Sovereign Debt Maturity Structure and Dilution *

Mohammad Hasan † **Rajesh Singh** ‡

[This Version: October 31, 2023]

ABSTRACT

The maturity structure of debt plays an important role in default probability and the resulting pricing of sovereign bonds. Using a calibrated model this paper shows that both the default probabilities and the prices get improved when the sovereign issues long-term bonds or both short- and long-term bonds instead of issuing only short-term bonds. This paper also shows that the inclusion of the compensation covenant mitigates the dilution problem of sovereign debt on a larger scale when the maturity of long-term bonds is sufficiently high.

JEL Classification: F34, H63

Keywords: sovereign debt, sovereign default, maturity structure, debt dilution, compensation covenant.

*We thank Juan Carlos Cordoba, Joydeep Bhattacharya, Helle Bunzel, Travis Sapp, Gary Lyn, and all members of the Macro-Environment Development group at Iowa State University for helpful comments and discussions. All remaining errors are our own and the usual disclaimers apply.

†(Corresponding Author) College of Business, Doane University, mohammad.hasan@doane.edu

‡Department of Economics, Iowa State University, rsingh@iastate.edu

I Introduction

An important mechanism of financing the fiscal deficit is issuing sovereign bonds which engendered several policy debates, over the last two decades, including recently debated optimal maturity structure, and debt dilution problem inherent to such sovereign debt. Moreover, it is evident from figure 1 (Beers and de Leon-Manlagnit, 2019) that advanced economies are also susceptible to default which adds to the dimension of the sovereign default problem that was supposedly pertinent to only emerging market economies. Severe financial crisis in Greece, Ireland, Italy, Portugal, and Spain (the GIIPS countries) beginning in 2009 led worldwide anxiety on the ability to refinance their outstanding debt. Moreover, the worsening creditworthiness propagated to the financial sector due to the banks' significant exposure to the sovereign bonds (Acharya et al., 2014; Acharya and Steffen, 2015). In expectation of higher returns, weakly capitalized banks bought risky assets such as distressed sovereign debt (Drechsler et al., 2016). Liquidity squeeze of banks further depressed investment, employment, and sales growth of firms affiliated with such affected banks (Acharya et al., 2018). Figure 1 shows the total sovereign debt in default in US dollar between the period of 1975 and 2018. The figure also depicts that sovereign default problem is not fading out by time rather coming back strongly in new dimensions since nearly 40 percent of sovereign default amount was coming from the advanced economies in the year of 2018. In this paper we show the linkage between the maturity structure of sovereign debt and the dilution problem. In particular, extending the study of Arellano (2008), I show the sovereign debt contract with only long-term maturity structure and both short- and long-term maturity structure. Default probabilities go down and therefore bond prices go up as we go from bonds with only short-term maturity to bonds with only long-term maturity and finally, to both maturity structure. As a potential solution of the debt dilution problem, we add the compensation covenant in Hatchondo et al. (2016) and find improvement in sovereign bond prices under the framework of Chatterjee and Eyigungor (2015).

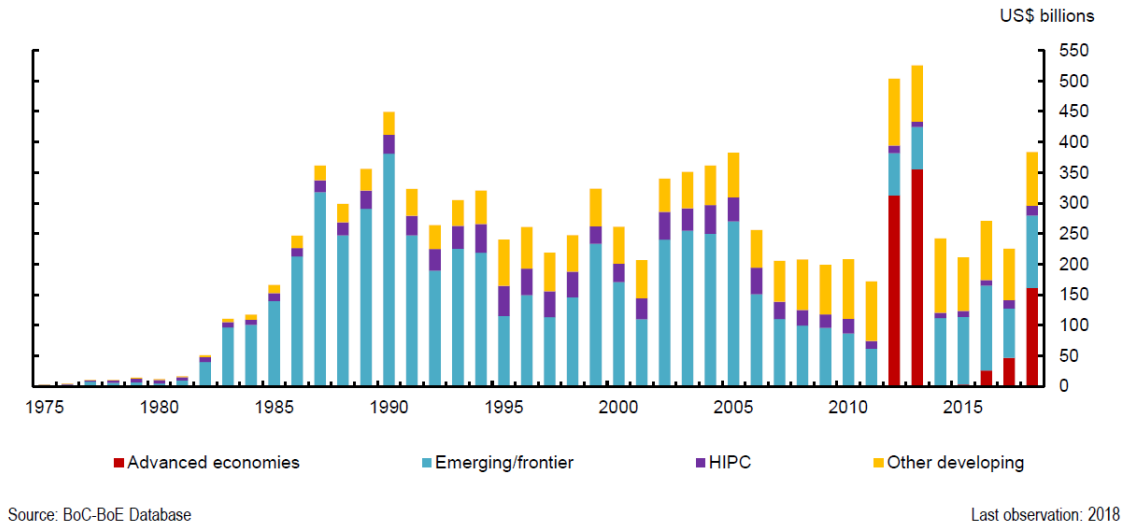


Figure 1: Sovereign Debt in Default

If a sovereign keeps increasing debt level then probability of default also increases and sometimes lead to eventual default. A higher default probability is adjusted through reduction in the value of bonds- a debt dilution faced by the sovereign. Debt dilution problem arises because current government cannot control the bond issuances by the future governments. It is not possible to restrict a sovereign from issuing bonds since doing so may lead to restricting the sovereign from consumption. Therefore, a covenant of "no further issuance of bonds" is never a feasible idea. However, any additional bond issued by the sovereign results in higher risk of default anticipated by the rational investors and therefore investors offer lower price for the bonds. Eventually, the sovereign faces a dilution in the value of their issued bonds in the market. A possible way of reducing the debt dilution problem is to shorten the maturity of the bonds. However, shortening the maturity of bonds would increase the rollover risk for the sovereign, i.e., the risk of higher borrowing cost. There is a trade-off between the short-term debt which has a relative incentive benefit and the long-term debt which has a hedging benefit. This trade-off of the maturity structure is reflected through the price functions. Therefore, there should be a suitable solution for the optimal maturity structure and the debt dilution problem.

