New optimization techniques for power system generation scheduling

by

Wei Sun

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

Program of Study Committee:
Chen-Ching Liu, Major Professor
Lizhi Wang
Venkataramana Ajjarapu
Dionysios Aliprantis
Manimaran Govindarasu

Iowa State University
Ames, Iowa
2011

Copyright © Wei Sun, 2011. All rights reserved.
TABLE OF CONTENTS

LIST OF FIGURES v

LIST OF TABLES vii

ABSTRACT x

CHAPTER 1. INTRODUCTION 1

1.1 Research Problem Statement 1
1.2 Background and Motivation 2
1.3 Literature Review 5
  1.3.1 Optimal Generator Start-Up Strategy for Bulk Power System Restoration 5
  1.3.2 Optimal Installation Strategy of Blackstart Capability in Power System Restoration 7
  1.3.3 Optimal Generation Scheduling in a Carbon Dioxide Allowance Market Environment 9
1.4 Contributions of this Dissertation 12
1.5 Thesis Organization 14

CHAPTER 2. OPTIMAL GENERATOR START-UP STRATEGY FOR BULK POWER SYSTEM RESTORATION 16

2.1 System Restoration Procedure 16
2.2 Maximizing Generation Capability during System Restoration 18
  2.2.1 Generator Characteristic and Constraints 18
  2.2.2 Optimal Generator Start-Up Strategy 21
2.3 The “Two-Step” Algorithm 23
  2.3.1 Problem Formulation 23
  2.3.3 Algorithm 26
2.4 The Mixed Integer Linear Programming-Based Strategy 29
  2.4.1 Objective Function 29
  2.4.2 Constraints 29
2.5 Optimal Transmission Path Search Module 36
2.6 Adding the Time of GRAs into the Optimization Modules 38
2.7 Connections of Optimization Modules 39
2.8 Numerical Results 41
  2.8.1 Case of Four-Generator System 41
  2.8.2 Case of PECO System 44
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER 5. CONCLUSIONS</td>
<td>142</td>
</tr>
<tr>
<td>5.1  Summary of Dissertation</td>
<td>142</td>
</tr>
<tr>
<td>5.2  Future Research Direction</td>
<td>143</td>
</tr>
<tr>
<td>APPENDIX A. PROOF OF LEMMA 1 in Section 2.3.1</td>
<td>145</td>
</tr>
<tr>
<td>APPENDIX B. THE SOLUTION METHODOLOGY OF MIBLP IN SECTION 3.2.3</td>
<td>148</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>152</td>
</tr>
<tr>
<td>LIST OF PUBLICATIONS</td>
<td>158</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1. Power system restoration strategy ................................................................. 17
Figure 2.2. Generation capability curve ........................................................................ 20
Figure 2.3. “Two-Step” generation capability curve ...................................................... 24
Figure 2.4. Flow chart of “Two-Step” algorithm .............................................................. 28
Figure 2.5. Generation capability function .................................................................. 30
Figure 2.6. Generator start-up power function ............................................................... 31
Figure 2.7. Two steps of generation capability curve of four-generator system .......... 43
Figure 2.8. Two steps of generation capability curve of PECO system ..................... 47
Figure 2.9. IEEE 39-bus system topology with optimal transmission paths ............... 51
Figure 2.10. Comparison of generation capability curves by using different modules..... 52
Figure 2.11. Progress of restoring power system ........................................................... 54
Figure 2.12. Generation capability curve of AEP case .................................................. 56
Figure 2.13. Generation capability curve of Entergy case ............................................. 57
Figure 3.1. Illustration of generation availability connecting three restoration stages .... 69
Figure 3.2. Updated restoration plan with additional blackstart capability .................. 72
Figure 3.3. Criteria of restoration time and blackstart capability ................................. 74
Figure 3.4. Restoration tool for updating restoration plans .......................................... 75
Figure 3.5. Flow chart of algorithm .............................................................................. 78
Figure 3.6. PJM 5-bus system ...................................................................................... 79
Figure 3.7. Comparison of system generation capability curves considering different optimization modules .......................................................... 82
Figure 3.8. Comparison of different installation strategies ............................................ 83
Figure 3.9. Comparison of system generation capability curves under different blackstart capabilities .................................................................................. 84
Figure 3.10. Comparison of different installation strategies ......................................... 86
Figure 3.11. Comparison of system generation capability curve under different installation strategies .................................................................................. 86
Figure 3.12. IEEE RTS 24-bus test system .................................................................. 87
Figure 3.13. Comparison of restoration times under different blackstart capabilities on Bus 22 .......................................................................................... 90
Figure 3.14. Comparison of restoration times with installation of one new blackstart unit at different buses .......................................................................................................................... 91
Figure 3.15. Comparison of restoration times at different buses with additional blackstart capability ................................................................................................................................. 93
Figure 3.16. Comparison of reduced restoration times and installed blackstart capability at each bus ................................................................................................................................. 94
Figure 3.17. Comparison of increased system generation capability with different blackstart capabilities ............................................................................................................................. 96
Figure 3.18. Comparison of reduced restoration time and increased system generation capability with installed blackstart capability at different buses .................. 97
Figure 4.1. GENCOs’ bidding offers .......................................................................................................................... 104
Figure 4.2. Market clearance ....................................................................................................................................... 105
Figure 4.3. New GSP in two markets .......................................................................................................................... 113
Figure 4.4. GENCOs’ interactions in electricity market and CO₂ allowance market........ 114
Figure 4.5. Time horizon of three-year GSP .............................................................................................................. 114
Figure 4.6. Structure of new GSP ........................................................................................................................................ 116
Figure 4.7. PJM 5-bus system ......................................................................................................................................... 130
Figure 4.8. One-week load data ....................................................................................................................................... 132
Figure 4.9. Comparison of the profit of G1 .................................................................................................................... 134
Figure 4.10. Comparison of the profit under different maintenance schedules ............................................................................. 135
Figure 4.11. Comparison of the reduced profit under different CO₂ bidding strategies..... 135
Figure 4.12. Comparison of the profit under different maintenance schedules and bidding strategies ............................................................................................................. 136
Figure 4.13. Comparison of the generation output of G1 .............................................................................................. 137
Figure 4.14. Comparison of the generation output under different maintenance schedules .......................................................................................................................... 138
Figure 4.15. Comparison of the reduced generation output under different maintenance schedules .......................................................................................................................... 138
Figure 4.16. Comparison of the generation output under different maintenance schedules .......................................................................................................................... 139
Figure 4.17. The profit under optimal maintenance scheduling and CO₂ allowance bidding strategy .......................................................................................................................... 139
LIST OF TABLES

Table 2.1 Time to Complete GRAs ................................................................. 39
Table 2.2 Data of Generator Characteristic .................................................. 41
Table 2.3 Generator Starting Time ................................................................. 42
Table 2.4 Generator Status for the Optimal Solution ..................................... 42
Table 2.5 Data of Generator Characteristic .................................................. 44
Table 2.6 Generator Starting Time ................................................................. 45
Table 2.7 Generator Status for the Optimal Solution ..................................... 47
Table 2.8 Data of Generator Characteristic .................................................. 48
Table 2.9 Data of Transmission System ......................................................... 49
Table 2.10 Comparison of Generator Start-up Time Considering Different
        Optimization Modules ............................................................................ 49
Table 2.11 Energized Time of All Buses ......................................................... 50
Table 2.12 Energized Time of All Lines ......................................................... 50
Table 2.13 Optimal Transmission Paths ......................................................... 51
Table 2.14 Actions to Restore Entire Power System ...................................... 53
Table 2.15 Generator Starting Times .............................................................. 55
Table 2.16 Data of Four Generators ............................................................... 57
Table 2.17 Generator Starting Times .............................................................. 57
Table 2.18 Performance for Different Test Cases .......................................... 58
Table 2.19 Comparisons with Other Methods .............................................. 59
Table 3.1 Data of Generator Characteristic .................................................. 80
Table 3.2 Data of Transmission System ......................................................... 80
Table 3.3 Comparison of Generator Start-up Time Considering Different
        Optimization Modules ............................................................................. 80
Table 3.4 Restoration Actions Considering GSS, TPS and GRAs ..................... 81
Table 3.5 Comparison of Restoration Time and System Generation Capability
        with Different Blackstart Capabilities .................................................... 82
Table 3.6 Comparison of Restoration Time and System Generation Capability under
        Different Installation Strategies ............................................................ 85
Table 3.7 Data of Generator Characteristic .................................................. 88
Table 3.8 Sequence of Restoration Actions ................................................................. 88
Table 3.9 Different Sequence of Restoration Actions Compared with Base Case ........ 89
Table 3.10 Sequence of Restoration Actions ............................................................... 92
Table 3.11 Comparison of Restoration Time and System Generation Capability with Different Blackstart Capabilities ......................................................... 95
Table 3.12 Performance Analysis ............................................................................. 99
Table 4.1 GENCOs’ Characteristics ........................................................................ 125
Table 4.2 Equilibrium Solutions ............................................................................. 125
Table 4.3 Sensitivity Analysis of Forecasted Electricity Price to CO₂ Allowance Price .... 126
Table 4.4 Sensitivity Analysis of Total Amount of CO₂ Allowance to CO₂ Allowance Price ................................................................. 127
Table 4.5 GENCOs’ Characteristics ......................................................................... 128
Table 4.6 Equilibrium Solutions ............................................................................. 128
Table 4.7 Equilibrium Solutions ............................................................................. 129
Table 4.8 Branch Data .......................................................................................... 130
Table 4.9 Date of Generator Characteristic ............................................................. 131
Table 4.10 GENCOs’ Electricity Bidding Offers/Production Cost ($/pu-hr).............. 131
Table 4.11 Maintenance Limit of GENCO 1 ............................................................. 131
Table 4.12 CO₂ Allowance Bidding Offers of GENCO 1 .......................................... 132
Table 4.13 GENCO 1’s Profit and Generation Output under Different Strategies ...... 133
ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my advisor, Dr. Chen-Ching Liu. I am deeply appreciative of his guidance, invaluable advice, and continuous support throughout my Ph.D. study at Iowa State University. I am very grateful to Professor Liu for providing me with the financial support to attend technical conferences and the opportunity to work in various research projects and workshop organization. His leadership, professional accomplishments, dedication to education, and admirable decision making and time management skills have been motivating and inspiring me to pursue excellence in my future career. I would like to give my best wishes to Professor Liu and his family.

I would also like to thank my minor professor, Dr. Lizhi Wang, for his invaluable advice and continuous encouragement throughout my research and career preparation. I am also grateful to my committee members, Dr. Venkataramana Ajjarapu, Dr. Dionysios Aliprantis, and Dr. Manimaran Govindarasu, for their valuable suggestions and comments.

It is my pleasure to thank Dr. Ron Chu from PECO Energy and Dr. Li Zhang from American Electric Power for sharing their industry experiences and providing enlightening discussions. I would also like to thank Dr. Yunhe Hou from The University of Hong Kong for the financial support to visit and work with him.

I would like to acknowledge the financial support of Power Systems Engineering Research Center (PSERC) and Electric Power Research Institute (EPRI).

Finally and foremost, my special and deepest thanks to my parents, Peishi Sun and Linmei Wang, and my dear wife, Qun Zhou. Only with your unconditional support, unending encouragement and amazing love, I can successfully accomplish this dissertation.
ABSTRACT

Generation scheduling in restructured electric power systems is critical to maintain the stability and security of a power system and economical operation of the electricity market. However, new generation scheduling problems (GSPs) are emerging under critical or new circumstances, such as generator starting sequence and black-start (BS) generator installation problems in power system restoration (PSR), and generation operational planning considering carbon dioxide (CO₂) emission regulation. This dissertation proposes new optimization techniques to investigate these new GSPs that do not fall into the traditional categories.

Resilience and efficient recovery are critical and desirable features for electric power systems. Smart grid technologies are expected to enable a grid to be restored from major outages efficiently and safely. As a result, power system restoration is increasingly important for system planning and operation. In this dissertation, the optimal generator start-up strategy is developed to provide the starting sequence of all BS or non-black-start (NBS) generating units to maximize the overall system generation capability. Then, based on the developed method to estimate the total restoration time and system generation capability, the optimal installation strategy of blackstart capabilities is proposed for system planners to develop the restoration plan and achieve an efficient restoration process. Therefore, a new decision support tool for system restoration has been developed to assist system restoration planners and operators to restore generation and transmission systems in an on-line environment. This tool is able to accommodate rapidly changing system conditions in order to avoid catastrophic outages.
Moreover, to achieve the goal of a sustainable and environment-friendly power grid, CO₂ mitigation policies, such as CO₂ cap-and-trade, help to reduce consumption in fossil energy and promote a shift to renewable energy resources. The regulation of CO₂ emissions for electric power industry to mitigate global warming brings a new challenge to generation companies (GENCOs). In a competitive market environment, GENCOs can schedule the maintenance periods to maximize their profits. Independent System Operator’s (ISO) functionality is also considered from the view point of system reliability and cost minimization. Considering these new effects of CO₂ emission regulation, GENCOs need to adjust their scheduling strategies in the electricity market and bidding strategies in CO₂ allowance market. This dissertation proposes a formulation of the emission-constrained GSP and its solution methodology involving generation maintenance scheduling, unit commitment, and CO₂ cap-and-trade. The coordinated optimal maintenance scheduling and CO₂ allowance bidding strategy is proposed to provide valuable information for GENCOs’ decision makings in both electricity and CO₂ allowance markets.

By solving these new GSPs with advanced optimization techniques of Mixed Integer Linear Programming (MILP) and Mixed Integer Bi-level Liner Programming (MIBLP), this dissertation has developed the highly efficient on-line decision support tool and optimal planning strategies to enhance resilience and sustainability of the electric power grid.
CHAPTER 1. INTRODUCTION

1.1 Research Problem Statement

Resilience and efficient recovery are critical and desirable features for electric power systems. Power system restoration is increasingly important for system planning and operation to restore a grid from major outages efficiently and safely. It is critical to start up all generators in an optimal sequence so that the overall system generation capability will be maximized. Moreover, to achieve a faster restoration process, installing new BS generators can be beneficial in accelerating system restoration. Power systems have to update the restoration plan and quantify the benefit based on appropriate criteria.

To achieve a sustainable and environment-friendly power grid, CO₂ mitigation policies help to reduce consumption in fossil energy and promote a shift to renewable energy resources. The regulation of CO₂ emissions from electric power industry to mitigate global warming brings a new challenge to GENCOs. Considering these new effects, GENCOs need to adjust their scheduling strategies in the electricity market and bidding strategies in CO₂ allowance market.

This research is intended to provide the optimal strategies for operation and planning to address these challenging issues and enhance power system resilience and sustainability. Specifically, the objective of this research is to develop an on-line decision support tool and optimal planning strategies for power system restoration and to provide generation companies with the optimal scheduling and bidding strategies that facilitate their decision makings in both electricity and CO₂ allowance markets.
1.2 Background and Motivation

Traditional GSPs can be categorized as real-time security analysis, short-time generation operation, i.e., security-constrained unit commitment (SCUC) and security-constrained optimal power flow (SCOPF), mid-term generation operation planning, i.e., maintenance scheduling, fuel allocation, emission allowance, optimal operation cost, etc., and long-term generation resource planning problems [1].

The short-term generation operation is to assure generation and load balance with emphasis on real-time power system security. In the day-ahead market, participants submit their hourly and block offers to the ISO, where SCUC and SCOPF are performed to decide the generation schedule and dispatch. Benders decomposition is an efficient method to solve the interaction between SCUC and SCOPF. SCUC is composed of a master problem of unit commitment (UC) and a subproblem for transmission security analysis in a steady state. UC determines a day-ahead (or weekly) generation schedule for supplying the system demand and meeting the security margin. If transmission security violations are not mitigated in the subproblem, Benders cuts will be added as constraints to the master problem for the next iteration until the iterations converge. While the master problem of SCOPF is represented by the subproblem of SCUC, the subproblem of SCOPF executes the contingencies evaluation [2].

The mid-term operation planning is coordinated with short-term operation to maintain the power system security (by ISO), extend the life span of existing generating units (by GENCOs), and maintain transmission security through proper maintenance (by TRANSCOs). The priority is the optimal maintenance scheduling of GENCOs and TRANSCOs and the optimal allocation of natural resources. The long-term resource planning
problem addresses the economic selection of generation and transmission additions necessary to meet projected load requirements. The tradeoff between economics and security is a major consideration in the restructured electricity planning. All of these GSPs are combinatorial optimization problems with a large number of binary, continuous, or discrete decision variables, and various equality and inequality constraints.

First, GSP in PSR following a blackout is to provide an initial generation starting sequence of all BS or NBS generating units. System restoration is one of the most important tasks for power system operators. During system restoration, generation availability is fundamental for all stages of system restoration: stabilizing the system, establishing the transmission path, and picking up load. Available BS units must provide cranking power to NBS units in such a way that the overall available generation capability is maximized. The generator start-up strategy is important to the system restoration plan. The corresponding generation optimization problem is combinatorial with complex practical constraints that vary with time, and it needs modern optimization techniques to provide efficient computer solutions.

Second, to achieve a faster restoration process, installing new BS generators can be beneficial in accelerating system restoration. While additional BS capability does not automatically benefit the restoration process, power systems have to update the PSR plan and quantify the benefit based on appropriate criteria. Blackstart capability assessment is also highly system dependent and lacks universal solutions. It requires a decision support tool to provide a systematic way to assess the optimal installation location and amount of BS capability.
Third, CO₂ emission regulation affects both short-term generation operation and mid-term generation operation planning. Generation scheduling problem considering CO₂ emission regulation is to investigate the effects of this new mechanism on current system operation and the corresponding adjustment of GENCOs’ decision makings. The regulation of greenhouse gas (GHG) emissions for electric power industry to mitigate global warming brings the challenge to GENCOs. Among various climate change policies, emission trading is an efficient market-based mechanism to regulate emission of CO₂, the principal human-caused GHG. Incorporating the CO₂ emission allowance market with the electricity market, GENCOs have to adjust their strategies to maximize the profit. The appropriate model of GSP considering CO₂ emission cap-and-trade needs to be developed.

Mathematically, GSPs can be formulated as optimization problems with different objective functions and constraints that are nonconvex, nonlinear, large-scale, and mixed-integer combinatorial optimization problems. In the literature, a number of methods, such as enumeration, dynamic programming (DP), Lagrangian relaxation (LR), mixed-integer programming (MIP), and heuristic methods (genetic algorithms, artificial neural networks, expert and fuzzy systems) have been proposed to achieve an optimal or near optimal solution. However, the high computational burden and high dimensionality are barriers to practical applications. MIP is a powerful optimization technique to solve combinatorial optimization problems. The optimization problem can be formulated in the MIP format and an optimal solution can be obtained without involving heuristics, enabling the application of MIP approach to large-scale power systems. Although the large number of binary variables brings a tremendous computation burden, the advanced optimization technique of branch-and-cut, which combines branch-and-bound and cutting plane methods, makes MIP method
more attractive and applicable together with the commercially available software, such as CPLEX, LINDO, etc.

The above three new generation scheduling problems will be investigated in this research using advanced optimization techniques.

1.3 Literature Review

1.3.1 Optimal Generator Start-Up Strategy for Bulk Power System Restoration

System restoration following a blackout of power grids, for example, the Aug. 14, 2003, blackout in USA and Canada, Aug. 28, 2003, blackout in London, U.K., Sept. 23, 2003, blackout in Sweden and Denmark, Sept. 28, 2003, blackout in Italy, and May 24-25, 2005, blackout in Moscow, Russia [3]-[4], is one of the most important tasks for power system planning and operation. The restoration process returns the system to a normal operating condition following an outage of the system. Dispatchers are guided by restoration plans prepared off line while they assess system conditions, start BS units, establish the transmission paths to crank NBS generating units, pick up the necessary loads to stabilize the power system, and synchronize the electrical islands [5]-[6]. Power system restoration is a complex problem involving a large number of generation, transmission and distribution, and load constraints. In [7], the restoration process is divided into three stages: preparation, system restoration, and load restoration. According to these restoration stages, PSR strategies can be categorized into six types [8], i.e., Build-Upward, Build-Downward, Build-Inward, Build-Outward, Build-Together and Serve-Critical. Nevertheless, one common thread linking these stages is the generation availability at each stage [9].
The general requirements of PSR are defined by a series of standards developed by North American Electric Reliability Corporation (NERC). However, there are various PSR strategies for systems with different characteristics. Nevertheless, most bulk power systems share some common characteristics and different restoration strategies share a number of common guidelines [10]. Therefore, dispatchers must be able to identify the available BS capabilities and use the BS power strategically so that the generation capability can be maximized during the system restoration period. However, power system dispatchers are likely to face extreme emergencies threatening the system stability [11]. They need to be aware of the situation and adapt to the changing system conditions during system restoration. Therefore, utilities in the NERC Reliability Council regions conduct system restoration drills to train dispatchers in restoring the system following a possible major disturbance. There are simulation-based training tools; for example, EPRI-OTS and PowerSimulator, offer training on system restoration for control center dispatchers. However, practically no system restoration decision support tool has been widely adopted in an on-line operational environment of the bulk transmission systems.

The system restoration problem can be formulated as a multi-objective and multi-stage nonlinear constrained optimization problem [12]. The combinatorial nature of the problem presents challenges to dispatchers and makes it difficult to apply restoration plan system-wide. To better support the dispatchers in the decision making process, several approaches and analytical tools have been proposed for system restoration strategies. Heuristic methods [13] and mathematical programming [12] are used to solve this optimization problem. However, either the optimality of the solution cannot be guaranteed or the complexity affects the effectiveness of the restoration procedures for large-scale systems.
Knowledge based systems (KBSs) [14]-[17] have been developed to integrate both dispatchers’ knowledge and computational algorithms for system analysis. However, KBSs require special software tools and, furthermore, the maintenance of large-scale knowledge bases is a difficult task. The technique of artificial neural networks [18] has been proposed for system restoration. By the duality theory, the coupling constraints are adjoined to the objective function with Lagrangian multipliers. Then the LR problem is decomposed into subproblems, and the optimal solution is obtained by choosing from the strategy sets of individual problems’ possible states by parallel computing technique. To properly adjust Lagrangian multiplier, the augmented LR [19] introduces quadratic penalty terms into the Lagrangian function, which helps improve the convexity of the problem and the convergence of LR algorithm.

1.3.2 Optimal Installation Strategy of Blackstart Capability in Power System Restoration

After a partial or complete system blackout, BS resources initiate the process of system restoration and load recovery to return the system to a normal operating condition. Blackstart is a process of restoring a power station to operation without relying on external energy sources. A typical BS scenario includes BS generating units providing power to start large steam turbine units located close to these units. It also involves the supply of auxiliary power to nuclear power stations and off-site power to critical service load, such as hospitals and other public health facilities, and military facilities. Transmission lines must be available to deliver cranking power to NBS units or large motor loads, and transformer units, including step-up transformers of BS units and steam turbine units, and auxiliary transformers serving
motor control centers at the steam plant [20].

During PSR, BS generation availability is fundamental for all restoration stages [10]: stabilizing the system, establishing the transmission path, and picking up load. Available BS generating units must provide cranking power to NBS generating units in such a way that the overall available generation capability is maximized [9]. It may be helpful to install additional BS generators to accelerate the restoration process. After new BS generating units are installed, system restoration steps, such as generator startup sequence, transmission path, and load pick-up sequence, will change. However, there is a point where benefits of additional BS capabilities will not increase further. Therefore, power systems need to evaluate the strategy of both placement and size of new BS generators [21].

To better support dispatchers in the decision making process, several approaches and analytical tools have been proposed for assessment of BS capability. The KBS system restoration tool is developed in an EPRI project [7] to integrate both dispatchers’ knowledge and computational algorithms for system analysis. In [22], the Critical Path Method is applied to estimate system restoration time based on pre-selected PSR strategies. In [23], the MILP-based optimal generator start-up strategy is proposed to provide the overall system generation capability and update the solution throughout the BS process. The concept of Generic Restoration Actions (GRAs) is proposed in [10] to generalize various restoration steps in different system restoration strategies. In [15], the Petri Net algorithm is proposed to coordinate the schedule of GRAs. While different system restoration strategies share some characteristics, Generic Restoration Milestones (GRMs) are proposed in [24] to generalize power system restoration actions. Utilizing the concept of GRMs, the System Restoration
Navigator (SRN) is developed by EPRI to serve as a decision support tool for evaluating system restoration strategies.

1.3.3 Optimal Generation Scheduling in a Carbon Dioxide Allowance Market Environment

Greenhouse gases are gases that permit sunlight to go through the earth’s atmosphere and absorb infrared radiation or heat which is supposed to be re-radiated back to the space but is trapped in the atmosphere [25]. Greenhouse gases include water vapor, CO₂, methane (CH₄), nitrous oxide (N₂O) and ozone (O₃). The primary GHG emitted by human activities in the United States was CO₂, representing approximately 81.3 percent of total GHG emissions. Electricity generators consumed 36 percent of U.S. energy from fossil fuels and emitted 42 percent of the CO₂ from fossil fuel combustion in 2007 [26]. Electric generators rely on coal for over half of their total energy requirements and accounted for 94 percent of all coal consumed for energy in the United States in 2007. The challenge to the electric power industry is to meet the nation’s energy needs with the environmental control of emissions from electric power plants.

There are various climate change policies to regulate GHG emissions from the electric power industry. The Kyoto Protocol is an international agreement to reduce CO₂ and other GHG emissions in an effort to reduce climate change. It required that all industrialized countries reduce the average annual emissions at least 5 percent below the 1990 levels during the 2008 to 2012 period [27]. The Kyoto Protocol also introduced three market-based mechanisms: emission trading, the clean development mechanism, and joint implementation. European Union (EU) is committed to cutting its emissions at least 20% below 1990 levels
by 2020 [28]. The European Union Greenhouse Gas Emission Trading Scheme (EU ETS) is developed to offer the most cost-effective way for EU members to meet their Kyoto obligations and transform towards a low-carbon economy. This scheme also creates incentives to develop technologies for emission reduction.

