

An Empirical Function to Describe Measured Water Distributions From Horizontal Infiltration Experiments

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To determine the soil water diffusivity, $D(\theta)$, by the horizontal infiltration method of R. R. Bruce and A. Klute (1956) the slope of the water distribution curve and the area under the curve must be evaluated. Experimental data often exhibit scatter, thus making the evaluation of the slope difficult. In this paper a rapid, simple method is described that introduces an empirical function that by linear least squares regression yields a curve that fits water distribution data for a wide range of soil textures. The function is differentiable, and its integral is easily calculated with a numerical technique available on programmable scientific calculators. The proposed method produces estimates of $D(\theta)$ that reach expected large values near water saturation. This result offers a clear advantage over other simple methods that assume an exponential relation between D and θ and thus describe $D(\theta)$ adequately only over the middle range of water contents.

INTRODUCTION

Unsaturated soil conditions predominate in the vadose zone, the zone between the ground surface and the water table. The advance of the wetting front during infiltration initiated by a rainfall event or by irrigation is driven by potential gradients in the vadose zone. Water flow through the vadose zone replenishes the groundwater, and the rate of replenishment determines drainage requirement and drainage system design. Soil water flowing in the vadose zone contains solutes such as nitrates, which greatly influence water quality. Thus unsaturated flow is an important phenomenon to consider in vadose zone hydrology and in water resource management.

The one-dimensional horizontal movement of water in unsaturated soil can be described by the equation [Bruce and Klute, 1956]

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[D(\theta) \frac{\partial \theta}{\partial x} \right] \quad 0 < x < \infty \quad (1)$$

where θ is water content, t , the time, x , the position, and D , the soil water diffusivity. The diffusivity parameter permits a relation of water flux to a water content gradient rather than to a water potential gradient. This simplifies the mathematical and experimental treatments of unsaturated flow.

One method to calculate diffusivity is that of Bruce and Klute [1956]. By introducing the Boltzmann transformation, $\lambda = x/t^{1/2}$, (1) becomes

$$-\frac{1}{2} \frac{\lambda}{d\lambda} \frac{d\theta}{d\lambda} = \frac{d}{d\lambda} \left[D(\theta) \frac{d\theta}{d\lambda} \right] \quad (2)$$

With the following boundary conditions,

$$\begin{aligned} \theta \rightarrow \theta_i & \quad \lambda \rightarrow \infty \\ \theta = \theta_s & \quad \lambda = 0 \end{aligned}$$

where θ_i is the initial water content, and θ_s is the saturated water content, the integration of (2) between the limits θ_i and θ yields

$$D(\theta) = -\frac{1}{2} \frac{d\lambda}{d\theta} \int_{\theta_i}^{\theta} \lambda \, d\theta \quad (3)$$

Bruce and Klute [1956] present a transient-flow experiment to determine $D(\theta)$. Water is allowed to infiltrate at a constant head into a horizontal air-dry soil column. $D(\theta)$ is then evaluated from the resulting water distribution curve, $\theta(\lambda)$, by estimation of the water distribution curve's slope and graphic integration according to (3). An example of this procedure is found in the work by Kirkham and Powers [1972, pp. 266-267]. Difficulties in determining the water distribution curve's slope often arise from the scatter of experimental data.

Several approaches to circumvent the difficulties of determining the slopes of water distribution curves from experimental data have been proposed. One approach is to use explicit functions composed of constants and soil parameters to describe experimental diffusivity data. Gardner and Mayhugh [1958] use

$$D = D_a \exp [\beta \Theta] \quad (4)$$

where D_a is the soil water diffusivity at the air-dry content water content, β is a constant, and

$$\Theta = \left[\frac{\theta - \theta_i}{\theta_s - \theta_i} \right] \quad (5)$$

Miller and Bresler [1977] expand (4) and introduce

$$D = \varepsilon \lambda_i^2 \exp [\beta \Theta] \quad (6)$$

where β and Θ are as previously defined, ε is a constant, and

$$\lambda_i = x_i/t^{1/2} \quad (7)$$

in which x_i is the distance to the wetting front at the conclusion of horizontal infiltration; $\varepsilon = 10^{-3}$ and $\beta = 8$ were suggested as "universal" constants. Clothier and White [1981] found $\beta = 3$ in a field soil, thereby disproving the contention of universality. Brutsaert [1979] provides another function based on (4) and (6):

$$D = [\gamma S^2 / (\theta_s - \theta_i)^2] \exp (\beta \Theta) \quad (8)$$

where γ is a parameter dependent on β , and S is the sorptivity. Although accurate in the middle range of water contents, the inadequacy of these approaches to describe the soil water diffusivity near saturation is shown clearly by Clothier and Wooding [1983].

