CALCULATION OF WIDEBAND ULTRASONIC FIELDS RADIATED BY WATER-COUPLED TRANSDUCERS INTO ANISOTROPIC MEDIA

Nicolas Gengembre, Alain Lhémery and Pierre Calmon
Commissariat à l'Énergie Atomique, CEREM,
CEA-Saclay Bat. 611, 91191 Gif-sur-Yvette *cedex*, France

INTRODUCTION

In nuclear power plants, welded components can be tested by ultrasonic non destructive means. Such media have peculiar elastic properties, being anisotropic because of the grain growth in the thermal directions during the solidification process, and being heterogeneous due to the succession of welding layers constituted of disoriented grains. Both properties modify the field radiated by a water (or solid wedge) coupled transducer into this medium, by deviating, distorting and scattering it. In the aim to assist settings and interpretations of ultrasonic tests in these media, the French Atomic Energy Commission (CEA) extends its model for the field radiated by arbitrary immersed transducers *Champ-Sons* [1] to such cases. This paper presents the derivation of the theoretical model for predicting the radiation into an arbitrary anisotropic homogeneous medium. The extension is derived by an asymptotic expansion nearby stationary phase points of an integral expressing the refracted angular spectrum (geometrical optics (GO) approximation). Such a method has proved to ensure both computing efficiency and accuracy, when applied to the case of isotropic medium [2]. Specific developments are required in the neighborhood of caustics (directions of higher intensity), for which the asymptotic expansion is performed to a higher order. Since NDT transducers are mostly wideband, an analytical Fourier transformation allows one to express the field in terms of a time dependent impulse-response.

ANGULAR SPECTRUM FORMULATION OF THE DISPLACEMENT FIELD

In order to treat arbitrary transducers, the source surface is divided into elementary ones (point sources). The integration over the whole surface is then performed numerically by summing each elementary contribution. Therefore, we will concentrate onto the field radiated by a point source in what follows. This surface integral is the Rayleigh equation [1], whose discretisation gives the elementary field $\Delta\varphi(\mathbf{M},\omega)$ (expressed in terms of velocity potential) generated by an elementary surface ΔS at a point $\mathbf{M}$ in the coupling medium, considered as a fluid. One has

$$\overline{\Delta\varphi}(\mathbf{M},\omega)=\frac{e^{jk_1 PM}}{2\pi PM}\overline{V_0}(\mathbf{P},\omega)\Delta S, \qquad (1)$$

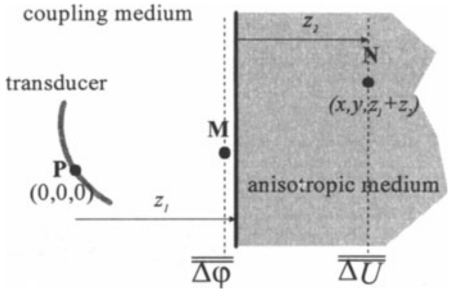

Figure 1. Configuration considered for our developments. The z-axis is the normal to the interface. **P** is the source in the fluid medium. **N** is the point in the anisotropic medium (in gray) where the field is calculated. Vertical dotted lines represent the planes where the different angular spectra are calculated.

where **P** is the point source (as shown in figure 1), k_1 the wave number in the fluid, and $V_0(\mathbf{P},\omega)$ the normal velocity on the radiating surface. The upper line denotes a time Fourier transform, according that we first take into account one frequency component ω of the wideband signal.

Then we can express one component of the angular spectrum, by means of Weyl's representation [3], allowing us to drop the Fourier double integration over x and y (the axes are parallel to the interface). That yields

$$\overline{\overline{\Delta\varphi}}(k_x,k_y,\omega) = j\frac{e^{jk_{1z}z_1}}{k_{1z}}\overline{V_0}(\mathbf{P},\omega)\Delta S, \tag{2}$$

where the double line notifies the additional spatial Fourier transform.