In the U.S., the Regional Greenhouse Gas Initiative (RGGI) is an agreement among the Governors of ten Northeastern and Mid-Atlantic states to reduce GHG emissions from power plants. RGGI operates the first mandatory cap-and-trade program to cap regional power plants’ CO₂ emissions, and the cap will be 10 percent lower by 2018 than at the start of the RGGI program in 2009. The initial regional emissions budget is approximately 188 million short tons of CO₂, and each ton of CO₂ will constitute an “allowance” [29]. The number of issued allowances is controlled to ensure that total emissions in the region will not exceed the cap. The initial auction will offer allowances through a single-round, uniform-price, sealed-bid auction format. RGGI allows market forces to determine the most efficient and cost-effective means to regulate emissions. It provides a market signal to incorporate the cost of emitting carbon into energy pricing.

In response to the Assembly Bill AB32 and Senate Bill 1368, the California Energy Commission (CEC) introduced the “GHG emissions performance standard.” This standard prohibits investor-owned utilities (and later municipal utilities) from entering into long-term contracts to purchase electricity from sources that emit more CO₂ than a combined-cycle natural gas plant. While other states have not yet adopted restrictions on emissions from electricity production as California did, six western states and two Canadian provinces have recently announced that they will collaborate in achieving California’s stated goal of reducing 15% of the 2005 GHG emissions by 2020.
Carbon capture and storage (CCS) is a method to mitigate climate change by capturing CO₂ emissions from large power plants and other sources and storing it instead of releasing it into the atmosphere. However, the capturing process is costly and energy intensive, and CCS technologies are not economically feasible at the present time. Shifting from high CO₂ emission power sources to non-CO₂ or low CO₂ emission power sources, for example, hydroelectric, nuclear, wind, solar, photovoltaic, geothermal, ocean, etc., is an effective method to reduce CO₂ emissions. However, all these alternative resources have their limitations. The available and economical hydro resources are being exhausted. The long-term construction period of nuclear power is not helpful to achieve short-term CO₂ emission reduction requirements. Other renewable energy resources have significant potential, but the widespread applications are limited currently due to either being economically infeasible or technically pre-mature. Emission trading is an efficient market-based mechanism to regulate CO₂ emission. CO₂ emission cap-and-trade helps to reduce consumption in fossil energy and promote a shift to renewable energy resources.

There have been various research projects about the effects of emission constraints on the electric power system. Reference [30] included emission constraints in classical economic dispatch (ED) by weights estimation technique to solve environmentally constrained economic dispatch problem. The work of [31] provided a set of dispatching algorithms to solve the constrained emission dispatch problem with SO₂ and NOₓ emission constraints. References [32]-[33] presented a short-term unit commitment approach based on Lagrangian relaxation technique to solve the emission constrained unit commitment problem. However, all these models are developed to solve SO₂ or NOₓ emission regulation problem and these models cannot provide insights to the CO₂ emission regulation without detailed
modeling of CO₂ allowance market. References [34]-[35] formulated the electrical power and NOₓ allowances market as complementarity problems by using Cournot game. In [34], a nonlinear complementarity model is used to investigate long-run equilibria of alternative CO₂ emissions allowance allocation systems in electric power market. However, the daily electricity market and quarterly CO₂ allowance auction market should be incorporated in an appropriate time framework.

1.4 Contributions of this Dissertation

This dissertation is focused on the development of on-line decision support tools for power system restoration and optimal scheduling and bidding strategies for generation companies in both electricity and CO₂ allowance markets. The original contributions of this dissertation are summarized as follows:

1. An on-line decision support tool that assists system restoration operators and planners to restore power systems from major outages efficiently and safely

The tool uses the developed optimization modules of generator start-up sequence, transmission path search and the time to take restoration actions, which outperforms other simulation-based training tools in industry. It provides an automated and “best adaptive strategy” procedure to identify restoration decisions that will reduce restoration time while maintaining system integrity. This tool can be used to assist both system operators by enabling them to adapt to changing system conditions, and system planners by preparing them updated system restoration plan to achieve an efficient restoration process. This tool is a great enhancement to power system resilience.
2. An optimal generator start-up strategy for bulk power system restoration in an on-line operational environment

This strategy utilizes the proposed transformation techniques on the nonlinear generation capability curves to obtain the global optimal solution in a highly efficient way. It provides an initial starting sequence of all generators and also updates the generation capability as system restoration progresses. The technique outperforms methods in the literature in quality of solutions and computational speed. The optimization formulation does not require special maintenance and support of software tools, and it is more practical and suitable for the long term development of the on-line decision support tool.

3. An optimal installation strategy of blackstart generators in order to achieve an efficient restoration process

This strategy introduces a systematic way to assess blackstart capability and quantify the benefit from new blackstart generators based on the appropriate criteria. By utilizing the decision support tool to estimate system restoration time and evaluate system generation capability, the optimal installation location and amount of blackstart capability is provided for system planners to prepare power system restoration plan and enhance resilience of the electric power grid.

4. A coordinated optimal strategy that assists generation companies to maximize their profits participating in both electricity market and CO₂ allowance market

Based on an industry model, CO₂ allowance market is analyzed using Cournot equilibrium model to achieve the goal of a sustainable and environment-friendly power grid. This dissertation is the first to analyze the emission-constrained
generation scheduling problem in the three-year CO2 allowance compliance period, involving generation maintenance scheduling, unit commitment and CO2 allowance cap-and-trade. The strategy facilitates generation companies in their decision makings of optimal mid-term generation maintenance scheduling and CO2 allowance bidding in two markets.

1.5 Thesis Organization

The remainder of this dissertation is organized as follows. Chapter 2 presents the optimal generator start-up strategy in power system restoration, based on the results of Sun et al. [9], [23]. The combinatorial optimization problem of generator start-up sequencing is formulated as a MILP problem. The simulation results of the IEEE 39-Bus system, American Electric Power (AEP), and Entergy test cases demonstrate the high efficiency of the proposed strategy. Chapter 3 presents the optimal installation strategy of black start capability in power system restoration, based on the results of Sun et al. [21], [41]. Based on the developed optimization modules of generator start-up sequencing and transmission path search, considering the time to take GRAs, the optimal installation of black start capability is formulated as a MIBLP problem. A novel solution methodology is proposed, and it is shown that power systems can benefit from new BS generators by reducing total restoration time and increasing system generation capability. However, there is a maximum amount beyond which system restoration time cannot be further reduced with additional BS capability. Chapter 4 presents the optimal generation scheduling in a CO2 allowance market environment, based on the results of Sun et al. [54]. The emission-constrained GSP involving generation maintenance scheduling, unit commitment and CO2 emission cap-and-trade is
formulated as a MIBLP problem. Simulation results based on the PJM 5-bus system test case demonstrate that the proposed MIBLP-based model is able to provide valuable information for GENCOs’ decision makings in both electricity and CO₂ allowance markets. Finally, Chapter 5 provides a summary of this dissertation’s contributions and discusses the proposed future research directions.
CHAPTER 2. OPTIMAL GENERATOR START-UP STRATEGY FOR BULK POWER SYSTEM RESTORATION

During system restoration, it is critical to utilize the available BS units to provide cranking power to NBS units in such a way that the overall system generation capability will be maximized. The corresponding optimization problem is combinatorial with complex practical constraints that can vary with time. This research provides a new formulation of generator start-up sequencing as a MILP problem. The linear formulation leads to an optimal solution to this important problem that clearly outperforms heuristic or enumerative techniques in quality of solutions or computational speed. The proposed generator start-up strategy is intended to provide an initial starting sequence of all BS or NBS units. The method can provide updates on the system MW generation capability as the restoration process progresses. The IEEE 39-Bus system, AEP, and Entergy test cases are used for validation of the generation capability optimization. Simulation results demonstrate that the proposed MILP-based generator start-up sequencing algorithm is highly efficient.

2.1 System Restoration Procedure

A comprehensive strategy to facilitate system restoration is to develop computational modules for the generation, transmission and distribution subsystems. The primary modules in Figure 2.1 are generation capability maximization, transmission path search, and constraint checking. The focus of this research is on the module for generation capability optimization. Other modules are developed by team members in the supported Power Systems Engineering Research Center (PSERC) project. Identification of generator start-up sequence in order to
maximize the MW generation capability is a complex combinatorial problem. The quality of solution depends on available blackstart capabilities, transmission paths, and technical and non-technical constraints.

![Power System Restoration Diagram](image)

**Figure 2.1. Power system restoration strategy**

The developed optimization modules shown in Figure 2.1 are not separate from each other. Rather, they interact with each other to develop a power system restoration plan that incorporates generation, transmission, distribution and load constraints. Specifically, *Generation Capability Optimization Module* (GCOM) first provides an optimal generator starting sequence, and *Transmission Path Search Module* (TPSM) identifies the paths for the cranking sequence from GCOM and energizes the transmission network. Then *Distribution Restoration Module* (DRM) provides the load pickup sequence to maintain the system stability and minimize the unserved load. The sequences from GCOM and DRM and the transmission paths from TPSM are checked by *Constraint Checking Module* (CCM) using
power system simulation software tools to ensure that various constraints are met. They interact with each other by receiving the input from and passing the output to other modules. Eventually, this will lead to the successful restoration of a power system.

If there is a constraint violation, the corresponding constraint will be added back to specific module and the restoration plan will be updated accordingly. For example, if the transmission path is unavailable, say due to a fault on a line, which causes a unit in the starting sequence to become unavailable, then the corresponding constraint will be added to *Generation Capability Optimization Module* to determine a new cranking sequence so that the unit can be cranked by other units that are available through another path. For another example, if one generating unit cannot be started, say, due to a generator transient stability limits violation, then *Generation Capability Optimization Module* will update the starting time of this generator, which will be delayed until after the planned starting.

### 2.2 Maximizing Generation Capability during System Restoration

#### 2.2.1 Generator Characteristic and Constraints

According to the start-up power requirement, generating units can be divided into two groups: BS generators and NBS generators. A blackstart generator, e.g., hydro or combustion turbine units, can be started with its own resources, while NBS generators, such as steam turbine units, require cranking power from outside. It is assumed that all available BS generators are started at the beginning of system restoration.

*Objective function*: The objective is to maximize the overall system MW generation capability during a specified system restoration period. The system generation capability is
defined as the sum of MW generation capabilities over all units in the power system minus the start-up power requirements.

**Constraints:** NBS generators may have different physical characteristics and requirements. The terms, “critical maximum time interval,” and “critical minimum time interval,” have been used in [7]. If a NBS unit does not start within the corresponding critical maximum time interval $T_{cmax}$, the unit will become unavailable after a considerable time delay. On the other hand, a NBS unit with the critical minimum time interval constraint $T_{cmin}$, is not ready to receive cranking power until after this time interval. Moreover, all NBS generators have their start-up power requirements. These units can only be started when the system can supply sufficient start-up power $P_{start}$. Based on these definitions, the generator start-up sequencing problem can be formulated as:

\[
\text{Max} \quad \text{Overall System Generation Capability}
\]

subject to

- Critical Minimum & Maximum Time Intervals
- Start-Up Power Requirements

The solution to this optimization problem will provide the optimal starting sequence for all BS and NBS units. The MW capability of a BS or NBS generator $P_{gen}$ is illustrated in Figure 2.2. The area between its generation capability curve and the horizontal axis represents the total MW capability over the duration of a system restoration period. In Figure 2.2, $P_{max}$ is maximum generator MW output, $t_{start}$ is generator starting time, $t_{ctp}$ is cranking time for generators to begin to ramp up and parallel with system, $R_r$ is the generator ramping rate, and $T$ is the specified system restoration period.
Normally, the cranking power for a NBS unit comes from a BS unit nearby. Then this limited BS resource can be treated as BS MW capability. In an unusual case, the BS units can be used to support NBS units further away. The proposed method can handle both scenarios. The available BS MW capability can be added to the constraint of MW Startup Requirement as a source for cranking power. The proposed strategy will then provide the starting sequence for the NBS units. By use of the shortest path search algorithm for transmission paths in the proposed method, a NBS unit will have priority to receive cranking power from the BS unit(s) nearby. The system condition in a blackout scenario may deviate from the assumption in the System Restoration Blackstart Plan, say, due to unavailability of the nearby BS units. Therefore, the actual cranking unit and its switching sequence may be different.

It is assumed that all available BS generators can be started at the beginning of system restoration. (Theoretically, all BS units can be started after the recognition of the system situation. In reality, however, it depends on the actual system situation, such as fuel
availability of the BS unit, success of load rejection of the Automatic Load Rejection units, and availability of the cranking path.) A different starting time of a BS unit can be incorporated in the proposed method by changing the starting time of the generation capability curve in Figure 2.1.

2.2.2 Optimal Generator Start-Up Strategy

In the above formulation, a complete shutdown of the power system is assumed. It is also assumed that each generator can be started and the cranking power can be delivered through the transmission network. During system restoration, it is likely that some BS, NBS units or transmission paths become unavailable due to, say, line faults or fuel problems. The following modifications have been incorporated into the proposed algorithm so that the proposed decision support tool can adapt to the actual system conditions.

**Critical generators:** If there is a critical generator $i$ that has to be started first, then the following constraint is added to ensure that unit $i$ has the earliest starting time, i.e.,

$$
t_{istart} = \min \{ t_{jstart}, \quad j = 1, \ldots, M \}$$

where $M$ is the number of NBS units.

**Generator cuts:** If a generator cannot be started due to the lack of cranking power, the algorithm will remove the generator and calculate a new start-up sequence. If there is a feasible solution, the one that results in the maximum generation capability among all possible combinations $C_M^i$ will be chosen. Otherwise, the algorithm will remove more generators, until feasible solutions are found. The number of total iterations is

$$N_{cut} \sum_{i=1}^{N_{cut}} C_M^i,$$

where $N_{cut}$ is the number of NBS generators that cannot be started.
No available transmission paths: Suppose that transmission paths are not available to deliver cranking power to start some NBS generator $G_i$. However, after another unit $G_j$ is started, the system will have cranking power to start $G_i$. In this case, the following constraint is added and the optimization problem is solved again to find the new optimal starting sequence.

$$t_{i\text{start}} > t_{j\text{start}}$$ (2.2)

Partial blackstart: If at the beginning of system restoration, the system has some power sources available, then this part of already existed power $P_{\text{source}}$ can be added to the constraint of MW Startup Requirement as a source for cranking power.

Voltage and reactive power have to be carefully considered during the development of System Restoration Blackstart Plan and the execution of a blackstart switching sequence. Voltage constraint at system and plant should be within the required range, e.g., 95-105% or 90-105% depending on the requirement of different systems. Factors related to voltage and reactive power need to be incorporated, such as real and reactive capability of generating units, line charging including underground cable charging, shunt capacitor and shunt reactor, and startup of large motors. In the proposed restoration procedure, the reactive power control and constraint checking are performed by the Constraint Checking Module in Figure 2.1. If any violation occurs, the corresponding constraint will be added and Generation Capability Optimization Module will be used to calculate a revised solution.
2.3 The “Two-Step” Algorithm

2.3.1 Problem Formulation

**Definition of quasiconcavity**: A function $f$ is quasiconcave if and only if for any $x, y \in \text{dom} f$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1-\theta) y \right) \geq \min \left\{ f(x), f(y) \right\}$$

(2.3)

In other words, the value of $f$ over the interval between $x$ and $y$ is not smaller than $\min \left\{ f(x), f(y) \right\}$.

With the above definition, one can obtain the following lemma:

**Lemma**: The generation capability function is quasiconcave.

The proof can be referred to Appendix A.

Convex optimization is concerned with minimizing convex functions or maximizing concave functions. Optimality cannot be guaranteed without the property of convexity or concavity. Due to the quasiconcavity property, one cannot directly use the general convexity-based or concavity-based optimization method for developing solutions.

Therefore, a “Two-Step” method is proposed to solve the quasiconcave optimization problem. For each generator, the generation capability curve is divided into two segments. One segment $P_{\text{igen1}}$ is from the origin to the “corner” point where the generator begins to ramp up, as shown by the red line in Figure 2.3. The other segment $P_{\text{igen2}}$ is from the corner point to point when all generators have been started, as shown by the blue line in Figure 2.3.
The quasiconcave function is converted into two concave functions. Then time horizon is divided into several time periods, and in each time period, generators using either first or second segment of generation capability curves. The quasiconcave optimization problem is converted into concave optimization problem, which optimality is guaranteed in each time period.

First, define the following sets:

- **ASG**: set of all already started generators;
- **BSG**: set of all BS generators;
- **NBSG**: set of all NBS generators;
- **NBSGMIN**: set of NBS generators with constraint of $T_{cmin}$;
- **NBSGMAX**: set of NBS generators with constraint of $T_{cmax}$.

The objective function can be written as

$$
\max \sum_{t=1}^{N_t} \sum_{i=1}^{N} \left[ P_{igen}(t) - u_i^t \left(1 - u_i^{t-1}\right) P_{isqrt}(t) \right] 
$$

(2.4)
where $N$ is the number of total generation units, binary decision variable $u'_i$ is the status of NBS generator at each time slot, which $u'_i=1$ means $i$th generator is on at time $t$, and $u'_i=0$ means $i$th generator is off. It is assumed that all BS generators are started at the beginning of restoration.

**Critical Time Constraints:** Generators with constraints of $T_{c_{\text{max}}}$ or $T_{c_{\text{min}}}$ should satisfy the following inequalities:

$$
\begin{align*}
& t_{\text{start}} \geq T_{c_{\text{min}}}, \quad i \in \text{NBSGMIN} \\
& t_{\text{start}} \leq T_{c_{\text{max}}}, \quad i \in \text{NBSGMAX}
\end{align*}
$$

(2.5)

**Start-Up MW Requirements Constraints:** NBS generators can only be started when the system can supply sufficient cranking power:

$$
\sum_{i=1}^{N} \left[ P_{\text{gen}}(t) - u'_i \left(1-u'^{-1}_i\right) P_{\text{start}}(t) \right] \geq 0, \quad t = 1, \ldots, N_T
$$

(2.6)

Generator capability function $P_{\text{gen}}(t)$ can be expressed as:

$$
P_{\text{gen}}(t) = P_{\text{gen1}}(t) + P_{\text{gen2}}(t)
$$

(2.7)

where,

$$
P_{\text{gen1}}(t) = 0 \quad 0 \leq t \leq t_{\text{start}} + T_{\text{icp}}
$$

(2.8)

$$
P_{\text{gen2}}(t) = R_{ni} \left(t - t_{\text{start}} - T_{\text{icp}}\right) \quad t_{\text{start}} + T_{\text{icp}} \leq t \leq T
$$

(2.9)

$$
P_{\text{gen2}}(t) \leq P_{\text{max}}
$$

(2.10)

**Generator Status Constraints:** It is assumed that once generator was restarted, it will not be tripped offline again, which is guaranteed by the following inequalities:

$$
u'^{-1}_i \leq u'_i, \quad i = 1, \ldots, N, \quad t = 2, \ldots, N_T
$$

(2.11)
Then the generator start-up sequencing problem can be formulated as a Mixed Integer Quadratically Constrained Program (MIQCP) problem:

$$\max \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ (P_{i1}(t) + P_{i2}(t)) - u'_i (1 - u'^{-1}_i) P_{istart} \right]$$

s.t. $t_{istart} \geq T_{min}, \quad i \in NBSGMIN$

$$t_{istart} \leq T_{max}, \quad i \in NBSGMAX$$

$$\sum_{i=1}^{N} \left[ (P_{i1}(t) + P_{i2}(t)) - u'_i (1 - u'^{-1}_i) P_{istart} \right] \geq 0$$

$$P_{i1}(t) = 0 \quad 0 \leq t < t_{istart} + T_{icp}$$

$$P_{i2}(t) = R_i \left( t - t_{istart} - T_{icp} \right) \quad t_{istart} + T_{icp} \leq t \leq T$$

$$P_{i2}(t) \leq P_{\text{max}}$$

$$u'^{-1}_i \leq u'_i, \quad t = 2, \ldots, N_t$$

$$i = 1, \ldots, N, \quad t = 1, \ldots, N_T$$

$$u'_i \in \{0,1\}, \quad t_{istart} \text{ integer}$$

### 2.3.3 Algorithm

Start solving the optimization problem with all generators using the first segment of generation capability function $P_{i1}(t)$. The restoration time $T$ at which all NBS generators (excluding nuclear generators that usually require restart time greater than the largest critical minimum time of all generators) have been restored, is discretized into $N_T$ equal time slots. Beginning at $t = 1$, the optimization problem is solved and the solution is recorded. Then at $t = 2$, the problem is solved again to update the solution. This iteration continues until $t = N_T$ by advancing the time interval according to the following criteria:

1) If generation capability function of every generator $\in ASG$ has been updated from $P_{i1}(t)$ to $P_{i2}(t)$, set $t = t + 1$;
2) If every generator $\in NBSGMAX$ have been started, set $t = \min \{T_{icmin}, i \in NBSMIN\}$;

3) If all generators have been started, set $t = T$;

4) Otherwise, set $t = \min \{t_{istart} + T_{ic3p}, i \in ASG\}$.

Then in the next iteration, if any generator reaches its maximum capability, update the generation capability function from $P_{igen1}(t)$ to $P_{igen2}(t)$. At this time, some generators are in their first segments of the capability curves and others are in the second segments. During the process, if any new generator was started, add it to the set $ASG$. Then the problem can be solved each time period by time period until all generators have been started. The number of total time periods is different for each individual case.

The flow chart of the proposed “two-step” algorithm is shown in Figure 2.4.
Generator Data
Set $t = 0$

$\text{ASG} = \text{BSG}$

$\text{ASG} = \emptyset$

$t = t + \min \{ t_{\text{start}} + T_{\text{up}}, i \in \text{ASG} \}$

$t = T$, solve MIQCP

All generators have been started up

$\text{ASG} = \emptyset$

$t = t + 1$

$t = t + \min \{ T_{\text{max}}, i \in \text{NBSGMIN} \}$

Solve MIQCP

Any new generator has been start up

$t = \min \{ t_{\text{start}} + T_{\text{up}}, i \in \text{ASG} \}$

Add to ASG

$t \leq t_{\text{start}} + T_{\text{up}}, i \in \text{ASG}$

$\text{ASG} = \emptyset$

$t = t_{\text{start}} + T_{\text{up}}, i \in \text{ASG}$

update $P_{\text{gov}}(t)$

$\text{ASG} = \text{ASG} \setminus \{i\}$

Stop

$t = t + 1$

Figure 2.4. Flow chart of “Two-Step” algorithm
2.4 The Mixed Integer Linear Programming-Based Strategy

2.4.1 Objective Function

The objective is to maximize the generation capability during the restoration period. The system generation capability $E_{sys}$ is the total system MW capability minus the start-up requirements [7], given by:

$$E_{sys} = \sum_{i=1}^{N} E_{igen} - \sum_{j=1}^{M} E_{jstart}$$

(2.13)

where $E_{igen}$ is MW capability of generator $i$, $E_{jstart}$ is start-up requirement of NBS generator $j$ and $N$ is the total number of generation units.

2.4.2 Constraints

**Critical minimum and maximum intervals:**

$$T_{j_{cmin}} \leq t_{jstart} \leq T_{j_{cmax}}, \quad j = 1, 2, \ldots, M$$

(2.14)

**Start-up power requirement constraints:**

$$\sum_{i=1}^{N} P_{igen}(t) - \sum_{j=1}^{M} P_{jstart}(t) \geq 0, \quad t = 1, 2, \ldots, T$$

(2.15)

where $P_{igen}(t)$ is the generation capability function of unit $i$, and $P_{jstart}(t)$ is the start-up power function of NBS unit $j$.

The above formulation leads to a nonlinear combinatorial optimization problem. The proposed formulation of a MILP problem relies on a 4-step transformation to be described in the following.
Step 1: Introduce binary decision variables $w_{i1}, w_{i2}$ and linear decision variables $t_{i1}', t_{i2}', t_{i3}'$ to define generator capability function $P_{gen}(t)$ (piecewise linear function) in linear and quadratic forms.

$$
P_{gen}(MW)
$$

Figure 2.5. Generation capability function

The point $(t_{start} + t_{icp}, 0)$, where generator begins to ramp up, and point $(t_{start} + t_{icp} + P_{max}/R_{ri}, P_{max})$, where generator reaches its maximum generation capability, separate the curve into three segments. The symbols $t_{i1}', t_{i2}', t_{i3}'$ represent the three segments, and $w_{i1}, w_{i2}$ restrict these three variables within the corresponding range. Then the MW capability of each generator $E_{gen}$, over the system restoration horizon is represented by the shaded area in Figure 2.5, i.e.,

$$
E_{gen} = \frac{1}{2} P_{max} \frac{P_{max}}{R_{ri}} + P_{max} \left[ T - \left( t_{start} + T_{icp} + \frac{P_{max}}{R_{ri}} \right) \right]
$$

(2.16)
Step 2: Introduce binary decision variables $w_{j3}$ and linear decision variables $t_{j4}, t_{j5}$ to define generator start-up power function $P_{j\text{start}}(t)$ (step function) in linear and quadratic forms.

$$P_{\text{start}} \text{ (MW)}$$

$P_{\text{jstart}}$ $t_{j\text{start}}$ $T$ $t$

$$t'_{j4} \quad t'_{j5}$$

Figure 2.6. Generator start-up power function

The point $(t_{j\text{start}}, 0)$, where NBS generator receives the cranking power for start-up, separates the curve into two segments. The symbols $t'_{j4}, t'_{j5}$ represent the segments and $w_{j3}$ restricts these variables within the corresponding range.