Government bonds are issued in the primary market and traded in the secondary market. The price is determined based on the default premium coming from the default risk of the sovereign at the time the bond is issued in the primary market. In the secondary market, the price reflects the current situation of the sovereign. Issuing more bonds increases the default risk of the sovereign and therefore default premium increases to reduce the price of the bond. The senior bondholders (who purchased bonds in earlier periods) are affected by the borrowing decision of the sovereign since the price at which they can sell the bonds they hold in the secondary market is lower than what would have been without more borrowing by the bond issuing sovereign. Even if the bondholders hold their bonds until maturity, they face an increased default risk due to increase in debt level through issuance of more bonds by the sovereign. Only the new bondholders get the compensation of the higher default risk through lower bond price. This requires to arrange a compensation package for the existing bondholders before any new issuance of sovereign bonds.

Several authors propose varying solutions to mitigate the debt dilution problem. While Hatchondo et al. (2016) propose imposing a maximum limit on either total foreign debt or debt-to-GDP ratio, Chatterjee and Eyigungor (2015) corroborate the idea of seniority of sovereign debt creditors. The idea proposed by Hatchondo et al. (2016) is supposed to reduce the probability of default and ensure the existing bondholders a compensation in case the sovereign wants to issue more bonds. Bond covenant can include the clause of compensation equal to the differential of the observed price and the price at which the bonds would be traded without new bond issues. To circumvent the difficulty of finding the counterfactual price, the authors study two covenants which set a penalty if the sovereign chooses a debt level above a threshold or borrows at bond prices below a threshold. However, a differential compensation is not enough for the existing bondholders when the sovereign defaults and all the creditors, both earlier and later, get repaid at the same proportion during the settlement. Moreover, Hatchondo et al. (2016) use debt-to-GDP ratio as a measure of debt level which requires continuous adjustment

of sovereign debt due to the change in GDP, if the ratio is close to the threshold.

To avoid the problem of continuous adjustment of sovereign debt, Chatterjee and Eyigungor (2015) propose a relative seniority arrangement where all the creditors lose an equal amount of outstanding debt but get paid based on pre-decided seniority for the remaining portion of debt in case of reorganization after default by the borrowing sovereign. This is for the resolution of dispute among the creditors under absolute seniority arrangement. A seniority arrangement specifies the allocation of repayments after default and therefore, such an arrangement is not a suitable candidate for the solution of the sovereign debt dilution problem. Even with a seniority arrangement, debt dilution problem exists. For instance, an investor A may buy a long-term sovereign bond and would like to sell it before the maturity. If the sovereign issues further long-term bonds then the default risk increases for the sovereign and the price goes down in the secondary market to adjust this risk. If the investor A is still holding the previously issued bond then he faces the downward price movement when selling the bond before maturity. Now, a seniority rule may act as an insurance for the investor for getting the investment back after the default by the sovereign. However, such arrangement fails to even reduce the problem of downward price movement faced by the investors like A who intend to sell the security before the maturity. Therefore, seniority arrangement cannot fully eliminate the debt dilution when the default probability is increased because of new borrowings. Bizer and DeMarzo (1992) also observe that a seniority arrangement is not sufficient to eliminate debt dilution when default probability is increased through new borrowings.

Following Arellano (2008), this paper initiates with the basic model of sovereign debt of only one period, i.e., short-term bond. Then, we study the model with only long-term bond followed by the model with multiple maturity structure which includes both the short- and long-term bonds. We also include the compensation covenant in Hatchondo et al. (2016) for both long-term bonds contract and contracts with short- and long-term bonds as a solution of the debt dilution problem. However, we follow the model struc-

ture of Chatterjee and Eyigungor (2015) with a probabilistic maturity of the bonds. In the appendix, we show the sovereign debt model with settlement of defaulted bonds where settlement happens in one of the later periods when the sovereign gets back to the international financial market after an exclusion due to the default decision on outstanding debt. Finally, we also explore the competing solutions of the debt dilution problem, e.g., debt threshold covenant, price threshold covenant, and income-contingent debt.

The paper proceeds from here with the literature review of sovereign debt maturity structure and the dilution problem in section II. In section III, there are basic models of only short-term sovereign bonds as in Arellano (2008), long-term bonds in Chatterjee and Eyigungor (2015) framework, and bonds with both maturity structure as in Arellano and Ramanarayanan (2012). Section IV shows the models without the debt dilution problem while introducing compensation covenant to the existing bondholders with both long-term bonds and maturity structure of short- and long-term bonds. Section V presents the comparative analyses of all the models used for different maturity structure of long-term bonds. Section VI concludes the paper .

II Literature Review

Sovereign debt has an extensive literature covering but not limited to existence and uniqueness of equilibrium, strategic structure of the debt market, contract and maturity choice, settlement following default, restructuring without default, reputation and partial default, debt dilution and added covenants, fiscal rules and default, default cost, and contagion and correlated defaults. The seminal paper in sovereign debt literature is Eaton and Gersovitz (1981) where the authors introduce the sovereign debt commitment problem and default decision of the sovereign in comparison to repayment based on relative welfare on either decisions. There is a substantial body of work in sovereign debt literature based on the Eaton and Gersovitz (1981) framework. See Aguiar et al. (2016) for a detailed quantitative literature on diverse issues of sovereign debt.

Arellano (2008), with a one period model, shows how default is more likely in recessions using business cycles data of Argentina. However, while the defaulting sovereign and the creditors resolve through settlements in most of the cases, this paper assumes that the sovereign never pay back the defaulted debt. Later, Arellano and Ramanarayanan (2012) study on the emerging markets term structure of interest rate spreads and the maturity composition of government debt and show that the sovereign has more incentive to repay in case of short-term debt whereas long-term debt is a hedge against future fluctuations in spread. Understanding the emerging market sovereign debt maturity structure requires comprehending this trade-off between the incentive benefits of short-term debt and hedging benefits of long-term debt. Broner et al. (2013) document that shorter-maturity debt is issued in crisis periods while long-term debt is issued in normal times by the emerging market sovereigns.