Ahuja and Swartzendruber [1972], intending to better de-

scribe D near saturation, present a power function form of soil water diffusivity:

$$D(\theta) = a\theta^n/(\theta_s - \theta)^m \quad (9)$$

where a , n , and m are constants for a particular soil. Following the approach of *Ahuja and Swartzendruber* [1972], one solves (3) for water distribution data with the criteria that the areas under the calculated and experimental $\theta(\lambda)$ curves are equal and that the sum of squared residuals between the two curves is minimized. To accomplish this, the integral in (3) is approximated by the trapezoidal rule, and the slope is estimated numerically between adjacent data points. Subsequently, a $\lambda(\theta)$ curve is determined by *Philip's* [1960] method [see *Kirkham and Powers*, 1972, pp. 287-320] and is then transformed to a $\theta(\lambda)$ curve. Once the criteria are met, the constants in (9) are determined.

Another approach to determine the slope of water distribution curves is to use the less complicated approach of fitting (by least squares regression) a function or group of functions directly to the water distribution data and then to evaluate the derivatives at the desired points. *Erh* [1972] and *Kimball* [1976] describe the use of cubic splines for this purpose. *DuChateau et al.* [1972] suggest fitting a piecewise parabolic curve (a quadratic spline or sliding parabolic). When interpolating with piecewise polynomials, there is the problem of concavity changes if the length of individual spline segments is too large. Such changes in concavity produce oscillations in the derivative and, consequently, meaningless values of soil water diffusivity.

Clothier et al. [1983] propose fitting a function chosen from those presented by *Philip* [1960] for which analytical relations between soil water diffusivity and soil water content can be developed. They select a function of the form

$$\lambda(\Theta) = \lambda_i[1 - \Theta]^\rho \quad (10)$$

where ρ is a variable calculated by setting $(\rho + 1)^{-1}$ equal to the observed curve shape parameter, P , defined as

$$P = S[\lambda_i(\theta_s - \theta_i)]^{-1} \quad (11)$$

where the variables are as defined previously.

In this paper we use various aspects of these approaches to develop a rapid and simple method of determining $D(\theta)$ from horizontal infiltration experiments. Specifically, we introduce a function, fit by linear least squares regression to water distribution data, which adequately describes soil water distribution curves. The function is differentiable and enables the use of simple integration schemes to determine the soil water diffusivity following (3).

MATERIALS AND METHODS

The empirical equation proposed to describe water distribution data is

$$\theta' = \theta_s' 10^{-[\alpha(\lambda_i - \lambda)]^{1/2}} \quad (12)$$

where the prime notation denotes a volumetric water content adjusted by a constant, α , such that $(\theta_{i+\alpha})/(\theta_{s+\alpha}) = 0.10$; b is the single unknown parameter; and λ and λ_i are as defined earlier. Adjusting the water contents by α is significant because the use of unadjusted water contents may result in positive values of the derivative near λ_i , thus yielding meaningless values of the soil water diffusivity in that domain. The function is defined over the interval $0 \leq \lambda \leq \lambda_i$. Also, only the positive square root is used.

Equation (12) may be written in the form

$$\log \log \frac{\theta_s'}{\theta'} = b(\lambda_i - \lambda)^{1/2} \quad (13)$$

A linear least squares regression through the origin, omitting data points where $\theta' > (\theta_s'/1.01)$, yields the single unknown parameter, b . Some of the data points are omitted because of their proximity to the asymptote at $\theta' = \theta_s'$.