Then the start-up requirement for each NBS generator $E_{j\text{start}}$, is represented by the shaded area in Figure 2.6. That is,

$$E_{j\text{start}} = P_{j\text{start}} (T - t_{j\text{start}})$$  \hspace{1cm} (2.17)

Using (2.16) and (2.17), (2.13) can be expressed as follows:

$$E_{\text{sys}} = \left\{ \sum_{i=1}^{N} \left[ \frac{(P_{\text{max}})^2}{2 \cdot R_{i}} + P_{\text{max}} \left( T - T_{\text{csp}} - \frac{P_{\text{max}}}{R_{i}} \right) \right] - \sum_{j=1}^{M} P_{j\text{start}} T \right\} - \left( \sum_{i=1}^{N} P_{\text{max}} t_{i\text{start}} - \sum_{j=1}^{M} P_{j\text{start}} t_{j\text{start}} \right)$$  \hspace{1cm} (2.18)
The above equation shows that system generation capability consists of two components. The first component (in braces) is constant, and the second component is a function of decision variable \( t_{\text{start}} \). Note that BS units are assumed to be started at the beginning of system restoration. Their starting times are zero. Therefore, the first summation of the second component in (2.18) can be reduced from \( N \) to \( M \). Based on the observation, the objective function can be simplified as:

\[
\max E_{\text{sys}} \iff \min \sum_{j=1}^{M} (P_{j_{\text{max}}} - P_{j_{\text{start}}}) t_{j_{\text{start}}}
\]  

(2.19)

In the equations derived in Steps 1 and 2, the quadratic component has the same structure, i.e., a product of one binary decision variable and one integer decision variable.

**Step 3:** Introduce new binary variables \( u_{jt} \) to transform the quadratic component into the product of two binary variables.

\[
w'_{jht} t_{j_{\text{start}}} \Rightarrow w'_{j} \left( \sum_{t=1}^{T} (1-u_{jt}) + 1 \right) \quad h=1,2,3
\]  

(2.20)

where \( u_{jt} \) is the status of \( NBS \) generator \( j \) at each time slot. The value \( u_{jt} = 1 \) means \( j_{\text{th}} \) generator is on at time \( t \), and \( u_{jt} = 0 \) means \( j_{\text{th}} \) generator is off. The symbol \( u_{jt} \) satisfies the following constraints.

\[
t_{j_{\text{start}}} = \sum_{t=1}^{T} (1-u_{jt}) + 1
\]  

(2.21)

\[
u_{jt} \leq u_{j(t+1)}
\]  

(2.22)

Each NBS generator’ starting time is the total number of its off-state plus one, which is denoted by (2.21). Moreover, it is assumed that once a generator is started, it will not be taken offline, as shown by (2.22).
Step 4: Introduce new binary variables \( \nu^i_{j3t} \), \( \nu^i_{j2t} \) and \( \nu^i_{jt} \) to transform the product of two binary variables into one binary variable.

\[
\nu^i_{jht} = w^i_{jht} \quad h = 1, 2, 3
\]

(2.23)

It can be seen that \( \nu^i_{jt} \), \( \nu^i_{j2t} \) and \( \nu^i_{j3t} \) satisfy the following constraints:

\[
\nu^i_{jht} \leq w^i_{jht}, \quad h = 1, 2, 3
\]

(2.25)

\[
\nu^i_{jht} \leq u^i_{jht}, \quad h = 1, 2, 3
\]

(2.26)

By taking the four Steps, generator capability function \( P_{igen(t)} \) can be written as:

\[
P_{igen}(t) = R_i \left( t - t^i_{11} - t^i_{12} \right)
\]

(2.27)

subject to the following constraints:

\[
w^i_{1t} T_{icp} \leq t^i_{1t} \leq T_{icp} \quad P_{igen}(t) = R_i \left( t - t^i_{11} - t^i_{12} \right)
\]

(2.28)

\[
(T + 1 + T_{icp}) w^i_{1t} - \sum_{t=1}^{T} \nu^i_{jht} \leq t^i_{1t} \leq t^i_{1t} = t_{jstart} + T_{jcp}
\]

(2.29)

\[
w^i_{12} \frac{P_{\max}}{R_i} \leq t - t^i_{12} - t^i_{12} \leq w^i_{11} \frac{P_{\max}}{R_i}
\]

(2.30)

\[
t^i_{12} \leq w^i_{12} \left( T - T_{icp} + \frac{P_{\max}}{R_i} \right)
\]

(2.31)

\[
t^i_{j2t} \leq \sum_{t=1}^{T} \nu^i_{j2t} - \left( T_{jcp} + \frac{P_{\max}}{R_j} - 1 \right) w^i_{j2t}
\]

(2.32)

\[
w^i_{j2t} \leq w^i_{j1}
\]

(2.33)
where \( t'_{ih} \in \{0,1,\cdots,T\} \), \( w'_{ih}, u_{ij}, v'_{jhl} \in \{0,1\} \), \( t \in (0,1,\cdots,T) \), \( i = 1,2,\cdots,N \), \( j = 1,2,\cdots,M \) and \( l = 1,2,\ldots,N-M, h = 1,2 \).

Inequality constraints (2.28)-(2.33) restrict each segment within the corresponding range of the piecewise linear function \( P_{i\text{gen}}(t) \).

The generator start-up power function \( P_{j\text{start}}(t) \) can be expressed as:

\[
P_{j\text{start}}(t) = w^j_{j3} P_{j\text{start}}
\]  

subject to the following constraints:

\[
w^j_{j3} T - \sum_{r=1}^{T} v^j_{j3r} \leq t^j_{j3} \leq t - 1
\]  

\[
w^j_{j3} \leq t - t^j_{j3} \leq \sum_{r=1}^{T} v^j_{j3r}
\]  

Inequality constraints (2.35) and (2.36) restrict each segment within the corresponding range of the step function \( P_{j\text{start}}(t) \).

Then (2.15) can be simplified as:

\[
\sum_{i=1}^{N} R_{i} \left( t - t'_{i1} - t'_{i2} \right) - \sum_{j=1}^{M} w^j_{j3} P_{j\text{start}} \geq 0 \quad t = 1,2,\ldots,T
\]  

Finally, the problem is transformed into a MILP problem. The global optimal starting sequence for all generators is obtained by solving optimization problem (2.38). The proposed method leads to global optimality for the formulated generator start-up optimization problem. Although the global optimality is true in a mathematical sense, it should be cautioned that when the developed module incorporates more constraints from other modules in Figure 2.1, the global optimality will be compromised.
\[
\min \sum_{j=1}^{M} \left( P_{j \text{max}} - P_{j \text{start}} \right) t_{j \text{start}}
\]

\begin{align*}
\text{Eq. (2.14)} & \quad \Leftarrow \text{constraints of critical time intervals} \\
\text{Eq. (2.37)} & \quad \Leftarrow \text{constraints of MW start-up requirement} \\
\text{Eq. (2.28–2.33)} & \quad \Leftarrow \text{constraints of generator capability function} \\
\text{Eq. (2.35–2.36)} & \quad \Leftarrow \text{constraints of generator start-up power function} \\
\text{Eq. (2.21–2.26)} & \quad \Leftarrow \text{constraints of decision variables}
\end{align*}

*Generation Capability Optimization Module* provides an initial starting sequence of all BS and NBS units. The feasibility of the sequence needs to be checked to ensure that transmission paths are available and various constraints are met. This is achieved through interactions with *Transmission Path Search* and *Constraint Checking Modules*, as shown in Figure 2.1. If a unit in the starting sequence cannot be started, say, due to the lack of a transmission path, the subsequence following that unit needs to be re-calculated by the *Generation Capability Optimization Module*. Also, the restoration process depends on switching of lines, bus bars, and load. The time to take each action depends on the actual scenario. These times must be added to the generation start up times in order to obtain an estimate of the restoration time. When there is a transmission violation, the corresponding capacity constraint will be added by *Generation Capability Optimization Module* so that the starting time of the generator in the previous step will be delayed until after the planned starting. Specific transmission constraint checking will be accomplished in *Transmission Path Search Module*. 


2.5 Optimal Transmission Path Search Module

By assigning each bus or line one integer decision variable \( u_{bus,m,n} \) to represent its status, which 1 represents energized and 0 represents de-energized, the optimal transmission path search problem can be formulated as a MILP problem to find the status of each bus or line at each time. The detailed formulations are shown as following.

First, define the following sets:

- \( \Omega_t = 1...T \) Set of Time
- \( \Omega_{t-1} = 1...T-1 \) Set of Time
- \( \Omega_{t-2} = 2...T \) Set of Time
- \( \Omega_{bus} = 1...N_{bus} \) Set of Bus Number
- \( \Omega_{line} = 1...N_{line} \) Set of Line Number
- \( \Omega_{BSU} = 1...N_{BSU} \) Set of BSU Number
- \( \Omega_{NBSU} = 1...N_{NBSU} \) Set of NBSU Number
- \( \Omega_{ALLU} = 1...N_{ALLU} \) Set of All Generators Number
- \( \Omega_{line,m} \) Set of Line connected with Bus \( m \)
- \( \Omega_{bus,BSU} \) Set of Bus connected with BSU

**Objective Function:** The objective is to maximize the total generation output.

\[
\text{Max} \sum_{t \in \Omega_t} \sum_{n \in \Omega_{NBSU}} u_{bus,n}^t P_{gen,n}^t
\]  

(2.39)

**Constraints:**

1. If both connected buses are de-energized, then the line is de-energized.
\[-M u_{busm}^i \leq u_{linemn}^i \leq M u_{busm}^i, \quad t \in \Omega, mn \in \Omega_{line}, m \& n \in \Omega_{bus}\] (2.40)

\[-M u_{busn}^i \leq u_{linemn}^i \leq M u_{busn}^i, \quad t \in \Omega, mn \in \Omega_{line}, m \& n \in \Omega_{bus}\] (2.41)

2. If both connected buses are de-energized at \(t\), then the line is de-energized at \(t+1\); if the line is energized at \(t+1\), then at least one of connected buses is energized at \(t\).

\[-M (u_{busm}^i + u_{busn}^i) \leq u_{linemn}^{i+1} \leq M (u_{busm}^i + u_{busn}^i), \quad t \in \Omega, mn \in \Omega_{line}, m \& n \in \Omega_{bus}\] (2.42)

3. For any bus that not connected with BSU, if all lines connected with this bus are de-energized, then this bus is de-energized.

\[-M \sum_{mn \in \Omega_{line}} u_{linemn}^i \leq u_{busm}^i \leq M \sum_{mn \in \Omega_{line}} u_{linemn}^i, \quad t \in \Omega, m \in \Omega_{bus}/\Omega_{bus-BSU}\] (2.43)

4. Once bus or line is energized, it won’t be de-energized again.

\[u_{linemn}^i \leq u_{linemn}^{i+1}, \quad t \in \Omega, mn \in \Omega_{line}\] (2.44)

\[u_{busm}^i \leq u_{busm}^{i+1}, \quad t \in \Omega, m \in \Omega_{bus}\] (2.45)

5. All the lines’ initiate states are de-energized; all the buses’ initiate states are de-energized, except the ones connected with BSU are energized.

\[u_{linemn}^i = 0, \quad t = 1, mn \in \Omega_{line}\] (2.46)

\[u_{busm}^i = 1, \quad t = 1, m \in \Omega_{bus-BSU}\] (2.47)

\[u_{busm}^i = 0, \quad t = 1, m \in \Omega_{bus}/\Omega_{bus-BSU}\] (2.48)

6. Transmission line thermal limit constraint

\[-(1-u_{linemn}^i)M \leq f_{mn}^i - B_{mn} (\theta_m^i - \theta_n^i) \leq (1-u_{linemn}^i)M, \quad t \in \Omega, m \in \Omega_{bus}, mn \in \Omega_{line}\] (2.49)

\[-u_{linemn}^i F_{mn} \leq f_{mn}^i \leq u_{linemn}^i F_{mn}, \quad t \in \Omega, m \in \Omega_{bus}, mn \in \Omega_{line}\] (2.50)

where, \(M\) is the arbitrarily large number, \(f_{mn}^i\) is the power flow on line \(mn\) at time \(t\), \(\theta_m^i\) is the
bus \( m \) voltage angle, \( B_{mn} \) is the susceptance of line \( mn \), \( F_{mn} \) is the thermal limit for real power flow on line \( mn \).

### 2.6 Adding the Time of GRAs into the Optimization Modules

The concept of GRAs is proposed in [10] to generalize various restoration steps in different system restoration strategies. The time to take restoration actions should be considered to achieve more accurate estimation of total restoration time. The (fictitious) time to complete each GRA is given in Table 2.1. Then each GRA time can be integrated into the optimization modules in the following way:

1. **BSU Module**
   - \( t=0 \), start BSU
   - \( t=\max\{T_{ctp}, T_{GRA1}=15\} \), BSU is ready
   - \( t= \max\{T_{ctp}, T_{GRA1}=15\}+5 \), bus connected with BSU is energized

2. **NBSU Module**
   - \( t=0 \), crank NBSU from bus
   - \( t=15 \), synchronize NBSU with bus
   - \( t=15+\max\{T_{ctp}, T_{GRA5}=20\} \), NBSU is ready

3. **Bus/Line Module**

   Each line needs the following time to energize:

   \((T_{GRA3}=5 \text{ mins}) + (T_{GRA8}=5 \text{ mins}) = 10 \text{ mins.}\)

4. **Load Module**
   - \( t=0 \), pick up load from bus
   - \( t=10 \), load is ready
5. If necessary, add $T_{GRA6}=25$ mins to connect tie line.

**Table 2.1 Time to Complete GRAs**

<table>
<thead>
<tr>
<th>GRA</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRA1: start_black_start_unit</td>
<td>$T_{GRA1}=15$</td>
</tr>
<tr>
<td>GRA2: find_path</td>
<td>$T_{GRA2}=N/A$</td>
</tr>
<tr>
<td>GRA3: energize_line</td>
<td>$T_{GRA3}=5$</td>
</tr>
<tr>
<td>GRA4: pick_up_load</td>
<td>$T_{GRA4}=10$</td>
</tr>
<tr>
<td>GRA5: synchronize</td>
<td>$T_{GRA5}=20$</td>
</tr>
<tr>
<td>GRA6: connect_tie_line</td>
<td>$T_{GRA6}=25$</td>
</tr>
<tr>
<td>GRA7: crank_unit</td>
<td>$T_{GRA7}=15$</td>
</tr>
<tr>
<td>GRA8: energize_bus</td>
<td>$T_{GRA8}=5$</td>
</tr>
</tbody>
</table>

**2.7 Connections of Optimization Modules**

The optimization modules of generator start-up sequencing and transmission path search, considering the time to take restoration actions, can be integrated by adding the following constraints:

1. NBSU will be started after the connected bus being energized.

   $$u_{bus}^m \geq u_j^t, \quad m \in \Omega_{bus-NBSU}, \quad j \in \Omega_{NBSU}, \quad t \in \Omega_t$$  \hspace{1cm} (2.51)

2. The starting time of NBSU equals the time when the bus connected with NBSU being energized plus the time of GRA7.

   $$\sum_{t \in \Omega_t} (1-u_{bus}^m) + 1 + T_{GRA7} = \sum_{t \in \Omega_t} (1-u_j^t) + 1$$

   $$m \in \Omega_{bus-NBSU}, \quad j \in \Omega_{NBSU}$$  \hspace{1cm} (2.52)
3. The time when the bus connected with BSU being energized equals the sum of the starting time of BSU (zero), the time of GRA8 and the larger value between the time of GRA1 and $T_{csp}$ of BSU.

$$\sum_{t \in \Omega} (1-u_{bus,m}^t) + 1 = \max \left\{ T_{GRA1}, T_{csp} \right\} + T_{GRA8} + 0$$

$$m \in \Omega_{bus-BSU}, i \in \Omega_{BSU}$$

(2.53)

Then change equation (2.47) to that all buses are de-energized at $t=1$.

Then all the developed optimization modules can be integrated into one MILP problem as following:

$$\text{Min} \quad \sum_{j \in \Omega_{\text{bus}}} \left( P_{j_{\text{max}}} - P_{j_{\text{start}}} \right) t_{j_{\text{start}}}$$

s.t. \quad \text{Critical Time Intervals Constraint - (2.14)}

\text{MW Start-up Requirement Constraint - (2.37)}

\text{Generator Capability Function Constraints - (2.28-2.33)}

\text{Generator Start-up Power Function Constraints - (2.35-2.36)}

(2.54)

\text{Decision Variables Constraints - (2.21-2.26)}

\text{Transmission Path Search Logic Constraints - (2.40-2.48)}

\text{Transmission Line Thermal Limit Constraints - (2.49-2.50)}

\text{GRAs Time and Modules Connection Constraints - (2.51-2.53)}

The developed modules provide the optimal generator start-up sequence and transmission path search in a very efficient way. When system conditions change, system dispatchers can update the restoration actions by utilizing the developed decision support tool.
2.8 Numerical Results

In this research, the software tool of ILOG CPLEX is used to solve the proposed MIQCP and MILP. CPLEX provides Simplex Optimizer and Barrier Optimizer to solve the problem with continuous variables, and Mixed Integer Optimizer to solve the problem with discrete variables. ILOG CPLEX Mixed Integer Optimizer includes sophisticated mixed integer preprocessing routines, cutting-plane strategies and feasibility heuristics. The default settings of MIP models are used with a general and robust branch & cut algorithm.

2.8.1 Case of Four-Generator System

A four-generator system with fictitious data is studied to illustrate the “Two-Step” algorithm. Table 2.2 gives the generator characteristic data.

<table>
<thead>
<tr>
<th>i</th>
<th>$T_{ctp}$</th>
<th>$T_{cmin}$</th>
<th>$T_{cmax}$</th>
<th>$Rr$</th>
<th>$P_{start}$</th>
<th>$P_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>N/A</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>N/A</td>
<td>4</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>N/A</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>N/A</td>
<td>N/A</td>
<td>1</td>
<td>N/A</td>
<td>3</td>
</tr>
</tbody>
</table>

In this system, there are 3 NBS generators and 1 BS generator. Among the 3 NBS generators, 2 units have $T_{cmax}$ and 1 unit have $T_{cmin}$. The total restoration time is set to be 12 time unit. The optimal starting time for all generating units is obtained after 5 iterations by applying proposed algorithm, as shown in Table 2.3.
Table 2.3 Generator Starting Time

<table>
<thead>
<tr>
<th>Unit</th>
<th>$t_{\text{start}}$ (p.u. time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.4 gives the generator status for the optimal solution:

Table 2.4 Generator Status for the Optimal Solution

<table>
<thead>
<tr>
<th>t=0</th>
<th>i=1</th>
<th>i=2</th>
<th>i=3</th>
<th>i=4</th>
<th>System Generation Capability (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>t=1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>t=2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>t=4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>t=6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>t=12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>39</td>
</tr>
</tbody>
</table>

The following stages summarize how the restoration process progresses:

1) In the beginning, BS generator G4 is started up $t=0$, and add it to $\text{ASG}$.

2) In time period 1, according to criterion (4), set $t=t_{\text{start}}+t_{\text{ctp}}=1$, and solve the problem. Update generation capability function of G4 to $P_{\text{gen}}(t)$.

3) In time period 2, by criterion (1), set $t=1+1=2$, and solve the problem. It is shown that NBS generator G1 is started, and add it to $\text{ASG}$. 
4) Then in time period 3, set \( t = t_{start} + t_{ctp} = 4 \) by criterion (4), and solve the problem again. It is shown NBS generator G3 is started, and add it to \( ASG \). Update generation capability function of G1 to \( P_{1gen2(t)} \).

5) In time period 4, set \( t = t_{3start} + t_{3ctp} = 6 \) according to criterion (4) and solve the problem. NBS generator G2 is started, and adds it to \( ASG \). Update generation capability function of G3 to \( P_{3gen2(t)} \).

6) In time period 4, according criterion (3), set \( t = T = 12 \), and solve the problem.

Figure 2.7 shows the time instants where generators change to the respective second segment of the capability function.
The red line is system total generation capability curve. As shown in Figure 2.7, there are a total of five time periods to calculate the optimal solution for four-generator system.

2.8.2 Case of PECO System

The “Two-Step” algorithm is applied to the generators in the PECO system. For simplicity, units at the same station with similar characteristics are aggregated into one [7]. Table 2.5 gives the generator characteristic data.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Type</th>
<th>( T_{cep} ) (hr)</th>
<th>( T_{cmin} ) (hr)</th>
<th>( T_{cmax} ) (hr)</th>
<th>( R_r ) (MW/hr)</th>
<th>( P_{start} ) (MW)</th>
<th>( P_{max} ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chester_4-6</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>120</td>
<td>N/A</td>
<td>39</td>
</tr>
<tr>
<td>Conowingo_1-11</td>
<td>Hydro</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>384</td>
<td>N/A</td>
<td>560</td>
</tr>
<tr>
<td>Cromby_1-2</td>
<td>Steam</td>
<td>1:40</td>
<td>N/A</td>
<td>N/A</td>
<td>148</td>
<td>8</td>
<td>345</td>
</tr>
<tr>
<td>Croydon_1</td>
<td>CT</td>
<td>0:30</td>
<td>5:00</td>
<td>N/A</td>
<td>120</td>
<td>6</td>
<td>384</td>
</tr>
<tr>
<td>Delaware_9-12</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>162</td>
<td>N/A</td>
<td>56</td>
</tr>
<tr>
<td>Eddystone_1-4</td>
<td>Steam</td>
<td>1:40</td>
<td>3:20</td>
<td>N/A</td>
<td>157</td>
<td>12</td>
<td>1341</td>
</tr>
<tr>
<td>Eddystone_10-40</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>168</td>
<td>N/A</td>
<td>60</td>
</tr>
<tr>
<td>Falls_1-3</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>135</td>
<td>N/A</td>
<td>51</td>
</tr>
<tr>
<td>Moser_1</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>90</td>
<td>N/A</td>
<td>51</td>
</tr>
<tr>
<td>Muddy Run_1-8</td>
<td>Hydro</td>
<td>0:30</td>
<td>N/A</td>
<td>N/A</td>
<td>246</td>
<td>13.2</td>
<td>1072</td>
</tr>
<tr>
<td>Richmond_91_92</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>288</td>
<td>N/A</td>
<td>96</td>
</tr>
<tr>
<td>Schuylkill_1</td>
<td>Steam</td>
<td>2:00</td>
<td>N/A</td>
<td>2:30</td>
<td>135</td>
<td>2.7</td>
<td>166</td>
</tr>
<tr>
<td>Schuylkill_10-11</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>84</td>
<td>N/A</td>
<td>30</td>
</tr>
<tr>
<td>Southwark_3-6</td>
<td>CT</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>156</td>
<td>N/A</td>
<td>52</td>
</tr>
<tr>
<td>CCU1</td>
<td>CC</td>
<td>2:40</td>
<td>N/A</td>
<td>3:20</td>
<td>108</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td>CCU2</td>
<td>CC</td>
<td>2:00</td>
<td>2:30</td>
<td>N/A</td>
<td>162</td>
<td>7.5</td>
<td>500</td>
</tr>
</tbody>
</table>
In this system, there are 7 NBS generators and 9 BS generators. Among 7 NBS generators, 2 units have \( T_{cmax} \) and 3 other units have \( T_{cmin} \). The total restoration time is set to be 15 hours, which is divided into 90 time slots with a 10 min length for each time slot. After a blackout, the optimal starting time for all generating units is obtained after 9 iterations by applying the proposed algorithm, as shown in Table 2.6.

**Table 2.6 Generator Starting Time**

<table>
<thead>
<tr>
<th>i</th>
<th>Unit</th>
<th>( t_{\text{start}} ) (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Cromby_1-2</td>
<td>0:10</td>
</tr>
<tr>
<td>4</td>
<td>Croydon_1</td>
<td>5:00</td>
</tr>
<tr>
<td>6</td>
<td>Eddystone_1-4</td>
<td>3:20</td>
</tr>
<tr>
<td>10</td>
<td>Muddy Run_1-8</td>
<td>0:10</td>
</tr>
<tr>
<td>12</td>
<td>Schuylkill_1</td>
<td>0:10</td>
</tr>
<tr>
<td>15</td>
<td>CCU1</td>
<td>0:10</td>
</tr>
<tr>
<td>16</td>
<td>CCU2</td>
<td>2:30</td>
</tr>
</tbody>
</table>

The following is a summary of the restoration process:

1) In the beginning, BS generators G1, G2, G5, G7, G8, G9, G11, G13 and G14 are started up \( t=0 \), and add them to \( \text{ASG} \).

2) In time period 1, since none of BS generators have the characteristic of \( T_{c_{tp}} \), according to criterion (1), set \( t=0+1=1 \), and solve the problem. It is shown that NBS generator G3, G10, G12 and G15 are started, and add them to \( \text{ASG} \).

3) In time period 2, by criterion (4), set \( t = t_{10_{\text{start}}} + t_{10_{c_{tp}}} = 4 \), and solve the problem. Update the generation capability curve of G10 to \( P_{10_{gen2}}(t) \).
4) Then in time period 3, set \( t = t_{3\text{start}} + t_{3\text{cp}} = 11 \) by criterion (4), and solve the problem again. Update the generation capability curve of G3 to \( P_{3\text{gen}2}(t) \).

5) In time period 4, set \( t = t_{12\text{start}} + t_{12\text{cp}} = 13 \) according to criterion (4), and solve the problem again. Update the generation capability curve of G12 to \( P_{12\text{gen}2}(t) \).

6) In time period 5, according to criterion (4), set \( t = t_{15\text{start}} + t_{15\text{cp}} = 17 \), and solve the problem again. It is shown that NBS generator G16 is started at \( t=15 \), and add it to ASG. Update the generation capability curve of G15 to \( P_{15\text{gen}2}(t) \).

7) In time period 6, by criterion (4), set \( t = t_{16\text{start}} + t_{16\text{cp}} = 27 \), and solve the problem again. It is shown that NBS generator G6 is started at \( t=20 \), and add it to ASG. Update the generation capability curve of G16 to \( P_{16\text{gen}2}(t) \).

8) In time period 7, set \( t = t_{6\text{start}} + t_{6\text{cp}} = 30 \) by criterion (4), and solve the problem again. It is shown that NBS generator G4 is started at \( t=30 \), and add it to ASG. Update the generation capability curve of G6 to \( P_{6\text{gen}2}(t) \).

9) In time period 8, by criterion (4), set \( t = t_{4\text{start}} + t_{4\text{cp}} = 33 \), and solve the problem. Update the generation capability curve of G4 to \( P_{4\text{gen}2}(t) \).