Although the solution of debt dilution has got attention recently, the literature on sovereign dent dilution is intensive. Borensztein et al. (2006) observe that emerging-market borrowers face a lengthy and costly period of restructuring due to debt dilution. The possibility of dilution induces the sovereign borrowers to opt for short-term bonds (Kletzer, 1984; Sachs and Cohen, 1982) since such bonds are difficult to dilute or debt those cannot be restructured easily to raise the cost of default and lower the likelihood of default (Dooley, 2000). The short-term debt introduces the rollover crisis driven by lenders' confidence (Giavazzi and Pagano, 1990; Cole et al., 1996), and hard-to-dilute bonds worsen the crisis inefficiently (Bolton and Jeanne, 2009).

Absolute priority rule in repayments of bondholders is explicitly studied in sovereign debt literature (Borensztein et al., 2006; Bolton and Skeel Jr, 2004; Bolton and Jeanne, 2009; Gelpern, 2004). However, absolute seniority is not a panacea of debt dilution problem and therefore there should be a relative seniority rule (Chatterjee and Eyigungor, 2015). There may also be a compensation package for the existing bondholders before issuing any new bonds by the sovereign (Hatchondo et al., 2016).

III The Basic Models

1 Sovereign Debt with Short-term Bonds

We have a small open economy where the sovereign government decides on both consumption and savings/borrowing on behalf of the domestic residents who face fluctuating endowment stream. The sovereign faces the given world risk-free interest rate and cannot change it through any decisions. The sovereign can borrow from the credit market where the lenders are risk-neutral. Sovereign debt contracts are not enforceable in the international financial market and therefore, a sovereign can take a default decision at any time on the outstanding debt. In case of default, the sovereign is assumed to be excluded temporarily from the international credit market and to incur output costs directly. Sovereign bond price available to the sovereign make creditors break even in expected value and reflect the default event likelihood.

The paper proceeds from here by describing the endowment and preference structure of the sovereign first. Then, we give description of the financial market and events timing. Afterwards, we describe both the sovereign's and the lenders' problem followed by the recursive equilibrium.

The economy has identical, risk averse households who have preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \;\; 0 < \beta < 1. \tag{1}$$

where, c is the consumption, β is the discount factor, and $u(\cdot)$ is increasing and strictly concave. y is the tradable good with a stochastic stream and received by the households. The output shock is a Markov process with a transition function $f(y', y)$ with a compact support. There is a lump sum transfer of goods from the government to the households. The former is benevolent with an objective of maximizing the utility of the later.

The government can sell one-period discount bonds b' at price $q(y, b')$ in the international financial markets and decides, at the end of the period, whether to default or repay

on its outstanding debt. The default probability depends on both the aggregate shock y, and the size of the bond b'. Therefore, the bond price function $q(y, b')$ also depends on both of these two and is endogenous to the default incentive of the government. Purchasing a discount bond with price $q(y, b')$, where $b' \geq 0$, means saving $q(y, b')b'$ units of goods in the current period to receive next period b' units of goods. Purchasing a discount bond with price $q(y, b')$, where $b' \leq 0$, means receiving $-q(y, b')b'$ units of goods in the current period to deliver b' units of goods the next period, conditional on not declaring default. The households receive all the proceedings of the international credit operations by the government in a lump sum way.

Being benevolent, the government, with an effort to smooth consumption, uses borrowing form the international credit market effectively. y induces an idiosyncratic income uncertainty, and the asset markets are incomplete due to two reasons: (i) an endogenous default risk, (ii) an uncertain set of available assets. The set of available bonds pay a state and time invariant amount which cannot insure away the idiosyncratic income uncertainty.

The cost of default has two components: direct output cost, $\phi(y)$ and exclusion from the international financial market. This specification follows the results of defaulting from recent events, i.e., lower aggregate output and a temporary loss of access to the international financial market. With this specification, all outstanding debts get erased from the budget constraint of the government and it is no longer allowed to participate in the international financial market, i.e., the government cannot save or borrow. Following a default, there is a stochastic number of period for the government in financial autarky and reenters the markets with an exogenous probability ξ.

Following is the government resource constraint of a small open economy:

$$c \leq \begin{cases} y - q(y, b')b' + b & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2}$$

The international credit market has a constant interest rate, $r^* > 0$, at which investors can lend or borrow as much as required. Investors can observe the income level every period and therefore, they possess perfect information about the endowment process of the economy. Foreign creditors are risk-neutral, i.e., in every bond contract offered they break even in expected value. Every period lenders choose loans b' to maximize expected profits π, taking prices as given:

$$\pi = q(y, b')b' - \frac{1 - d(y, b')}{1 + r^*} b' \tag{3}$$

where $d(y, b')$ is the probability of default which depends on the level of income, y and level of foreign asset holdings, b'.

The probability of default, $d(y, b')$ is zero for positive level of foreign asset holdings, $b' \geq 0$. In this case, the price of a bond is equal to the opportunity cost of the creditors. If the level of foreign asset holdings in negative, $b' < 0$, then the equilibrium bond price equals to the opportunity cost after risk-adjustment. This bond prices are, therefore, required to satisfy

$$q(y, b') = \frac{1 - d(y, b')}{1 + r^*} \tag{4}$$

Following is the the timing of events within each period. The government, foreign creditors, and households act sequentially. The government enters each period with asset level b to repay at the end of the period. The income shock y is the observed by the government, who decides whether to default or repay its outstanding debt obligations at the end of the period. If the government decides to repay, then chooses b' subject to resource constraint, given the bond price schedule $q(y, b')$. Otherwise, the government defaults on its debt and exits the international financial market for a stochastic period of time. In case of repayment by the government, creditors choose b', taking q as given. Following these events, consumption c takes place by the households. Consumption includes the endowment and the government transfer from the international financial market credit operations.

The economy is characterized by a recursive equilibrium where the government's policy function b', bonds' price function q, and consumers' policy functions c determine the equilibrium, given aggregate states $s = (y, b)$. The government chooses whether to repay or default, given initial foreign assets b and after observing the income shock y. With the choice of repayment of debt obligations, the government then decides the new level of foreign assets b', given the price of new borrowing $q(y, b')$, that depends on the states y and the choice of b'.