Equation (2) represents an harmonic plane wave in the fluid, which propagation up to the point **N** can easily be formulated. The transmission through the interface is taken into account by keeping the k_x and k_y components of the wave vector (according to Snell-Descartes' law) and multiplying by the harmonic plane wave refraction coefficient. We have chosen coefficients linking the velocity potential in the fluid to the displacement in the anisotropic solid for writing convenience. Moreover, propagation in a single medium leads to multiply the spectrum component by the propagator $e^{jk_z z}$, z being the distance covered along the z-axis. In the anisotropic medium, the dependency of k_z on k_x and k_y (*i.e.* the slowness surface in cartesian coordinates) is given by Christoffel's equation :

$$\left|\rho_0\omega^2\delta_{ik} - C_{ijkl}k_j k_l\right| = 0, \tag{3}$$

where ρ_0 is the density, C_{ijkl} the stiffness constants, and δ_{ik} Kronecker's symbol. The Einstein summation convention for repeated subscripts is followed.

Finally, one can write the formulation of the elementary field at point **N** by applying the inverse Fourier transform upon the variables k_x and k_y, in order to sum every plane wave contributions. In order to separate spatial and frequential dependencies, the wave vector is replaced with the corresponding slowness vector $\mathbf{s} = \mathbf{k} / \omega$. We get

$$\overline{\Delta U}(\mathbf{N}) = \frac{j\omega \overline{V}_0 \Delta S}{4\pi^2} \int\int_{-\infty}^{+\infty} \frac{\overline{\overline{T^{\alpha}_{\varphi \to U}}}}{s_{1z}} e^{j\omega\left(s_x x + s_y y + s_{1z} z_1 + s_{2z} z_2\right)} ds_x ds_y \, , \tag{4}$$

where $\overline{\overline{T^{\alpha}_{\varphi \to U}}}$ is the transmission coefficient as defined previously, α denotes the kind of wave selected (quasi-longitudinal, slow quasi-transverse or fast quasi-transverse wave).

This exact solution for the transmission problem from a single point source involves a double infinite integral and is therefore not acceptable for the computer performances expected.

STATIONARY PHASE APPROXIMATION (GEOMETRICAL OPTICS)

If one assumes that ω is not too small, the integrand of the double infinite integral (Eq. (4)) is rapidly oscillating. Hence, this integral can be evaluated by means of the stationary phase method (SPM) approximation [4], consisting in considering that only stationary phase points (SPP) and their neighborhood make significant contribution to the elementary displacement field. In a fluid / isotropic solid configuration [1], a SPP corresponds to one plane wave following a path between the source and the calculation point defined by Fermat's principle (the GO path). In the configuration we deal with, the paths of stationary phase are obtained by solving the following system of equations

$$\begin{cases} z_1 \dfrac{V^e_{1x}}{V^e_{1z}} + z_2 \dfrac{V^e_{2x}}{V^e_{2z}} = x \\ z_1 \dfrac{V^e_{1y}}{V^e_{1z}} + z_2 \dfrac{V^e_{2y}}{V^e_{2z}} = y \end{cases} , \tag{5}$$

where $\mathbf{V}^e_i$ is the energy velocity in the medium i. This set of equations is permitted since the energy velocity vector is normal to the slowness surface, thus parallel to the vector

$$\mathbf{n}^e = \begin{pmatrix} -\partial s_{iz} / \partial s_x \\ -\partial s_{iz} / \partial s_y \\ 1 \end{pmatrix} . \tag{6}$$

Some algebraic transformations of Eq. (5) show that the GO ray between two points in two arbitrary media follows the energy path (see Fig. 2). Let us notice here that, for anisotropic media, this ray is not in the incident plane in general, so that a 3D modeling is always required.

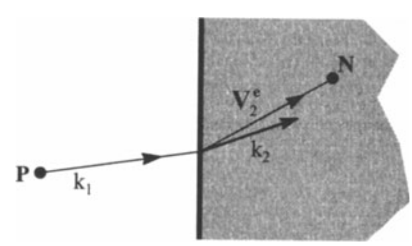

Figure 2. The geometrical optics ray for fluid / anisotropic solids configuration.