10) In time period 9, according criterion (3), set \( t=T = 90 \), and solve the problem.

Table 2.7 gives the generator status for the optimal solution. Figure 2.8 shows the time instants where generators change to the respective second segment of the capability function. The red line is system total generation capability curve.
Table 2.7 Generator Status for the Optimal Solution

<table>
<thead>
<tr>
<th></th>
<th>i=3</th>
<th>i=4</th>
<th>i=6</th>
<th>i=10</th>
<th>i=12</th>
<th>i=15</th>
<th>i=16</th>
<th>System Generation Capability (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t=0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>t=1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>264.5</td>
</tr>
<tr>
<td>t=4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>623.1</td>
</tr>
<tr>
<td>t=11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1166.1</td>
</tr>
<tr>
<td>t=13</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1297.4</td>
</tr>
<tr>
<td>t=17</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1642.6</td>
</tr>
<tr>
<td>t=27</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2460.8</td>
</tr>
<tr>
<td>t=30</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2718.8</td>
</tr>
<tr>
<td>t=33</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2926.3</td>
</tr>
<tr>
<td>t=90</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5084.3</td>
</tr>
</tbody>
</table>

Figure 2.8. Two steps of generation capability curve of PECO system
2.8.3 Case of IEEE 39-Bus System

The IEEE 39-Bus system is used for illustration of the Generation Capability Optimization Module and its interaction with transmission path search and constraint checking. Generator data and transmission system data are shown in Table 2.8 and Table 2.9. There are 10 generators and 39 buses. The scenario of a complete shutdown is assumed. Unit G10 is a black-start unit (BSU) while G1 – G9 are non-black-start units (NBSUs). The restoration actions are checked and updated every 10 minutes.

<table>
<thead>
<tr>
<th>Gen.</th>
<th>$T_{cp}$ (hr)</th>
<th>$T_{cmin}$ (hr)</th>
<th>$T_{cmax}$ (hr)</th>
<th>$R_r$ (MW/hr)</th>
<th>$P_{start}$ (MW)</th>
<th>$P_{max}$ (MW)</th>
<th>Connected Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>0:35</td>
<td>0:40</td>
<td>N/A</td>
<td>215</td>
<td>5.5</td>
<td>572.9</td>
<td>39</td>
</tr>
<tr>
<td>G2</td>
<td>0:35</td>
<td>N/A</td>
<td>N/A</td>
<td>246</td>
<td>8</td>
<td>650</td>
<td>31</td>
</tr>
<tr>
<td>G3</td>
<td>0:35</td>
<td>N/A</td>
<td>2:00</td>
<td>236</td>
<td>7</td>
<td>632</td>
<td>32</td>
</tr>
<tr>
<td>G4</td>
<td>0:35</td>
<td>1:10</td>
<td>N/A</td>
<td>198</td>
<td>5</td>
<td>508</td>
<td>33</td>
</tr>
<tr>
<td>G5</td>
<td>0:35</td>
<td>N/A</td>
<td>2:00</td>
<td>244</td>
<td>8</td>
<td>650</td>
<td>34</td>
</tr>
<tr>
<td>G6</td>
<td>0:35</td>
<td>N/A</td>
<td>N/A</td>
<td>214</td>
<td>6</td>
<td>560</td>
<td>35</td>
</tr>
<tr>
<td>G7</td>
<td>0:35</td>
<td>N/A</td>
<td>N/A</td>
<td>210</td>
<td>6</td>
<td>540</td>
<td>36</td>
</tr>
<tr>
<td>G8</td>
<td>0:35</td>
<td>N/A</td>
<td>N/A</td>
<td>346</td>
<td>13.2</td>
<td>830</td>
<td>37</td>
</tr>
<tr>
<td>G9</td>
<td>0:35</td>
<td>N/A</td>
<td>N/A</td>
<td>384</td>
<td>15</td>
<td>1000</td>
<td>38</td>
</tr>
<tr>
<td>G10</td>
<td>0:15</td>
<td>N/A</td>
<td>N/A</td>
<td>162</td>
<td>0</td>
<td>250</td>
<td>30</td>
</tr>
</tbody>
</table>

The optimal starting times for all generating units are calculated considering different optimization modules, such as, generator start-up sequence (GSS), transmission path search (TPS) and the time to take GRAs. The results are shown in Table 2.10.
Table 2.9 Data of Transmission System

<table>
<thead>
<tr>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>2</td>
<td>17</td>
<td>5</td>
<td>8</td>
<td>33</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>18</td>
<td>8</td>
<td>9</td>
<td>34</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>39</td>
<td>19</td>
<td>6</td>
<td>31</td>
<td>35</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
<td>9</td>
<td>20</td>
<td>6</td>
<td>11</td>
<td>36</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>25</td>
<td>21</td>
<td>12</td>
<td>11</td>
<td>37</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>2</td>
<td>22</td>
<td>12</td>
<td>13</td>
<td>38</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>26</td>
<td>23</td>
<td>11</td>
<td>10</td>
<td>39</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>3</td>
<td>24</td>
<td>10</td>
<td>32</td>
<td>40</td>
<td>35</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>3</td>
<td>25</td>
<td>13</td>
<td>10</td>
<td>41</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>17</td>
<td>26</td>
<td>26</td>
<td>27</td>
<td>42</td>
<td>20</td>
<td>34</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>4</td>
<td>27</td>
<td>26</td>
<td>29</td>
<td>43</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>14</td>
<td>28</td>
<td>26</td>
<td>28</td>
<td>44</td>
<td>19</td>
<td>33</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>5</td>
<td>29</td>
<td>28</td>
<td>29</td>
<td>45</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>6</td>
<td>30</td>
<td>29</td>
<td>38</td>
<td>46</td>
<td>23</td>
<td>36</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>7</td>
<td>31</td>
<td>27</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>7</td>
<td>8</td>
<td>32</td>
<td>17</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.10 Comparison of Generator Start-up Time Considering Different Optimization Modules

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consider GSS</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Consider GSS and TPS</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Consider GSS, TPS and GRAs</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>
The energized time of each bus and line are shown in Table 2.11 and Table 2.12.

Table 2.11  Energized Time of All Buses

<table>
<thead>
<tr>
<th>Bus</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{start}$</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Bus</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Bus</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Bus</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.12  Energized Time of All Lines

<table>
<thead>
<tr>
<th>Line</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{start}$</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>26</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Line</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Line</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>8</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Line</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td>41</td>
<td>42</td>
<td>43</td>
<td>44</td>
<td>45</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{start}$</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.13 and Figure 2.9 show the transmission paths for the available generators to provide cranking power to NBSUs.

Based on the steady state analysis and power flow calculation tools, Constraint Checking is performed with the following two functions: pick up load according to generation capability to maintain system frequency and balance reactive power to control bus voltage and branch MVA.
### Table 2.13 Optimal Transmission Paths

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>G10</td>
<td>Bus: 30→2→1→39</td>
</tr>
<tr>
<td>G2</td>
<td>G10</td>
<td>Bus: 30→2→3→4→5→6→31</td>
</tr>
<tr>
<td>G3</td>
<td>G10</td>
<td>Bus: 30→2→3→4→14→13→10→32</td>
</tr>
<tr>
<td>G4</td>
<td>G10</td>
<td>Bus: 30→2→3→18→17→16→19→33</td>
</tr>
<tr>
<td>G5</td>
<td>G10</td>
<td>Bus: 30→2→3→18→17→16→19→20→34</td>
</tr>
<tr>
<td>G6</td>
<td>G10</td>
<td>Bus: 30→2→3→18→17→16→21→22→25</td>
</tr>
<tr>
<td>G7</td>
<td>G10</td>
<td>Bus: 30→2→3→18→17→16→21→22→23→36</td>
</tr>
<tr>
<td>G8</td>
<td>G10</td>
<td>Bus: 30→2→25→37</td>
</tr>
<tr>
<td>G9</td>
<td>G10</td>
<td>Bus: 30→2→25→26→29→38</td>
</tr>
</tbody>
</table>

*Figure 2.9. IEEE 39-bus system topology with optimal transmission paths*
By the cooperation of the generation capability maximization together with constraint checking and transmission path search, the entire system is restored back to normal state. Figure 2.10 shows the comparison of system generation capability curves by incorporating different techniques, where the time per unit is 10 minutes.

![Generation Capability Curve](image)

**Figure 2.10. Comparison of generation capability curves by using different modules**

Table 2.14 shows the restoration actions at each time slot. Figure 2.11 shows the restoration progress at each major time slot:
### Table 2.14 Actions to Restore Entire Power System

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Action</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>t=0:15</td>
<td>Energize</td>
<td>Bus 30</td>
</tr>
<tr>
<td>t=0:20</td>
<td>Energize</td>
<td>Bus 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 30-2</td>
</tr>
<tr>
<td>t=0:25</td>
<td>Energize</td>
<td>Bus 25,1,3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 2-25,2-1,2-3</td>
</tr>
<tr>
<td>t=0:30</td>
<td>Energize</td>
<td>Bus 37,39,26,4,18</td>
</tr>
<tr>
<td></td>
<td>Energize</td>
<td>Branch 25-37,1-39,25-26,3-4,3-18</td>
</tr>
<tr>
<td></td>
<td>Connect</td>
<td>G10</td>
</tr>
<tr>
<td>t=0:35</td>
<td>Energize</td>
<td>Bus 27,5,14,17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 26-27,4-5,4-14,18-17</td>
</tr>
<tr>
<td>t=0:40</td>
<td>Energize</td>
<td>Bus 6,13,16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 5-6,14-13,17-16</td>
</tr>
<tr>
<td>t=0:45</td>
<td>Energize</td>
<td>Bus 10,19,21,24,28,29,31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 13-10,16-19,16-21,16-26,26,26-29,6-31</td>
</tr>
<tr>
<td>t=0:50</td>
<td>Energize</td>
<td>Bus 20,22,23,32,33,38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 19-20,21-22,24-23,10-32,19-33,29-38</td>
</tr>
<tr>
<td></td>
<td>Crank</td>
<td>G8,G1</td>
</tr>
<tr>
<td>t=0:55</td>
<td>Energize</td>
<td>Bus 34,35,36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 20-34,22-35,23-36</td>
</tr>
<tr>
<td></td>
<td>Crank</td>
<td>G9</td>
</tr>
<tr>
<td>t=1:00</td>
<td>Crank</td>
<td>G2,G3,G5,G6,G7</td>
</tr>
<tr>
<td>t=1:10</td>
<td>Crank</td>
<td>G4</td>
</tr>
<tr>
<td>t=1:25</td>
<td>Connect</td>
<td>G1,G8</td>
</tr>
<tr>
<td>t=1:30</td>
<td>Connect</td>
<td>G9</td>
</tr>
<tr>
<td>t=1:35</td>
<td>Connect</td>
<td>G2,G3,G5,G6,G7</td>
</tr>
<tr>
<td>t=1:45</td>
<td>Energize</td>
<td>Bus 9,8,7,11,15,12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch 39-9,5-8,6-7,6-11,14-15,13-12,22-23</td>
</tr>
<tr>
<td>t=1:40</td>
<td>Connect</td>
<td>G4</td>
</tr>
<tr>
<td>t=1:50</td>
<td>Energize</td>
<td>Branch 29-28,10-11,17-27,16-15,9-8,8-7,11-12</td>
</tr>
</tbody>
</table>
Figure 2.11. Progress of restoring power system
2.8.4 Case of AEP System

According to the AEP system restoration plan, generating units that have successfully rejected all but auxiliary load should be in a state of readiness to provide startup power to other units. It is vital to quickly restart these generators based on the restoration steps to pick up (cold) load or to parallel with a restored portion of the system. Therefore, the proposed algorithm is used to find the optimal starting sequence of subcritical units that should be capable of load rejection. The scenario of a total blackout is hypothesized for the AEP system. With 24.04 seconds of computational time, Generation Capability Optimization Module provides the optimal solution, as shown in Table 2.15.

<table>
<thead>
<tr>
<th>Generator</th>
<th>$T_{start}$ (hr)</th>
<th>Generator</th>
<th>$T_{start}$ (hr)</th>
<th>Generator</th>
<th>$T_{start}$ (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>0:10</td>
<td>Unit 14</td>
<td>0:00</td>
<td>Unit 27</td>
<td>0:10</td>
</tr>
<tr>
<td>Unit 2</td>
<td>0:00</td>
<td>Unit 15</td>
<td>0:10</td>
<td>Unit 28</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 3</td>
<td>3:20</td>
<td>Unit 16</td>
<td>0:00</td>
<td>Unit 29</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 4</td>
<td>0:10</td>
<td>Unit 17</td>
<td>0:00</td>
<td>Unit 30</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 5</td>
<td>2:30</td>
<td>Unit 18</td>
<td>2:30</td>
<td>Unit 31</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 6</td>
<td>0:10</td>
<td>Unit 19</td>
<td>0:10</td>
<td>Unit 32</td>
<td>0:10</td>
</tr>
<tr>
<td>Unit 7</td>
<td>0:10</td>
<td>Unit 20</td>
<td>0:10</td>
<td>Unit 33</td>
<td>2:30</td>
</tr>
<tr>
<td>Unit 8</td>
<td>0:10</td>
<td>Unit 21</td>
<td>0:10</td>
<td>Unit 34</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 9</td>
<td>0:00</td>
<td>Unit 22</td>
<td>3:20</td>
<td>Unit 35</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 10</td>
<td>0:10</td>
<td>Unit 23</td>
<td>0:00</td>
<td>Unit 36</td>
<td>0:00</td>
</tr>
<tr>
<td>Unit 11</td>
<td>0:00</td>
<td>Unit 24</td>
<td>2:30</td>
<td>Unit 37</td>
<td>0:10</td>
</tr>
<tr>
<td>Unit 12</td>
<td>0:00</td>
<td>Unit 25</td>
<td>0:10</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Unit 13</td>
<td>5:00</td>
<td>Unit 26</td>
<td>3:20</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure 2.12 provides the system generation capability curve over a period of 10 hours. The developed module is able to quickly provide the initial starting sequence of all generating units. The AEP generation system can be restored efficiently with the maximum system generation capability.

![Generation Capability Curve](image)

**Figure 2.12. Generation capability curve of AEP case**

### 2.8.5 Case of Western Entergy Region

A weather related outage occurred in the Western Region of the Entergy System in June 2005, four generators were tripped off line. It is assumed that the 4 generators were ready to be started and synchronized, and that there was blackstart power from outside to start 1 generator. The generator data are shown in Table 2.16. With a computational time of 0.15 seconds, *Generation Capability Optimization Module* provides the optimal solution, as shown in Table 2.17.
### Table 2.16 Data of Four Generators

<table>
<thead>
<tr>
<th>Generator</th>
<th>( T_{ctp} ) (hr)</th>
<th>( T_{cmin} ) (hr)</th>
<th>( T_{cmax} ) (hr)</th>
<th>( R_r ) (MW/hr)</th>
<th>( P_{start} ) (MW)</th>
<th>( P_{max} ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>2:40</td>
<td>N/A</td>
<td>3:00</td>
<td>N/A</td>
<td>5</td>
<td>N/A</td>
</tr>
<tr>
<td>G2</td>
<td>2:40</td>
<td>N/A</td>
<td>3:00</td>
<td>N/A</td>
<td>6</td>
<td>N/A</td>
</tr>
<tr>
<td>G3</td>
<td>2:00</td>
<td>N/A</td>
<td>2:30</td>
<td>N/A</td>
<td>3.3</td>
<td>N/A</td>
</tr>
<tr>
<td>G4</td>
<td>1:40</td>
<td>N/A</td>
<td>3:20</td>
<td>N/A</td>
<td>8</td>
<td>N/A</td>
</tr>
</tbody>
</table>

### Table 2.17 Generator Starting Times

<table>
<thead>
<tr>
<th>Generator</th>
<th>( T_{start} ) (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>3:00</td>
</tr>
<tr>
<td>G2</td>
<td>2:50</td>
</tr>
<tr>
<td>G3</td>
<td>3:10</td>
</tr>
<tr>
<td>G4</td>
<td>2:40</td>
</tr>
</tbody>
</table>

**Figure 2.13.** Generation capability curve of Entergy case
Figure 2.13 provides the system generation capability curve. The generation system is successfully restored in 8 hours. The fast starting time of all generating units facilitates the restoration tasks to return the Western Entergy Region to a normal operating condition.

### 2.8.6 Performance Analysis of MILP Method

From the simulation results shown in Table 2.18, it is seen that the computational time is within the practical range for both system restoration planning and on-line decision support environments.

<table>
<thead>
<tr>
<th>Number of NBS Generators</th>
<th>3</th>
<th>7</th>
<th>9</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of All Generators</td>
<td>4</td>
<td>16</td>
<td>10</td>
<td>37</td>
</tr>
<tr>
<td>Total Restoration Time (hr)</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Number of Decision Variables</td>
<td>429</td>
<td>5173</td>
<td>6327</td>
<td>84651</td>
</tr>
<tr>
<td>Number of Constraints</td>
<td>1119</td>
<td>14259</td>
<td>17847</td>
<td>245532</td>
</tr>
<tr>
<td>Max Generation Capability</td>
<td>167.5</td>
<td>60683</td>
<td>167403</td>
<td>316914</td>
</tr>
<tr>
<td>Computational Time (sec.)</td>
<td>0.15</td>
<td>5.05</td>
<td>7.61</td>
<td>24.04</td>
</tr>
</tbody>
</table>

### 2.8.7 Comparison with Other Methods

Table 2.19 gives the computational time for the proposed MILP method and other available techniques. The tools are used to determine the generator starting times for the IEEE 39-Bus system. The enumerative algorithm searches from the combination of all possible starting times. Although global optimality can be achieved by searching all possibilities, the extremely high computation burden prevents its application in reality. By breaking the entire problem into stages, DP [38] tries to find the optimal path connecting each state. However, the complexity affects the effectiveness of the restoration procedures.
for large-scale systems. Two-Step algorithm [9] solves the problem for discretized times with optimality guaranteed at each step. However, MIQCP method cannot guarantee the global optimality due to the quadratic components that exist in both objective function and constraints. The MILP method proposed in this paper is able to obtain the optimal solution in an efficient way.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Global Optimality</th>
<th>Computational Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Enumeration</td>
<td>Yes</td>
<td>1 hour and 53 minutes</td>
</tr>
<tr>
<td>2. Dynamic Programming</td>
<td>No</td>
<td>55 minutes</td>
</tr>
<tr>
<td>3. Two-Step</td>
<td>No</td>
<td>4 minutes</td>
</tr>
<tr>
<td>4. MIQCP</td>
<td>No</td>
<td>35 minutes</td>
</tr>
<tr>
<td>5. MILP</td>
<td>Yes</td>
<td>8 seconds</td>
</tr>
</tbody>
</table>

2.9 Summary

The generator starting sequence problem has been successfully formulated into a MIQCP optimization problem for determining an optimal generator start-up strategy for power system restoration following a blackout. Incorporating the proposed “Two-Step” algorithm to take advantage of the quasiconcave property of generation capability curve, the optimization problem can be solved with available convexity-based optimization tools. The numerical results demonstrate the accuracy of the models and effectiveness of the algorithm. Compared to the empirical solutions based on heuristic methods or other knowledge-based approaches, optimal solutions are obtained at each time step. Added to that, the specific formulation does not depend on highly special software tools, the optimization formulation
appears to be free of special maintenance and support requirements and is more practical and attractive for the long term development of a decision support tool.

Moreover, an optimal generator start-up strategy for bulk power system restoration following a blackout is proposed. Using the proposed transformation techniques on the nonlinear generation capability curves, a MILP model is developed. The numerical results demonstrate the accuracy of the models and computational efficiency of the MILP algorithm. Global optimality is achieved by the proposed strategy. While the solution provides system operators an optimal start-up sequence of the generators at the start of the system restoration, system operators need to identify transmission paths and pick up critical loads as the restoration effort continues.

More practical constraints need to be incorporated, such as switching transients, generating station voltage limits, generator transient stability limits. In the future work, the under-excitation capability of generators, load rejection and low frequency isolation scheme should also be incorporated. It can be accomplished by integrating the developed module with power system simulation software tools. To provide an adaptive decision support tool for power system restoration, the data and implementation issues for an on-line operational environment need to be investigated in the future.
CHAPTER 3. OPTIMAL INSTALLATION STRATEGY OF BLACKSTART CAPABILITY IN POWER SYSTEM RESTORATION

Blackstart capability is important for system planners to prepare the PSR plan. To achieve a faster restoration process, installing new BS generators can be beneficial in accelerating system restoration. While additional BS capability does not automatically benefit the restoration process, power systems have to update the PSR plan and quantify the benefit based on appropriate criteria. In this research, a decision support tool using GRMs-based strategy is utilized to provide a quantitative way for assessing BS capability. Then, based on the developed optimization modules of generator start-up sequencing and transmission path search, considering the time to take GRAs, the optimal installation of BS capability is formulated as a MIBLP problem. By solving this combinatorial problem with the advanced solution methodology, the benefit from additional BS capability is quantified in terms of reduced restoration time and increased generation capability. The PJM 5-bus, IEEE Reliability Test System (RTS) 24-bus and IEEE 39-bus test systems are used for validation of the proposed strategy. It is shown that power systems can benefit from new BS generators to reduce the restoration time. However, there is a maximum amount beyond which system restoration time cannot be further reduced with additional BS capability. Economic considerations should be taken into account when assessing additional BS capabilities.
3.1 Background of Blackstart

3.1.1 Definition of Blackstart

A blackstart is the process of restoring a power station to operation without relying on external energy sources. Following an outage of the power system, power stations usually rely on the electric power provided from the station’s own generators. For example, small diesel generators can provide electric power to start larger generators (of several MW capacity), which in turn can be used to start the main power station generators. However, steam turbine generators require station service power of up to 10% of their capacity for boiler feed water pumps, boiler forced-draft combustion air blowers, and fuel preparation [20]. It is not economical to provide such a large standby capacity at each station, so BS power must be provided over the transmission network from other stations. If part or all of the plant’s generators are shut down, station service power is also supplied from the grid.

After a partial or complete system blackout, dispatchers rely on off-line restoration plans and available BS capabilities to restore system back to normal operation conditions. The typical BS scenario includes BS generating units providing power to start large steam turbine units located electrically close to these units, the supply of auxiliary power to nuclear power stations and off-site power to critical service load, such as hospitals and other public health facilities, military facilities, transmission lines that transport the cranking power to NBS units or large motor loads, and transformer units, including step-up transformers of the BS units and steam turbine units, and auxiliary transformers serving motor control centers at the steam plant [20].
According to the start-up power requirement, generating units can be divided into two groups: BS generators and NBS generators. A blackstart generator, e.g., hydro or combustion turbine units, can be started with its own resources, while NBS generators, such as steam turbine units, require cranking power from outside. The typical BS generators are:

**Hydroelectric generating units:** These generators need very little initial power to open the intake gates, and have fast response characteristics to provide power to start fossil fueled or nuclear stations.

**Diesel generating units:** Diesel generators usually require only battery power and can be started quickly to supply the power to start up larger generating units. They are small in size, and generally cannot be used to pick up any major transmission system elements.

**Gas turbine generating units:** Aero-derivative gas turbine generators can be started remotely with the help of local battery power. Large gas turbine generators are coupled with on-site diesel generator sets, which are started and used to energize plant auxiliary buses and start either the gas turbine or steam turbine. Gas turbine generators can be started and pick up load within a short time. Time to restart and available ramping capability are functions of the duration when the unit was off-line.

The typical generators that are contracted for BS service are 10 to 50 MW small hydro or gas turbine units, and in a few cases even 200 to 400 MW steam units. The bus voltage values can be 6.9 kV for hydro units, 12.8 or 13.8 kV for gas turbine units, and 22 kV for steam turbine units. However, not all generating plants are suitable for BS service. For example, wind turbines are connected to induction generators which are incapable of providing power to a de-energized network, and mini-hydro or micro-hydro plants rely on a power network connection for frequency regulation and reactive power supply. Therefore,
BS units must be stable when operated with the large reactive load of a long transmission line. Also traditional high-voltage direct current converter (HVDC) stations cannot operate without the commutation power from the system at the load end.

### 3.1.2 Constraints during Blackstart

Power system restoration following a blackout begins with the BS units and then restores the system outward toward critical system loads. The objective of system restoration includes BS generating units providing power to start large steam turbine units located electrically close to these units, the supply of auxiliary power to nuclear power stations and off-site power to critical service load, such as hospitals and other public health facilities, and military facilities. The typical BS scenario includes BS units, transmission lines that deliver cranking power to NBS units or large motor loads, and transformer units, including step-up transformers of the BS units and steam turbine units, and auxiliary transformers serving motor control centers at the steam plant [20]. Among various BS restoration steps, the key concern is the control of voltage and frequency, both of which must be kept within a tight band around nominal values to guarantee no equipment failure will severely hinder the restoration process.

Voltage stress is a major concern for a blackstart during power system restoration. Blackstart units are required to be able to absorb the produced reactive power from charging current by energizing the unloaded generator step-up transformer and transmission lines. When energizing a transmission line, the produced charging currents can be large enough to result in the BS generating unit absorbing reactive power, which may cause self-excitation. The self-excitation will result in an uncontrolled rise in voltage or equipment failure. Thus, it
is important to verify the reactive power capacity of the BS unit when operated at a leading power factor. The installation of shunt reactors, synchronization of generating units as a block and minimizing the time between paralleling units online will help to reduce the probability of exposing a generating unit to the condition of self-excitation.

During the BS process, starting up generators and picking up large blocks of load will perturb the system frequency, which can be prevented by picking up loads in increments that can be accommodated by system inertia and response of already started-up generators. However, when re-energizing the load that has been de-energized for several hours or longer, the generated inrush current can be as high as eight to ten times normal, known as the phenomenon of cold load pickup. The large inrush current brought by picking up loads need to be carefully considered in a BS plan.

During the implementation of the BS plan, voltage, rotor angle and frequency stability have to be maintained as important components in the stability assessment of the plan. Therefore, BS plans must be validated by tests or simulation in terms of both steady state and transient operating conditions. A step-by-step simulation is required to verify the BS plan’s feasibility and compliance with required operational limits on voltage and power flows. Also, the robustness of BS plan needs to be verified to ensure its ability to compensate for some equipment unavailability.