Let $V^o(y, b)$ be the lifetime utility of a sovereign that enters a period with $V^o(y, b)$ and maintains a good standing (not in default state) and $V^d(y)$ be the lifetime utility of a sovereign in default.

Let $V^o(y, b)$ be the sovereign's lifetime utility with good standing (not in default) while entering a period with (y, b) and $V^d(y)$ be the sovereign's lifetime utility in default. When the government decides to default, the economy goes into a financial autarky for a short period of time and income falls by $\phi(y)$. Then the value of default is given by,

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1 - \xi)V^d(y') + \xi V^o(y', 0)] \tag{5}$$

Where, ξ is the probability of reentry in the international financial market.

The payoff from repaying the debt and remaining in the contract, denoted $V^c(y, b)$, is given by

$$V^c(y, b) = \max_{b' \in \mathbb{B}}[u(y - q(y, b')b' + b) + \beta E_{\{y'|y\}}V^o(y', b')] \tag{6}$$

The government solves the utility maximization problem by choosing the optimal level of new debt b' in each period. The expected value of remaining in the contract incorporates the fact that default choice could be made by the government at any time in the future. To prevent any Ponzi schemes, there is a lower bound on the level of new debt that government may choose, $b' \geq -Z$ which is not binding in equilibrium otherwise.

Finally, given the default option, $V^o(y,b)$ satisfies

$$V^o(y,b) = \max\{V^c(y,b), V^d(y)\} \tag{7}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b)$ be the set of income levels for which the optimal decision is repayment when assets are b, such that

$$R(b) = \{y \in Y : v^c(y,b) \geq v^d(y)\}$$

and let $D(b) = \tilde{R}(b)$ be the set of income levels for which the optimal decision is default for assets level of b:

$$D(b) = \{y \in Y : v^c(y,b) < v^d(y)\}$$

The equilibrium bond price function $q(y,b')$ should correctly assess the probability of default by the government and therefore, needs to be consistent with the optimization of the government and with the expected zero profits for the lenders. Default probabilities $d(y,b')$ and default sets $D(b')$ are therefore related in the following way:

$$d(y,b') = \int_{D(b')} f(y',y)dy' \tag{8}$$

The equilibrium default probabilities $d(y,b')$ are equal to zero when there are empty default sets, $D(b') = \emptyset$ because the optimal choice is to repay for all realizations of the endowment shocks, with assets b'. Default probabilities $d(y,b')$ are equal to one when $D(b') = Y$ because for any realization of the endowment shocks it is optimal to default.

Definition 1: *The definition of the recursive equilibrium of the economy is given by a set of policy functions (i) government's asset holdings $b'(s)$, default sets $D(b)$, and repayment sets $R(b)$, (ii) consumption $c(s)$, and the price function for bonds $q(y,b')$ such that*

1. Given the bond price function $q(y,b')$, the government's policy functions $b'(s)$, default sets $D(b)$, and repayment sets $R(b)$ satisfy the government optimization problem.

2. Given the government policies, households' consumption $c(s)$ satisfies the resource constraint.

3. Bond prices $q(y, b')$ are consistent with both the default probabilities of the government and the expected zero profits by the creditors.

2 Sovereign Debt without Exclusion

This section is a continuation of the previous one except for the exclusion of the sovereign from the international credit market following a default of sovereign debt. We assume here that the sovereign is not excluded from participating in the international credit market if the default is led by a lower level of output. This is a rational assumption since the international market does not want to exclude a sovereign only because it could not produce enough output. Moreover, in the presence of sufficient international trades with the defaulting sovereign, no other nations would stop trades because that would have a negative impact on the social welfare of that nation.

The question is why would the international credit market allow a defaulting sovereign to keep continue the participation. The answer is that the price of the sovereign debt already adjusted the default probability and therefore, a default is already priced and the defaulting sovereign should not suffer from another punishment of exclusion from the international credit market and further output loss. In contrary, the sovereign should be allowed to borrow more from the market without paying off the outstanding debt. However, such a default should not come at no cost.

There is a policy cost $\phi(b')$ that should be sufficiently high in the channel of resource cost of policy intervention from the international financial market so that the sovereign does not experience similar low level of output due to lack of experience, knowledge or simply intention. A resource cost through policy intervention from the international credit market is a rational assumption since we observe similarity of this with International Monetary Fund (IMF) credit to nations and its influence on the borrowing nations' internal monetary policies. The policy cost depends on the level of current borrowing b' since the international credit market imposes stricter policy guidelines to implement for

higher level of borrowing. If the sovereign decides to default regardless of the output level then it faces an output cost which depends on the output level. The output and default scenarios are as follows:

$$c \leq \begin{cases} y - q(y,b')b' + b & \text{Participation | High output} \\ y - q(y,b')b' + \max\{0,b\} - \phi(b') & \text{Participation | Low output} \\ y - \phi(y) & \text{Autarky} \end{cases} \tag{9}$$

The output cost should be sufficiently high to exclude the possibility of a moral hazard of sovereign default during high output level. In particular, $\phi(y)$ should be greater than or equal to $[q(y,b')b' - b]$ so that, being in autarky is never a preferable option during high output states. This condition ensures that the net repayment of current outstanding debt after new borrowing should be less than or equal to the output cost in default. For low output level, if the sovereign has positive outstanding financial assets (b) in current period, then receives the repayment. Otherwise, with a negative credit balance, the sovereign gets what is called forgive and forget and repays nothing. However, the sovereign is allowed to borrow (b') at price $q(y,b')$ and at policy cost $\phi(b')$. To restrict the sovereign from defaulting during low output states, the output cost $\phi(y)$ should also be higher than or equal to $[q(y,b')b' + \phi(b')]$. This condition explains that the net policy cost after new borrowing is less than or equal to the default output cost. To encompass both the conditions, we must have the following:

$$\phi(y) \geq q(y,b')b' + \max\{\phi(b'), -b\} \tag{10}$$

3 Sovereign Debt with Long-term Bonds

This section differs from the previous one in terms of the maturity of the debt contracts. Here, long-term debt contracts mature probabilistically (Leland, 1998; Chatterjee and Eyigungor, 2015). Specifically, there is a probability λ that determines the maturity of each

unit of outstanding debt next period. In case, the debt does not mature, there is an associated coupon payment z. Debt service requires to pay λb for matured portion of debt and $(1-\lambda)zb$ for the periodic coupon payment for the portion of debt that is not matured. Therefore, total debt service is $[\lambda + (1-\lambda)z]b$. Debt level of the next period is denoted b' where the debt that is not matured in the current period is $(1-\lambda)b$. Hence, total new issue of bonds is $(1-\lambda)b - b'$ and with bond price $q(y, b')$, total credit funding for the consumption is $q(y, b')[(1-\lambda)b - b']$.