Second Order Expansion in the Vicinity of SPP

The approximation of double integrals such as Eq. (4) is based on a Taylor development of the phase function (say f) at the SPP to the second order ; less precision is required for the module function, hence it is only developed to the first order. The method, given in [4], has been applied to our case, leading to

$$\overline{\Delta U}(\mathbf{N}) = \frac{j\overline{V_0}\Delta S\,\mathrm{sgn}(\omega)\overline{\overline{T^{\alpha}_{\varphi\to u}}}e^{j\sigma\,\mathrm{sgn}(\omega)\pi/4}}{2\pi|H|^{1/2}s_{1z0}}e^{j\omega f(s_{x0},s_{y0})}, \tag{7}$$

where the 0 subscript stands for a value evaluated at SPP. H is the so called Hessian, or local gaussian curvature of f, evaluated at SPP, and defined as follows :

$$H = \frac{\partial^2 f}{\partial s_x{}^2}\frac{\partial^2 f}{\partial s_y{}^2} - \left(\frac{\partial^2 f}{\partial s_x \partial s_y}\right)^2 . \tag{8}$$

The σ variable describes the shape of f nearby the SPP, equaling -2 for convex shape, +2 for concave shape, and 0 for saddle shape. Note that our phase function

$$f(s_x, s_y) = s_x x + s_y y + s_{1z} z_1 + s_{2z} z_2 \tag{9}$$

depends on the slowness surfaces. Thus, although in the isotropic case f is always convex, it is generally not true in anisotropic media. Another difference in this latter case is that more than one SPP can coexist. When these SPP are separated, the field can be approximated for the sum of each SPP contribution. When two SPP tend to coalesce, a second order development is no longer valid, because one cannot assume anymore that the phase is a quadric. The coalescence of two SPP happens at an inflexion point, where H=0, implying the divergence of Eq. (7). The energy velocity vector being normal to the slowness surface, this occurs at cuspidal edges of the wavefront, as shown Fig. 3. The points $\mathbf{N}(x, y, z_1 + z_2)$ leading to these properties are on caustics [5]. The actual field near caustics must be calculated using specific developments, which are presented in the next paragraph.

Taking account of the neighborhood of SPP to the second order means that we calculate the divergence of the pencil rounding the GO ray, as a function of the local curvature of the wavefront.

Third Order Expansion Nearby Caustics

Special attention has been taken in caustics, because they are zones of high intensity [5]. An approximation of single integrals with a third order rapidly varying phase can be found in the literature [4]. Two methods are available : a transitional one and a uniform one. We based our extension to double integrals on the uniform method for two main reasons. First, it ensures continuity with the second order method when the two SPP go far one from the other. Second, it requires the same values as those required for the second order approximation, that are related to pencils (the Hessians), so that it will ensure computation ease. Let us write the integral to be calculated in the form

$$I = \iint g(s_x, s_y) e^{j\omega f(s_x, s_y)} ds_x ds_y \,. \tag{10}$$

We assume that the phase function contains two neighboring SPP, implying they are one extremum and one saddle point (according that the proximity of two extrema would require at least another close SPP [6]). Values taken at these points will be denoted by a (1) or (2) superscript, point (1) corresponding to the lowest phase. Then we can define a change of variables as

$$f(s_x, s_y) = h(u,v) = A - Bu + \frac{1}{3}u^3 + \mu v^2, \tag{11}$$

the shape of the phase being well represented by the right side of this equation.