### 3.1.3 Blackstart Service Procurement

Blackstart service is an ancillary service that is procured for power system restoration after a complete or partial outage. These BS resources must be able to energize buses and have on-site diesel or gas turbine generators to provide power for the auxiliary systems of the
generating unit, which then can be used to start the unit. In North America, ISOs identify and contract resources with BS capability and forms financial agreements with them to provide this obligatory service. Traditionally, BS service costs were rolled into a broad tariff for cost recovery from ratepayers. In the deregulated environment, some ISOs, for example, Electric Reliability Council of Texas (ERCOT) has shifted this Cost-of-Service provision to a competitive procurement. There are three methods of procuring BS service.

- The most common one is Cost of Service, in which generating units are identified for BS resources and the costs are rolled into a tariff for cost recovery. This simple and traditional method is currently used by the California Independent System Operator (CAISO), the PJM Interconnection and the New York Independent System Operator (NYISO).

- The second method is a new methodology that uses a flat rate in $/kWyr to increase BS remuneration to encourage provision. Then the monthly compensation paid to a generator is determined by multiplying this flat rate by the unit's Monthly Claimed Capability for that month. The new method is aimed at simplifying the procurement process and incentivizing the provision of BS, which is currently used by the Independent System Operator of New England (ISO-NE).

- The last method is a competitive procurement as used in ERCOT, which runs a market for BS services. In the market, interested participants submit an hourly standby cost in $/hr, which is named an availability bid that is unrelated to the capacity of the unit. Then, based on various criteria, ERCOT evaluates these bids and the selected units are paid as bid. Each BS unit must be able to demonstrate its ability to startup another unit.
There are other different procurement methods. The New Zealand System Operator procures the BS service through a competitive tender. Other jurisdictions also have some sort of competitive procurement, although not as structured as ERCOT. Alberta Electric System Operator and Independent Electric System Operator of Ontario, use a long-term "Request for Proposals" approach, which is similar to but not as structured as in ERCOT.

In ERCOT, BS service is awarded through a competitive annual bidding process where market participants submit bids for hourly standby price for their generators to provide the service. ERCOT then select the capable resources of providing BS service that meet the BS selection reliability criteria at a minimum cost. The criteria include the required amount of load to be recovered and the minimum time to recover that load. The general process in each year is [36]:

1. On April 1st, ERCOT posts the BS Request for Proposal (REP) on the website;
2. In the following two months, market participants formulate and submit their bids for entering BS service market;
3. Based on the criteria listed as above, ERCOT evaluates and analyzes the bids to develop a list of preliminary BS units;
4. The preliminary units must undergo physical tests in a “real” BS scenario, including Basic Start test, Line Energizing test, Load Carrying test, and Next Start Resource test, to prove their ability to provide BS service;
5. On successful completion the BS tests, BS resources are awarded the BS service contracts for the next calendar year;
6. At the same time, the complete BS plan is formulated with the Regional Transmission Operators (RTOs) and is made available to the system operator for training and use.

After BS service resources are selected, ERCOT pays an hourly standby fee at their bid price, with an adjustment for reliability based on a six-month rolling availability equal to 85% in accordance with the BS Agreement.

In the deregulated environment, Independent Power Producers (IPPs) are able to provide BS service, which is not possible in the previous regulated power system. Generators owned by IPPs are usually located near or within industrial areas. These resources can quickly supply power to adjacent users [37]. While more and more BS resources are available, it is important for system dispatchers to provide adequate but not redundant BS capabilities considering BS costs. The BS capability assessment is required to assist PSR plans.

3.1.4 Blackstart Capability Assessment

Power system restoration is a complex problem involving a large number of generation, transmission and distribution, and load constraints [8]. A common approach to simplify this task is to divide the restoration process into stages (e.g. preparation, system restoration and load restoration stages) [10]. According to these restoration stages, PSR strategies can be categorized into six types [10], i.e., Build-Upward, Build-Downward, Build-Inward, Build-Outward, Build-Together and Serve-Critical. Nevertheless, one common thread linking each of these stages is the generation availability at each restorative stage for
stabilizing the system, establishing the transmission path and restoring load [9], as shown in Figure 3.1.

![Diagram of generation availability connecting three restoration stages]

**Figure 3.1. Illustration of generation availability connecting three restoration stages**

Following a system blackout, some fossil units may require cranking power from outside in order to start the unit. Some units may have time constraints within which the unit can be started successfully or else they have to be off line for an extended period of time before they can be restarted and re-synchronized to the grid. As a result, it is important that, during system restoration, available BS generating units must provide cranking power to NBS generating units in such a way that the overall available generation capability is maximized [23]. Given limited BS resources and different system constraints on different generating units, the maximum available generation can be determined by finding the optimal start-up sequence of all generating units in the system.

### 3.2 Installation of New Blackstart Generators

To achieve a faster restoration, additional BS generators might be useful to accelerate the restoration process. After new BS generating units are installed, system restoration steps,
such as, generator start-up sequence, transmission path, and load pick-up sequence, will change. Then each system restoration stage will adjust the restoration steps to accommodate the additional BS capability.

1. In Preparation stage, there are three tasks for updating restoration plans:

   • **Task 1**: Update generator start-up sequence. With the help of additional BS generating units, more cranking power can be provided to start up NBS generators in the earlier time to increase system generation capability.

   • **Task 2**: Update the transmission path to deliver the cranking power. With the updated generator start-up sequence, a new transmission path search is required to implement this updated sequence.

   • **Task 3**: Update critical load pick-up sequence. Since the critical load needs to be picked up to maintain system stability, the critical load pick-up sequence will be updated according to the new generator start-up sequence.

By completing these three tasks, the updated restoration plan will proceed to the next stage.

2. In System Restoration stage, three tasks needed to be performed to update restoration plans:

   • **Task 4**: Update transmission paths to energize and build the skeleton of the transmission system. The critical restoration actions, such as energization of high voltage lines and switching actions, need to follow the updated transmission path search.
• **Task 5**: Update the dispatchable load pick-up sequence. Sufficient loads need to be restored to stabilize generation and voltage. Larger or base-load units are prepared for load restoration in the nest stage.

• **Task 6**: Update the resynchronization of electrical islands. Many system parameters, such as voltage stability, VAR balance, and voltage/frequency response, need to be checked and monitored to synchronize islands in a reliable way.

3. In **Load Restoration** stage, there are one tasks required for restoration plans:

• **Task 7**: Updated load pick-up sequence. This is different from load pick-up in the previous two stages that are aimed at stabilizing the power system. The objective in this stage is to minimize the unserved load according to the total system generation capability.

The seven tasks in three restoration stages help to update the restoration plan with installation of additional BS capabilities. This is illustrated in Figure 3.2.

While restoration plans are updated by installing new BS generators, the benefits of additional BS capabilities need to be evaluated. There are multiple options for installation sizes and locations. Each option will lead to a different restoration time. Therefore, the restoration time can be used as a criterion to quantify the benefit. However, there is a point where the benefits will not increase further. Therefore, power systems need to evaluate the strategy of both placement and size of new BS generators and quantify the benefits with the appropriate criteria.
3.2.1 Criteria of Restoration Time and Blackstart Capability

In the literature, there are various objectives or criteria to develop PSR strategies. For example, to maximize the total system generation capability [9], to minimize the unserved energy [38], to maximize the total (or certain percentage of) restored load within the given restoration time [39], to minimize the total restoration time [40], etc. However, there are obstacles for the direct applications of previous work on the BS capability assessment. The KBSs system restoration tool [7] has been developed to integrate both dispatchers’ knowledge and computational algorithms for system analysis. However, KBSs require special software tools and, furthermore, the maintenance of large-scale knowledge bases is a difficult task. The Critical Path Method [22] is able to estimate system restoration time, which requires the pre-selected restoration strategies. However, for different installation strategies of new BS generators, it is difficult to provide and compare the updated PSR
strategies. The MILP-based optimal generator start-up strategy [23] is able to provide the overall system generation capability and update the solution throughout the BS process. Without considering system topology and power flow constraints, the solution will deviate from the actual system generation capability. Therefore, the appropriate criteria and solution methodology are required to assess blackstart capability to provide the optimal installation strategy of new BS generators, which is proposed in this research.

One of the objectives of system restoration is to restart as much load as possible within the shortest time. After installing additional BS generating units, the reduced restoration time can be obtained from the updated restoration plan. Each installation strategy, including different placement and size of new BS generators, will bring different restoration time. The value of additional BS capability will be evaluated in terms of the system restoration time. However, from the cost-benefit point of view, the cost of installing additional BS capability is another criterion to evaluate the strategy. These two criteria of reduced restoration time and cost of installing BS capability will decide the installation plan, as shown in Figure 3.3. These two criteria together provide information on the benefit based on the cost of installation, for example, installing a certain amount of BS capability will reduce the restoration time. Then power systems can develop their best installation strategies according to their own perspectives. Therefore, based on the criteria of the total restoration time and installed additional BS capability, power systems can make decisions on the optimal installation strategy of both location and amount of BS capability.
3.2.2 Optimal Installation Strategy of Blackstart Capability

The seven tasks will provide the updated restoration plan. However, these tasks will change with different systems or different installation strategies. Among different PSR strategies, there are several general actions to perform these seven restoration tasks, such as:

- Generator Start-up Sequencing
- Transmission Path Search
- Load Pick-up Sequencing
- Optimal Power Flow Check

Then restoration tools, for example, a GRM-based restoration tool, can be utilized to provide the algorithms of generic restoration actions to perform the seven tasks, as shown in Figure 3.4.
In this way, a method will be developed to determine the optimal sizes and locations of new BS units:

- **Step 1**: Select the installation location and amount for the additional BS capability.
- **Step 2**: Based on restoration tools, the seven tasks are performed to obtain the updated restoration plan.
- **Step 3**: Based on the criteria of Restoration Time and Blackstart Capability, the installation choice is evaluated.
- **Step 4**: Update the installation strategy, continue the previous steps to obtain the optimal installation strategy.

Based on the developed MILP modules of generation and transmission system restoration, the optimal installation of BS capabilities is formulated as the following MIBLP problem [41].
\[
\max_{p_{\text{max}}, R_i, t_{\text{start}}} \left\{ \sum_{j \in \Omega_{\text{ALLU}}} \left[ \left( P_{\text{max}} \right)^2 \left( T - T_{\text{icp}} - P_{\text{max}} / R_i \right) \right] \right. \\
- \sum_{j \in \Omega_{\text{ASU}}} P_{j\text{start}} T - \sum_{j \in \Omega_{\text{ASU}}} \left( P_{j\text{max}} - P_{j\text{start}} \right) t_{j\text{start}} \\
\text{s.t.} \quad \min_{t_{\text{start}}} \sum_{j \in \Omega_{\text{ASU}}} \left( P_{j\text{max}} - P_{j\text{start}} \right) t_{j\text{start}} \\
\text{s.t.} \quad \text{Critical Time Intervals Constraint - (2.14)} \\
\quad \text{MW Start-up Requirement Constraint - (2.37)} \\
\quad \text{Generator Capability Function Constraints - (2.28-2.33)} \quad (3.1) \\
\quad \text{Generator Start-up Power Function Constraints - (2.35-2.36)} \\
\quad \text{Decision Variables Constraints - (2.21-2.26)} \\
\quad \text{Transmission Path Search Logic Constraints - (2.40-2.48)} \\
\quad \text{Transmission Line Thermal Limit Constraints - (2.49-2.50)} \\
\quad \text{GRAs Time and Modules Connection Constraints - (2.51-2.53)} \\
\]

By solving this problem, system generation capability equals the optimal objective value, and the estimated total restoration time can be obtained as following:

\[
t_{\text{sys}} = \max_{i=1, \ldots, N} \left( t_{i\text{start}} + T_{\text{icp}} + P_{i\text{max}} / R_i \right) \quad (3.2)
\]

### 3.2.3 Algorithm

In the literature, there are a few methods for a restricted class of Bi-Level Linear Programming (BLP) problems. For example, no integer decision variable is involved in the lower level problem. There are no direct applications to MIBLP problems from the previous work. In this research, based on Benders decomposition and transformation procedure, a novel methodology is proposed. The detailed derivations are shown in Appendix B. The algorithm is described as follows:

**Step 1:** Divide the MIBLLP problem into one Restricted Master Problem (RMP) and several Slave Problems (SPs) by fixing binary variables. In the initial step, RMP will only
have the objective, and constraints are added in future iterations from the cut of solving SP. RMP will provide an upper bound.

**Step 2:** Transfer SP problem to LPCC problem, and solve it by “θ-free” algorithm. SP is the restricted MIBLLP, and it provides a lower bound. The decomposition technique allows parallel computation of solving multiple SPs.

**Step 3:** From the solution of LPCC problem, construct and solve the corresponding Linear Programming (LP) problem:

1) If solution is unbounded, add the **Feasibility Cut** and go to Step 4;

2) If solution is bounded, which provides a lower bound and restricts RMP, add the **Optimality Cut** and go to step 4;

3) If solution is bounded, which provides a lower bound but does not restrict RMP, add the **Integer Exclusion Cut** and go to step 4.

**Step 4:** Solve RMP with added cut, and get an updated upper bound. Find the difference between upper bound and lower bound. If it is within the tolerance, stop; otherwise, update the SP by setting constraint of current binary variable and go back to step 2.

The flow chart of the proposed algorithm is shown in Figure 3.5. The proposed solution methodology leads to an optimal solution in an efficient way. The decomposition technique is essential for large-scale problems, and the transformation procedure validates the use of Karush-Kuhn-Tucker (KKT) conditions and transforms the MIBLP into two single level problems. Therefore, the proposed algorithm outperforms traditional enumeration or reformulation techniques in both quality and computational efficiency.
In this section, numerical results are presented to validate the proposed installation strategy of additional BS capability. After installing new BS generators, the restoration process needs to be analyzed using the restoration tool. The System Restoration Navigator (SRN), developed by EPRI, is able to compute the total restoration time and provide detailed restoration steps, which facilitate the investment strategy of additional BS capability.

In first two case studies, this GRMs-based restoration tool is utilized to compute the restoration time and update restoration plan. From the numerical results, the following conclusions can be made for BS capability assessment based on estimation of restoration time:

- Increasing BS capability, power system can benefit to reduce restoration time.
There is one maximum amount, beyond which the system restoration time no longer benefits from additional BS capability.

Each bus has a different threshold and reduced restoration time.

GRM-based algorithm can provide power systems with an optimal installation strategy for both location and amount of additional BS capability.

### 3.3.1 Case of PJM 5-Bus System

The PJM 5-Bus System [42] is used for illustration of the proposed model and solution methodology, as shown in Figure 3.6. There are 4 generators, 5 buses and 6 lines. The generator and transmission system information is given in Table 3.1 and Table 3.2. The scenario of a complete shutdown is assumed. Unit G4 is a BSU while G1 – G3 are NBSUs. The restoration actions are checked and updated every 10 minutes.

![Figure 3.6. PJM 5-bus system](image)
Table 3.1 Data of Generator Characteristic

<table>
<thead>
<tr>
<th>Gen.</th>
<th>$T_{cp}$ (hr)</th>
<th>$T_{cmin}$ (hr)</th>
<th>$T_{cmax}$ (hr)</th>
<th>$R_r$ (MW/hr)</th>
<th>$P_{start}$ (MW)</th>
<th>$P_{max}$ (MW)</th>
<th>Connected Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>2</td>
<td>N/A</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>G2</td>
<td>1</td>
<td>5</td>
<td>N/A</td>
<td>4</td>
<td>1</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>G3</td>
<td>2</td>
<td>N/A</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>G4</td>
<td>1</td>
<td>N/A</td>
<td>N/A</td>
<td>1</td>
<td>N/A</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.2 Data of Transmission System

<table>
<thead>
<tr>
<th>Branch</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Reactance X</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.0281</td>
<td>2.50</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0.0108</td>
<td>3.5</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>0.0297</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5</td>
<td>0.0297</td>
<td>2.4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>0.0064</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4</td>
<td>0.0304</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The optimal starting times for all generating units are calculated using different optimization modules, for example, only considering GSS, considering GSS and TPS, and considering GSS, TPS and the time to take GRAs. The results are shown in Table 3.3.

Table 3.3 Comparison of Generator Start-up Time Considering Different Optimization Modules

<table>
<thead>
<tr>
<th></th>
<th>$t_{start1}$ (hr)</th>
<th>$t_{start2}$ (hr)</th>
<th>$t_{start3}$ (hr)</th>
<th>$t_{start4}$ (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consider GSS</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Consider GSS and TPS</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Consider GSS, TPS and GRAs</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
The restoration actions considering GSS, TPS and the time to take GRAs are shown in Table 3.4. The comparison of system generation capability curves considering different optimization modules is shown in Figure 3.7.

**Table 3.4 Restoration Actions Considering GSS, TPS and GRAs**

<table>
<thead>
<tr>
<th>Time</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=0$</td>
<td>Start G4</td>
</tr>
<tr>
<td>$t=1$</td>
<td>N/A</td>
</tr>
<tr>
<td>$t=2$</td>
<td>Energize Bus 5</td>
</tr>
<tr>
<td>$t=3$</td>
<td>Energize Bus 4, Line 4</td>
</tr>
<tr>
<td>$t=4$</td>
<td>Energize Bus 1, Line 5, Line 6, Start G3</td>
</tr>
<tr>
<td>$t=5$</td>
<td>Energize Bus 2, Line 1, Start G2</td>
</tr>
<tr>
<td>$t=6$</td>
<td>Energize Bus 3, Line 2, Line 3</td>
</tr>
<tr>
<td>$t=7$</td>
<td>Start G1</td>
</tr>
</tbody>
</table>

In the base case, there is only 1 BSU, G4, on Bus 5, with ramping rate of 1 MW/hr and maximum generation output of 3 MW. The comparison of installation strategy when only considering GSS and considering GSS, TPS and GRAs are shown in the following.

**(I) Only considering GSS**

After increasing system BS capability by three times than the original, generator starting sequence and time will not change any more. The comparison of generator start-up time, system generation capability and total restoration time with three different BS capabilities are shown in Table 3.5.
Figure 3.7. Comparison of system generation capability curves considering different optimization modules

Table 3.5 Comparison of Restoration Time and System Generation Capability with Different Blackstart Capabilities

<table>
<thead>
<tr>
<th></th>
<th>( t_{\text{start}}^1 ) (hr)</th>
<th>( t_{\text{start}}^2 ) (hr)</th>
<th>( t_{\text{start}}^3 ) (hr)</th>
<th>( t_{\text{start}}^4 ) (hr)</th>
<th>Total Restoration Time (hr)</th>
<th>System Generation Capability (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original BS</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>11</td>
<td>245</td>
</tr>
<tr>
<td>Double BS</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>9</td>
<td>309</td>
</tr>
<tr>
<td>Triple BS</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>9</td>
<td>350.5</td>
</tr>
</tbody>
</table>
The BS capability assessment should consider the factor of installed new BS capability, increased system generation capability and reduced total system restoration time. Figure 3.8 shows the comparison and provides guidance to system operators for the optimal installation strategy.

![Figure 3.8. Comparison of different installation strategies](image)

The comparison of system generation capability curves under different BS capabilities is shown in Figure 3.9.
(2) Considering GSS, TPS and GRAs

When increasing the BS capability by two times than the base case, the generator starting sequence and time will change. However, if one continues increasing the BS capability, there will be no further improvements. Therefore, the optimal amount of new BS capability will be 3 MW. The comparisons of generator starting time after installing new BS capability at different buses are shown in Table 3.6.
Table 3.6 Comparison of Restoration Time and System Generation Capability under Different Installation Strategies

<table>
<thead>
<tr>
<th>Installation Strategy</th>
<th>$t_{start1}$ (hr)</th>
<th>$t_{start2}$ (hr)</th>
<th>$t_{start3}$ (hr)</th>
<th>$t_{start4}$ (hr)</th>
<th>Total Restoration Time (hr)</th>
<th>System Generation Capability (MWhr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>13</td>
<td>210.5</td>
</tr>
<tr>
<td>Install at Bus 5</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>11</td>
<td>259</td>
</tr>
<tr>
<td>Install at Bus 4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>10</td>
<td>284</td>
</tr>
<tr>
<td>Install at Bus 3</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>11</td>
<td>273</td>
</tr>
<tr>
<td>Install at Bus 2</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>11</td>
<td>266</td>
</tr>
<tr>
<td>Install at Bus 1</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>11</td>
<td>259</td>
</tr>
</tbody>
</table>

Also, the comparison of installed BS capabilities, increased system generation capability and reduced total restoration time with three different installation strategies of BS capabilities are shown in Figure 3.10. The comparison of system generation capability curves under different installation strategies of BS capability is shown in Figure 3.11. The optimal installation strategy will be to install 3 MW BS capabilities at Bus 4 to reduce restoration time by 2 hours and increase system generation capability by 63.5 MWh.
Figure 3.10. Comparison of different installation strategies

Figure 3.11. Comparison of system generation capability curve under different installation strategies
3.3.2 Case of IEEE RTS 24-Bus Test System

In this case study, IEEE RTS 24-bus test system [44], as shown in Figure 3.12, is used to illustrate the proposed installation strategy of new BS generators. In this system, there are 1 BSU and 9 NBSUs. Based on the generator data in PECO system [9], the characteristics of generators are shown in Table 3.7.

![IEEE RTS 24-bus test system](image)

**Figure 3.12. IEEE RTS 24-bus test system**

In the base case, the time to energize branch or transformer (GRA3 [15]) is set as 2 minutes. By utilizing the SRN tool, the time to restore all the generators is:

\[ T_{\text{restore}} = 36 \text{ (mins)} \]

The steps to restore all the generators are shown in Table 3.8.
### Table 3.7 Data of Generator Characteritise

<table>
<thead>
<tr>
<th>Gen</th>
<th>Bus</th>
<th>BSU/NBSU</th>
<th>$T_{ctp}$ (min)</th>
<th>$T_{cmin}$ (hr)</th>
<th>$T_{cmax}$ (hr)</th>
<th>$R_r$ (MW/hr)</th>
<th>$P_{start}$ (MW)</th>
<th>$P_{max}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>BSU</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>138</td>
<td>0</td>
<td>138</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>NBSU</td>
<td>30</td>
<td>N/A</td>
<td>N/A</td>
<td>120</td>
<td>6.6</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>NBSU</td>
<td>30</td>
<td>N/A</td>
<td>N/A</td>
<td>346</td>
<td>13.2</td>
<td>660</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>NBSU</td>
<td>100</td>
<td>N/A</td>
<td>N/A</td>
<td>157</td>
<td>12</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>NBSU</td>
<td>120</td>
<td>N/A</td>
<td>N/A</td>
<td>150</td>
<td>30</td>
<td>252</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>NBSU</td>
<td>160</td>
<td>N/A</td>
<td>N/A</td>
<td>30</td>
<td>2.7</td>
<td>135</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
<td>NBSU</td>
<td>100</td>
<td>N/A</td>
<td>3.3</td>
<td>120</td>
<td>6</td>
<td>300</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>NBSU</td>
<td>120</td>
<td>N/A</td>
<td>3.5</td>
<td>100</td>
<td>9</td>
<td>300</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>NBSU</td>
<td>30</td>
<td>N/A</td>
<td>4</td>
<td>148</td>
<td>12</td>
<td>345</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>NBSU</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>120</td>
<td>0</td>
<td>302</td>
</tr>
</tbody>
</table>

### Table 3.8 Sequence of Restoration Actions

<table>
<thead>
<tr>
<th>Restoration Action</th>
<th>Time (min.)</th>
<th>Path</th>
<th>Dispatchable Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restart BSU on Bus 22</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 18</td>
<td>12</td>
<td>22-17-18</td>
<td>N/A</td>
</tr>
<tr>
<td>Pick up critical loads on 19</td>
<td>12</td>
<td>17-16-19</td>
<td>N/A</td>
</tr>
<tr>
<td>Pick up critical loads on Bus 14</td>
<td>12</td>
<td>16-14</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 15</td>
<td>20</td>
<td>22-21-15</td>
<td>N/A</td>
</tr>
<tr>
<td>Pick up critical loads on Bus 9</td>
<td>20</td>
<td>14-11-9</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 16</td>
<td>20</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 2</td>
<td>22</td>
<td>9-4-2</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 7</td>
<td>26</td>
<td>9-8-7</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 21</td>
<td>26</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 23</td>
<td>30</td>
<td>19-20-23</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 13</td>
<td>32</td>
<td>11-13</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 1</td>
<td>36</td>
<td>11-10-5-1</td>
<td>4,10,11,17</td>
</tr>
</tbody>
</table>
**Study 1: Install New Blackstart Generating Units at Bus 22**

If a new BS generator is to be installed, which is the same as the current BS unit at Bus 22, the benefit of installing this new BS unit will be analyzed using the SRN tool. The system now has 2 BS units and 9 NBS units. The time to restore all generators is:

$$T_{\text{restore}} = 28 \text{ (mins)}$$

The steps to restore all the generators are shown in Table 3.9:

**Table 3.9 Different Sequence of Restoration Actions Compared with Base Case**

<table>
<thead>
<tr>
<th>Restoration Action</th>
<th>Time (min.)</th>
<th>Path</th>
<th>Dispatchable Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restart BSU on Bus 22</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 15</td>
<td>8</td>
<td>22-21-15</td>
<td>N/A</td>
</tr>
<tr>
<td>Pick up critical loads on 19</td>
<td>8</td>
<td>22-17-16-19</td>
<td>N/A</td>
</tr>
<tr>
<td>Pick up critical loads on Bus 14</td>
<td>8</td>
<td>16-14</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 18</td>
<td>12</td>
<td>17-18</td>
<td>N/A</td>
</tr>
</tbody>
</table>

After increasing BS capability at Bus 22, it provides more cranking power to first start NBSU on Bus 15 and then to crank NBSU on Bus 18, both at earlier times than in the base case. Compared with NBSU on Bus 18, NBSU on Bus 15 requires more cranking power but has a higher ramping rate and generation capacity. With the help of additional BS capability, it is started at an earlier time and helps to start other NBS generating units. It is shown that the installation of additional BS generators can benefit system restoration by shortening the total restoration time.

