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Srinivasan, R., Somani, Arun

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On the Blocking Performance of Tree Establishment in Time-Space Switched Optical Networks

R. Srinivasan and Arun K. Somani
Dependable Computing & Networking Laboratory
Department of Electrical and Computer Engineering
Iowa State University, Ames, IA 50011
E-Mail: {rsrini, arun}@iastate.edu

ABSTRACT

Multicasting in the optical layer has gained significant importance in the recent years due to several factors. Most of the research work in this area concentrate either on minimizing the number of wavelengths required to meet a given static demand or on multicast route selection algorithms to achieve efficient utilization of fiber bandwidth. Very few significant research has been found, to the best of authors' knowledge, on developing an analytical model for evaluating the blocking performance of tree establishment in optical networks, which motivates this research.

In this paper, an analytical model for evaluating the blocking performance of multicast tree establishment in time-space switched optical networks is developed. The performance of different switch architectures are then studied using the analytical model. It is observed that if the multicast tree has very low degree of branching, the blocking probability of establishing the tree is the same as that of establishing a path with same number of links.

Keywords: Optical networks, WDM-TDM switching, Multicast, Blocking performance

1. INTRODUCTION

Wavelength Division Multiplexing (WDM) has emerged as an efficient mechanism for information transport in all-optical networks. WDM divides the available fiber bandwidth into a set of wavelengths (WDM channels). Early research in optical networks focussed on single-fiber multi-wavelength wavelength-routed networks. Nodes in these networks can switch wavelengths across ports. Wavelength Converter (WC) is a device that allows the optical signal on a wavelength to be converted into another wavelength. If wavelength converters are not available, a call arriving at a node on a certain wavelength has to be switched to the same wavelength at the output. Although wavelength converters improve network blocking performance, the high cost of wavelength converters have made it impractical to employ full-wavelength conversion at all nodes. The role of wavelength converters in wavelength-routed networks has been studied extensively in the literature.[1–4] Subramaniam, Azizoğlu, and Somani[5] evaluate the impact of sparse-wavelength conversion, where only a few nodes in the network have full-wavelength conversion capability. The effect of limited-wavelength conversion,[6,7] where a given input wavelength can be converted into a set of (but not all) output wavelengths, has also been studied. Multi-fiber multi-wavelength wavelength-routed networks have been shown to offer blocking performance similar to that of networks employing limited- or sparse-wavelength conversion.[8–10]

WDM offers bandwidth granularity close to the peak electronic transmission speed. The bandwidth of a single wavelength is too large for certain traffic requirements. While some traffic may have a requirement of fractional wavelength, another traffic that is already using a full-wavelength might want to expand its capacity, but not want to pay for an entire new wavelength. This motivates the need for providing fractional wavelength capacity to the network traffic.

Provisioning of fractional wavelength capacity is achieved by dividing a wavelength into time slots and multiplexing traffic on the wavelength. The resulting multiwavelength optical time division multiplexed networks (WDM-TDM networks) can be classified into two categories[11]: dedicated-wavelength TDM (DW-TDM) networks and shared-wavelength TDM (SW-TDM) networks. In DW-TDM networks, each source-destination pair is connected by a *lightpath*, where a lightpath is defined as an all-optical connection between two nodes. Calls between a source and destination are multiplexed on the lightpaths. If the bandwidth required by a new call at a node is not available on any of the existing lightpaths to the destination, a new lightpath is established. On the other hand, in SW-TDM networks, if a call cannot be accommodated on an existing lightpath to the

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destination, it is allowed to be multiplexed onto an existing lightpath to an intermediate node. The call is then switched from the intermediate node to the final destination either directly or through other nodes. However, if none of the existing lightpaths from the node can accommodate the call, a new lightpath to the destination is established.

The performance of SW-TDM networks depend on efficient merging of fractional wavelength requirements of the nodes into a full- or almost-full wavelength requirement. This merging of traffic from different source-destination pairs is called *traffic grooming*. Nodes that can groom traffic are capable of multiplexing/de-multiplexing lower rate traffic onto a wavelength and switching them from one lightpath to another. The grooming of traffic can be either static, where the source-destination pairs whose requirements are combined are pre-determined, or dynamic, where the connection requests from different source-destination pairs are combined dynamically depending on the existing lightpaths.

Recent advances optical switching technology[12–14] have shown the possibility of realizing fast all-optical switches with switching time less than a nanosecond. The use of such fast switches along with fiber delay lines as time-slot interchangers[15,16] have opened up the possibility to realize multiwavelength optical time switched networks. These networks will be referred to as *WDM-TDM Switched* networks in the rest of this paper. Connection between a source and destination in a WDM-TDM switched network is realized by assigning a time slot on every link of a chosen path, with the constraint that the slot on one link can be switched to the successive link by the intermediate node. WDM-TDM switched network can be considered as a special case of SW-TDM network, where all the nodes in network are capable of grooming traffic and lightpaths to the neighboring nodes are established permanently. The bandwidth granularity offered by a WDM-TDM network is determined by the duration of a time slot which, in turn, depends on the speed at which the switching can be accomplished. In general, a WDM-TDM switched network is a multi-fiber, multi-wavelength, TDM-switched all-optical network.