The first step in incorporating (13) with (3) is to solve for λ . Given (13):

$$\lambda = \lambda_i - \left(\frac{\log \log \frac{\theta_s'}{\theta'}}{b} \right)^2 \quad (14)$$

From (14):

$$\frac{d\lambda}{d\theta'} = \left(\frac{\log \log \frac{\theta_s'}{\theta'}}{2.65(b)^2(\theta') \log \frac{\theta_s'}{\theta'}} \right) \quad (15)$$

Substituting (14) and (15) into (3), one derives

$$D(\theta) = D(\theta') = - \left(\frac{\log \log \frac{\theta_s'}{\theta'}}{5.30(b)^2(\theta') \log \frac{\theta_s'}{\theta'}} \right) \int_{\theta_i}^{\theta'} \left[\lambda_i - \left(\frac{\log \log \frac{\theta_s'}{\theta'}}{b} \right)^2 \right] d\theta' \quad (16)$$

The definite integral in (16) can be calculated by using numerical techniques. We used the key-defined algorithm available on a Hewlett-Packard 15C programmable scientific calculator.

To test our proposed method, water distribution data were extracted for a range of soil textures from figures presented in the literature. In addition, a horizontal infiltration experiment was conducted with two subsoil materials classified as Fayette silty clay loam and Nicollet sandy clay loam. The respective soils were air-dried, ground, and passed through a 2-mm sieve. They were then packed into a sectioned acrylic cylinder (0.038 m diameter and 0.10 m long) with a porous ceramic plate at one end while the cylinder was vibrated. After the soil column was placed horizontally, 0.01N CaCl₂ was allowed to infiltrate at a constant head of -50 Pa relative to the bottom of the soil column, such that cumulative infiltration was linearly related to the square root of time in accordance with the criterion of the Boltzmann transformation. After a desired time had elapsed, the soil column was quickly sectioned in 0.01-m increments, and the volumetric water content was determined.

RESULTS AND DISCUSSION

Composite Figure 1 presents the fit of (12) to water distribution data for Hagener sand [*Selim et al.*, 1970], Manawatu fine sandy loam [*Clothier et al.*, 1983], Hayden sandy loam [*Whisler et al.*, 1968], Pachappa loam [*Jackson*, 1963], Edina silt loam [*Selim et al.*, 1970], Adelanto loam [*Jackson*, 1963], Brookston clay loam [*Elrick et al.*, 1979], Panoche clay loam [*Reichardt et al.*, 1972], Pine silty loam [*Jackson*, 1963], and Yolo clay [*Nofziger*, 1978]. For the range of soil types presented, (12) approximates closely the shapes of the curves established by the data while avoiding the subjectivity of drawing a free-hand curve.

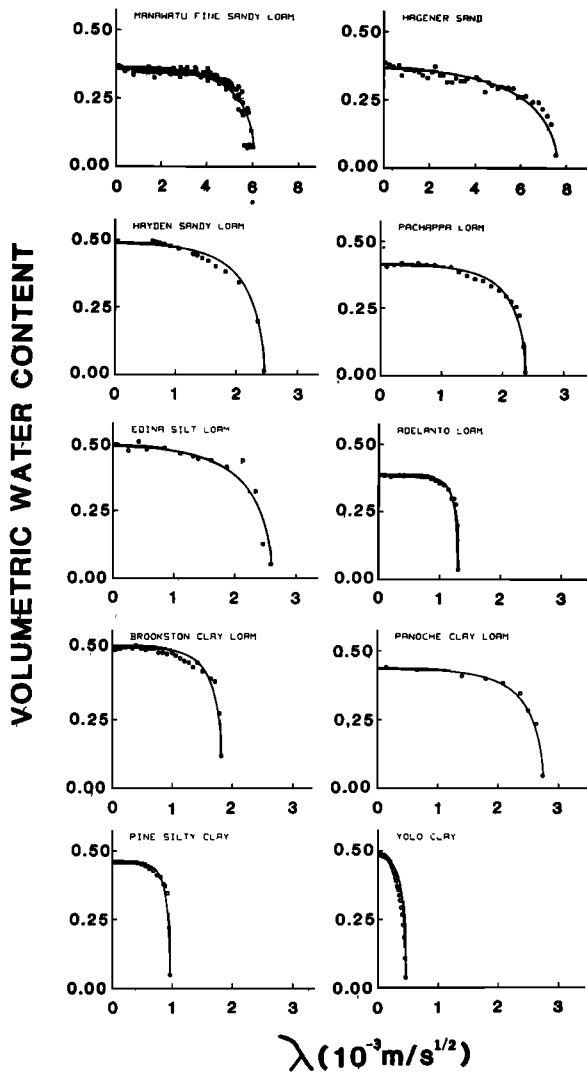


Fig. 1. Observed and fitted water distribution data for a range of soils. The observed data were taken from the literature (see text for references).

