtually no shift due to the effect of fluid conductivity.

A quantitative way to characterize the frequency shift is to plot fd of the modes against values of conductivity. Figure 4 and 5 show the locations of frequency minima as a
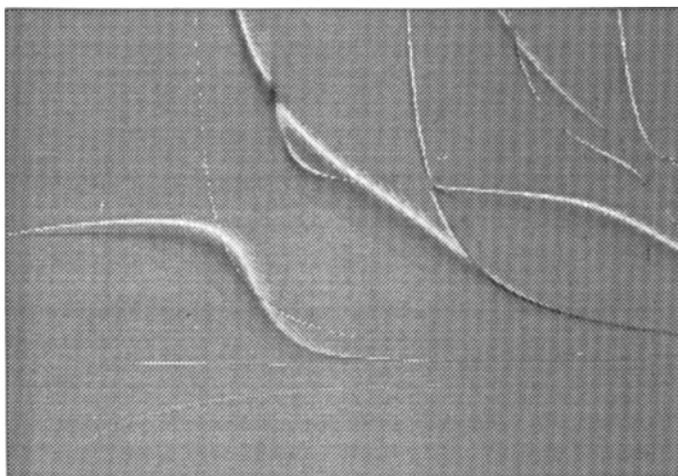

Figure 3: Image showing the shifting of the reflection minima caused by conductivity.

function of conductivity for both theoretical predictions and experimental measurements Figure 5 is the *fd* minima corresponding to A_1 mode for an X-cut $LiNbO_3$ plate of an azimuthal angle of $\Phi=45°$, and the incident angle, θ, of 13°. Figure 5 is the minima corresponding to S_0 mode for the incident angle θ of 16°.

For both Figure 4 and 5, we observe that the experiment follows the same trend of frequency shifting in reflection minima. As the conductivity increases, the frequency minima shifts down with a decreasing velocity and tends to saturate with the decreasing. Secondly, for both Figure 4 and 5, at the conductivity of 8 $\mu S/cm$, the experimentally measured minima are higher than the theoretical predictions. This can due to slightly misalignment for the zero of θ. But the discrepancy is very small.

For both Figure 4 and 5, the measured frequency is always lower than the predictions (except for the case of pure deironized water), i.e. theory always underestimates the frequency shift in all cases. As a matter of fact the underestimation of the theory is reasonable. Since the conductivity for the theoretical calculations is based upon the conductivity measurements using the conductivity meter, which measures static conductivity. However, in the
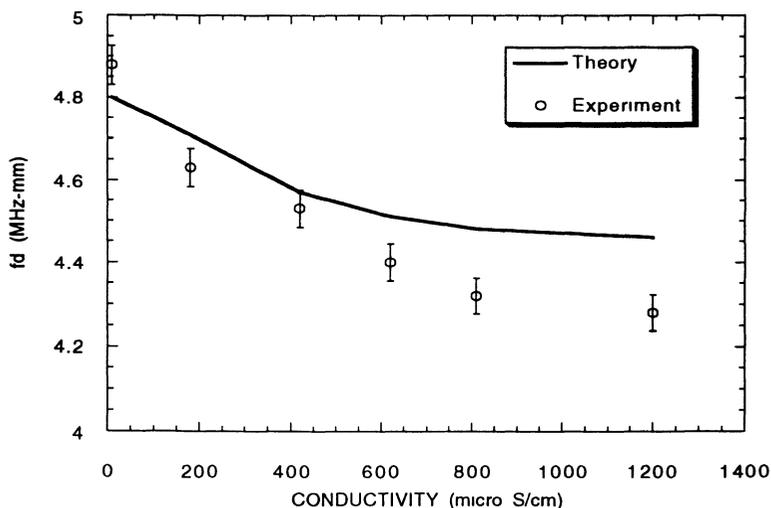

Figure 4: Comparison of theoretical predicted frequency minima and the experimental results for X-cut $LiNbO_3$, $\Phi=45°$, $\theta=13°$, A_1 mode.

experiments, the acoustic field is never static, but with high frequency AC field of several Mhz. Conductivity at high frequency is usually higher than that at static case[7]. And also the dielectric constants are also different between static case and the case at high frequency.

In the point of view as a sensor to probe the conductivity of a fluid, the sensitivity of the LLW system is very good. The solution with the conductivity of 420 μS/cm is 0.0004 M, which is only 0.023 gm of sodium in one liter of deironized water. For the application of conductivity sensor, apparently the sensitivity is very high.

CONCLUSIONS

This research include both theoretical modeling and experimental measurements for the acoustic wave propagation in piezoelectric plate influenced by the conductivity of the surrounding fluid. Using an image-based scheme for the display of dispersion relation, we observe not only the expected phenomenon in frequency shift, but also we see the change of mode topology. Our experimental results with the measurements of frequency shift due to the fluid conductivity demonstrates the same trend as the theory. There is minor discrepancy between the theory and experiment. The reason for the discrepancy is that the theoretical prediction is based
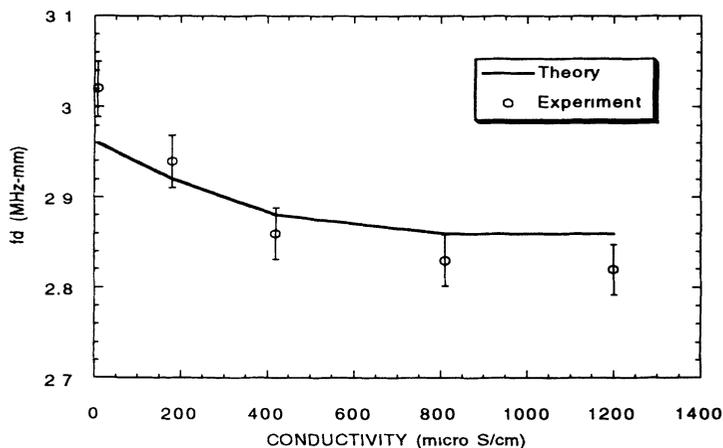

Figure 5: Comparison of theoretical predicted frequency minima and the experimental results for X-cut LiNbO$_3$, Φ=45°, θ=16°, S$_0$ mode.

on the static conductivity of the fluid, while the fluid conductivity in the experiments is not static, but with significantly high frequency. And also, dielectric constants at high frequency are also different from those at static condition. At last, the piezoelectric X-cut $LiNbO_3$ demonstrates very good sensitivity for the application as a conductivity sensor.

ACKNOWLEDGMENT

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