THREE DIMENSIONAL RESPONSE OF A CROSS-PLY COMPOSITE PLATE WITH IMPERFECT INTERFACES

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INTRODUCTION

Composite materials are widely used in many industries now. Usually, they are fabricated as laminated structures, where two adjacent laminae are bonded by a layer of adhesive material. Frequently, the failure of the whole structure is initiated from the failure of the weak interface layers. For this reason the strength of interface/interphase layers and its effects on the dynamic response of laminated structures have been a focus of attention from researchers in nondestructive evaluation. Several studies have been reported on this topic. Rokhlin and Maron [1] studied the reflection coefficient of an oblique incident wave to evaluate the interface properties and monitor the curing process in thin adhesive layers. Alers and Thompson [2] considered trapped modes in layered media for the nondestructive test of the quality of adhesion. The problems considered in both papers were simple, namely a layered half-space and a sandwich plate. However the structures made of composites used in industries are usually composed of many plies, which are anisotropic. These cause difficulties in interpreting the signals to measure the bond strength or to detect the imperfect interface. In this paper we present a model study of the effect of interface layers on the guided wave modes.

In an earlier study Xu and Datta [3] investigated the interface effects in a sandwich plate on cutoff frequencies and dispersion curves. In their study each layer was taken to be isotropic. In this paper, we consider the dynamic response of a four layered cross-ply graphite/epoxy plate with two interface bond layers. Numerical results are presented showing the significant changes in dispersion behavior with decreasing stiffnesses of the bond layers.

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FORMULATION

We consider a laminated composite plate containing N layers. Each layer is assumed to be homogeneous and transversely isotropic. A transversely isotropic material has five independent elastic stiffnesses, namely $C_{11}, C_{33}, C_{12}, C_{13}$ and C_{55}. All deformations are assumed to be small so that the problem can be analyzed using linear elastodynamic theories. The layers are assumed to be perfectly bonded to one another, so that the displacement and stress vectors are continuous across the interfaces. It will be assumed that the Cartesian coordinate axes x, y, z, are parallel to the principal axes of each lamina. The Greens functions for a given medium $G_{ki}(\underset{\sim}{x}, \underset{\sim}{x}')$ are defined as the displacements in the i^{th} direction produced at a field point $\underset{\sim}{x}$, usually called *receiver*, due to a unit impulsive load in the k^{th} direction applied to a point $\underset{\sim}{x}'$, called *source*.

In this paper we consider a cross–ply composite plate, the fiber direction in each ply being along either x or y axis. Thus, the xz (or, yz) plane is a plane of symmetry. The dynamic response of the plate can be written as [4]

$$\underset{\sim}{U}(x,y,z,t) = \frac{1}{8\pi^3} \int \int \int \bar{\underset{\sim}{U}}(k,\zeta,z,\omega)\; e^{ikx+i\zeta y-i\omega t}\; dk d\zeta d\omega \qquad (1)$$

where $\underset{\sim}{U}(x,y,z,t)$ is the displacement at the receiver located at $\underset{\sim}{x}(x,y,z)$ at time t and $\bar{\underset{\sim}{U}}(k,\zeta,z,\omega)$ is the Fourier transform of $\underset{\sim}{U}$ with respect to x, y, and t. $\bar{\underset{\sim}{U}}$ can be interpreted as the displacement at depth z as a function of the spatial wavenumbers k, ζ and the frequency ω. This depends on the frequency dependence of the force, $\underset{\sim}{S}(\omega)$, and the position vector of its application, $\underset{\sim}{x}'$. If the force is applied at $(0,0,0)$ in the z–direction, then we can write

$$\bar{\underset{\sim}{U}}(k,\zeta,z,\omega) = \underset{\sim}{S}(\omega)\frac{\underset{\sim}{F}(k,\zeta,z,\omega)}{D(k,\zeta,\omega)} \qquad (2)$$

The roots of $D(k,\zeta,\omega) = 0$ define the dispersive characteristics of the guided waves which will be studied in this paper.