One feature of (12) is that the function is forced to terminate at the point (λ_s, θ_s) . Another feature is that the difference between θ_s and the predicted θ_s will depend on how close the exponent, $b(\lambda_i - \lambda)^{1/2}$, comes to the value of $\log \log \theta_s'/\theta'$ as θ' approaches θ_s' . The soil water diffusivity function calculated with (16) will asymptote at θ_i and θ_s because the slope of (12) goes to $-\infty$ as θ approaches θ_i and to zero as θ approaches θ_s . Consequently, the soil water diffusivity reaches expected large values near saturation. Thus (12) should be used only to fit water distribution data obtained from infiltrating water into a horizontal column of air-dry soil at a soil water pressure potential slightly less than 0 Pa. This will provide a large water content range and assure that the experimental θ_s is equal to or nearly equal to the saturated water content of the soil.

In addition to using a linear least squares regression method to determine the b parameter in (13), a nonlinear regression technique, which provides a slightly better fit, can be employed to fit (12). For the 10 soils in composite Figure 1, the nonlinear regression technique yielded a value of b an average of 6.0% different from b determined with linear regression. Although the sum of squared residuals was smaller for the nonlinear regression than for the linear regression, the

latter is judged an effective regression method to determine b because the b parameters determined from both techniques are nearly equal.

The functional relationship between θ and λ presented in (12) can be made more general if θ_s is considered to be an unknown parameter rather than a known constant. The two parameters θ_s and b can be determined simultaneously with a nonlinear regression method. A problem is inherent with this approach because $D(\theta)$ becomes large near θ_s . An overestimate of $D(\theta)$ results if the calculated θ_s is less than the actual θ_s and, conversely, an underestimate of $D(\theta)$ results if the calculated θ_s is greater. To avoid this problem we recommend that the b parameter be determined with a fixed θ_s .

Another approach to determine the b parameter in (12) or (13) is to use weighted least squares regression. Weighted regression could be considered because a given error in λ near the more vertical portion of the water distribution curve results in a larger error increment of θ than would occur for the same error in λ near the more horizontal portion of the curve. Thus the $\theta(\lambda)$ values in the more horizontal portion of the curve should receive more weight in the regression calculation than the values in the more vertical portion. Weights can be computed as the inverse of the slope squared at the points. We think that in most cases, weighted regression will not be a significant improvement over linear regression because most data sets have fewer measured values in the more vertical portion of the curve than in the more horizontal portion. Thus

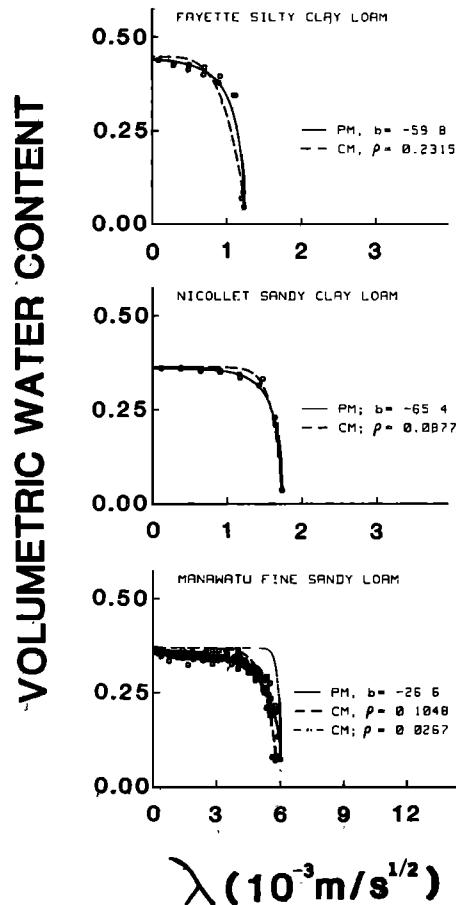


Fig. 2. Comparison between fitted water distribution data as calculated by using the method in this paper (PM) and by the method of Clothier et al. [1983] (CM) for observed water distribution data from three soils (see text for details).