Equation (1) will still hold since it does not depend on the maturity structure of bonds.

Government resource constraint for issuing long-term bonds only is as follows:

$$c \leq \begin{cases} y + q(y, b')[(1-\lambda)b - b'] + [\lambda + (1-\lambda)z]b & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{11}$$

The value function of a sovereign in a state of default is:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0)] \tag{12}$$

Let $V^c(y, b)$ be the payoff from repaying the debt which is given by

$$V^c(y, b) = \max_{\{b', c\}}[u(c) + \beta E_{\{y'|y\}} V^o(y', b')] \tag{13}$$

Finally,

$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \tag{14}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b)$ be the set of income levels for which the optimal decision is repayment when long-term debt is b, such that

$$R(b) = \{y \in Y : v^c(y, b) \geq v^d(y)\}$$

and let $D(b) = \tilde{R}(b)$ be the set of income levels for which the optimal decision is default for the debt level of b:

$$D(b) = \{y \in Y : v^c(y, b) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q(y, b') = \frac{1}{1 + r^*} \int_{R(b')} [\lambda + (1 - \lambda)\{z + q(y', \tilde{b}')\}] f(y', y) dy' \tag{15}$$

where,

$$b' = \tilde{b}(y, b)$$

4 Sovereign Debt with Multiple Maturity Structure

In this section we assume that the sovereign can issue both short- and long-term bonds as in Arellano and Ramanarayanan (2012). The sovereign now start a period with b_S level of short-term bonds and b_L level of long-term bonds. There are two prices now- $q_S(y, b'_S, b'_L)$ for short-term bonds and $q_L(y, b'_S, b'_L)$ for long-term bonds. The price of short-term bonds does not only depend on the its own level of debt level chosen for the next period but also on the long-term debt level b'_L. This is because with each unit of bond of any maturity structure, the default probability of the sovereign goes up and the prices of both short- and long-term bonds depend on the default probability. Revenue collected from issuing short-term debt is $-q_S(y, b'_S, b'_L)b'_S$ and from long-term debt is $q_L(y, b'_S, b'_L)[(1 - \lambda)b_L - b'_L]$.

Government resource constraint for issuing both short- and long-term bonds is as follows:

$$c \leq \begin{cases} y - q_S(y, b'_S, b'_L)b'_S + q_L(y, b'_S, b'_L)[(1 - \lambda)b_L - b'_L] + b_S + [\lambda + (1 - \lambda)z]b_L & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{16}$$

The value of default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0, 0)] \tag{17}$$

Let $V^c(y, b_S, b_L)$ be the payoff of the sovereign from repaying the debt which is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'_S, b'_L)] \tag{18}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{19}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b_S, b_L)$ be the set of income levels for which the optimal decision is repayment when short- and long-term debt are b_S and b_L, such that

$$R(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \geq v^d(y)\}$$

and let $D(b_S, b_L) = \tilde{R}(b_S, b_L)$ be the set of income levels for which the optimal decision is default for the debts level of b_S, and b_L:

$$D(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q_S(y, b'_S, b'_L) = \frac{1}{1+r^*} \int_{R(b'_S, b'_L)} f(y', y) dy', \tag{20}$$

$$q_L(y, b'_S, b'_L) = \frac{1}{1+r^*} \int_{R(b'_S, b'_L)} [\lambda + (1-\lambda)\{z + q_L(y', \tilde{b}'_S, \tilde{b}'_L)\}] f(y', y) dy' \tag{21}$$

where,

$$b'_S = \tilde{b}_S(y, b_S, b_L)$$

$$b'_L = \tilde{b}_L(y, b_S, b_L)$$

5 Quantitative Analysis

In this section we describe the functional forms and set all required parameters to calibrate the model. Simulation results follow the calibration of the model.

5.1 Calibration and Functional Forms

We make the following specifications of functional form and distributional assumptions for the quantitative analysis:

Utility function: $u(c) = c^{1-\gamma}/(1-\gamma)$

Endowment process: $\ln y_t = \rho \ln y_{t-1} + \epsilon_t$, where $0 < \rho < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

For the numerical specification of the model we need 9 parameter values. There are (i) two endowment process parameters, ρ and σ_ϵ^2; (ii) two preference parameters, β and γ; (iii) two parameters describing the bond, the maturity parameter λ, and the coupon payment z; (iv) the probability of reentry following default, ξ, and the output cost parameter during the default, $\phi(y)$; and (v) the risk-free rate r^*.