In order to focus our attention on the effects of thin adhesive bond layers on these characteristics we consider, as an example, the particular case of a 4–layered cross–ply plate when there are thin adhesive bond layers between the outermost 0 degree plies and the adjacent inner 90 degree plies.

DISPERSION EQUATION

A stiffness method has been used in this paper to derive an approximate algebraic eigenvalue problem to solve for the roots of $D(k,\zeta,\omega) = 0$. In this method the plate is divided into many sublayers and within each sublayer the displacement $\underset{\sim}{U}(x,y,z,t)$ is written as

$$\{\underset{\sim}{U}(x,y,z,t)\} = [N(z)]\{q(x,t)\}\; e^{i\zeta_0 y} \qquad (3)$$

where $\{q\}$ is a nine–vector, representing the displacements at the top, middle and bottom of each sublayer. $[N(z)]$ is the matrix of shape functions (see Karunasena, *et al.* [5] for these expressions and further details).

The equation of motion is then obtained by minimizing the Lagrangian as,

$$[K_1]\{Q''\} + [K_2^*]\{Q'\} - [K_3]\{Q\} - [M]\{\ddot{Q}\} = 0 \qquad (4)$$

where $\{Q\}$ is the vector of all nodal displacement components, which are functions of x and t. The matrices $[K_1], [K_2^*], [K_3]$, and $[M]$ are functions of ζ and depend on the material properties of the sublayers. The primes denote derivatives with respect to x and the overdots represent time derivatives.

To derive the dispersion equation we assume a solution of (4) in the form

$$\{Q(x,t)\} = \{Q_0\}e^{i\gamma x - i\omega t} \tag{5}$$

Then (4) gives

$$\Big[-\gamma^2[K_1] + i\gamma[K_2^*] - [K_3] + \omega^2[M]\Big]\{Q_0\} = 0 \tag{6}$$

Equation (6) is the eigenvalue problem, which van be solved for γ for given ζ and ω. Our earlier studies have shown that these eigenvalues can be found as close to the exact wavenumbers k as desired by increasing the number of sublayers. In the following we present some representative numerical results that show the significant changes in the phase velocities, $c = \omega/\sqrt{\gamma^2 + \zeta^2}$, as functions of ω for fixed ζ.

NUMERICAL RESULTS

The present study is focussed on a $[0/90]_s$ graphite/epoxy laminated plate with two interface bond layers between the outer 0 degree and inner 90 degree plies. The thickness of each ply is taken to be 0.4764 mm and that of each adhesive layer is 0.0464 mm. Thus the total thickness of the plate is 2 mm. Changes in the dispersion of phase velocities of Rayleigh–Lamb modes with decreasing moduli of the adhesive layers have been studied. As initial starting values these moduli are taken to be those of an epoxy material, viz., Youngs modulus E $= 0.528$ GPa, Poisson's ratio $\nu = 0.354$, density $\rho = 1.2\ g/cm^3$. The properties of the graphite/epoxy zero degree ply are: $C_{11} = 160.72\ GPa$, $C_{22} = C_{33} = 13.92\ GPa$, $C_{13} = C_{12} = 6.42\ GPa$, $C_{23} = 6.92\ GPa$, $C_{55} = C_{66} = 7.07\ GPa$, $\rho = 1.8\ g/cm^3$. Because the plate is symmetric the Rayleigh–Lamb waves separate into symmetric and antisymmetric modes. In this paper we present results for the symmetric modes only. Figures 1–3 show the variations of phase velocity with frequency when the bond stiffnesses are given by the values noted above (Fig. 1) and when they are reduced by a factor of 10 (Fig. 2) and 100 (Fig. 3). These results are for the case $\zeta = 0$. In this case the in–plane motion is uncoupled from the SH motion and Figures 1–3 show the dispersion behavior of the in–plane modes. Figure 3 shows remarkable differences from both figures 1 and 2. Of particular interest is the behavior of the first two modes. It is seen that as the frequency increases the first one tends to resemble the a_o mode of the top zero degree ply for a certain range of frequency (0.25–1.2 MHz) and then it drops rapidly to the shear velocity of the bond layer. The second mode, on the other hand, tends to the S_o mode of the middle 90 degree ply in the frequency range 0.2 - 2.4 MHz. However, between 2.4–2.6 MHz the group velocity becomes negative, indicating that this mode does not propagate. Above 2.6 MHz this mode propagates with a velocity that is less than the shear velocity of the bond. Another noticeable feature is the plateau that appears in the higher branches at about 9.3 km/s, which corresponds to the longitudinal plate velocity of the top ply. The behavior of the branches as the bond stiffnesses decrease is explained if one examines Figure 4, which shows the branches of the guided modes in the top and middle plies. These branches are marked by superscripts t and m. Comparison of Figures 4 and 3 shows that the modes of the plate become asymptotic to those of the top or middle plies in certain bands of frequency. This feature becomes highly pronounced as the bond becomes very weak (soft) and this is clearly seen in Figure 5, which shows the results when the bond stiffnesses are