TABLE 1. Parameter Values for the Soils Shown in Figure 2

Soil	θ_i	θ_s	$\lambda_p, \text{m/s}^{1/2}$	$S, \text{m/s}^{1/2}$
Fayette silty clay loam	0.048	0.448	1.24×10^{-3}	4.04×10^{-4}
Nicollet sandy clay loam	0.038	0.364	1.74×10^{-3}	5.22×10^{-4}
Manawatu fine sandy loam, <i>Clothier and Scotter</i> [1982]	0.08	0.36	6.05×10^{-3}	1.65×10^{-3}
Manawatu fine sandy loam, <i>Clothier et al.</i> [1983]	0.08	0.36	5.8×10^{-3}	1.47×10^{-3}

the linear regression already places more emphasis on the more horizontal portion of the curve than on the more vertical portion of the curve.

Elaborate computer-assisted empirical curve-fitting methods exist [e.g., *Erh*, 1972; *Kimball*, 1976; *DuChateau et al.*, 1972] that can produce a curve through water distribution data with a lower sum of squared residuals than the method proposed in this paper. Among less complicated methods, the approach outlined in *Clothier et al.* [1983] seems to offer the ease of calculation that our method does, with the advantage that the soil water diffusivity can be described by an analytical function.

In Figure 2, curves determined by our method are compared with those calculated by the method of *Clothier et al.* [1983] for the two sets of water distribution data (Fayette silty clay loam and Nicollet sandy clay loam) obtained in our laboratory and for the Manawatu fine sandy loam data taken from *Clothier and Scotter* [1982] and from *Clothier et al.* [1983]. Table 1 presents the values of θ_i , θ_s , λ_p , and S used. For Manawatu fine sandy loam the values were taken as the observed experimental values from *Clothier and Scotter* [1982] and were estimated from the free-hand curve shown in Figure 1 of *Clothier et al.* [1983].

The ρ parameter in the method of *Clothier et al.* [1983] for the Fayette soil was 0.2315, which yielded a sum of squared residuals of 0.0553. Our proposed method gave a sum of squared residuals of 0.0202, and the integral in (14) produced a value of $4.17 \times 10^{-4} \text{ m/s}^{1/2}$ for sorptivity; this was only 3.2% greater than the observed value. With the Nicollet soil a ρ parameter of 0.0877 gave a sum of squared residuals of 0.0048, whereas our method yielded one of 0.0020 and a sorptivity value of $5.13 \times 10^{-4} \text{ m/s}^{1/2}$, 1.8% lower than observed value.

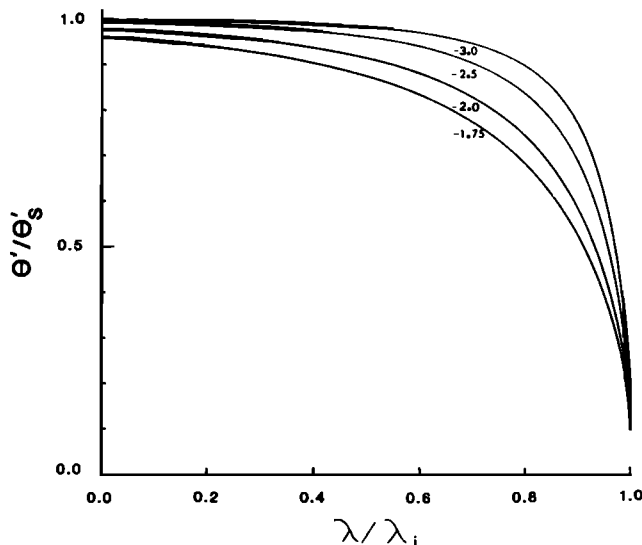


Fig. 3. Family of scaled water distribution curves produced for several values of b .