Routing individual slot dynamically requires processing information in optical domain. However, the technology in optical processing and storage has not matured to achieve run-time routing decisions at high-speeds. Therefore, WDM-TDM switched networks are expected to be *circuit-switched* in nature. As the information in a time slot is not read by an intermediate node at run-time, the switching employed here is also referred to as *transparent optical switching*.

Yates et al.[11] analyze a single-fiber multi-wavelength TDM-switched network for blocking performance by extending the link-independence model[3,4] proposed for wavelength-routed network. Wauters and Deemester[9,10] show the equivalence of single-fiber multi-wavelength TDM-switched network to multi-fiber multi-wavelength wavelength-routed networks.

Srinivasan and Somani[17] introduce a generalized network model, called Trunk Switched Network (TSN). In this model, every node views a link as a set of trunks and channels. Analytical model for evaluating path blocking probabilities has also been developed for a class of TSN's where all the nodes have similar view of the links. The analytical models proposed earlier in literature for wavelength-routed optical networks, WDM-TDM networks, etc. can be derived from this generalized framework.

1.1. Multicasting in Optical Networks

Supporting multicast connections in WDM networks has gained importance in recent years due to the increasing number of distributive services. Sahasrabuddhe and Mukherjee[18] discuss the benefits of supporting multicast traffic at the WDM layer. Yang, Wang, and Qiao[19] introduce various multicast models along with methods to implement different multicast capable switch architectures.

The analytical models proposed thus far in literature mostly consider unicast connections and evaluate path blocking probabilities. Earlier work on the analysis of multicasting in the optical domain concentrate on two main areas: (1) minimizing the number of wavelengths required to support a static traffic demand[18,20] and (2) multicast route selection algorithms to achieve efficient utilization of the fiber bandwidth when dynamic setup and tear down of multicast traffic is considered.[21,22] Iannone, Listanti, and Sabella[23] study the blocking performance of establishing multicast trees with a source reaching the destinations in a two-link path with a branching after the first link by assuming statistical independence of link loads. To the best of authors' knowledge, there has been no other significant work that analyzes the blocking performance for establishing a multicast tree in an optical network.

In this paper, an analytical model for evaluating the blocking performance of establishing a multicast tree in time-space switched optical networks is developed based on the analytical model proposed for homogeneous TSN's.[17] The paper is organized as follows: Section 2 describes a WDM-TDM switched network. Section 3 introduces the concept of a trunk-switched network, the modeling of a WDM-TDM switched network as a TSN, and *intra-trunk copying* which enables copying of a signal from one channel to another. An analytical model for evaluating the blocking performance of tree establishment in a class of TSN's is developed in Section 4. The performance of different switch architectures are studied using the analytical model on two different kinds of networks. Section 5 discusses the performance results. Section 6 concludes the paper.

2. WDM-TDM SWITCHED NETWORKS

A WDM-TDM switched network consists of switching nodes interconnected by one or more optical fibers. Each fiber carries a certain number of wavelengths. Each wavelength is divided into frames which are further sub-divided into time slots. Let L denote the number of links at a node, F denote the number of fibers per link, W denote the number of wavelengths per fiber, and T denote the number of time slots per frame on a wavelength.

Every slot within a frame can be denoted by a 4-tuple, (l, f, w, t), where $1 \leq l \leq L$, $1 \leq f \leq F$, $1 \leq w \leq W$, and $1 \leq t \leq T$. For example, the tuple $(1, 1, 2, 1)$ (read from right to left) denotes first time slot in a frame on the second wavelength of the first fiber on the first link. A *channel* on a link is defined as a collection of a particular time slot across successive frames. Hence, the number of channels in a link is the same as the number of slots in a frame, $F \times W \times T$. Each channel is also represented by a 4-tuple, (l, f, w, t), similar to the representation of a slot. It can be observed that if a frame has only one time slot, $T = 1$, a WDM-TDM switched network reduces to a multi-fiber multi-wavelength wavelength-routed network. A switch at a node maps an input channel to an output channel. The constraints on the mapping of an input channel to an output channel are determined by the nature of the switch.

The simplest switch architecture is a space switch. In this switch, an input channel, $(l, f, w, t)_i$, could be mapped to an output channel, $(l, f, w, t)_o$, if and only if $t_i = t_o$, $w_i = w_o$, and $f_i = f_o$. With a time slot interchanger (TSI), an input channel $(l, f, w, t)_i$ could be can be mapped to an output channel $(l, f, w, t)_o$, where $t_i \neq t_o$, by delaying the signals. A combination of time and space switching can be employed in multiple stages to realize more permutation of space and time. If the switches do not employ wavelength conversion, then the wavelength of the input and output channels must be the same, hence $w_i = w_o$. In networks with multiple fibers connecting two nodes an input channel, $(l, f, w, t)_i$, can be mapped to an output channel $(l, f, w, t)_o$, where $f_i \neq f_o$.