By gradually increasing the total BS capability on Bus 22, the SRN tool is used to calculate the restoration time. The comparison of restoration time under different BS
capability is shown in Figure 3.13. It can be seen that when increasing somewhat the BS capability, the system restoration will not benefit. There is a maximum amount under which restoration time is reduced due to the additional BS capability. However, beyond the maximum amount, the extra BS capability will not help further to reduce the restoration time. In this case study, the threshold at Bus 22 is to install additional 50 MW BS capabilities to reduce restoration time by 8 minutes, about 20% of the restoration time in the base case. Therefore, when making decisions on installing additional BS units, the benefit analysis needs to be conducted to find the optimal capability of installed BS units.

![Figure 3.13. Comparison of restoration times under different blackstart capabilities on Bus 22](image)

**Study 2: Install One New Blackstart Unit at Different Buses**

When installing new additional blackstart units, system restoration steps will change. At different installation locations, the new restoration strategy can be made using GRMs-
based restoration tool. In this study, one new BS unit, same with the BS unit on Bus 22, is installed at different buses. The times to restore all the generators are shown in Figure 3.14. From the comparison, it is shown that after installing one new BS generating unit at different buses, some benefit from the additional BS capability to reduce the restoration time, while others have the same restoration time. In this case study, the optimal locations for installing a new BS generating unit are Bus 8 and Bus 12. The restoration steps of installing an additional BS generating unit at Bus 8 are shown in Table 3.10. Compared with the restoration steps by increasing BS capability at Bus 22, NBSU on Bus 7 and Bus 21 are started earlier, which brings a shorter restoration time.

Figure 3.14. Comparison of restoration times with installation of one new blackstart unit at different buses
Table 3.10  Sequence of Restoration Actions

<table>
<thead>
<tr>
<th>Restoration Action</th>
<th>Time(min.)</th>
<th>Path</th>
<th>Dispatchable Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restart BSU on Bus 22</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Restart BSU on Bus 8</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Crank NBSU on Bus 15</td>
<td>6</td>
<td>22-21-15</td>
<td>10</td>
</tr>
<tr>
<td>Pick up critical loads on 19</td>
<td>6</td>
<td>22-17-16-19</td>
<td>4,10</td>
</tr>
<tr>
<td>Pick up critical loads on Bus 14</td>
<td>6</td>
<td>16-14</td>
<td>4,10</td>
</tr>
<tr>
<td>Pick up critical loads on Bus 9</td>
<td>6</td>
<td>14-11-9</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 18</td>
<td>12</td>
<td>17-18</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 16</td>
<td>12</td>
<td>17-16</td>
<td>10,11,17</td>
</tr>
<tr>
<td>Crank NBSU on Bus 2</td>
<td>14</td>
<td>9-4-2</td>
<td>10,11,17</td>
</tr>
<tr>
<td>Crank NBSU on Bus 7</td>
<td>14</td>
<td>9-8-7</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 21</td>
<td>14</td>
<td>N/A</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 23</td>
<td>18</td>
<td>19-20-23</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 13</td>
<td>20</td>
<td>11-13</td>
<td>4,10</td>
</tr>
<tr>
<td>Crank NBSU on Bus 1</td>
<td>24</td>
<td>11-10-5-1</td>
<td>4,10,11,17</td>
</tr>
</tbody>
</table>

**Study 3: Optimal Installation Strategy of Additional Blackstart Capability**

The installation of additional BS generating unit does not automatically benefit system restoration. In the previous study 1 and 2, it is shown that when installing new BS generators, both the location and amount of BS capability need to be decided to obtain the optimal installation strategy. In this study, the developed GRMs-based algorithm is utilized to calculate the optimal installation location and amount. Increasing the amount of additional BS capability at each bus, the restoration times are obtained.

The comparisons of restoration times are shown in Figure 3.15.
Figure 3.15. Comparison of restoration times at different buses with additional blackstart capability

It can be seen that, after installing new BS generating units, the restoration time decreases in the most cases, while it stays the same at some buses. For each situation with reduced restoration time, there is a threshold at which restoration time reaches the minimum value. However, for some buses, such as Bus 2 and Bus 16, there are multiple break points. They provide multiple choices for increasing BS capability to reduce the restoration time.

However, the optimal installation strategy of additional BS capability needs to consider both the reduced restoration time and installed BS capability. The comparisons are shown in Figure 3.16. When installing 50 MW additional BS capabilities at Bus 8, the restoration time is reduced by 12 minutes. When installing 100 MW additional BS capabilities at Bus 18, the restoration time is reduced by 10 minutes. When installing 150 MW additional BS capabilities at Bus 12, the restoration time is down by 12 minutes. When installing 200 MW additional BS capabilities at Bus 13, the restoration time is down by 12
minutes. Therefore, in this system, the optimal installation strategy is to install additional 50 MW BS generating unit at Bus 8 to reduce restoration time by 12 minutes, or 33% of the restoration time in the base case.

![Figure 3.16. Comparison of reduced restoration times and installed blackstart capability at each bus](image)

3.3.3 Case of IEEE 39-Bus System

In this case study, the IEEE 39-Bus system is used for illustration of the proposed MIBLP model and its solution methodology.

1) Only considering GSS

Increase system BS capability, and get the generator start-up time, total restoration time and system generation capability, as shown in Table 3.11. The comparison of increased system generation capability with different BS capabilities is shown in Figure 3.17.
Table 3.11 Comparison of Restoration Time and System Generation Capability with Different Blackstart Capabilities

<table>
<thead>
<tr>
<th>BS Capability</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>Total Restoration Time</th>
<th>System Generation Capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>26</td>
<td>94002</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>26</td>
<td>97995</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>26</td>
<td>100346</td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>26</td>
<td>103785</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>26</td>
<td>106969</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>26</td>
<td>109769</td>
</tr>
<tr>
<td>70</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>26</td>
<td>112480</td>
</tr>
<tr>
<td>80</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>26</td>
<td>115287</td>
</tr>
<tr>
<td>90</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>26</td>
<td>117805</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>26</td>
<td>120438</td>
</tr>
<tr>
<td>110</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>26</td>
<td>123322</td>
</tr>
<tr>
<td>120</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>26</td>
<td>125672</td>
</tr>
</tbody>
</table>

It is shown that when the increased amount is small (10MW) for the BS capability, the system will benefit from the extra BS capability significantly. Although the total restoration time does not change, due to the time constraint of G4, system generation capability increases until the amount of installed BS capability exceeds the threshold, which is 110 MW in this case. And the more installed BS capability, the smaller increasing rate of system generation capability.
2) **Considering GSS, TPS and GRAs**

Install 10 MW BS capabilities at different buses, and calculate the total restoration time and system generation capability. Continue increasing the BS capability, and it is observed that there is no more improvement. The comparisons are shown in Figure 3.18.

It is shown that several optimal installation strategies are available. Installing at Bus 14 will bring the maximally reduced restoration time of 3 p.u. time and second largest increased system generation capability. Installing at Bus 15 will bring the third largest reduced restoration time and second largest increased system generation capability. Installing at Bus 17 will bring the third largest reduced restoration time and largest increase in system generation capability. System planners can utilize the information provided by the proposed method to make the optimal installation decisions.
If installing at Bus 14 to reduced restoration time by 3 p.u. time, which is saving 30-minute of outage of power systems, it contributes substantially to enhancement of power system reliability. Reliability is a high priority in power system operation. Following an outage, it is critical to restore system back to a normal operation condition as efficiently as possible. According to the Mid-Atlantic Area Council (MAAC) Reliability Principles and Standards, “sufficient megawatt generating capacity shall be installed to ensure that in each year for the MAAC system the probability of occurrence of load exceeding the available generating capacity shall not be greater, on the average, than one day in ten years” [44]. Based on this “one day in ten year” loss-of-load expectation (LOLE) principle, North American Electric Reliability Corporation (NERC) standard of “Planned Resource Adequacy Assessment” is established to analyze and assess the resource adequacy for load in the
Reliability First Corporation (RFC) region [45]. Therefore, it is important to reduce the duration of outages in order to meet this NERC reliability standard. The reduced outage time is an essential performance index of a power system’s resilience.

Moreover, from the economic point of view, saving 30-min could save millions of dollars. The cost increases exponentially during the scale and duration of an outage. A major power outage makes a significant impact on people’s daily lives and the economy. The Aug. 14, 2003, blackout in USA and Canada affected an area with an estimated 50 million people and 61,800 MW of load. The duration of the outage is about two days, and the estimated of total costs ranges between $4 billion and $10 billion [46]. After the blackout, industrial sectors, public transportation, financial and other physical systems were severely affected.

NERC reliability standard requires a system to establish a Blackstart Capability Plan (BCP) to ensure that the quantity and location of system blackstart generators are sufficient to provide blackstart service [47]. Therefore, it is mandatory for power systems to have sufficient blackstart capability to increase its resilience against disturbances or outages. Installing new blackstart capability can reduce the total restoration time and achieve an efficient restoration process.

Typical blackstart generating units are diesel, hydro or combustion turbine units, which are expensive and are often used to serve the peak load. A rolling blackout often happens during the periods of peak energy demand. By installing more blackstart generators based on the system reliability requirement, these peaking units can serve the peak load and enhance power system security.
The performance of the proposed optimization modules is shown in Table 3.12. From the simulation results, it is seen that the computational time is within the practical range for both system restoration planning and on-line decision support environments.

Table 3.12 Performance Analysis

<table>
<thead>
<tr>
<th></th>
<th>PJM 5-Bus Case</th>
<th>IEEE 39-Bus Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of NBS Generators</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Number of All Generators</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Number of Buses</td>
<td>5</td>
<td>39</td>
</tr>
<tr>
<td>Number of Lines</td>
<td>6</td>
<td>46</td>
</tr>
<tr>
<td>Total Restoration Time (hr)</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Number of Decision Variables</td>
<td>480</td>
<td>10642</td>
</tr>
<tr>
<td>Number of Constraints</td>
<td>1521</td>
<td>36253</td>
</tr>
<tr>
<td>Computational Time (sec.)</td>
<td>0.27</td>
<td>3.20</td>
</tr>
</tbody>
</table>

3.4 Summary

This research developed a new method to estimate total restoration time and system generation capability and proposed a restoration time based BS capability assessment methodology. Blackstart capability assessment is an important task, and it is highly system dependent and lacks universal solutions. The proposed methodology provides a systematic way to assess the optimal installation location and amount of BS capability for power systems to make the installation strategy.

It is shown that power systems can benefit from new BS generators to reduce the restoration time. However, there is a maximum amount beyond which system restoration time cannot be further reduced with additional BS capability. Economic considerations should be taken into account when assessing additional BS capabilities. The proposed
strategy is able to provide the optimal location and amount of additional BS capability to assist system planners in decision makings of the optimal installation strategy.

In the future work, more optimization modules, e.g., load pick-up, optimal power flow, voltage stability check, can be included in the model to provide more accurate solutions.
CHAPTER 4. OPTIMAL GENERATION SCHEDULING IN A CARBON DIOXIDE ALLOWANCE MARKET ENVIRONMENT

In a competitive market environment, GENCOs can schedule the maintenance periods to maximize their profits. ISO’s functionality is also considered from the view point of system reliability and cost minimization. Carbon Dioxide mitigation policies, such as CO₂ emission cap-and-trade, help to reduce consumption in fossil energy and promote a shift to renewable energy resources. Considering these new effects, GENCOs need to adjust their scheduling strategies in the electricity market and bidding strategies in CO₂ allowance market. In this paper, the emission-constrained GSP involving generation maintenance scheduling, unit commitment and CO₂ emission cap-and-trade is formulated as a MIBLP problem. A novel solution methodology is proposed, and simulation results based on the PJM 5-bus system test case demonstrate that the proposed MIBLP-based model is able to provide valuable information for GENCOs’ decision makings in both electricity and CO₂ allowance markets.

4.1 The Optimization Model of Carbon Dioxide Allowance Market

4.1.1 Background of Cournot Equilibrium Model

As the electric power industry becomes market driven, the development of power market design provides an opportunity for GENCOs and other market participants to exercise least-cost or profit-based operations. The equilibrium model of generator competition can be used to investigate the ability of GENCOs to unilaterally manipulate prices (market power).
Most models are based upon a general approach of defining a market equilibrium as a set of prices, producer input and output decisions, transmission flows, and consumption that simultaneously satisfy each market participant’s first-order conditions for maximization of their net benefits (KKT conditions) while clearing the market (supply = demand) [49]. The complete set of KKT and market clearing conditions defines a mixed complementarity problem (MCP). If a market solution exists that satisfies the optimality conditions for each market player along with the market clearing conditions, no participant would want to alter their decision unilaterally (as in a Nash equilibrium).

The equilibrium of Nash games is defined as Nash Equilibrium:

**Definition:** Let $X_f \in X_f$ be strategies under the control of firm $f$; $X_f$ the space of feasible strategies for $f$; $X_{-f}$ the space of feasible strategies for all firms except $f$; $\Pi_f(X_f, X_{-f}^*)$ the payoff to $f$ given the decisions of all firms. Then, $\{X_f^*, \forall f\}$ is a Nash Equilibrium in $X$ if $\Pi_f(X_f^*, X_{-f}^*) \geq \Pi_f(X_f, X_{-f}^*)$ for all $X_f \in X_f, \forall f$.

For Cournot games, $X_f = q_f$. There are several types of strategic interactions; they differ in how each generating firm $f$ anticipates how rivals will react to its decisions concerning either prices $p$ or quantities $q$.

- **Pure Competition (No Market Power) / Bertrand:** Only $q_f$ is the decision variable and $p$ is fixed.

- **Generalized Bertrand Strategy (“Game in Prices”):** firm $f$ acts as if its rivals’ prices, $p_{-f}^*$, will not change in reaction to changes in $f$’s prices.
• **Cournot Strategy ("Game in Quantities"):** firm \( f \) acts as if its rivals’ quantities, \( q^* \), will not change in reaction to changes in \( f \)'s quantities.

• **Collusion:** If \( f \) colludes with another supplier, then they would maximize their joint profit.

• **Stackelberg:** It defines a “leader” whose decisions correctly take into account the reactions of “followers,” who do not recognize how their reactions affect the leader’s decisions.

• **General Conjectural Variations (CVs):** Output from firms other than \( f \), \( q_f(q_f) \), is assumed to be a function of \( q_f \).

• **Conjectured Supply Function (CSF):** Output by rivals is anticipated to respond to price according to function \( q_f(p) \).

• **Supply Function Equilibria (SFE):** The decision variables for each firm \( f \) are the parameters \( \phi_f \) of its bid function \( q_f(p|\phi_f) \).

In this research, the Cournot model is used with bidding on quantities to analyze the CO\(_2\) emission allowance market.

### 4.1.2 Formulation of the EPEC Module

The CO\(_2\) allowance market is formulated as the Cournot equilibrium model based on the market rules in RGGI. In RGGI, the primary market offer initial allowances through a single-round, uniform-price, sealed-bid auction, which the price paid by all bidders with the highest bids for the available units is equal to the highest rejected bid. The characteristics of allowance banking, auction limit, CO\(_2\) reserve price, offset limit, etc. are considered in the developed model. The model is formulated as an equilibrium problem with equilibrium
constraints (EPEC), and the EPEC formulation is transformed to a nonlinear complementarity problem (NCP) and nonlinear programming problem (NLP), which can be solved by AMPL/MINOS commercial solver.

4.1.2.1 Bid and Auction

Each GENCO submit its bidding offer \((\lambda_i, q_i)\) to the CO\(_2\) allowance market, which \(\lambda_i\) is the bidding price and \(q_i\) is the bidding amount. The example of three GENCOs’ bidding offers is shown in Figure 4.1. After the market clearance, the CO\(_2\) allowance price \(\lambda^{CO2}\) and the allowance dispatch \(A_i\) is achieved, as shown in Figure 4.2.
4.1.2.2 Problem Formulation

Each GENCO solves the following optimization problem to decide its bidding strategy:

\[
\max_{P, a_i, b_i, \lambda^{CO2}_i, A_i} \quad \lambda^{CO2}_i P_i - (a_i P_i + b_i P_i^2) - \lambda^{CO2}_i A_i - h_i OS_i
\]

s.t.

\[
0 \leq k_i P_i \leq A_i + OS_i \quad \text{(4.2)}
\]

\[
OS_i \leq 0.033 A_i \quad \text{(4.3)}
\]

\[
0 \leq P_i \leq P_{i}^{\max} \quad \text{(4.4)}
\]

where, \( \lambda^{e} \) is the forecasted electricity price, \( P \) is generation output, \( a \& b \) are coefficients of generator’s cost function, \( h \) is the cost rate of offsets, \( OS \) is the offset, \( k \) is the CO\(_2\) emission rate, \( P_{i}^{\max} \) is generation capacity.
Cournot competition is to describe an industry structure in which companies compete on the amount of output they produce, which they decide on independently of each other and at the same time. Each GENCO has the bidding price of CO₂ allowance to its own expectation. Therefore, in the developed model, the bidding price is the parameter and the bidding amount is the decision variable.

The objective is to maximize the profit, which is equal to the revenue from selling power to electricity market minus the cost of generation, buying allowances from CO₂ market and using offsets. The first constraint requires each GENCO to have enough allowances to cover its generated CO₂. The second constraint requires that the use of CO₂ offset allowances is constrained to 3.3% of a unit’s total compliance obligation during a control period. Offsets referred to the project-based emissions reductions outside the capped sector [48].

The market clearing price is obtained from solving the following optimization problem:

\[
\max_A \sum_{j=1}^n \lambda_j A_j \tag{4.5}
\]

\[
s.t. \quad \sum_{j=1}^n A_j = CAP^{CO2} \tag{4.6}
\]

\[
A_j \leq 0.25CAP^{CO2} \tag{4.7}
\]

\[
0 \leq A_j \leq q_j \tag{4.8}
\]

where, \(CAP^{CO2}\) is the total amount of allowances in the auction.
The first constraint is based on the assumption that all allowances will be sold to assure there is one CO\textsubscript{2} allowance price. The second constraint is based on the auction rules in RGGI that it established a total limit for the number of allowances that entities may purchase in a single auction, equivalent to 25\% of the allowance offered for sale in any single auction. The third constraint restricts that each GENCO’s purchased allowances should not exceed its bidding allowances and should be nonnegative.

Given \((\lambda_j, q_j) \, j = 1, \ldots, n\), the optimal solution \((A_j, \lambda^{CO2}) \, j = 1, \ldots, n\) of the concave optimization problem (4.5)-(4.8) can be obtained by solving its KKT conditions as following:

\[
\sum_{j=1}^{n} A_j - CAP^{CO2} = 0 \tag{4.9}
\]

\[
0 \leq A_j \quad \perp - \lambda_j + \lambda^{CO2} + w_j^1 - w_j^2 \geq 0, \quad j = 1, \ldots, n \tag{4.10}
\]

\[
0 \leq w_j^1 \quad \perp q_j - A_j \geq 0, \quad j = 1, \ldots, n \tag{4.11}
\]

\[
0 \leq w_j^2 \quad \perp A_j \geq 0, \quad j = 1, \ldots, n \tag{4.12}
\]

\[
0 \leq w_j^3 \quad \perp 0.25CAP^{CO2} - A_j \geq 0, \quad j = 1, \ldots, n \tag{4.13}
\]

Then KKT conditions (4.9)-(4.13) are added to each GENCO’s maximization problem (4.1)-(4.4), and each GENCO’s optimization problem is formulated as the following mathematical problem with equilibrium constraints (MPEC) problem (4.14).

Each GENCO solves the above MPEC problem and all the GENCOs together will get an equilibrium point of this EPEC.
4.1.3 Formulation of NCP and NLP Modules

In the literature, several methods are available to solve the EPEC problem:

1) Diagonalization techniques such as Gauss-Jacobi and Gauss-Seidel type methods. Such methods solve a cyclic sequence of MPEC until the decision variables of all participants reach a fixed point.

2) Sequential nonlinear complementarity problem (SNCP) approach. The approach is related to the relaxation approach used in MPEC that relaxes the complementarity condition of each player and drives the relaxation parameter to zero.

3) Deriving a NCP formulation of the EPEC based on the equivalence between the KKT conditions of the MPEC and strong stationarity. Then EPEC will be solved by standard NCP solvers to MPEC.

The traditional way in the third method is to replace the complementarity condition, such as $0 \leq y \perp s \geq 0$, by $y \geq 0, s \geq 0, y^T \cdot s = 0$, and this equivalent NLP can be solved by

$$
\max_{P_i, \phi_i, OS_i, \lambda_i^{CO2}, A_i, w_i^1, w_i^2, w_i^3} \lambda^T P_i - (a_i P_i + b_i P_i^2) - \lambda_i^{CO2} A_i - h_i OS_i
$$

s.t.

$$
\begin{align*}
0 & \leq k_i P_i \leq A_i + OS_i \\
OS_i & \leq 0.033 A_i \\
0 & \leq P_i \leq P_i^{max} \\
\sum_{j=1}^n A_j - CAP^{CO2} &= 0 \\
0 & \leq A_j \perp - \lambda_j + \lambda_i^{CO2} + w_j^1 - w_j^2 \geq 0 \\
0 & \leq w_j^1 \perp q_j - A_j \geq 0 \\
0 & \leq w_j^2 \perp A_j \geq 0 \\
0 & \leq w_j^3 \perp 0.25 CAP^{CO2} - A_j \geq 0
\end{align*}
$$

(4.14)
using standard NLP solvers. Unfortunately, this NLP violated the Mangasarian-Fromovitz constraints qualification (MFCQ) at any feasible point [50]. Instead, it is proposed to use the method of [51]. Based on that, strong stationarity is equivalent to the KKT conditions of the equivalent NLP.

First, define:

\[ x_i = (P_i, q_i, OS_i) \]  
\[ f_i(x_i, y_i) = \lambda^x_i P_i - a_i P_i - b_i P_i^2 - \lambda^{CO2}_i A_i - h_i OS_i \]  
\[ g_i(x_i, y_i) = \begin{pmatrix} k_i P_i - A_i - OS_i \\ -k_i P_i \\ OS_i - 0.033 A_i \\ P_i - P_i^{\text{max}} \\ -P_i \end{pmatrix} \]  
\[ H_j = \begin{pmatrix} \sum_{j=1}^{n} A_j - CAP^{CO2} \\ -\lambda_j + \lambda^{CO2}_j + w_j^1 - w_j^2 \\ q_j - A_j \\ A_j \geq 0 \\ 0.25 CAP^{CO2} - A_j \geq 0 \end{pmatrix} \]  
\[ y_j = \begin{pmatrix} \lambda^{CO2}_j, A_j, w_j^1, w_j^2, w_j^3 \end{pmatrix} \]

Then the MPEC problem (4.14) is rewritten in the following compact format:

\[
\begin{align*}
\max_{x, y, s} & \quad f(x, y) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
& \quad H_j - s_j = 0 \\
& \quad 0 \leq y_j \perp s_j \geq 0 \quad j = 1, \ldots n
\end{align*}
\]
where, $s$ is the introduced slack variable.

The NCP formulation is derived by introducing new multipliers:

$$
\nabla_x f(x_i, y_i) + \nabla_x g(x_i, y_i) \mu_i + \nabla_x h(x_i, y_i) \xi_i - \chi_i = 0 \quad (4.21)
$$

$$
\nabla_y f(x_i, y_i) + \nabla_y g(x_i, y_i) \mu_i + \nabla_y h(x_i, y_i) \xi_i + s \eta_i = 0 \quad (4.22)
$$

$$
\xi_i - \sigma_i + y \xi_i = 0 \quad (4.23)
$$

$$
0 \leq g(x_i, y_i) \perp \mu_i \geq 0 \quad (4.24)
$$

$$
h(x_i, y_i) - s_i = 0 \quad (4.25)
$$

$$
0 \leq x_i \perp \chi_i \geq 0 \quad (4.26)
$$

$$
0 \leq \psi_i + s_i \perp y \geq 0 \quad (4.27)
$$

$$
0 \leq \sigma_i + s \perp s \geq 0 \quad (4.28)
$$

$$
0 \leq \psi_i + \sigma_i \perp \eta_i \geq 0 \quad (4.29)
$$

**Definition**[51]: A solution of problem (4.20) is called an equilibrium point of the EPEC. A solution $(x^*, y^*, s^*, \chi^*, \mu^*, \xi^*, \psi^*, \sigma^*, \eta^*)$ of (4.21)-(4-29) is called a strongly stationary point of the EPEC.

**Proposition**[51]: If $(x^*, y^*, s^*)$ is an equilibrium of the EPEC and if every MPEC of (4.20) satisfies an MPEC linear independence constraints qualification (MPEC-LICQ), then there exist multipliers $(\chi^*, \mu^*, \xi^*, \psi^*, \sigma^*, \eta^*)$ such that (4.21)-(4-29) hold.