No approximation is made by this change of variables, but there will appear an unknown Jacobian in the integral. Nevertheless, we are able to express this Jacobian as a function of the values of f and its second derivatives at SPP. In the same way, A and B can be found by evaluating Eq. (10) at SPP. Depending on whether we have a minimum and a saddle point, or a saddle point and a maximum, μ equals +1 or -1 respectively. One has

$$A = \frac{[f^{(1)} + f^{(2)}]}{2} \quad \text{and} \quad B = \left[\frac{3\left(f^{(2)} - f^{(1)}\right)}{4}\right]^{2/3}. \tag{12}$$

After some algebraic manipulations, it is shown that at SPP (i), the Jacobian is expressed as

$$(J^{(i)})^2 = \frac{H_h^{(i)}}{H_f^{(i)}}, \tag{13}$$

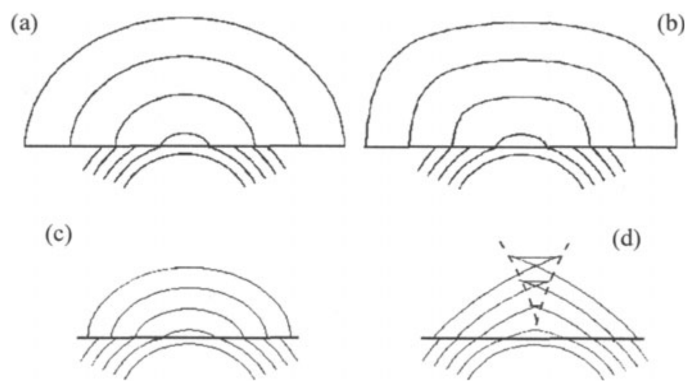

Figure 3. Wavefronts at various times generated by a point source in a fluid (lower) medium into a solid (upper) medium. (a) & (c) : isotropic solid. (b) & (d) : anisotropic solid. (a) & (b) : Longitudinal or quasi-longitudinal wave. (c) & (d) : Shear or quasi-shear waves. The dotted lines correspond to caustics.

where $H_h^{(i)} = -(-1)^i \mu B^{1/2}$ is the Hessian of function h at SPP (i). The next steps of the developments are similar to those given in [4], using the integral formulation of the Airy function Ai to evaluate the integral with respect to u. The integral with respect to v is performed by means of the second order approximation. Defining

$$\zeta = \frac{g^{(1)}B^{1/4}}{\left|H_f^{(1)}\right|^{1/2}} + \frac{g^{(2)}B^{1/4}}{\left|H_f^{(2)}\right|^{1/2}} \quad , \quad \xi = \frac{g^{(1)}B^{-1/4}}{\left|H_f^{(1)}\right|^{1/2}} - \frac{g^{(2)}B^{-1/4}}{\left|H_f^{(2)}\right|^{1/2}} \quad , \tag{14}$$

the expression of the displacement field for a point source and a pulsation ω becomes :

$$\overline{\Delta U}(\mathbf{N}) = \frac{\overline{V_0}\Delta S}{2\sqrt{2\pi}} e^{j\omega A}\left(j\,\mathrm{sgn}(\omega) - \mu\right)\left[\zeta|\omega|^{1/6}\,\mathrm{Ai}\left(-B\omega^{2/3}\right) - j\xi|\omega|^{-1/6}\,\mathrm{Ai}'\left(-B\omega^{2/3}\right)\right]. \tag{15}$$

One can demonstrate that this formula tends to the formula found with our model at separated SPP, by means of an asymptotic development of the Airy function Ai [4]. It appears that the third order approximation becomes different from the second order one (with separated SPP) for small values of $-B\omega^{2/3}$. In other words, the time difference between the arrival of both contributions (on which B depends) has to be compared with the period of the signal. As a consequence, for wideband signals, a filtering of the lowest frequencies will occur close to caustics.

Such a model takes into account the third order shape of the phase surface. Nevertheless, it still diverges when the point **N** is exactly on the caustic. Although our computational algorithm will probably never have to treat such a case, let us notice that it can be solved by means of the transitional method described in [4].