3. TRUNK SWITCHED NETWORKS

A trunk switched network (TSN) consists of switching nodes interconnected by links. Each link has a set of channels. The number of channels in a link, denoted by C, is the same on all the links in the network. A node in a TSN views a link as a set of K trunks with S channels per trunk, where $KS = C$. Fig. 1 shows the node architecture in a TSN. The node has four links connected to it. Each link is viewed as a set of 4 trunks by the node. Switching at every node obeys the following two conditions:

- A full-channel interchanger (FCI) is employed at the input for every trunk, as shown in Fig. 1.
- Switching at a node obeys trunk-continuity constraint, i.e., the channels cannot be switched across trunks.

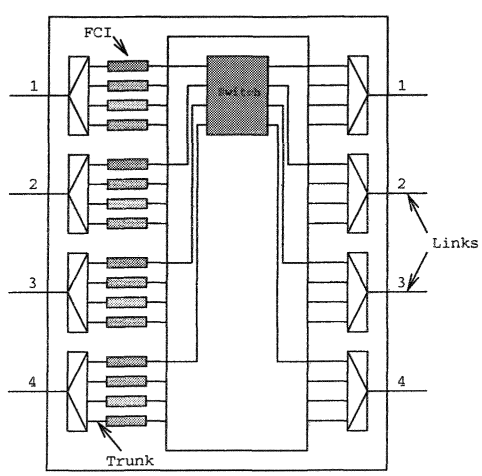


Figure 1. Node architecture in a TSN.

The definition of a trunk could be different across nodes. For example, one node could view a link as K_1 trunks with S_1 channels per trunk while another node could view the link as K_2 trunks with S_2 channels per trunk, where $K_1 S_1 = K_2 S_2 = C$.

Fig. 2 shows two nodes in a TSN connected by a link. The input to switch from other links at the node are not shown in the figure. Fig. 2(a) shows two nodes that view the link as a set of 4 trunks. In Fig. 2(b), the first node views the link as 3 trunks while the second node views the link as 4 trunks.

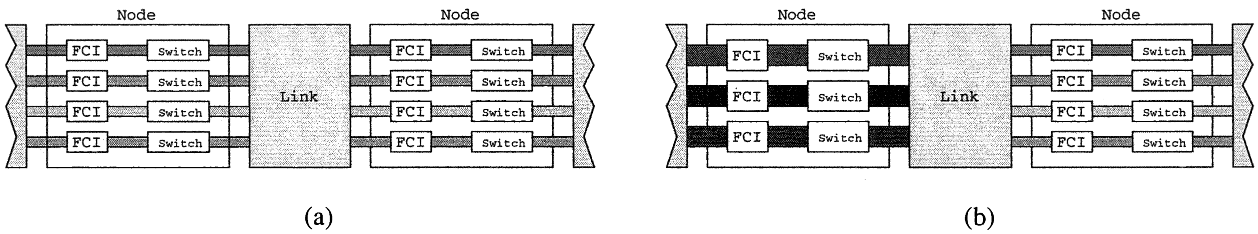


Figure 2. Two nodes connected by a link in a TSN. (a) The link is viewed as 4 trunks by both the nodes. (b) The link is viewed as 3 and 4 trunks by the first and second node, respectively.

A TSN is said to be *homogeneous* if the collection of channels that constitute a trunk at a node is the same for all the nodes in the network. Otherwise, it is said to be *heterogeneous*. The nodes shown in Fig. 2(a) could form a part of homogeneous TSN if the channels that constitute a trunk for the two nodes are the same as that at the rest of the nodes. On the other hand, the nodes shown in Fig. 2(b) form a part of a heterogeneous TSN. Note that two nodes could view a link as a set of K trunks but could still be heterogeneous if the channels within the trunks are not identical. Although the trunk definition is the same for all the nodes in a homogeneous TSN, the switching employed within a trunk could be different at different nodes. As channels cannot be switched across trunks, by the definition of a trunk at the node, a homogeneous TSN imposes an *end-to-end trunk-continuity constraint* on the connections.

A trunk on a link, as viewed by a node, is said to be *busy* if all the channels in the trunk are busy, otherwise it is said to be *free*. Consider the link shown in Fig. 2(a). The link is viewed as a set of 4 trunks by the nodes. The number of channels busy on a trunk at the input of a node is the same as the number of channels busy on the trunk at the input to the switch at the node. However, the distribution of the busy channels on the trunk at the input of the node is different from that at the input of the switch. The number of trunks busy at the input of a node is the same as the number of trunks busy at the input of the switch at that node.