The estimated values of the endowment parameters are $\rho = 0.945$ and $\sigma_\epsilon^2 = 0.025$. Using a quadrature based procedure (Tauchen and Hussey, 1991), there is a 51-state Markov chain for the shock. The time preference parameter, β is set to 0.953, and the risk aversion coefficient, σ is set to 2, which is a standard practice in real business cycle studies. The value of λ is set to 0.1 so that average maturity becomes 10 quarters. However, according to OECD, average sovereign bond maturity has risen to almost 8 years or 32 quarters in 2019. Therefore, there is a comparative analysis later with $\lambda = 0.03$. Based on the data on Argentine bonds reported in Broner et al. (2013), the value of z is set to 0.03. The probability to reenter the financial market after default is set to 0.282. Following Arellano (2008), the output cost of default, $\phi(y)$ is set to $0.969E(y)$. The risk-free interest rate, r^* is set to 1.0 percent, corresponding to an annual rate of 4.0 percent, which is the average nominal yield on three-month US Treasury bills during the period of 1980 to 2001. Table

Table 1: Parameters

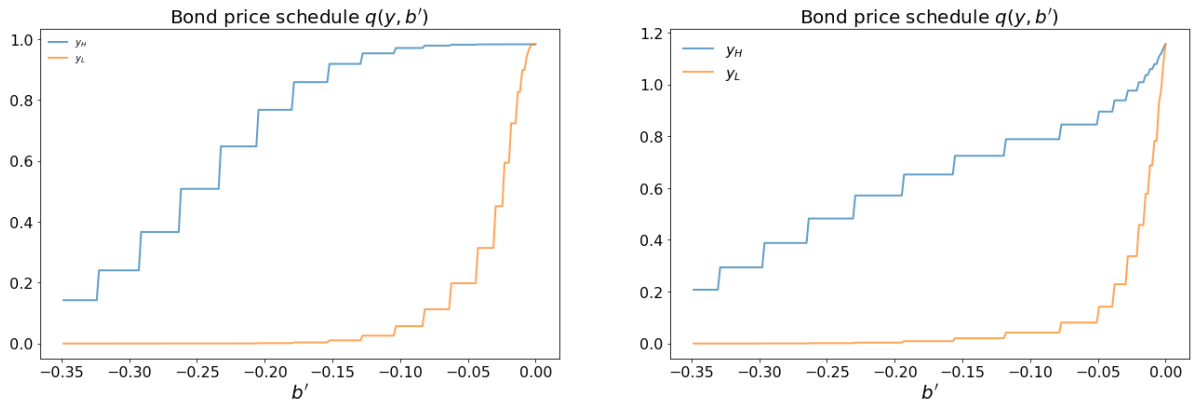
Parameter	Description	Value
β	Discount factor	0.953
γ	Risk aversion	2
ρ	Persistence of output process	0.945
σ_ϵ	Standard deviation of output shock	0.025
λ	Reciprocal of average maturity	0.10, 0.03
z	Coupon rate of long-term bonds	0.03
ξ	Probability of reentry in credit market	0.282
$\phi(y)$	Output costs	0.969 $E(y)$
r^*	Risk-free rate	0.01

1 summarizes the parameter values.

5.2 Simulation Results

This section analyzes the bond price schedules, value functions, default probabilities, simulation results of output, bond issuance, and the associated prices for previously discussed three methods of bond issuance: only short-term bonds, only long-term bonds, and both short- and long-term bonds.

Figure 2 shows the bond price schedule with respect to the choice of assets, b' (as ratio of mean output) for two different income shocks with 5 percent below and above trend while the sovereign issues either short-term bonds or long-term bonds. It is evident from the figure that there is a sharp decline in bond prices during recessions (low output) for a small amount of increase in borrowings regardless the maturity structure of bonds, and the strict borrowing constraint restricts the sovereign from further borrowing after reaching the borrowing capacity. During economic boom, bond prices remain steady for lower short-term borrowings but decline rapidly for long-term borrowings. However, after a certain level of borrowing, prices decline moderately for long-term bonds but sharply for



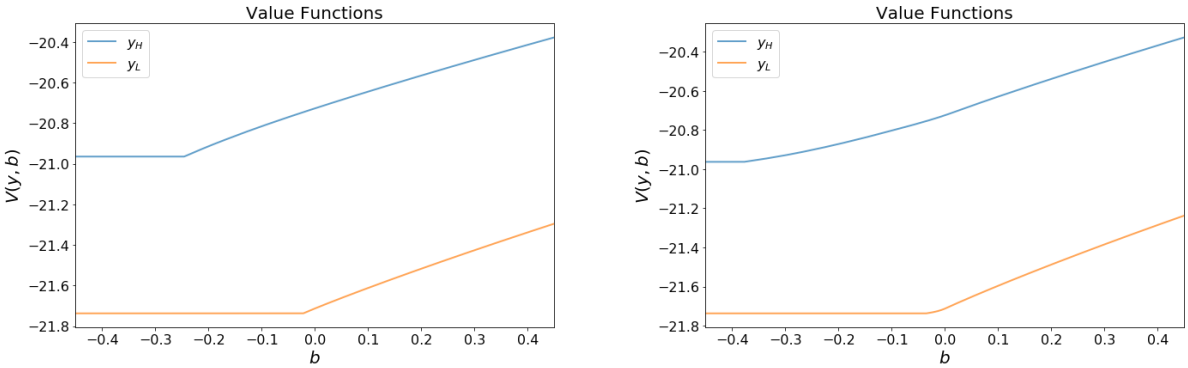
(a) Only Short-term Bond

(b) Only Long-term Bond

Figure 2: Bond Price Schedule

short-term borrowings. As in Arellano and Ramanarayanan (2012), there is a trade-off between the hedging benefit of long-term debt and the relative incentive benefit of short-term debt. During economic boom because of shock persistence sovereign is expected not to default in a short period of time but may default in the long run and therefore, short-term bonds have more value than long term bonds for small amount of borrowings. Here incentive benefit of short-term debts exceeds the hedging benefit of long-term debts. However, as borrowing increases, the value of short-term debts and its incentive benefit reduce sharply and the value is eventually lower than that of long-term debts at extreme borrowing level where hedging benefit of long-term debts exceed the incentive benefit of short-term debts. Therefore, the sovereign can generate more welfare through short-term bonds when the requirement of borrowing is lower and through long-term bonds when it is higher.