TABLE 2. Intermediate Values (Columns) Used to Calculate the Soil Water Diffusivity, $D(\theta)$

$\theta, \text{m}^3/\text{m}^3$	$\theta', \text{m}^3/\text{m}^3$	$\log \log \theta_s'/\theta'$	$d\lambda/d\theta', \text{m/s}^{1/2}$	$\int_{\theta_i}^{\theta'} \lambda d\theta'$	$D(\theta), \text{m}^2/\text{s}$
0.05	0.0482	-0.0575	-1.20×10^{-4}	2.09×10^{-5}	1.25×10^{-9}
0.10	0.0982	-0.2465	-3.90×10^{-4}	1.08×10^{-4}	2.11×10^{-8}
0.15	0.1482	-0.4111	-6.30×10^{-4}	1.93×10^{-4}	6.08×10^{-8}
0.20	0.1982	-0.5820	-9.88×10^{-4}	2.78×10^{-4}	1.37×10^{-7}
0.25	0.2482	-0.7848	-1.70×10^{-3}	3.59×10^{-4}	3.05×10^{-7}
0.30	0.2982	-1.0734	-3.76×10^{-3}	4.37×10^{-4}	8.21×10^{-7}
0.35	0.3482	-1.7665	-2.61×10^{-2}	5.02×10^{-4}	6.55×10^{-6}

Values were calculated for $\theta_i = 0.038$, $\theta_s = 0.364$, $\alpha = -0.0018$, $\lambda_i = 1.74 \times 10^{-3} \text{ m/s}^{1/2}$, and $b = -65.4$ with (16).

From *Clothier and Scotter's* [1982] data for the Manawatu soil, ρ was calculated as 0.0267 and produced a sum of squared residuals of 0.5280; for data from *Clothier et al.* [1983], values were 0.1048 and 0.1512, respectively. These values compare with a sum of squared residuals of 0.0920 for our proposed method used with the data of *Clothier and Scotter* [1982] and with a calculated value of $1.47 \times 10^{-4} \text{ m/s}^{1/2}$ for sorptivity, 11% less than the observed values. *Clothier et al.* [1983] reported a ρ parameter value of 0.15, but that seems based on the sorptivity determined from the free-hand curve and the measured value of $6.05 \times 10^{-3} \text{ m/s}^{1/2}$ for λ_i from *Clothier and Scotter* [1982]. This is not a correct application of their method because they did not use the measured value of sorptivity directly.

Our proposed method fits the measured values better than does the method proposed by *Clothier et al.* [1983] for the three sets of water distribution data presented in Figure 2. Scaling the water distribution curve to the measured S and λ_p as is recommended in their method, overlooks the influence of

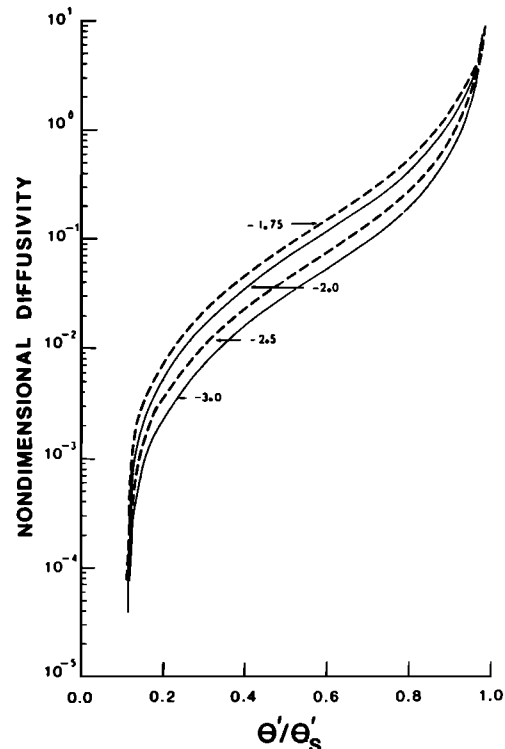


Fig. 4. Family of scaled soil water diffusivity curves produced for several values of b .