Consider a trunk on a two-link path, eg: a trunk at the input and output of a node shown in Fig. 2(a). The trunk is said to be *available* on the two-link path if there is a free channel in the trunk on the first link that can be switched by the node to a free channel in the second link, subjected to the constraints of the switch. Hence, if a trunk is free on two links individually, it does not necessarily imply that the trunk is available on the two-link path. For example, consider a scenario when the switch at the node shown in Fig. 2(a) is a space-only switch*. Hence channel continuity constraint is imposed by the switch. Also, assume that there are 5 channels per trunk. Let channels 1 and 2 on a trunk be busy at the input of the switch and channels 3, 4, and 5 be busy at the output of the switch. The free channels at the input of the switch (hence, at the node) cannot be switched to the free channels at output of the switch. Hence, the trunk is not available on the two-link path.

A connection between a source and destination is established over a path. Each path consists of a set of links and the number of links in a path denotes the length of the path. The selection of a path in the network could be either static or dynamic. A connection between a source and a destination over a path is realized by assigning a channel on each link on the path such that every node on the path can switch the channel assigned on its input link to the channel assigned on its output link. A call is said to be blocked if such a channel assignment is not possible.

Multicast connections are established in these networks by copying the signal in an input channel and switching the individual copies to multiple output channels, whenever a branching in the tree occurs. It is assumed that the copies of the input signal is constrained to remain within the same trunk. Hence, this copying is also referred to as *intra-trunk* copying. The maximum number of copies that can be made from an input signal is limited by the number of channels within a trunk. The number of copies that are made from an input signal is referred to as *degree of branching*.

*Although the switch at a node is space-only the node behaves like a channel-space switch.

3.1. Modeling a WDM-TDM switched network as a TSN

A WDM-TDM switched network can be modeled as a TSN. Although a trunk can be defined as an arbitrary collection of channels, only a few such collections make a meaningful trunk definition in reality. Some possible trunk definitions at a node are discussed here with an example.

Consider a link with one fiber, 4 wavelengths per fiber and 5 time slots per wavelength ($F = 1, W = 4, T = 5$). Each slot on a link l is denoted by a 4-tuple tuple (l, f, w, t), where $f = 1$, $1 \leq w \leq 4$, and $1 \leq t \leq 5$.

- If time slot interchange and wavelength conversion are not permitted, then, for any link l, each wavelength and time slot combination can be treated as a trunk, i.e., $Tr_{(w,t)} = \{(l, 1, w, t)\}$. In this case, a link is viewed as WT trunks with one channel per trunk.
- If time slot interchange is permitted and wavelength conversion is not, then for a given link l, every wavelength can be considered as a trunk, i.e., $Tr_w = \{(l, 1, w, t) | 1 \leq t \leq T\}$. Thus, a link is viewed as W trunks with T channels per trunk. Note that, the switch at a node need not provide full-permutation switching capability.
- If full-wavelength conversion is permitted and time slot interchange is not, then for a given link l, a time slot on all the wavelengths can be grouped to form a trunk, i.e., $Tr_t = \{(l, 1, w, t) | 1 \leq w \leq W\}$. Thus, a link is viewed as T trunks with W channels per trunk.
- If both full-wavelength conversion and time slot interchange are permitted, then the entire link is treated as one trunk with WT channels.

A multi-fiber multi-wavelength wavelength-routed network with F fibers and W wavelengths with no wavelength conversion can be viewed as W trunks with F channels per trunk. If full-wavelength conversion is available, then a link can be viewed as a single trunk with FW channels. However, networks that employ limited-wavelength conversion[6,7] cannot be modeled easily or effectively as a TSN as full-permutation wavelength-conversion is not employed.

4. ANALYSIS

Consider a homogeneous TSN with K trunks per link and S channels per trunk. The analytical model developed in this section for evaluating the blocking performance of establishing a multicast tree is based on the following assumptions[†]:

- The call arrival at every node follows a Poisson process with rate λ_n and is equally likely to be destined to any other node. The choice of Poisson traffic is to keep the analysis tractable.
- The bandwidth requirement of every call is assumed to be of one channel capacity.
- The holding time of every call follows an exponential distribution with mean $\frac{1}{\mu}$. The Erlang load offered by a node is $\rho_n = \frac{\lambda_n}{\mu}$.
- The path selection is pre-determined (fixed-path routing), eg: shortest-path routing.
- Blocked calls are not re-attempted.
- A call is assigned a channel randomly from a set of available channels.

Consider a tree, denoted by $\mathcal{T}$, that needs to be established in the network. Let $P_{\mathcal{T}}(T_f)$ denote the probability that exactly T_f trunks are available to establish the tree. $P_{\mathcal{T}}(0)$, therefore, denotes the blocking probability of tree establishment. To compute $P_{\mathcal{T}}(T_f)$, assume that T_y trunks are free on the first link and T_x trunks among them are available for establishing the tree. $P_{\mathcal{T}}(T_f)$ can be written as,

$$P_{\mathcal{T}}(T_f) = \sum_{T_x=T_f}^{K} \sum_{T_y=T_x}^{K} P_{\mathcal{T}}(T_f | T_x, T_y) P_1(T_x, T_y) \tag{1}$$