Moreover, to utilize the standard NCP solvers, two alternative formulations will be adopted. The first one is to force the EPEC to identify the basic or minimal multiplier for
each player by minimizing the $l_1$-norm of the multiplier on the complementarity constraint, as shown in (4.30)-(4.40).

$$\max_{x, y, s, \mu, \xi, \psi, \sigma, \eta} \sum_{i=1}^{n} e^T \eta_i$$

s.t.

$$\nabla_{x_i} f(x_i, y) + \nabla_{x_i} g(x_i, y) \mu_i + \nabla_{x_i} h(x_i, y) \xi_i - \chi_i = 0$$

$$\nabla_{y_i} f(x_i, y) + \nabla_{y_i} g(x_i, y) \mu_i + \nabla_{y_i} h(x_i, y) \xi_i - \psi_i + s_i \eta_i = 0$$

$$\xi_i - \sigma_i + y_i \xi_i = 0$$

$$0 \leq g(x_i, y) \perp \mu_i \geq 0$$

$$h(x_i, y) - s_i = 0$$

$$0 \leq x_i \perp \chi_i \geq 0$$

$$0 \leq y \perp \psi_i \geq 0$$

$$0 \leq s \perp \sigma_i \geq 0$$

$$0 \leq y \perp s \geq 0$$

$$\eta_i \geq 0$$

The second one penalizes the complementarity constraints and results in a well-behaved nonlinear optimization problem by introducing new slack variables $t_i$, as shown in (4.41)-(4.49).

$$\min_{x, y, s, \mu, \xi, \psi, \sigma, \eta} C_{p,n} = \sum_{i=1}^{n} \left( x_i^T \chi_i + t_i^T \mu_i + y_i^T \psi_i + s_i^T \sigma_i \right) + y^T s$$

s.t.
\[ \nabla_x f(x_i, y) + \nabla_x g(x_i, y) \mu_i + \nabla_x h(x_i, y) \xi_i - \chi_i = 0 \quad (4.42) \]
\[ \nabla_y f(x_i, y) + \nabla_y g(x_i, y) \mu_i + \nabla_y h(x_i, y) \xi_i - \psi_i + s_i \eta_i = 0 \quad (4.43) \]
\[ \xi_i - \sigma_i - y \xi_i = 0 \quad (4.44) \]
\[ g(x_i, y) = t_i \quad (4.45) \]
\[ h(x_i, y) = s_i \quad (4.46) \]
\[ y \geq 0, \quad s \geq 0 \quad (4.47) \]
\[ x_i \geq 0, \quad \mu_i \geq 0, \quad \psi_i \geq 0, \quad \sigma_i \geq 0, \quad \eta_i \geq 0 \quad (4.48) \]
\[ t_i \geq 0 \quad (4.49) \]

**Theorem**[51]: If \( (x^*, y^*, s^*, t^*, \chi^*, \mu^*, \xi^*, \psi^*, \sigma^*, \eta^*) \) is a local solution of (4.41)-(4.49) with \( C_{pen}=0 \), then it follows that \( (x^*, y^*, s^*) \) is a strongly stationary point of (4.20).

Two-GENCO and multi-GENCO case studies are learned for the sensitivity analysis of GENCOs' bidding prices, electricity price and total amount of CO2 allowances to CO2 allowance price and dispatch. The simulation results demonstrate the joint effect of electricity market and CO2 allowance market, which require appropriate model for further investigation. In the next section, the model of this new GSP in both markets is proposed.

### 4.2 New Generation Scheduling Problem in Two Markets

Under the new emerging circumstance of CO2 Emission Regulation, GENCOs need to participate in both electricity market and CO2 allowance market. They need to purchase enough allowances from CO2 allowance market to cover emitted CO2 from producing electricity, while at the same time, they bid to the electricity market. In order to maximize the
profit, GENCOs need to adjust and coordinate their strategies in both markets. An appropriate model is needed to analyze this new GSP, as shown in Figure 4.3.

![Diagram](image)

**Figure 4.3. New GSP in two markets**

GENCOs participate in the electricity market daily and they also auction in the CO₂ emission allowance market quarterly. They need to know the amount and price of CO₂ allowances to make decisions about how to bid in the electricity market, while they bid to CO₂ allowance market based on the information of electricity price and scheduled generation commitment and dispatch, as shown in Figure 4.4.

Moreover, during the three-year CO₂ allowance compliance period, the coordination of midterm generation maintenance scheduling with short-term unit commitment has to be considered to maintain the adequacy in midterm planning and security in short-term operation planning [52],[53]. Therefore, in this three-year time framework, as shown in Figure 4.5, generation scheduling problem involving generation maintenance scheduling, unit commitment and CO₂ allowance cap-and-trade need to be investigated.
Figure 4.4. GENCOs’ interactions in electricity market and CO₂ allowance market

Figure 4.5. Time horizon of three-year GSP
4.2.1 Problem Structure

GENCOs’ decision makings will be based on the following optimization problem:

Max \quad \text{Total Profit during Time Period } T

subject to

Generation Maintenance Scheduling Constraints

SCUC and SCOPF Constraints

CO\textsubscript{2} Allowance Cap-and-Trade Market Constraints

By solving this optimization problem, GENCOs are able to decide the following

**Decision Variables:**

\( A_{ft} \) \quad \text{Amount of allowances distributed to firm } f \text{ in interval } t’

\( OS_{ft} \) \quad \text{Offsets used by firm } f \text{ in interval } t’

\( p_{it}^{CO2} \) \quad \text{CO}_2 \text{ allowance price in interval } t’ \text{ [$/p.u.]}

\( p_{it}^{E} \) \quad \text{LMP at node } i \text{ in period } t \text{ [$/MWh]}

\( g_{it} \) \quad \text{Power output of generator } i \text{ in period } t \text{ [MW]}

\( u_{it} \) \quad \text{Binary variable of the commitment of generation } i \text{ in period } t

\( x_{it} \) \quad \text{Binary variable of the maintenance schedule of generation } i \text{ in period } t

Then this new GSP is formulated as a bi-level optimization problem, as shown in Figure 4.6 [54]. In the upper level problem, GENCOs make decisions to maximize their own profit. And in the lower level problem, after receiving the bids from GENCOs, ISO and CO\textsubscript{2} allowance market operator will clear both markets and make available the electricity price, generator commitment, generation dispatch level, CO\textsubscript{2} allowance price and cleared demand.
4.2.2 Upper Level Optimization Problem Formulation

**Objective function**: GENCO’s profit is equal to its revenue from selling power to the electricity market minus its cost of maintenance, fuel production, startup, shutdown and CO₂ allowance. GENCO maximizes its profit by solving the following problem:

$$
\max \sum_{i',d'} \left\{ P_{i',d'} g_{i',d'} - C_{i',d'} \left( g_{i',d'} \right) - C_{SU} - C_{SD} - \sum_{i'} C_{i'} \right\} - \sum_{i'} \left\{ P_{i'} A_{i'} + C_{i} OS_{i'} \right\} \tag{4.50}
$$

s.t.

$$
u_{i',d'} - u_{i',d'-1} \leq x_{i',w}, \quad \forall i',t^d \in \{t^w\} \tag{4.51}$$

$$x_{i',w} + u_{i',d'} \leq 1, \quad \forall i',t^d \in \{t^w\} \tag{4.52}$$

$$\sum_{i',w} x_{i',w} = T_{1,i}, \quad \forall i \tag{4.53}$$

$$x_{i',w} - x_{i',w-1} \leq x_{i',w+(T_{1,i}-1)}, \quad \forall i,t^w \tag{4.54}$$
\[ \sum_{i} x_{i}^w \leq N_{i}, \quad \forall t^w \quad (4.55) \]

\[ \sum_{i, t^d} g_{it}^d \leq \sum_{i, t^d} (G_{i}^{MAX} H_{it}^d), \quad \forall i \in \{ \text{hydroelectric} \} \quad (4.56) \]

\[ G_{i}^{MIN} \left(1 - x_{i}^w \right) \leq g_{it}^d, \quad \forall i, t^d \in \{ t^w \} \quad (4.57) \]

\[ g_{it}^d \leq G_{i}^{MAX} \left(1 - x_{i}^w \right), \quad \forall i, t^d \in \{ t^w \} \quad (4.58) \]

\[ \sum_{i, t^d, t^w} R_{i}^{CO2} g_{it}^d \leq A_{it}^w + OS_{it}^w + A_{it}^{LN}, \quad \forall t^q \quad (4.59) \]

\[ OS_{it}^w \leq \sum_{i, t^d, t^w} 0.033 R_{i}^{CO2} g_{it}^d, \quad \forall t^q \quad (4.60) \]

\[ \forall g_{it}^d, x_{i}^w, q_{iv}, OS_{it}^w \geq 0 \quad (4.61) \]

Where

\[ C_{i}^{M} = MT_{i} \left(1 - x_{i}^w \right) \quad (4.62) \]

\[ C_{i}^{SU} = SU_{i} \left( u_{it}^d - \sum_{j=1}^{k} u_{it^d-1}^d \right), \quad \forall i, t^d \quad (4.63) \]

\[ C_{i}^{SU} \geq 0, \quad \forall i, t^d \quad (4.63) \]

\[ C_{i}^{SD} = SD_{i} \left( u_{it}^d - u_{it^d} \right), \quad \forall i, t^d \quad (4.64) \]

\[ C_{i}^{SD} \geq 0, \quad \forall i, t^d \quad (4.65) \]

\[ A_{it}^{LN} = A_{it}^w + A_{it}^{LN} + OS_{it}^w - \sum_{i, t^d, t^w} R_{i}^{CO2} g_{it}^d \quad (4.66) \]

\[ C_{i}^{PP} \left( g_{it}^d \right) = a_{i} u_{it}^d + b_{i} g_{it}^d + c_{i} g_{it}^{2} \]

\[ = \left[ a_{i} + b_{i} G_{i}^{MIN} \right] + c_{i} \left( G_{i}^{MIN} \right)^{2} + \sum_{j=1}^{k} s_{it}^j \delta_{it}^j, \quad \forall i, t^d \quad (4.67) \]
\[ g_{it}^d = G_i^{MIN} + \sum_{j=1}^{k} \delta_{it}^j, \quad \forall i, t^d \]  

(4.68)

\[ \delta_{it}^1 \leq g_{it}^1 - G_i^{MIN}, \quad \forall i, t^d \]  

(4.69)

\[ \delta_{it}^j \leq g_{it}^j - g_{it}^{j-1}, \quad \forall i, t^d, j \]  

(4.70)

\[ \delta_{it}^k \leq G_i^{MAX} - g_{it}^{k-1}, \quad \forall i, t^d \]  

(4.71)

\[ \delta_{it}^j \geq 0, \quad \forall i, t^d, j = 1, \ldots, k \]  

(4.72)

And \( C_{it}^{SU}/C_{it}^{SD}/C_{it}^{M} \) is start-up/shut-down/maintenance cost function of generator \( i \) of time \( t \), \( NM_t \) is maximum number of units on simultaneous maintenance of time \( t \), \( HE_{it} \) is hydro energy availability factor for a hydro unit \( i \) of time \( t \), \( A_{it}^{JA} \) is the amount of allowances initially owned by firm \( i \) of time \( t \), \( SU_{it}/SD_{it}/MT_{it} \) is start-up/shut-down/maintenance cost of generation \( i \) of time \( t \), and \( S_{it} \) and \( \delta_{it} \) are variables of linearized production cost function.

**Constraints:**

- Maintenance resources availability
- Maintenance must be completed within the windows between the starting and ending times
- Coupling constraints between generation maintenance and unit commitment, which a unit cannot be online if it is on maintenance;
- Maximum outage duration constraint, which ensure that each unit is on maintenance outage for a pre-specified period over the year;
- Continuous maintenance constraint, which required that the maintenance must be completed once it begins;
- Maximum number of units simultaneously in maintenance
• Seasonal limitations, such as hydro energy constraint
• Maintenance resources availability
• Generation capability constraint

4.2.3 Lower Level Optimization Problem Formulation

1) ISO Maintenance Clearing Subproblem: GENCOs are independently responsible for generation maintenance, and they submit the maintenance schedule to ISO, which coordinates with market participants to improve the security of electricity services and reduce the likelihood of blackouts. ISO solves the following optimization problem to minimize the cost of maintenance and unserved energy, while maintaining the balance between generation and load.

\[
\begin{align*}
\min_{g_{it}, \, \text{it}} & \sum_{t} \left( C_{it}^{M} + C_{it}^{UE} \right) \\
\text{s.t.} & \sum_{t} g_{it} + UE_{it} = D_{it}, \quad \forall \text{it} \quad (4.74)
\end{align*}
\]

where, \( C_{it}^{UE} \) is cost of unserved energy, \( UE_{it} \) is unserved energy, and \( D_{it} \) is total load at node \( i \) of time \( t \).

2) ISO UC and OPF Subproblem: The short-term (daily/weekly) UC problem can be formulated in MILP formulation [55], as follows:

\[
\begin{align*}
\min_{g_{it}, \, \text{it}} & \sum_{i} \sum_{t} \left( C_{it}^{P} \left( g_{it} \right) + C_{it}^{SU} + C_{it}^{SD} \right) \\
\text{s.t.} & \sum_{t} g_{it} + UE_{it} = D_{it}, \quad \forall \text{it} \quad (4.75)
\end{align*}
\]
\[
\sum_i g_{it^d} = \sum_i D_{it^d}, \quad \forall t^d \tag{4.76}
\]

\[
\sum_i r_{it^d}^S \geq R_{it^d}^S, \quad \forall t^d \tag{4.77}
\]

\[
\sum_i r_{it^d}^O \geq R_{it^d}^O, \quad \forall t^d \tag{4.78}
\]

\[
G_{it}^{MIN} u_{it^d} \leq g_{it^d}, \quad \forall i, t^d \tag{4.79}
\]

\[
g_{it^d} + r_{it^d}^S + r_{it^d}^O \leq G_{it}^{MAX} u_{it^d}, \quad \forall i, t^d \tag{4.80}
\]

\[
r_{it^d}^S \leq R_{it^d}^S u_{it^d}, \quad \forall i, t^d \tag{4.81}
\]

\[
r_{it^d}^O \leq R_{it^d}^O u_{it^d}, \quad \forall i, t^d \tag{4.82}
\]

\[
g_{it^d} - g_{it^d-1} \leq \text{MaxInc}_i, \quad \forall i, t^d \tag{4.83}
\]

\[
g_{it^d} - g_{it^d-1} \geq -\text{MaxDec}_i, \quad \forall i, t^d \tag{4.84}
\]

\[
\left(\frac{Y_{it}^{ON}}{u_{it^d-1}} - T_{it}^{ON}\right)\left(u_{it^d-1} - u_{it^d}\right) \geq 0, \quad \forall i, t^d \tag{4.85}
\]

\[
\left(\frac{Y_{it}^{OFF}}{u_{it^d-1}} - T_{it}^{OFF}\right)\left(u_{it^d} - u_{it^d-1}\right) \geq 0, \quad \forall i, t^d \tag{4.86}
\]

\[
\sum_k PTDF_{ki}\left(g_{it^d} - D_{it^d}\right) \leq F_k^{max}, \quad \forall i, t^d \tag{4.87}
\]

where, \(r_{it}^S/r_{it}^O\) are spinning/operating reserve at node \(i\) of time \(t\), \(R_{it}^S/R_{it}^O\) are required spinning/operating reserve at node \(i\) of time \(t\), MaxInc/MaxDec\(i\) are maximum ramping rate for increasing/decreasing generation \(i\) output, \(Y_{it}\)/\(Y_{it-1}\) are time duration for generator \(i\) to stay ON/OFF from beginning of time \(t-1\). Also, \(T_{i}^{on}/T_{i}^{off}\) are required time duration after generator \(i\) startup/shutdown, \(PTDF_{ki}\) is the power transfer distribution factor, and \(F_k^{max}\) is the power flow limit of branch \(k\).
The constraints in UC problem include:

- Power balance
- System spinning and operating reserve requirements
- Generation unit capacity limits
- Maximum spinning and operating reserve limits
- Ramping rate limits
- Minimum ON/OFF time limits
- Transmission flow limits

The solutions of UC problem will provide the commitment of generation units. Based on this, solution of the OPF problem hourly will lead to information on the generation dispatch and locational marginal price (LMP).

\[
\min \sum_i \left( \alpha_i g_{it} + \frac{1}{2} \beta_i g_{it}^2 \right) \quad (4.88)
\]

s.t.

\[
\sum_i g_{it} - \sum_i D_{it} = 0, \quad (\lambda_i) \quad (4.89)
\]

\[
\sum_i GSF_{k-i} \left( g_{it} - D_{it} \right) \leq F_{k}^{\text{max}}, \quad \forall k \quad (\mu_k) \quad (4.90)
\]

\[
G_i^{\text{MIN}} u_{it} \leq g_{it} \leq G_i^{\text{MAX}} u_{it}, \quad \forall \ g_{it} \geq 0 \quad (4.91)
\]

Where

\[
p_{it}^E = LMP_{it} = LMP_{it}^{\text{energy}} + LMP_{it}^{\text{ congest }} = \lambda_i + \mu_{it} \sum_i \ GSF_{k-i} \quad (4.93)
\]

And GSF_{k-i} is generator shift factor, \( \mu_{it} \) is dual variable of branch power flow.
limit constraint, and $\lambda_t$ is dual variable of power balance constraint.

3) **CO$_2$ Allowance Market Clearing Subproblem**: The CO$_2$ allowance market clearing price is obtained by solving the optimization problem.

$$\max_{A_{rt}} \sum_i B_{rt}^{\text{CO}_2} A_{rt}$$  \hspace{1cm} (4.94)

s.t.

$$\sum_i A_{rt} - \text{CAP}^{\text{CO}_2}_{rt} = 0, \quad \forall t^q \quad (P_{rt}^{\text{CO}_2})$$  \hspace{1cm} (4.95)

$$A_{rt} \leq 0.25 \text{CAP}^{\text{CO}_2}_{rt}, \quad \forall i, t^q$$  \hspace{1cm} (4.96)

$$0 \leq A_{rt} \leq q_{rt}$$  \hspace{1cm} (4.97)

$$\forall A_{rt}, q_{rt} \geq 0$$  \hspace{1cm} (4.98)

Then, the three-year GSP is formulated as a MIBLP problem.

$$\max_{g_{rt}, x_{rt}, q_{rt}, OS_{rt}, \text{t}} \sum_{rt} \{ \begin{array}{l} p_{rt}^E g_{rt} - C_{rt}^P (g_{rt}) - C_{rt}^{SU} - C_{rt}^{SD} \} - \sum_{rt} C_{rt}^M \
- \sum_{rt} \{ p_{rt}^{\text{CO}_2} A_{rt} + C_{rt}^{\text{OS}_r} \} \end{array}$$  \hspace{1cm} (4.99)

s.t. Generation Maintenance Scheduling Constraint - (4.51-4.61)
ISO Maintenance Clearing Subproblem - (4.73-4.74)
ISO Unit Commitment Subproblem - (4.75-4.87)
ISO Economy Dispatch Subproblem - (4.88-4.92)
CO$_2$ Allowance Market Clearing Subproblem - (4.94-4.98)

The global optimal solutions of continuous variables of CO$_2$ allowance price $p_{rt}^{\text{CO}_2}$, LMP $p_{rt}^E$ and generation dispatch $g_{rt}$, binary variables of unit commitment $I_{rt}$ and generation maintenance schedule $X_{rt}$, and integer variables of allowance dispatch $A_{rt}$ and offsets usage $OS_{rt}$ are obtained by solving this MIBLP problem with the methodology developed in Section 3.2.3.
For profit-seeking GENCOs, full consideration of both markets can provide detailed signals to guide their strategic behaviors. It provides the information of power flow, electricity price, and generation output level, etc., which significantly enhance the awareness of GENCOs of the system condition and provide better support for their decision makings in two markets. This model is not intended to give precise prediction of electricity prices, but market traders may use the results as a reference to make the coordinated strategies. If simplifying the model using forecasted electricity price, the detailed interactions between two markets cannot be explored, then GENCOs can only produce one-sided strategies.

First, in the formulation of Cournot Equilibrium module for CO\textsubscript{2} allowance market, GENCO solves the optimization problem to decide its bidding offer in order to maximize its own profit, which equals to its revenue from selling energy to electricity market minus its cost of production, purchasing CO\textsubscript{2} allowances, and using offsets. The revenue obtained from electricity market is decided by the electricity price and GENCO’s generation output level. Accurate information can only be achieved by solving the short-term unit commitment and economic dispatch in electricity market.

Second, this new problem includes several generation scheduling problems under different time horizons. In the three-year CO\textsubscript{2} allowance compliance period, GENCOs need to consider the quarterly auction in CO\textsubscript{2} allowance market, the midterm generation maintenance scheduling and short-term unit commitment to maintain the adequacy in midterm planning and security in short-term operation planning. Therefore, within the time horizon of three-month CO\textsubscript{2} allowance auction, the proposed algorithm is focused on the midterm generation maintenance scheduling. Short-term unit commitment and economic dispatch are considered to provide the accurate solution of electricity price and generation
output level. But the bidding strategies of GENCOs participation in electricity market are neglected, since our focus is on the midterm time horizon.

However, detailed formulation could be possibly simplified by using forecasted electricity prices. In the current model, the accurate information of electricity price and generation output level is obtained by solving the unit commitment and economy dispatch problems, which leads to much complexity due to the involvement of various integer and binary decision variables. If the electricity price is achieved using forecasting tools, then generation output level can be calculated based on GENCOs’ production cost curve, which assumes that GENCOs bid based on their true cost curves. Based on these two assumptions, four subproblems in the lower lever of the proposed module can be reduced to two subproblems of ISO maintenance schedule clearing and CO₂ allowance market clearing. Then the complexity of the problem will be greatly reduced to one group of binary decision variables for generator maintenance scheduling. The simplification leads to loss of insight into GENCOs’ interactions between the two markets.

4.3 Numerical Results

4.3.1 Case of Two-Gенко System

It is assumed that there are two GENCOs participating in the CO₂ allowance auction. The illustrative parameters are shown in Table 4.1. In the base case, the reserved bidding prices are assumed to be $2 and $1.5 for GENCO 1 and 2, respectively (given the CO₂ allowance price in RGGI now is about $2). Also, the market has a reserved price for CO₂ allowance, which is set to be $1 in this simulation. The total allowance number is set to be 700 units. The equilibrium solutions are shown in Table 4.2.
Table 4.1 GENCOs’ Characteristics

<table>
<thead>
<tr>
<th>GENCO</th>
<th>$a$</th>
<th>$b$</th>
<th>$\lambda^e$ ($$/MW)$</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>Bidding Price ($$/p.u.$)</th>
<th>$k$ (ton/MW)</th>
<th>$h$ ($$/p.u.$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>0.005</td>
<td>20</td>
<td>500</td>
<td>2</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>0.004</td>
<td>20</td>
<td>800</td>
<td>1.5</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.2 Equilibrium Solutions

<table>
<thead>
<tr>
<th>GENCO</th>
<th>$P$ (MW)</th>
<th>$A$ (p.u.)</th>
<th>$q$ (p.u.)</th>
<th>$\lambda^{CO_2}$ ($$/p.u.$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
</tbody>
</table>

From the result, it is shown that two GENCOs receive the same amount of allowances, and they have different reserved bidding prices but same bidding amounts of allowances. GENCO 2 does not use its entire bought allowances and it is able to bank some allowances for the future use.

The above is the base case, and it proves that the model can be used to obtain an equilibrium point of CO2 allowance price and each participant’s bought allowances. The following is the result of a sensitivity analysis of bidding price, forecasted electricity price and total amount of allowances to the CO2 allowance price and allowance distribution level:

- Sensitivity analysis of GENCOs’ bidding price to CO2 allowance price

Both GENCOs’ bidding prices are changed in discrete values from $1 to $3. The result shows that CO2 allowance is always $1 and two GENCOs receive the same amount of allowances. This is due to the fact that there are only two players in this Cournot competition.
However, this is not a general conclusion, and the result is different in the multi-GENCO case.

- Sensitivity analysis of forecasted electricity price to CO\(_2\) allowance price

GENCO 2’s forecasted electricity price is fixed and GENCO 1’s forecasted electricity price changes. The result is shown in Table 4.3. If GENCO 1 forecasts a lower electricity price, it will bid less amount of allowance, but receive the same amount of allowance while banking some allowances. If GENCO 1 forecasts a higher electricity price, it will bid more allowances and receive more. Since the electricity price is high, it will use all the allowances to maximize its profit. Also, the CO\(_2\) allowance price is higher. Therefore, it is important for GENCOs to correctly forecast the electricity price to participate in the CO\(_2\) allowance market.

### Table 4.3 Sensitivity Analysis of Forecasted Electricity Price to CO\(_2\) Allowance Price

<table>
<thead>
<tr>
<th>GENCO</th>
<th>(\lambda^e) ($/MW)</th>
<th>(P) (MW)</th>
<th>(A) (p.u.)</th>
<th>(q) (p.u.)</th>
<th>Profit of Gen. 1 ($)</th>
<th>(\lambda^{CO2}) ($/p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>100</td>
<td>300</td>
<td>300</td>
<td>150</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>250</td>
<td>400</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>787.5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>250</td>
<td>400</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>22</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>1428</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>250</td>
<td>280</td>
<td>392</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Sensitivity analysis of total amount of CO\(_2\) allowances to CO\(_2\) allowance price

When the allowance is insufficient, GENCO 1 always receives more allowances and CO\(_2\) allowance price is higher, as shown in Table 4.4.
Table 4.4 Sensitivity Analysis of Total Amount of CO₂ Allowance to CO₂ Allowance Price

<table>
<thead>
<tr>
<th>GENCO</th>
<th>Total Allowance</th>
<th>P (MW)</th>
<th>A (p.u.)</th>
<th>q (p.u.)</th>
<th>Profit of Gen. 1 ($)</th>
<th>λ\textsuperscript{CO₂} ($/p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>600</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>600</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>613</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>800</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>800</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>400</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is caused by the fact that GENCO 1 has a higher bidding price than GENCO 1. And market clearing problem tries to maximize the CO₂ allowance market surplus, then GENCO1 with higher bidding price will get the more allowance than GENCO 2 with lower bidding price. When the allowance is superfluous, different bidding prices don’t have this impact. And GENCO 2 becomes the marginal price and set the CO₂ allowance price. Some allowances will be banked (it is assumed that all allowances will be sold; otherwise, the clearing price will be zero).

4.3.2 Case of Multiple-GENCO System

Four GENCOs participate in the CO₂ allowance auction and the total amount of allowances is 1500 units. Their parameters are shown as following:
Table 4.5 GENCOs’ Characteristics

<table>
<thead>
<tr>
<th>GENCO</th>
<th>$a$</th>
<th>$b$</th>
<th>$\lambda^c$($/MW)$</th>
<th>$P_{\text{max}}$(MW)</th>
<th>Bidding Price $\lambda$ ($/\text{p.u.}$)</th>
<th>$k$ (ton/MW)</th>
<th>$h$ ($$/\text{p.u.}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>0.005</td>
<td>20</td>
<td>500</td>
<td>2</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>0.004</td>
<td>20</td>
<td>800</td>
<td>1.5</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.005</td>
<td>20</td>
<td>800</td>
<td>2.5</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>0.004</td>
<td>20</td>
<td>800</td>
<td>2.3</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

The results in Table 4.6 show that GENCO 3 with highest bidding price gets most allowances (each player cannot buy more than 60% of total allowances), while GENCO 2 with the lowest bidding price does not get any allowance.