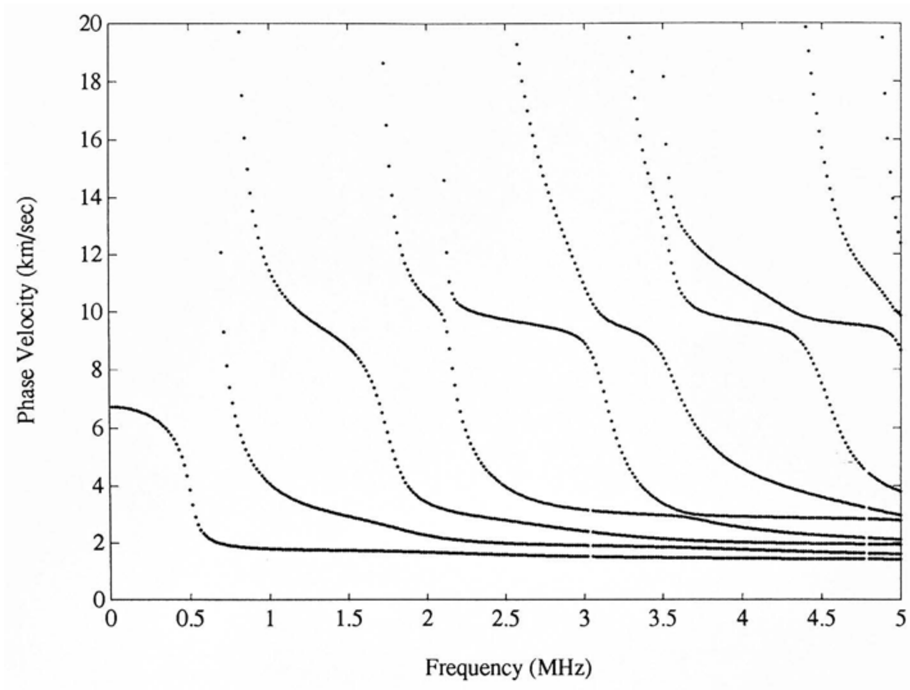

Fig. 1. Dispersion of guided waves with strong bond layers.