Degenerated Shear Waves

In arbitrary anisotropic media, two shear waves generally coexist. Nevertheless, there may be some directions in which they degenerate (corresponding to intersections of their slowness surfaces). There, the polarization (*i.e.* displacement direction) has an infinity of possible direction, each implying a different direction of energy propagation. One can demonstrate that the intersection of the two slowness sheets are mostly points, specially in orthotropic media, which interest us at most (austenitic steel). At those points, the resulting intensity is low, as far as the slowness surface possesses great variations [5]. However, when this intersection is a line (as for example in transversely isotropic media), we can separate the sheets into two smooth ones, and apply our model separately to each of them. Then both fields may be added up whenever linear assumption holds.

IMPULSE-RESPONSE METHOD : TRANSIENT FIELDS

The wide use of broadband transducers in ultrasonic NDT leads us to develop a time-dependent model in terms of impulse responses. Hence we have to perform a time Fourier transformation of the previous field formulas. When the second order approximation is made, the only frequency dependencies are that of $\overline{V_0}(\mathbf{P})$ and a sgn(ω) coefficient. The radiated signal at point **N** will be composed of $\overline{V_0}(\mathbf{P})$ and its Hilbert's transform. As emphasized by [1] in the isotropic case, a pulse distortion due to Hilbert's transform appears when one or more evanescent waves are generated at the interface, because in that case, the transmission coefficient owns a non zero phase proportional to sgn(ω). In the anisotropic case, the same phenomenon will happen when evanescent waves are created too, but also when the SPP is saddle shaped. Indeed, if we write

$$\overline{\overline{T^{\alpha}_{\varphi\to U}}} = Te^{j\theta\,\mathrm{sgn}(\omega)} \quad , \tag{16}$$

where T is the modulus and $\theta\,\mathrm{sgn}(\omega)$ is the phase of the transmission coefficient, the elementary transient displacement field, calculated by means of a Fourier transformation of Eq. (7), is given by

$$\Delta U(\mathbf{N}) = \frac{T\Delta S}{\left|H_f\right|^{1/2} s_{1z0}}\delta\left(t - \frac{r_1}{V_1^{\varphi}} - \frac{r_2}{V_2^{e}}\right) \otimes \left[\cos\left(\theta + \frac{\sigma\pi}{4}\right)\mathrm{TH}\{V_0(t)\} - \sin\left(\theta + \frac{\sigma\pi}{4}\right)V_0(t)\right], \tag{17}$$

where $\delta(t)$ is the Dirac pulse, r_i the length of the energy path in medium i, and $\mathrm{TH}\{V_0(t)\}$ the Hilbert's transform of the input signal.

In the vicinity of caustics, the calculation of Fourier transform of Eq. (15) can be derived with the help of generalized functions theory, specially their procedure of derivation at infinite discontinuities, and of some formulas involving Bessel and hypergeometric functions. For lack of place, the derivation is not given here, but may be found in [7]. It leads to the following results. Defining $a = 2B^{3/2}/3$ and

$$u_p(\tau) = -(-1)^p\sqrt{3\pi}\,[\delta(\tau-1)+\delta(\tau+1)] + \begin{cases} \bullet\, 2\sqrt{2}\left(\dfrac{\Gamma\left(\frac{9+2p}{12}\right)}{\Gamma\left(\frac{3+2p}{12}\right)} - (-1)^p\dfrac{\Gamma\left(\frac{9-2p}{12}\right)}{\Gamma\left(\frac{3-2p}{12}\right)}\right) {}_2F_1\left(\dfrac{9-2p}{12}, \dfrac{9+2p}{12}; \dfrac{1}{2}; \tau^2\right) & \text{for } |\tau| < 1 \\[2ex] \bullet -\dfrac{\sqrt{2\pi}}{6}\left[\dfrac{(3+2p)\Gamma\left(\frac{9+2p}{12}\right)}{\Gamma\left(\frac{3+p}{3}\right)\Gamma\left(\frac{9-2p}{12}\right)}|\tau|^{-\frac{9+2p}{6}} {}_2F_1\left(\dfrac{9+2p}{12}, \dfrac{15+2p}{12}; \dfrac{3+p}{3}; \tau^{-2}\right) \right. & \\ \left. \qquad + \dfrac{\Gamma\left(\frac{9-2p}{12}\right)}{\Gamma\left(\frac{3-p}{3}\right)\Gamma\left(\frac{9+2p}{12}\right)}|\tau|^{-\frac{9-2p}{6}} {}_2F_1\left(\dfrac{9-2p}{12}, \dfrac{15-2p}{12}; \dfrac{3-p}{3}; \tau^{-2}\right)\right] & \text{for } |\tau| > 1 \end{cases} , \tag{18}$$