Table 4.6 Equilibrium Solutions

<table>
<thead>
<tr>
<th>GENCO</th>
<th>$P$(MW)</th>
<th>$A$ (p.u.)</th>
<th>$q$ (p.u.)</th>
<th>Profit ($$$)</th>
<th>$\lambda_{\text{CO2}}$ ($$/\text{p.u.}$$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>262.5</td>
<td>262.5</td>
<td>262.5</td>
<td>574.2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>594</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>3600</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>437.5</td>
<td>437.5</td>
<td>437.5</td>
<td>765.6</td>
<td></td>
</tr>
</tbody>
</table>

The market clearing price is set by GENCO 2. If GENCO 2’s reserved price increases but does not exceed $1.9 (the second lowest bidding price), the market clearing price is still set by GENCO2, as shown in Table 4.7. It demonstrates that this model is capable of finding the equilibrium points of the Cournot competition.
Table 4.7  Equilibrium Solutions

<table>
<thead>
<tr>
<th>GENCO</th>
<th>P(MW)</th>
<th>A (p.u.)</th>
<th>q (p.u.)</th>
<th>Profit ($)</th>
<th>$\lambda^{CO2}$ ($/p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>310</td>
<td>310</td>
<td>310</td>
<td>480.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>313.3</td>
<td>0.225</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>3280</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>387.5</td>
<td>387.5</td>
<td>387.5</td>
<td>600.625</td>
<td></td>
</tr>
</tbody>
</table>

The above are the simulation results of the NLP formulation of the EPEC model of CO2 allowance market. It is shown that the model is capable of finding the equilibrium point. When the number of participants in the auction increases, it will become a large-scale optimization problem.

4.3.3  Case of PJM 5-Bus System

The PJM 5-Bus System is used for illustration of the proposed MIBLP model and solution methodology, as shown in Figure 4.7. It is assumed all five GENCOs participate in the electricity market and CO2 allowance market. The seven-week generation maintenance scheduling of GENCO 1, daily unit commitment and hourly economic dispatch, and one CO2 allowance market auction with three bidding strategies of GENCO 1 are considered in this case. It is assumed GENCO 1 makes its own decision, taking as given other GENCOs’ decisions on the bidding quantity in CO2 allowance market.

The system topology, branch data, generation data, load data, GENCOs’ bidding offers in electricity market, maintenance limit and CO2 bidding offer of GENCO 1 are given in Table 4.8 - Table 4.12, and Figure 4.8.
Figure 4.7. PJM 5-bus system

Table 4.8 Branch Data

<table>
<thead>
<tr>
<th>Branch</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Reactance X</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.0281</td>
<td>2.50</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0.0304</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>0.0064</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>0.0108</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
<td>0.0297</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>5</td>
<td>0.0297</td>
<td>2.4</td>
</tr>
</tbody>
</table>
Table 4.9 Date of Generator Characteristic

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>Fixed cost ($/hr)</th>
<th>Startup cost ($)</th>
<th>Shutdown cost ($)</th>
<th>Ramp up limit</th>
<th>Ramp down limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>50</td>
<td>100</td>
<td>20</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>60</td>
<td>150</td>
<td>20</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>70</td>
<td>200</td>
<td>20</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>150</td>
<td>400</td>
<td>20</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>50</td>
<td>120</td>
<td>20</td>
<td>1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 4.10 GENCOs’ Electricity Bidding Offers/Production Cost ($/pu-hr)

<table>
<thead>
<tr>
<th>Generator</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Price 1</th>
<th>Price 2</th>
<th>Price 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>12</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>1</td>
<td>0.4</td>
<td>15</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>1</td>
<td>0.4</td>
<td>16</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 4.11 Maintenance Limit of GENCO 1

<table>
<thead>
<tr>
<th>Equipment</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Windows</th>
<th>Duration (hrs)</th>
<th>Cost ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>/</td>
<td>Mon. – Sun.</td>
<td>24</td>
<td>84</td>
</tr>
</tbody>
</table>
Table 4.12  CO₂ Allowance Bidding Offers of GENCO 1

<table>
<thead>
<tr>
<th>Strategy</th>
<th>q (p.u.)</th>
<th>λ^{CO₂} ($/p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11000</td>
<td>1.60</td>
</tr>
<tr>
<td>2</td>
<td>12000</td>
<td>1.62</td>
</tr>
<tr>
<td>3</td>
<td>13000</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Figure 4.8. One-week load data

The profit and generation output of GENCO 1 utilizing three bidding strategies in CO₂ allowance market are shown in Table 4.13. Each column represents the seven-day maintenance activity during that week. The comparison of GENCO 1’s profit under different maintenance schedules and bidding strategies is shown in Figure 4.9.
### Table 4.13 GENCO 1’s Profit and Generation Output under Different Strategies

<table>
<thead>
<tr>
<th>Maintenance Schedule</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Case</strong></td>
<td>Profit/ $10^3$</td>
<td>942.9</td>
<td>968.45</td>
<td>1016.54</td>
<td>989.1</td>
<td>773.92</td>
<td>770.28</td>
</tr>
<tr>
<td></td>
<td>Generation/ $10^3$ MW</td>
<td>51.94</td>
<td>53.41</td>
<td>55.86</td>
<td>54.39</td>
<td>43.61</td>
<td>43.12</td>
</tr>
<tr>
<td><strong>Strategy 1</strong></td>
<td>Profit/ $10^3$</td>
<td>754.6</td>
<td>780.15</td>
<td>847.56</td>
<td>800.31</td>
<td>598.22</td>
<td>593.25</td>
</tr>
<tr>
<td></td>
<td>Generation/ $10^3$ MW</td>
<td>48.58</td>
<td>49.96</td>
<td>53.17</td>
<td>50.88</td>
<td>40.79</td>
<td>40.33</td>
</tr>
<tr>
<td><strong>Strategy 2</strong></td>
<td>Profit/ $10^3$</td>
<td>805</td>
<td>831.32</td>
<td>880.04</td>
<td>852.6</td>
<td>636.79</td>
<td>633.22</td>
</tr>
<tr>
<td></td>
<td>Generation/ $10^3$ MW</td>
<td>51.94</td>
<td>53.41</td>
<td>55.86</td>
<td>54.39</td>
<td>43.61</td>
<td>43.12</td>
</tr>
<tr>
<td><strong>Strategy 3</strong></td>
<td>Profit/ $10^3$</td>
<td>791.56</td>
<td>817.88</td>
<td>866.46</td>
<td>839.02</td>
<td>623.35</td>
<td>619.78</td>
</tr>
<tr>
<td></td>
<td>Generation/ $10^3$ MW</td>
<td>51.94</td>
<td>53.41</td>
<td>55.86</td>
<td>54.39</td>
<td>43.61</td>
<td>43.12</td>
</tr>
</tbody>
</table>

In the base case, GENCO 1 only participates in the electricity market, and obtains the highest profit. When GENCO 1 participates in both electricity market and CO₂ allowance market, it receives less profit and the profits are different using various bidding strategies. This is due to the fact that G1 will spend more money in paying for the CO₂ allowance to cover the emission from generating electricity. Also, seven maintenance schedules and three bidding strategies result in different profits.
The comparison of the profit under different maintenance schedules is shown in Figure 4.10. Under different maintenance schedules, the profits are different and they change in the similar pattern with different CO2 allowance bidding strategies.

The comparison of the reduced profit under different CO2 bidding strategies is shown in Figure 4.11. When G1 has small number of CO2 allowance, which is under bidding strategy 1, different maintenance schedule brings different profits. However, when G1 has enough CO2 allowances, different maintenance schedules will not have a significant impact on the reduced profit.
Figure 4.10. Comparison of the profit under different maintenance schedules

Figure 4.11. Comparison of the reduced profit under different CO₂ bidding strategies
As shown in Figure 4.12, neither Strategy 1 nor Strategy 3, which represents bidding too conservatively or aggressively, will bring the highest profit. In contrast, the medium bidding strategy can bring the optimal profit. Also optimal maintenance schedule will change with the different CO₂ allowance bidding strategy.

![Comparison of the profit under different maintenance schedules and bidding strategies](image)

**Figure 4.12. Comparison of the profit under different maintenance schedules and bidding strategies**

The comparison of G1’s generation outputs under different bidding strategies and maintenance schedules is shown in Figure 4.13. When G1 has enough allowances under bidding strategy 2 or 3, it will have similar generation output. However, if G1 does not have enough CO₂ allowances under bidding strategy 1, its generation output will decrease a lot compared to the base case.
As shown in Figure 4.14 and Figure 4.15, similar to the result of G1’s profit, its generation output will change in different patterns under different maintenance schedules and different bidding strategies. There is also a difference that when G1 has enough CO₂ allowance, it can actively participate in the electricity market and have the similar dispatch level as in the base case. However, G1 has to consider the cost of purchasing the CO₂ allowances, which will affect its profit, as shown in Figure 4.16.

The profit under optimal maintenance scheduling and CO₂ allowance bidding strategy is shown in Figure 4.17. It is shown that strategy 2 is the best bidding strategy, which means G1 should not bid too many allowances (strategy 3) or too few allowances (strategy 1). The optimal bidding strategy can be obtained by solving the proposed optimization problem.
Figure 4.14. Comparison of the generation output under different maintenance schedules

Figure 4.15. Comparison of the reduced generation output under different maintenance schedules
Figure 4.16. Comparison of the generation output under different maintenance schedules

Figure 4.17. The profit under optimal maintenance scheduling and CO$_2$ allowance bidding strategy
The following conclusions can be obtained based on the simulation results:

- Optimal maintenance scheduling will be changed considering different CO₂ allowance bidding strategies.
- Under three bidding strategies, which correspond to small, medium and large (can be used in secondary market) amount of allowances, GENCOs will have different profits. Bidding strategy of either small or large amount of allowances does not optimize the profit. The optimal bidding strategy is connected with optimal maintenance scheduling. GENCOs need to consider the maintenance scheduling and CO₂ allowance bidding together in order to maximize their profits.
- Based on the proposed model, GENCOs will be able to determine their optimal mid-term generation maintenance scheduling and CO₂ emission allowance bidding strategy participating in both electricity market and CO₂ allowance market.

**4.4 Summary**

Carbon mitigation policies, such as CO₂ allowance cap-and-trade market, help to reduce consumption in traditional energy and promote to shift to renewable energy resources. This dissertation addresses the challenging issue of generation scheduling taking into account new environmental considerations. The CO₂ emission allowance market is formulated as the Cournot equilibrium model. Practical market rules, such as those in RGGI, are considered in the developed model. The sensitivity of GENCOs’ bidding price, electricity price and total amount of CO₂ allowances to CO₂ allowance price and dispatch are analyzed. Then, the
emission-constrained GSP in the three-year CO₂ allowance compliance period, involving generation maintenance scheduling, unit commitment and CO₂ allowance cap-and-trade, is investigated. Based on the proposed model, GENCOs are able to know the amount and price of CO₂ allowances to make bidding decisions in the electricity market, while they bid to CO₂ allowance market based on the information of electricity price and scheduled generation commitment and dispatch. With this information, GENCOs will be able to determine their optimal mid-term operation planning and short-time operation schedules participating in both electricity market and CO₂ allowance market.

In the future work, the development of an accurate CO₂ emission model related with generation output is critical. The current emission model is to simply multiply generation output by constant emission rate to obtain the emitted CO₂. With more insight into the physical mechanisms, a more accurate model will be beneficial. Also, the proposed model will be tested with data and scenarios from large-scale power systems.
CHAPTER 5. CONCLUSIONS

5.1 Summary of Dissertation

The proposed research addresses three challenging issues of generation scheduling problem under critical and new circumstances. The first issue is the optimal generator start-up strategy for bulk power system restoration following a major power outage. The second issue is the optimal installation strategy of blackstart capability in power system restoration planning. The third issue is the emission-constrained generation scheduling problem involving generation maintenance scheduling, unit commitment and CO$_2$ allowance cap-and-trade.

Chapter 2 investigates the generator start-up sequencing in power system restoration. A “Two-Step” algorithm, which takes advantage of the quasiconcave property of generation capability curve, and a MILP-based optimal generator start-up strategy using the proposed transformation techniques on the nonlinear generation capability curves are proposed. The case studies on industry systems, such as PECO-Energy, AEP and Western Entergy Region, demonstrate the accuracy of the models and the computational efficiency of the algorithms. Moreover, based on the developed optimization modules of generator start-up sequence, transmission path search and the time to take restoration actions, an on-line decision support tool for power system restoration is proposed. The developed tool can be used to assist system operators and enable them to adapt to changing system conditions.

Chapter 3 constructs the blackstart capability assessment in power system restoration. This dissertation provides a systematic way to assess the optimal installation location and
amount of blackstart capability for system planners to make decisions of the installation strategy. It is shown that power systems can benefit from new blackstart generators to reduce the restoration time. However, there is a maximum amount beyond which system restoration time cannot be further reduced with additional blackstart capability. Economic considerations should be taken into account when assessing additional blackstart capabilities.

Chapter 4 analyzes the emission constrained generation scheduling problem considering the new mechanism of CO₂ emission cap-and-trade. The CO₂ allowance market is formulated as the Cournot equilibrium model. The practical market rules, such as, in RGGI, are considered in the developed model. The new generation scheduling problem in the three-year CO₂ allowance compliance period, involving generation maintenance scheduling, unit commitment and CO₂ allowance cap-and-trade, is investigated. Based on the proposed model, GENCOs will be able to determine their optimal mid-term operation planning, short-time operation schedules and bidding strategies when they participate in both electricity market and CO₂ allowance market.

5.2 Future Research Direction

The system restoration decision support tool now only considers the steady state analysis. More practical constraints need to be incorporated, such as switching transients, generating station voltage limits, and generator transient stability limits. In the future work, the under-excitation capability of generators, load rejection and low frequency isolation scheme should also be incorporated. It can be accomplished by integrating the developed module with power system simulation software tools. To provide an adaptive decision support tool for power system restoration, the data and implementation issues for an on-line
operational environment need to be investigated in the future. Moreover, more optimization modules, i.e., load pick-up, optimal power flow, voltage stability check, etc., can be included into the tool to provide more accurate solutions.

In the emission-constrained GSP, the development of an accurate CO₂ emission model related with generation output is critical. The current emission model is to simply multiply generation output by constant emission rate to obtain the emitted CO₂. With more insight into the physical mechanisms, a more accurate model will be beneficial. Moreover, if all GENCOs solve the new generation scheduling problem using the proposed module, and when none of them deviate from maintenance scheduling and bidding strategies, both markets reach the equilibrium. Then the objective function of CO₂ allowance market clearing problem, which is to maximize CO₂ market surplus, will be the collected tax. The comparison of CO₂ allowance market with carbon tax can be analyzed to develop a hybrid approach to take the advantage of both methods, such as a capped system with carbon tax, or the initial grandfathered permits with carbon tax. In the future work, the proposed model will be tested with data and scenarios from large-scale power systems.
APPENDIX A. PROOF OF LEMMA 1 in Section 2.3.1

First, divide \( \text{dom } f \) to three consecutive sets, and \( \text{dom } f = S_1 \cup S_2 \cup S_3 \):

\[
S_1 = \left\{ t : 0 \leq t < t_{\text{start}} + t_{\text{cep}} \right\} \quad (A.1)
\]

\[
S_2 = \left\{ t : t_{\text{start}} + t_{\text{cep}} \leq t < t_{\text{start}} + t_{\text{cep}} + P_{\text{max}} / R_r \right\} \quad (A.2)
\]

\[
S_3 = \left\{ t : t_{\text{start}} + t_{\text{cep}} + P_{\text{max}} / R_r \leq t \leq T \right\} \quad (A.3)
\]

Then, consider all possible cases:

1. If for any \( x, y \in S_1 \) and \( 0 \leq \theta \leq 1 \),

\[
f\left( \theta x + (1-\theta)y \right) = f(x) = f(y) \geq \min \left\{ f(x), f(y) \right\} \quad (A.4)
\]

2. If for any \( x \in S_1, y \in S_2 \) and \( 0 \leq \theta \leq 1 \),

\[
f\left( \theta x + (1-\theta)y \right) \geq f(x) \quad (A.5)
\]

Since \( f(x) \leq f(y) \),

\[
f\left( \theta x + (1-\theta)y \right) \geq \min \left\{ f(x), f(y) \right\} \quad (A.6)
\]

3. If for any \( x \in S_1, y \in S_3 \) and \( 0 \leq \theta \leq 1 \),

\[
f\left( \theta x + (1-\theta)y \right) \geq f(x) \quad (A.7)
\]

Since \( f(x) \leq f(y) \),

\[
f\left( \theta x + (1-\theta)y \right) \geq \min \left\{ f(x), f(y) \right\} \quad (A.8)
\]

4. If for any \( x \in S_2, y \in S_1 \) and \( 0 \leq \theta \leq 1 \),

\[
f\left( \theta x + (1-\theta)y \right) \geq f(y) \quad (A.9)
\]
Since $f(y) \leq f(x)$,

$$f \left( \theta x + (1 - \theta)y \right) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.10)

5. If for any $x, y \in S_2$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1 - \theta)y \right) = f(x) = f(y) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.11)

6. If for any $x \in S_2, y \in S_3$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1 - \theta)y \right) \geq f(x)$$ \hspace{1cm} (A.12)

Since $f(x) \leq f(y)$,

$$f \left( \theta x + (1 - \theta)y \right) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.13)

7. If for any $x \in S_3, y \in S_1$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1 - \theta)y \right) \geq f(y)$$ \hspace{1cm} (A.14)

Since $f(y) \leq f(x)$,

$$f \left( \theta x + (1 - \theta)y \right) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.15)

8. If for any $x \in S_3, y \in S_2$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1 - \theta)y \right) \geq f(y)$$ \hspace{1cm} (A.16)

Since $f(y) \leq f(x)$,

$$f \left( \theta x + (1 - \theta)y \right) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.17)

9. If for any $x, y \in S_3$ and $0 \leq \theta \leq 1$,

$$f \left( \theta x + (1 - \theta)y \right) = f(x) = f(y) \geq \min \{ f(x), f(y) \}$$  \hspace{1cm} (A.18)
From all above, for any $x, y \in \text{dom } f$ and $0 \leq \theta \leq 1$,

$$f(\theta x + (1 - \theta) y) \geq \min \{ f(x), f(y) \}$$

Therefore, the generation capability function is quasicave.
APPENDIX B. THE SOLUTION METHODOLOGY OF MIBLP

IN SECTION 3.2.3

Based on Benders decomposition and transformation procedure [56], a novel methodology is proposed. The algorithm is described in the following:

The MIBLP problem can be written in a compact format, i.e.,

$$\begin{align*}
\max_{x,y} & \quad C_x + d_i y \\
\text{s.t.} & \quad A_i x + B_i y \leq b_i \\
& \quad y \in \arg \max_w C_w x + d_w w \\
& \quad \text{s.t.} \quad A_w x + B_w w \leq b_w \\
& \quad w \geq 0 \quad w_j \text{ integer} \\
& \quad x \geq 0 \quad x_j \text{ integer}
\end{align*}$$

(B.1)

**Step 1:** Divide the MIBLP problem into one Restricted Master Problem (RMP) (B.7) and several Slave Problems (SPs) (B.2) by fixing binary variables.

1) Fix the values of the binary variables $z = \bar{z}$, and solve the BLP problem:

$$\begin{align*}
\max_{x,y,z} & \quad D_i x + E_i y + F_i \bar{z} \\
\text{s.t.} & \quad A_i x + B_i y \leq b_i - C_i \bar{z} \\
& \quad \text{max}_{y,z} D_w x + E_w y + F_w \bar{z} \\
& \quad \text{s.t.} \quad A_w x + B_w y \leq b_w - C_w \bar{z} \quad (w_1) \\
& \quad -x \leq 0 \quad (w_2) \\
& \quad -y \leq 0 \quad (w_3)
\end{align*}$$

(B.2)

where $w_1, w_2, w_3$ are the dual variables of the constraint.

2) In the initial step, RMP will only have the objective, and constraints are added in future iterations from the cut of solving SP. Then, in each step, the solution of RMP will provide an upper bound of the original problem, and solution SP
will provide a lower bound. Indeed, RMP is a relaxation of the original problem whereas the SP represents a restriction.

**Step 2:** Transform the SP problem (B.2) to a Linear Problem with Complementarity Constraints (LPCC) (B.3), and solve it by “θ-free algorithm” [57].

1) Based on KKT conditions, replace the lower level problem of (B.2) by complementarity constraints and add them to the upper level problem. One obtains the following LPCC:

\[
\max_{x,y} D_1 x + E_1 y + F_1 z
\]

\[\text{s.t. } A_1 x + B_1 y \leq b_1 - C_1 z \]

\[
0 \leq b_2 - C_2 z - A_2 x - B_2 y \quad \perp \quad w_1 \geq 0
\]

\[
0 \leq A_2 w_2 - D_2 \quad \perp \quad x \geq 0
\]

\[
0 \leq B_2 w_3 - E_2 \quad \perp \quad y \geq 0
\]

(B.3)

2) Solve (B.3) using the “θ-free algorithm”.

SP is the restricted MIBLP, and it provides a lower bound. The decomposition technique allows parallel computation of solving multiple SPs.

**Step 3:** From the solution of LPCC problem, construct the LP problem (B.4 and B.5).

1) From the solution of (B.3), it is known which constraint in (B.3) is active. Then formulate the following linear programming problem by removing the optimality constraint of the lower level problem and set the active constraint (here randomly assume one for illustration purpose) to be the equality:

\[
\max_{x,y} D_1 x + E_1 y + F_1 z
\]

\[\text{s.t. } A_1 x + B_1 y = b_1 - C_1 z \]

\[A_2 x + B_2 y \leq b_2 - C_2 z \]

\[x, y \geq 0 \]

(B.4)

2) Introduce slack variables to transform (B.4) into the following compact form:
\[ \text{max}_s \quad Hs \]
\[ \text{s.t.} \quad Ms = b - Cz \quad (u) \quad (B.5) \quad s \geq 0 \]

The dual of (B.5) is:

\[ \text{min}_u \quad u(b - Cz) \]
\[ \text{s.t.} \quad uM \geq H \quad u \text{ free} \quad (B.6) \]

where, \( u \) is the dual variable of constraint in (B.5).

3) Based on Farkas Lemma, the necessary and sufficient condition for (B.5) to have at least one non-empty solution for \( z \) is: Problem (B.5) has a solution \( s \) if and only if \( u^*(b - C^*z) \geq 0 \) and \( u^*M \geq 0 \) for all \( u \).

When choosing \( z \) arbitrarily, there is a finite number of possibilities: \( z_1, z_2, ..., z_n \). For each \( z_i \), there is a corresponding inequality constraint. Then to make sure (B.5) has at least one nonempty solution, solve the Master Problem (MP) (B.7). Since the values of all \( z_i \) are obtained during the iteration process, only some constraints in (B.7) are known explicitly.

\[ \text{max} \quad \xi \]
\[ \text{s.t.} \]
\[ u_{i1}^*(b - Cz_1) \geq \xi \quad u_{i2}^*(b - Cz_2) \geq \xi \quad \ldots \quad u_{in}^*(b - Cz_n) \geq \xi \]
\[ u_{p1}^*(b - Cz_1) \geq \xi \quad u_{p2}^*(b - Cz_2) \geq \xi \quad \ldots \quad u_{pn}^*(b - Cz_n) \geq \xi \]
\[ u_{i1}^*(b - Cz_1) \geq 0 \quad u_{i2}^*(b - Cz_2) \geq 0 \quad \ldots \quad u_{in}^*(b - Cz_n) \geq 0 \]
\[ u_{p1}^*(b - Cz_1) \geq 0 \quad u_{p2}^*(b - Cz_2) \geq 0 \quad \ldots \quad u_{pn}^*(b - Cz_n) \geq 0 \]
\[ \xi \text{ free, } \quad z \text{ binary} \quad (B.7) \]

4) Solve (B.5), and based on the solution to add the cut:
i. If the dual problem (B.6) is unbounded, add the following *Feasibility Cut* to RMP, and go to Step 4;  
\[ \bar{u}(b - Cz) \geq 0 \]  
where \( \bar{u} \) is the extreme ray of the dual problem (B.6).

ii. If the optimal value of dual problem (B.6) is bounded, which provides a lower bound, and restrict RMP \( (\bar{u}(b - Cz) < \xi) \), add the following *Optimality Cut* to RMP, and go to Step 4;  
\[ \bar{u}(b - Cz) \geq \xi \]  
where \( P \) is the set of \( z \)'s value is 1, and \( Q \) is the set of \( z \)'s value is 0. \( |P| \) is the number of variables \( z \) that has value of 1.

iii. If the optimal value of dual problem (B.6) is bounded, which provides a lower bound, and does not restrict RMP \( (\bar{u}(b - Cz) \geq \xi) \), but the difference between upper and lower bound exceeds the threshold, add the following *Integer Exclusion Cut* to RMP, and go to step 4.  
\[ \sum_{i \in P} z_i - \sum_{j \in Q} z_j \leq |P| - 1 \]  
where \( P \) is the set of \( z \)'s value is 1, and \( Q \) is the set of \( z \)'s value is 0. \( |P| \) is the number of variables \( z \) that has value of 1.

**Step 4:** Solve RMP with an added cut, and obtain an updated upper bound. Find the difference between upper bound and lower bound. If it is within the tolerance, stop; otherwise, update the SP by setting constraint of current binary variable \( z \) and go back to Step 2.
BIBLIOGRAPHY


[42] PJM Training Materials-LMP 101, PJM.


LIST OF PUBLICATIONS