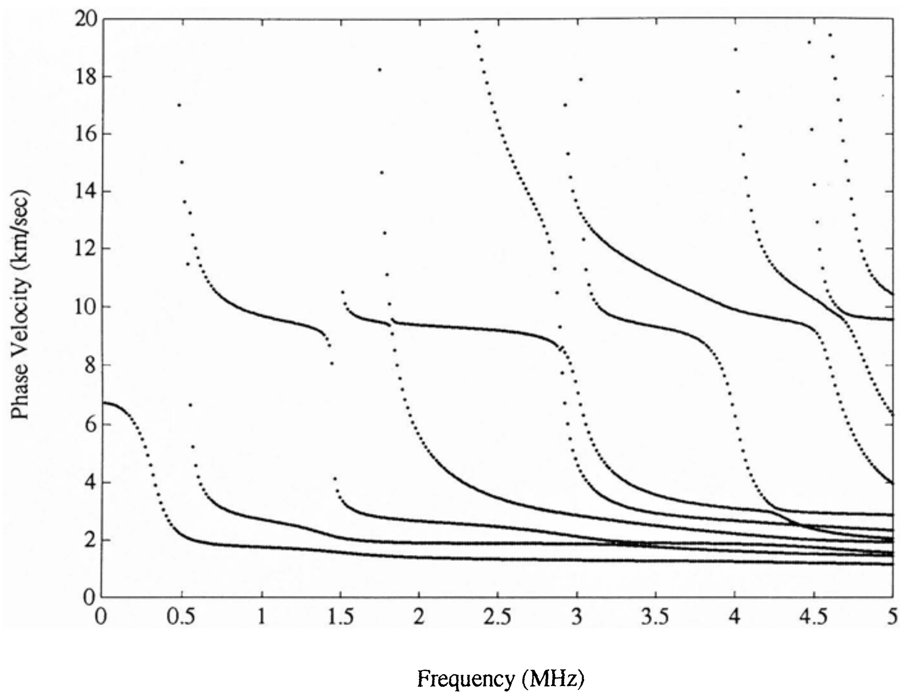

Fig. 2. Dispersion of guided waves when the bond layer stiffnesses are reduced 10.

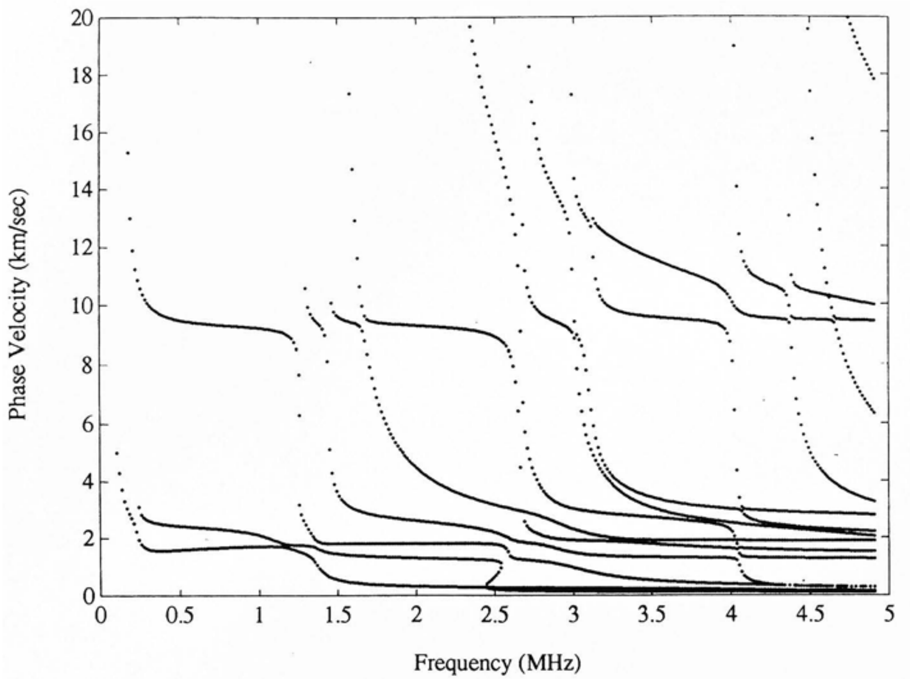

Fig. 3. Dispersion of guided waves when the bond layer stiffnesses are reduced by 100.