[†]The scope of the analytical model presented in this paper is limited to homogeneous trunk switched networks.

where $P_{\mathcal{T}}(T_f|T_x,T_y)$ denotes the probability of T_f trunks being available to establish the tree given that T_y trunks are free on the first link with T_x among them being available. $P_1(T_x,T_y)$ denotes the probability that T_y trunks are free on the first link with T_x among them being available. $P_1(T_x,T_y)$ can be written as,

$$P_1(T_x,T_y) = \begin{cases} P(T_y) & \text{if } T_x = T_y \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

where $P(T_y)$ denotes the probability of T_y trunks being free on a link. $P_{\mathcal{T}}(T_f|T_x,T_y)$ is computed by considering two cases, (1) the tree does not have any branching and (2) the tree has a branching.

Case 1:
If the tree $\mathcal{T}$ does not have any branching, then it is merely a path. If the path consists of z links, then $P_{\mathcal{T}}(T_f|T_x,T_y)$, denoted as $P_z(T_f|T_x,T_y)$ for a path, can be expressed as:

$$P_z(T_f|T_x,T_y) = \sum_{T_l=T_f}^{K} P_z(T_f,T_l|T_x,T_y) \tag{3}$$

where $P_z(T_f,T_l|T_x,T_y)$ denotes the probability of having T_f trunks available on a z-hop path with T_l trunks free on the last link given that T_y trunks are free on the first link with T_x among them available.

A z-link path is analyzed as a two-hop path by considering the first $z-1$ links as the first hop and last two links as the second hop, as shown in Fig. 3.

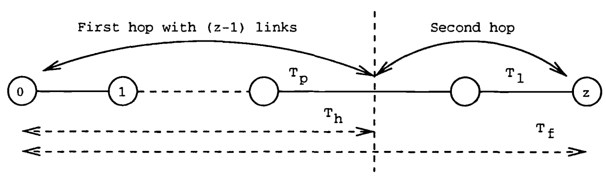


Figure 3. A z-link path model.

Let T_h and T_p denote the number of trunks available on the first hop and number of trunks that are free on the last link of the first hop (link $z-1$), respectively. $P_z(T_f,T_l|T_x,T_y)$ can then be recursively computed as:

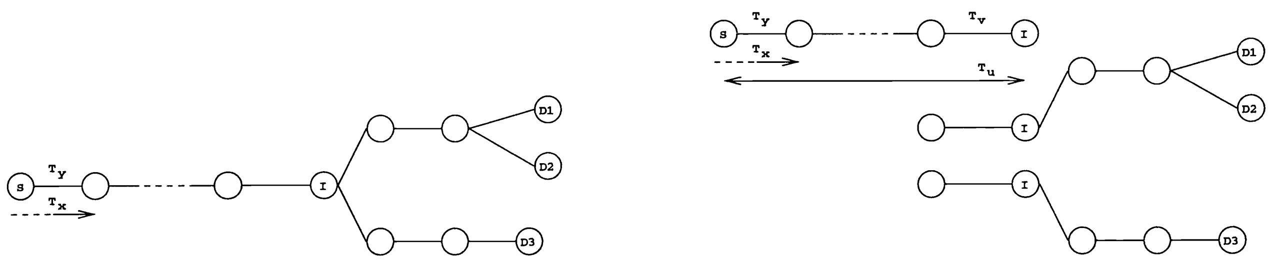
$$P_z(T_f,T_l|T_x,T_y) = \sum_{T_h=T_f}^{K} \sum_{T_p=T_h}^{K} P_{z-1}(T_h,T_p|T_x,T_y)\, P(T_f,T_l|T_h,T_p) \tag{4}$$

where $P(T_f,T_l|T_h,T_p)$ denotes the probability of T_f trunks being available on the z-link path with T_l trunks free on the last link given that T_h trunks are available on the first hop with T_p trunks free on the last link of the first hop. The starting point of the recursion $P_1(T_f,T_l|T_x,T_y)$ is given by:

$$P_1(T_f,T_l|T_x,T_y) = \begin{cases} 1 & \text{if } T_f = T_x \text{ and } T_l = T_y \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Case 2:
If the tree has a branching at an intermediate node, then the computation of the desired probability is carried out by splitting the tree into a combination of a path and subtrees. Consider an example tree as shown in Fig. 4(a) to be established. As the tree has branching, it is split into a path upto the intermediate node I, denoted by $\mathcal{P}$, and a set of subtrees that branch out at the intermediate node. Let s denote the number of subtrees at the branching point and $\mathcal{T}_1, \mathcal{T}_2, ..., \mathcal{T}_s$ denote the subtrees. The splitting of the tree into a path and a set of subtrees are shown in Fig. 4(b). Note that the last link of the path and first link of the subtrees are the same.