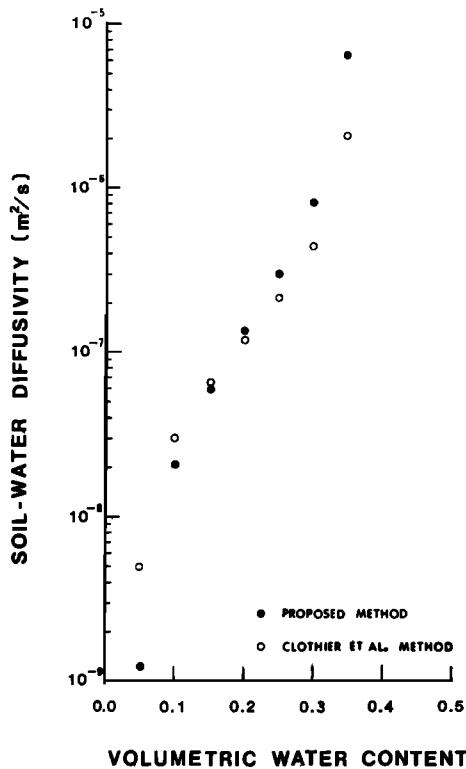


Fig. 5. Comparison between soil water diffusivity as calculated by the method in this paper and by the method of Clothier *et al.* [1983] for a Nicollet sandy clay loam soil.

the individual data points. Furthermore, if S is calculated as the area under the free-hand curve as indicated by Clothier *et al.* [1983], the same problem of fitting a curve through scattered data points is incurred.

Another way to utilize (12) is to make use of graphs with scaled x and y axes. The water distribution graph is created by scaling the y axis from 0.1 to 1.0 (θ'_i to θ'_s) and the x axis from 0 to 1.0 (0 to λ_i) and plotting curves produced by various values of b (Figure 3). The b parameter in this case is dimensionless because of the scaling of axes. Correspondingly, the diffusivity graph is created by scaling the x axis from 0.1 to 1.0 (θ'_i to θ'_s) and plotting curves obtained with various b values (Figure 4). In practice, Figure 3 can be used to find the b value for the curve with the best fit through measured, but scaled, water distribution data. Once a value of b is determined, the diffusivity can be read from the diffusivity graph (Figure 4). This value multiplied by λ_i^2 will give $D(\theta)$ for the soil sample.

Table 2 shows the steps taken to calculate $D(\theta)$ for selected values of θ . The water distribution data for the Nicollet soil are used to make these calculations. In this example, $\theta' = \theta - 0.0018$, and $b = -65.4$. The plot of $D(\theta)$ from Table 2 is presented in Figure 5 along with a plot of $D(\theta)$ calculated by the method of Clothier *et al.* [1983]. For the Nicollet soil the two methods yield similar $D(\theta)$ curves because both methods describe the water distribution curves similarly. On the other hand, for the Fayette and Manawatu soils, there will be less agreement for $D(\theta)$ values to the extent that the fitted water distribution curves differ.

CONCLUSIONS

This paper introduces a function that can be used to describe soil water distribution curves produced from horizontal infiltration experiments. The function's flexibility enables it to

fit (by linear least squares regression) water distribution data for a wide range of soil textures. The function is easy to differentiate and to integrate. Therefore it is useful in the calculation of soil water diffusivity by the method of Bruce and Klute [1956]. The proposed rapid, simple method compares favorably with the method of Clothier *et al.* [1983], a method similar in ease of calculation.

The proposed method yields calculated values of $D(\theta)$ that reach expected large values near saturation. This is a clear advantage over other available simple methods of determining $D(\theta)$ from horizontal infiltration. Most of these methods assume that soil water diffusivity is exponentially related to water content. Accordingly, they describe $D(\theta)$ adequately in the midrange of water contents but miss the important values near saturation.

The use of scaled axes, the other application of the method presented, offers a quick and easy procedure to obtain values of $D(\theta)$. This approach reduces calculations to a minimum. A set of overlays of water distribution curves can be used to determine b , the corresponding scaled $D(\theta)$ read from another graph, and finally, one calculation (multiplication by λ_i^2) is performed to obtain the actual $D(\theta)$ values.