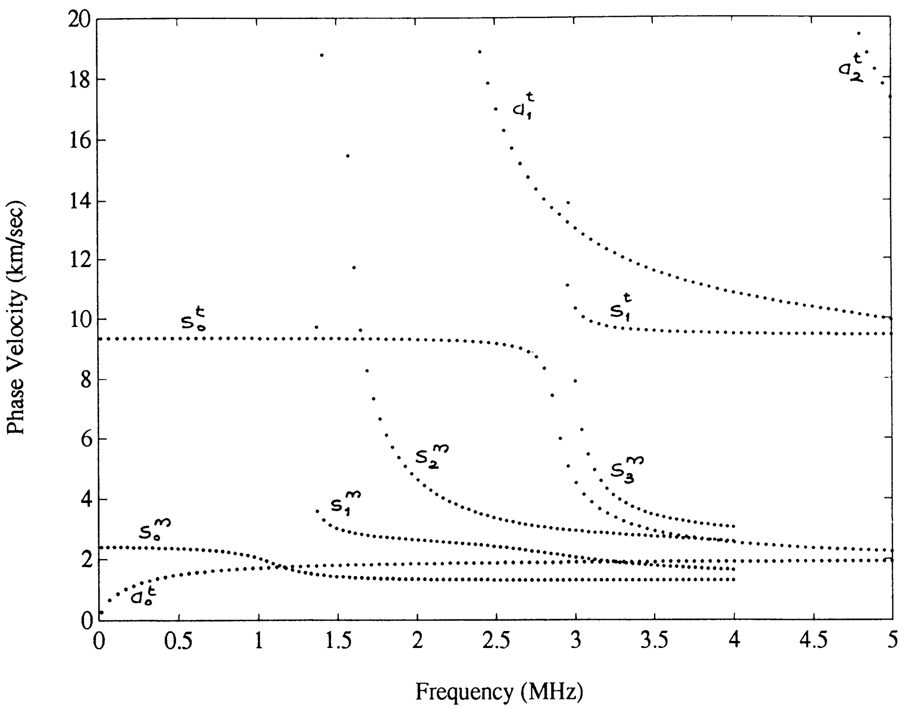

Fig. 4. Modes of guided waves in the top and middle plies.

reduced by 10^4. These figures also show that the decrease in the bond stiffnesses causes lowering of the cutoff frequencies which can be used as a measure of the weakening of the bond. Figure 6 shows that changes in the cutoff frequencies of the modes as h/C_{55} increases. Here h is the thickness of the bond layer. It is seen that the cutoff frequencies of the low order modes tend to zero as h/C_{55} increases. Those of the higher order modes, however, tend to be asymptotic to the cutoff frequencies of the top or middle plies. This is consistent with behavior of the dispersion curves.