Let $P_{\mathcal{P}}(T_u,T_v|T_x,T_y)$ denote the probability that T_u trunks are available to reach the intermediate node with T_v trunks free on the last link of the path given that the first link has T_y free trunks with T_x among them available. This probability is computed as described above in case 1. The probability of T_f trunks being available to establish the given tree can be obtained



(a) (b)

Figure 4. (a) Example multicast tree considered for analysis. (b) Decomposition of the tree into a path and a set of sub-trees for analysis.

by summing over all possible values of T_u, T_v, and number of trunks available to establish paths on each of the s subtrees. Hence, it follows:

$$P_{\mathcal{T}}(T_f|T_x,T_y) = \sum_{T_u=T_f}^{T_x} \sum_{T_v=T_u}^{K} \sum_{(t_1,t_2,...,t_s)} P_{\mathcal{P}}(T_u,T_v|T_x,T_y)\; P(t_1,t_2,...,t_s|T_u,T_v)P(T_f|t_1,t_2,...,t_s,T_u) \tag{6}$$

where $P(t_1,t_2,...,t_s|T_u,T_v)$ denotes the joint probability that subtree i has exactly t_i trunks available given that the first link in the subtree has T_v trunks free with T_u among them available. $P(T_f|t_1,t_2,...,t_s,T_u)$ denotes the probability of T_f trunks being available for establishing the tree given that T_u trunks are available to establish the path and t_i trunks are available to establish subtree $\mathcal{T}_i$. Note that this probability does not depend on the number of free trunks on the first link of the subtrees (T_v) as any trunk that is available to establish a subtree must be within the set of available trunks in the first link of the subtree (T_u). It can be also observed that each subtree should have atleast T_f trunks available, i.e., $t_i \geq T_f$, where $1 \leq i \leq s$.

It is assumed that the distribution of the channels across different subtrees are independent of one another. Similar to link correlation, there is also a correlation factor that is introduced due to the intra-channel copying. Hence, if a channel is occupied in a subtree, then there is a positive probability that a channel in the same trunk can be occupied in another subtree, as they could be part of a tree established earlier. However, as the offered load due to multicast traffic is expected to be much smaller compared to the unicast connections, the effect of this correlation is neglected in this paper. Assuming the channel distribution across the subtrees are independent of each other, $P(t_1,t_2,...,t_s|T_u,T_v)$ can be written as:

$$P(t_1,t_2,...,t_s|T_u,T_v) = \prod_{i=1}^{s} P_{\mathcal{T}_i}(t_i|T_u,T_v). \tag{7}$$

$P_{\mathcal{T}_i}(t_i|T_u,T_v)$ for each subtree $\mathcal{T}_i$ is computed recursively by considering if the subtree has a branching or not until the splitting of the subtrees result in paths.

Let $P_s(T_f|t_1,t_2,...,t_s,T_u,T_r)$ denote the probability of T_f trunks being available to establish the tree given that t_i trunks are available to establish subtree $\mathcal{T}_i$ with $T_u + T_r$ trunks available on the first link of the subtrees with the constraint that a trunk that is available to establish the tree must fall within the T_u set of trunks. It follows that $P_s(T_f|t_1,t_2,...,t_s,T_u)$ is same as $P_s(T_f|t_1,t_2,...,t_s,T_u,0)$. $P_s(T_f|t_1,t_2,...,t_s,T_u,T_r)$ is computed recursively by considering one subtree at a time and updating the number of trunks available to establish the tree depending on the number of available trunks on the subtree that is considered.

$$P_s(T_f|t_1,t_2,...,t_s,T_u,T_r) = \sum_{t=T_f}^{\min(t_1,T_u)} P_1(t|t_1,T_u,T_r)P_{s-1}(T_f|t_2,t_3,...,t_s,t,T_r+T_u-t) \tag{8}$$

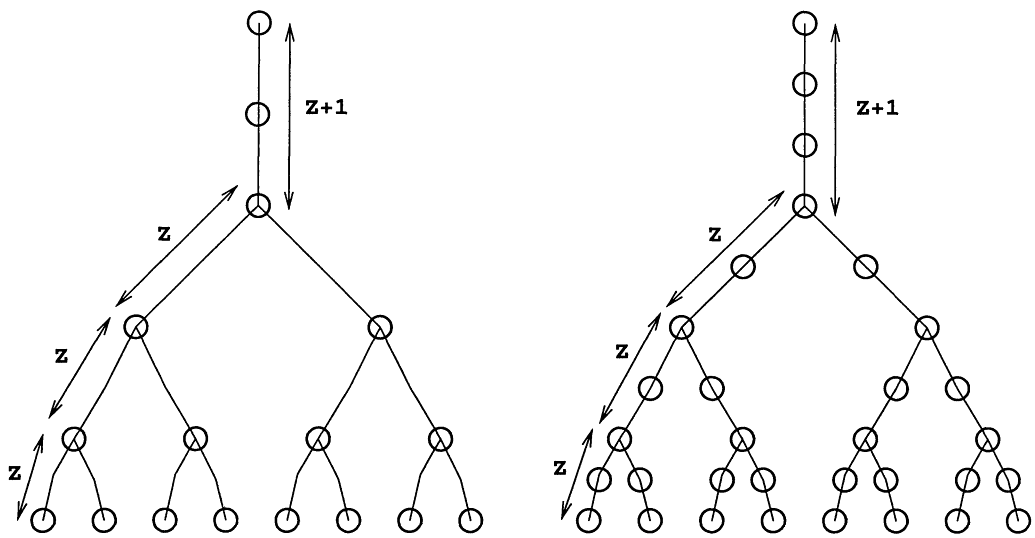
where,

$$P_1(t|t_1,T_u,T_r) = \frac{\binom{T_u}{t}\binom{T_r}{t_1-t}}{\binom{T_u+T_r}{t_1}} \tag{9}$$