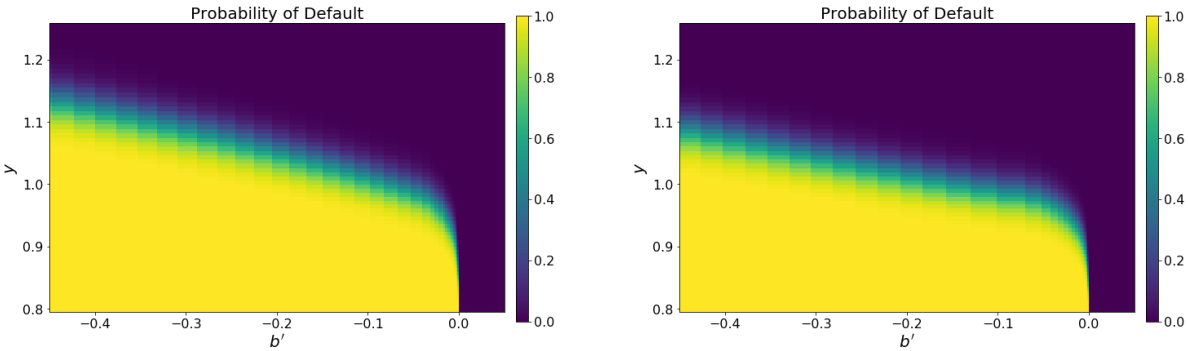
Figure 3 shows the relative comparison of value functions under two different maturity structure of bonds issuance- short-term bonds and long-term bonds when output is either high or low. During recessions the outcome is the same- after a very small amount of borrowing default becomes an obvious choice in both the cases. However, during boom the sovereign can generate higher welfare with long-term debt since the sovereign



(a) Only Short-term Bond

(b) Only Long-term Bond

Figure 3: Value Functions



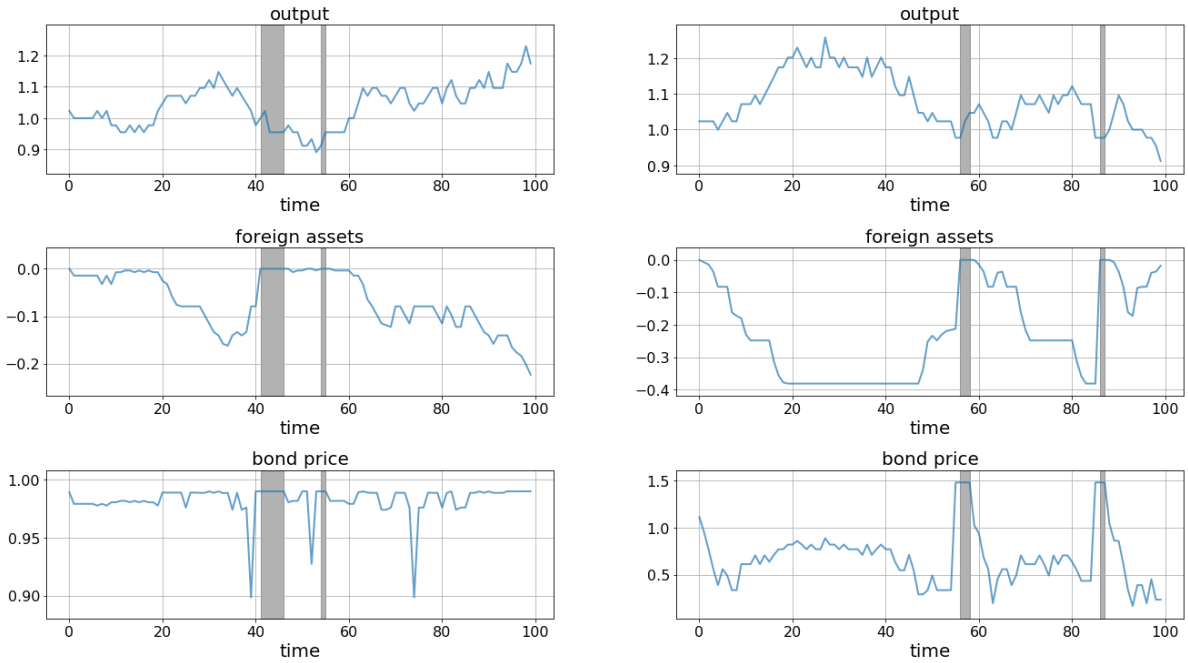
(a) Only Short-term Bond

(b) Only Long-term Bond

Figure 4: Default Probabilities

can accumulate more borrowings with long-term bonds than with short-term bonds before it needs to go for default.

Figure 4 shows the default probability as function of output, y and choice of bonds for next period, b' in heatmap when the sovereign issues short-term bonds or long-term bonds only. The yellow areas are for default and blue areas represent no default combination of output, y and bond level next period, b'. The figure shows that default area is higher for short-term bonds than for long-term bonds largely because of the rollover risk of short-term bonds. In case of short-term bonds, the sovereign needs to roll over the ex-



(a) Only Short-term Bond

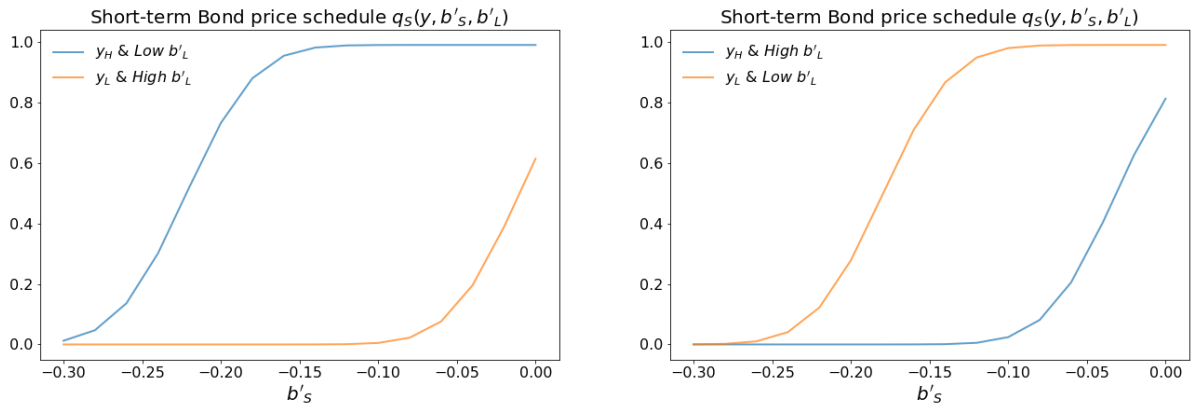
(b) Only Long-term Bond

Figure 5: Simulation Results of Output, Foreign Assets, and Bond Price

isting bonds by issuing new bonds whereas rollover is not required in case of long-term bonds. Therefore, default probability is higher for short-term bonds.