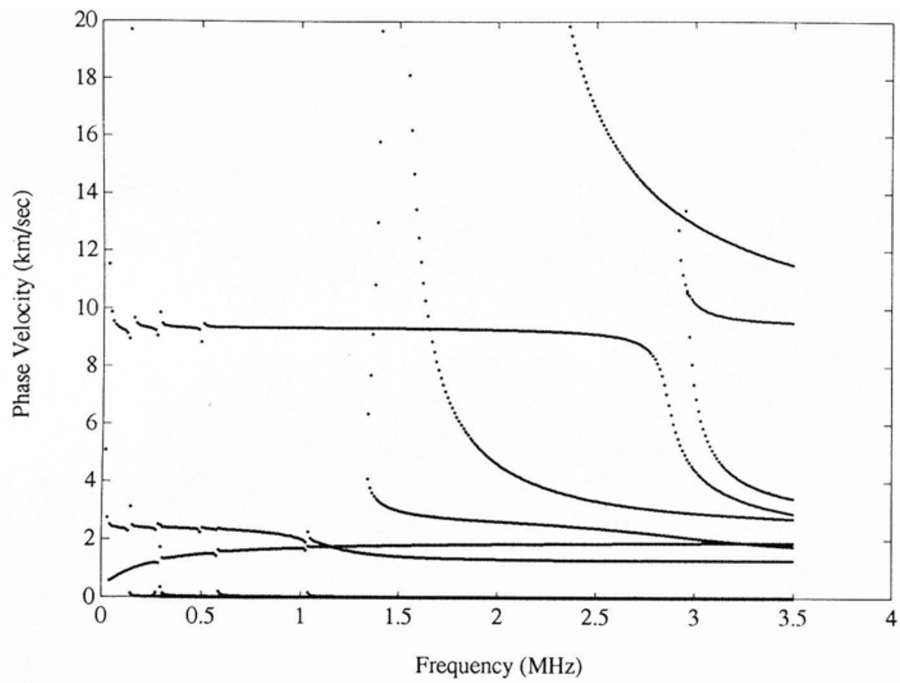

Fig. 5. Dispersion of guided waves when the bond layer stiffnesses are reduced by 10000.

All the above results are for the case $\zeta = 0$. When $\zeta \neq 0$ the in–plane and SH motions are coupled and the dispersion curves are much more complicated. Figures 7 and 8 show these when $\zeta = 1$ and 5 respectively. The bond layer properties are those of an epoxy material mentioned before. Note that the velocity of each branch starts from that of the mode propagating along the y–axis. Then as the frequency increases the velocities of these modes behave quite differently than when $\zeta = 0$. Comparison of these figures with Figure 1 shows the coupling of the SH motion with the in–plane motion and how the branches pinch one another at certain frequencies.

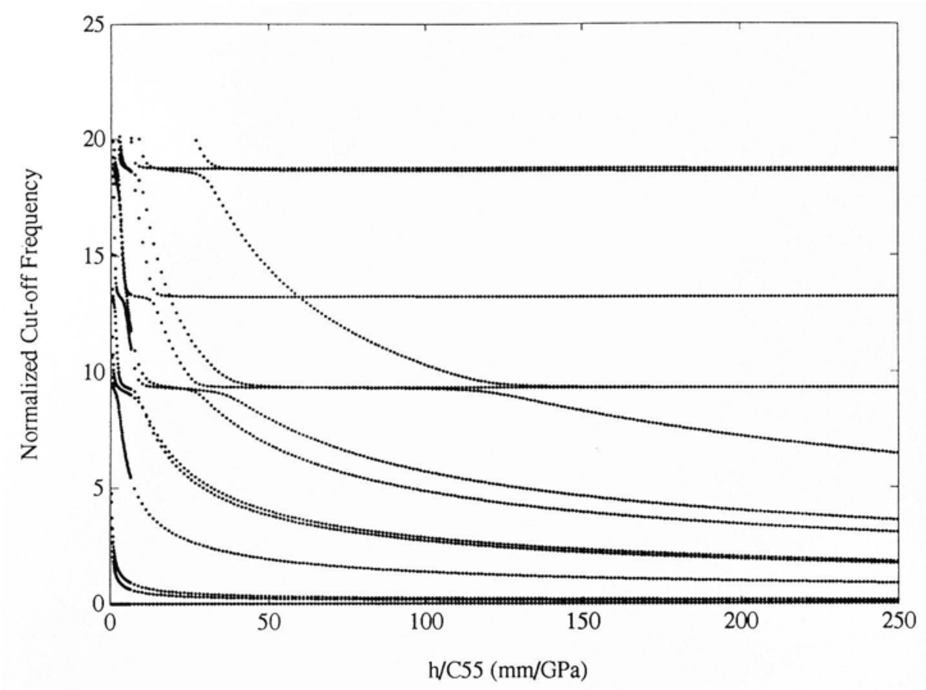

Fig. 6. Cutoff frequencies of the modes as the stiffnesses of the bond decrease.