$P(T_f)$ and $P(T_f,T_l|T_h,T_p)$ form the basis for the analysis developed in this section. These probabilities are computed using a two-link correlation model. Computing these probabilities are not explained in this paper due to space constraints. Readers are referred to the earlier work by the authors[17] for a detailed description on the computation of the above probabilities.

5. PERFORMANCE EVALUATION

Two regular k-ary tree structures are considered for performance evaluation: (1) binary ($k = 2$) and (2) ternary ($k = 3$). The number of levels in the tree is denoted by L, with the source reaching 2^L destination nodes. The root of the tree is assumed to the source and the leaf nodes are assumed to be the destinations. The number of hops between two successive branching nodes is denoted by Z. The distance between the source and the first branching node is set as $Z + 1$. Fig. 5(a) and (b) show binary trees with the distance between the branching nodes as 1 and 2, respectively. Note that as Z increases, the number of links in the tree increases.



(a) $Z = 1$ (b) $Z = 2$

Figure 5. Example binary trees considered for performance evaluation. (a) Distance between two branching nodes is 1. (b) Distance between two branching nodes is 2.

The blocking performance of multicast tree establishment are evaluated on two networks: (1) 100×100 bi-directional mesh-torus and (2) 12-dimensional hypercube network. The choice of these networks are due to the medium and low correlation of link-loads. Each link in the network is assumed to have 20 channels. Two different trunk and channel combinations are considered: (1) 4 trunks and 5 channels per trunk and (2) 1 trunk and 20 channels per trunk. A node that views a link as K trunks with S channels per trunk is referred to as a $K \times S$ node. A node that employs full-permutation switching is denoted by FP and that which provides channel-space switching is denoted by CS.

Tables 1 and 2 show the blocking performance for establishing 3-level binary trees and 3-level ternary trees, respectively, in a 100×100 bi-directional mesh-torus network. Tables 3 and 4 show the blocking performance versus link load for establishing 2-level binary trees and 2-level ternary trees, respectively, in a 12-dimensional hypercube network. The performance trends observed in both the networks are similar. The blocking probability with 4×5 CS switches is two to four orders or magnitude higher than that with 1×20 FP switches with increasing link load. The blocking probability with 4×5 FP switches is at-most one order of magnitude higher than that with 1×20 FP switches. These results indicate that a significant performance could be obtained even when the channels in a link are split into more trunks and employing full-permutation switching.

It is also observed that the blocking probability of establishing a tree is almost the same as that of establishing a path having the same number of links as in the tree for a given switch architecture at the nodes in the network. The path blocking probabilities are not reported separately as they exactly match the values of the tree blocking performance to the accuracy reported in the tables. The difference between the blocking probabilities of establishing a binary or ternary tree and a path with same number of links was observed to be less than 1% of the path blocking performance. In general, if the degree of branching at each node in the network decreases, the tree blocking performance can be approximated to a path blocking performance with

			Link	load	
Node type	Z	1E	2E	5E	7E
4×5 CS	1	1.4×10^{-16}	6.1×10^{-11}	1.8×10^{-5}	6.7×10^{-2}
	2	8.1×10^{-16}	5.3×10^{-10}	1.7×10^{-4}	2.9×10^{-1}
	3	3.3×10^{-15}	2.3×10^{-09}	6.9×10^{-4}	5.4×10^{-1}
4×5 FP	1	2.8×10^{-18}	1.1×10^{-12}	1.9×10^{-7}	1.4×10^{-3}
	2	5.4×10^{-18}	2.2×10^{-12}	4.4×10^{-7}	5.4×10^{-3}
	3	8.1×10^{-18}	3.4×10^{-12}	8.2×10^{-7}	1.4×10^{-2}
1×20 FP	1	2.4×10^{-18}	9.3×10^{-13}	1.3×10^{-7}	4.8×10^{-4}
	2	4.7×10^{-18}	1.8×10^{-12}	2.6×10^{-7}	9.2×10^{-4}
	3	7.0×10^{-18}	2.7×10^{-12}	3.8×10^{-7}	1.4×10^{-3}

Table 1. Blocking probability versus link load for establishing a 3-level binary tree in a 100×100 bi-directional mesh-torus network.