where Γ stands for the Euler's Gamma function, p=1,2 and ${}_2F_1(x,y;z;t)$ is the hypergeometric function (or Gauss series) [8], we have :

$$\Delta U(t) = \frac{\Delta S\delta(t-A)}{4a\sqrt{3\pi}} \otimes \left[\mathrm{TH}\{V_0(t)\} \otimes \left(\zeta B^{-1/4}u_1(t/a) + \mu\xi B^{1/4}u_2(t/a)\right) + V_0(t) \otimes \left(\xi B^{1/4}u_2(t/a) - \mu\zeta B^{-1/4}u_1(t/a)\right)\right] . \tag{19}$$

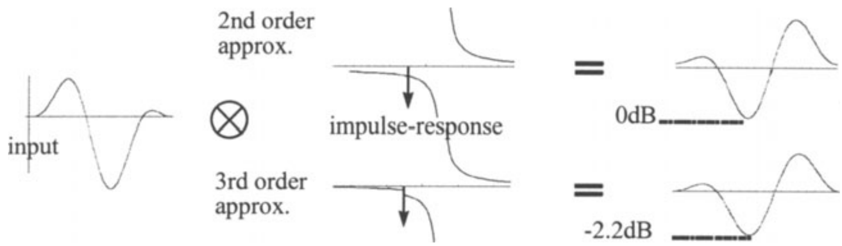

Figure 4. Comparison between the second and the third order approximations nearby caustics, in terms of impulse-responses and after a convolution with a synthetic signal. The main difference consists of a lower amplitude predicted by the third order approximation.

An illustration of this result is given in Fig. 4, and compared with the second order approximation in the same configuration, that is the neighborhood of a caustic.

CONCLUSION

In this paper, we have presented the theoretical derivation of a model for computing the field radiated by a water or solid wedge coupled arbitrary transducer into anisotropic media. It constitutes an extension of the existing model and software *Champs-Sons* developed at CEA which until now dealt with isotropic media. Because we want a computationally efficient and accurate model, the geometrical optics approximation (pencil method) has been used, which is equivalent to the stationary phase method applied to angular spectrum. Specific developments have been made in order to treat the case of caustic by means of a higher order expansion (third instead of second order) nearby stationary phase points. An impulse response formulation has been established, since a transient model is more appropriate to wideband transducer simulation.

ACKNOWLEDGEMENTS

This work is supported by *Électricité de France* and *Framatome*.

REFERENCES

1. M. El Amrani, P. Calmon, O. Roy, D. Royer and O. Casula, in *Review of Progress in QNDE*, Vol. 14, eds. D.O. Thompson and D.E. Chimenti (Plenum, New York, 1995), p. 1075.
2. P. Calmon, A. Lhémery and J. Nadal, in *Review of Progress in QNDE*, Vol. 15, *op. cit.* (1996), p. 1019.
3. A. Walther, *The ray and wave theory of lenses*, (Cambridge University Press, Cambridge, 1995).
4. J.J. Stamnes, *Waves in focal region*, Chap. VIII & IX, (A. Hilger, Boston, 1986).
5. G.A. Northrop and J.P.Wolfe, Phys. Rev. B-22, 6196, (1980).
6. P.R. Prentice, Proc. Roy. Soc. Lond. A-446, 341, (1994).
7. N. Gengembre, A. Lhémery, P. Calmon, in preparation for the J. Acoust. Soc. Am.
8. M. Abramowitz and I.A. Stegun, *Handbook of mathematical functions*, Chap. X & XV, (Dover, New York, 1970).