NOTATION

- a dimensionless constant defined in (9).
- b parameter defined in (12).
- D soil water diffusivity, L^2/T .
- D_d soil water diffusivity at the air-dry water content.
- m dimensionless constant defined in (9).
- n dimensionless constant defined in (9).
- P parameter defined in (11).
- S sorptivity, $L/T^{1/2}$.
- t time after initiation of infiltration, T .
- x horizontal position, L .
- x_i distance to the wetting front at the conclusion of infiltration, L .
- α constant, L^3/L^3 .
- β dimensionless constant defined in (4).
- γ dimensionless constant defined in (8).
- ε dimensionless constant defined in (6).
- θ water content of soil, L^3/L^3 .
- θ_i initial water content.
- θ_s saturated water content.
- θ' water content adjusted by α .
- Θ dimensionless water content defined in (5).
- λ transformed position, $L/T^{1/2}$.
- λ_i transformed distance to wetting front at the conclusion of infiltration.
- ρ dimensionless constant defined in (10).

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REFERENCES

- Ahuja, L. R., and D. Swartzendruber, An improved form of soil-water diffusivity function, *Soil Sci. Soc. Am. Proc.*, 36, 9-14, 1972.
- Bruce, R. R., and A. Klute, The measurement of soil-moisture diffusivity, *Soil Sci. Soc. Am. Proc.*, 20, 458-462, 1956.
- Brutsaert, W., Universal constants for scaling the exponential soil water diffusivity?, *Water Resour. Res.*, 15, 481-483, 1979.
- Clothier, B. E., and D. R. Scotter, Constant-flux infiltration from a hemispherical cavity, *Soil Sci. Soc. Am. J.*, 46, 696-700, 1982.
- Clothier, B. E., and I. White, Measurement of sorptivity and soil-water diffusivity in the field, *Soil Sci. Soc. Am. J.*, 45, 241-245, 1981.
- Clothier, B. E., and R. A. Wooding, The soil-water diffusivity near saturation, *Soil Sci. Soc. Am. J.*, 47, 636-640, 1983.

- Clothier, B. E., D. R. Scotter, and A. E. Green, Diffusivity and one dimensional absorption experiments, *Soil Sci. Soc. Am. J.*, *47*, 641-644, 1983.
- DuChateau, P. C., D. L. Nofziger, L. R. Ahuja, and D. Swartzendruber, Experimental curves and rates of change from piecewise parabolic fits, *Agron. J.*, *64*, 538-542, 1972.
- Elrick, D. E., K. B. Laryea, and P. H. Groenevelt, Hydrodynamic dispersion during infiltration of water into soil, *Soil Sci. Soc. Am. J.*, *43*, 856-865, 1979.
- Erh, K. T., Application of the spline function to soil science, *Soil Sci.*, *114*, 333-338, 1972.
- Gardner, W. R., and M. S. Mayhugh, Solutions and tests of the diffusion equation for the movement of water in soils, *Soil Sci. Soc. Am. Proc.*, *22*, 197-201, 1958.
- Jackson, R. D., Porosity and soil-water diffusivity relations, *Soil Sci. Soc. Am. Proc.*, *27*, 123-126, 1963.
- Kimball, B. A., Smoothing data with cubic splines, *Agron. J.*, *68*, 126-129, 1976.
- Kirkham, D., and W. L. Powers, *Advanced Soil Physics*, 534 pp., Wiley-Interscience, New York, 1972.
- Miller, R. D., and E. Bresler, A quick method for estimating soil-water diffusivity functions, *Soil Sci. Soc. Am. J.*, *41*, 1020-1022, 1977.
- Nofziger, D. L., Errors in gamma-ray measurements of water content and bulk density in nonuniform soils, *Soil Sci. Soc. Am. J.*, *42*, 845-850, 1978.
- Philip, J. R., General method of exact solution of the concentration-dependent diffusion equation, *Aust. J. Phys.*, *13*, 1-12, 1960.
- Reichardt, K., D. R. Nielsen, and J. W. Biggar, Scaling of horizontal infiltration into homogeneous soils, *Soil Sci. Soc. Am. Proc.*, *36*, 241-245, 1972.
- Selim, H. M., D. Kirkham, and M. Amemiya, A comparison of two methods for determining soil-water diffusivity, *Soil Sci. Soc. Am. Proc.*, *34*, 14-18, 1970.
- Whisler, F. D., A. Klute, and D. B. Peters, Soil-water diffusivity from horizontal infiltration, *Soil Sci. Soc. Am. Proc.*, *32*, 6-11, 1968.

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