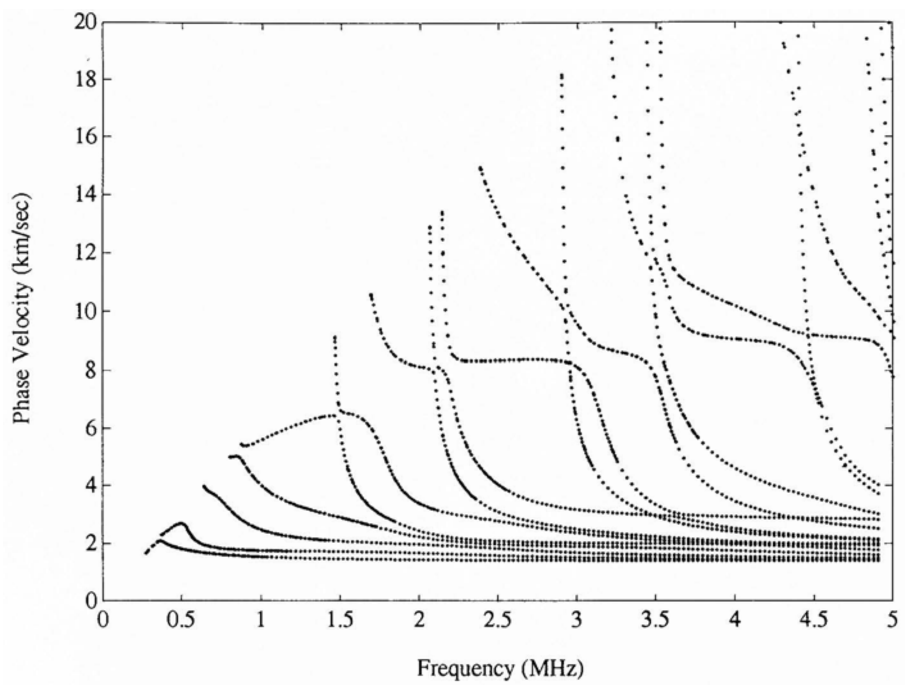

Fig. 7. Dispersion of guided modes when $\zeta = 1$.

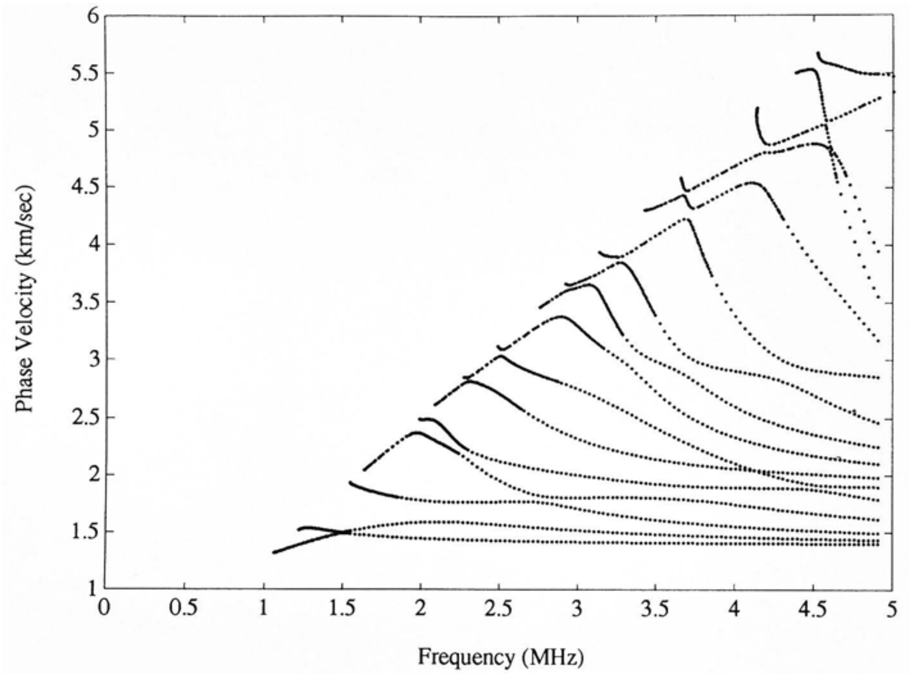

Fig. 8. Dispersion of guided modes when $\zeta = 5$.

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