			Link	load	
Node type	Z	1E	2E	5E	7E
4×5 CS	1	2.2×10^{-15}	1.5×10^{-9}	4.6×10^{-4}	4.6×10^{-1}
	2	2.8×10^{-14}	1.9×10^{-8}	4.8×10^{-3}	8.8×10^{-1}
	3	1.3×10^{-13}	9.2×10^{-8}	1.8×10^{-2}	9.8×10^{-1}
4×5 FP	1	7.2×10^{-18}	3.0×10^{-12}	6.8×10^{-7}	1.0×10^{-2}
	2	1.5×10^{-17}	6.5×10^{-12}	2.5×10^{-6}	5.5×10^{-2}
	3	2.2×10^{-17}	1.1×10^{-11}	7.2×10^{-6}	1.4×10^{-1}
1×20 FP	1	6.2×10^{-18}	2.4×10^{-12}	3.4×10^{-7}	1.2×10^{-3}
	2	1.2×10^{-17}	4.7×10^{-12}	6.7×10^{-7}	2.4×10^{-3}
	3	1.8×10^{-17}	7.1×10^{-12}	1.0×10^{-6}	3.6×10^{-3}

Table 2. Blocking probability versus link load for establishing a 3-level ternary tree in a 100×100 bi-directional mesh-torus network.

Node type	Z	1E	Link 2E	load 5E	7E
4×5 CS	1	6.9×10^{-17}	3.5×10^{-11}	7.7×10^{-6}	2.6×10^{-2}
	2	5.2×10^{-16}	2.9×10^{-10}	6.9×10^{-5}	1.4×10^{-1}
	3	2.1×10^{-15}	1.2×10^{-09}	2.7×10^{-4}	3.1×10^{-1}
4×5 FP	1	1.2×10^{-18}	4.8×10^{-13}	7.3×10^{-8}	3.9×10^{-4}
	2	2.3×10^{-18}	9.1×10^{-13}	1.5×10^{-7}	1.2×10^{-3}
	3	3.4×10^{-18}	1.4×10^{-12}	2.6×10^{-7}	2.8×10^{-3}
1×20 FP	1	1.2×10^{-18}	4.7×10^{-13}	6.6×10^{-8}	2.4×10^{-4}
	2	2.3×10^{-18}	8.7×10^{-13}	1.2×10^{-7}	4.5×10^{-4}
	3	3.3×10^{-18}	1.3×10^{-12}	1.8×10^{-7}	6.6×10^{-4}

Table 3. Blocking probability versus link load for establishing a 2-level binary tree in a 12-dimensional hypercube network.

Node type	Z	1E	Link 2E	load 5E	7E
4×5 CS	1	4.1×10^{-16}	2.3×10^{-10}	5.5×10^{-5}	1.7×10^{-1}
	2	4.5×10^{-15}	2.6×10^{-9}	5.7×10^{-4}	4.4×10^{-1}
	3	2.0×10^{-14}	1.2×10^{-8}	2.3×10^{-3}	7.1×10^{-1}
4×5 FP	1	2.1×10^{-18}	8.5×10^{-13}	1.4×10^{-7}	1.0×10^{-3}
	2	4.2×10^{-18}	1.7×10^{-12}	3.5×10^{-7}	4.5×10^{-3}
	3	6.4×10^{-18}	2.7×10^{-12}	7.4×10^{-7}	1.2×10^{-2}
1×20 FP	1	2.1×10^{-18}	8.2×10^{-13}	1.2×10^{-7}	4.2×10^{-4}
	2	4.1×10^{-18}	1.6×10^{-12}	2.2×10^{-7}	8.1×10^{-4}
	3	6.0×10^{-18}	2.3×10^{-12}	3.3×10^{-7}	1.2×10^{-3}

Table 4. Blocking probability versus link load for establishing a 2-level ternary tree in a 12-dimensional hypercube network.

the number of links in both being the same. Note that, in an extreme case when the degree of branching at the intermediate node is the same as the number of destination nodes, the analytical model emulates a statistical link-independence model. Hence, its comparison with the blocking performance of a path with the same number of links would indicate the difference between the blocking probabilities obtained using a link-independence and link-correlation model, which could be significant for longer paths and at high network loads.

6. CONCLUSION

In this paper, an analytical model for evaluating the blocking performance for establishing multicast trees in time-space switched optical networks has been developed. The blocking performance for establishing regular binary and ternary trees are evaluated on two different network architectures and three different switch architectures. It is observed that for establishing trees with low degree of branching (2 or 3) the blocking probability of establishing the tree match that of establishing a path with same number of links as in the tree.

While it can be concluded from the above results that path blocking probabilities can approximate tree blocking probabilities, with the same number of links with low degree of branching, the limit on the degree of branching above which the the difference exceeds a given factor remains to be studied. In this paper, a random channel assignment algorithm has been assumed for the analysis. The effect of other channel assignment algorithms such as first-fit, best-fit, etc. is yet to be studied. Also, the impact of having limited capability on the intra-trunk copying that allows the signal in only a few, but not all, trunks to be copied is also to be evaluated.

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