Figure 5 shows time series comparison between issuing either short-term bonds or long-term bonds in terms of output, foreign asset levels and bond price. The gray bars represent the default periods. The simulation results are almost the same for both the cases- when output is high and the economy is in boom, the borrowing capacity and borrowing levels are high with higher bond prices. The opposite case is also true for both the cases- when output is low and the economy is in recession, the borrowing capacity and the borrowing levels are low.

Figure 6 shows the short-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term. The price schedule is shown as a function of the level of choice of short-term bonds, b'_S by the sovereign for the next period,

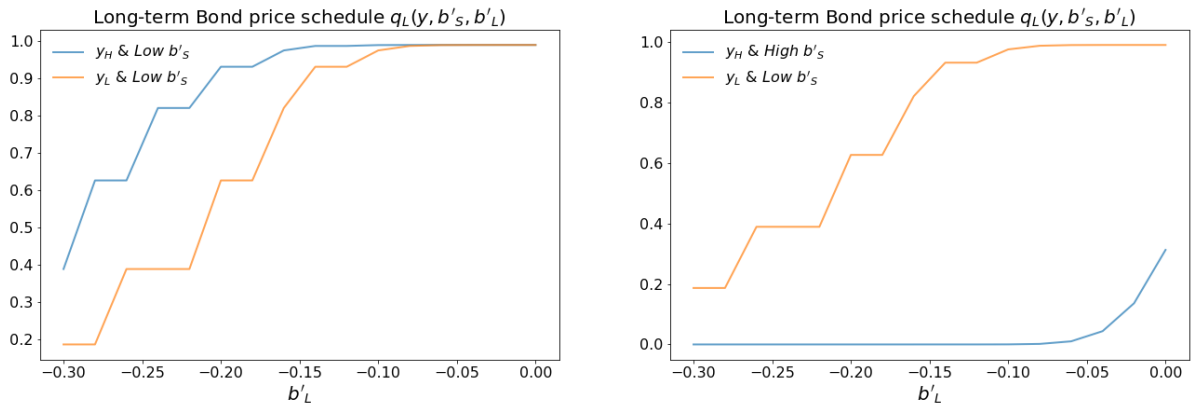


(a) (b)

Figure 6: Short-term Bond Price Schedule

given levels of output, y and the choice of long-term bonds, b'_L. The left panel shows the price schedule when output is high but long-term bond level is low and when output is low but level of long-term bond is high. During a recession the sovereign chooses to default after the price goes to zero after a small amount of bonds issuance. This is also because the sovereign has high level of long-term bonds already and further borrowing capacity is low. During economic boom, the price is close to one until debt-to-output ratio reaches 15 percent after which the price declines rapidly. The right panel shows the price schedule when both output and the level of long-term bonds are either high or low. This is interesting because the price is higher during a recession than during an economic boom due to a lower level of long-term bonds which frees up borrowing capacity with short-term bonds. Even in economic boom the sovereign goes to default with more than 10 percent short-term debt-to-output ratio since the borrowing capacity is low due to higher level of long-term debts.

Figure 7 shows the long-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term. The price schedule is shown as a function of the level of choice of long-term bonds, b'_L by the sovereign for the next period, given levels of output, y and the choice of short-term bonds, b'_S. The left panel shows the



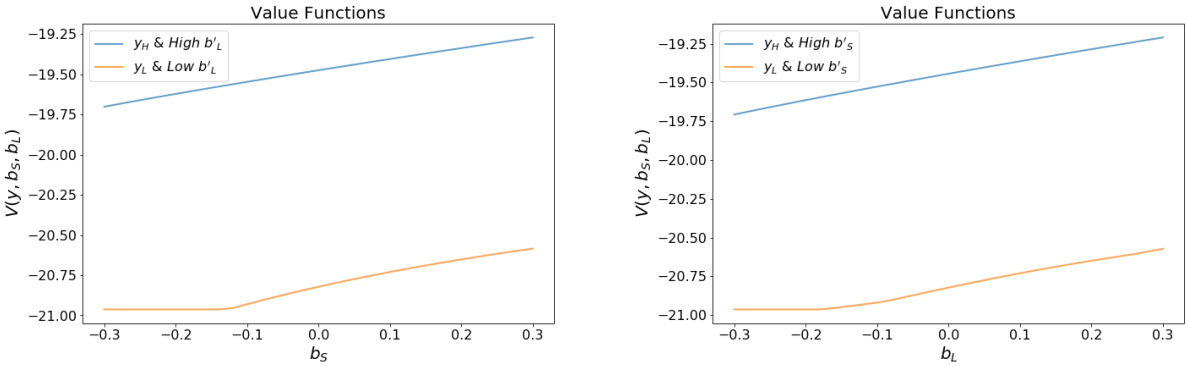
(a) (b)

Figure 7: Long-term Bond Price Schedule

price schedule when output is high or low but the level of short-term bond is always low. The price goes down gradually after a certain level of long-term bonds regardless of state of the economy. However, the price goes down more during recessions which is expected because of lower borrowing capacity and more default risk. The right panel shows the price schedule when both output and the level of short-term bonds are either high or low. Even the economy in boom, with a high level of short-term debt-to-output ratio, the sovereign cannot borrow without default after 5 percent long-term debt-to-output ratio.

Figure 8 shows the value functions with issuance of bonds of both maturity structure-short-term and long-term. The left panel shows the value function in term of short-term bonds when output, y and the level of choice of long-term bonds, b'_L are both high or low. The left panel shows the value function in term of long-term bonds when output, y and the level of choice of short-term bonds, b'_S are both high or low. We do not find any noticeable difference in the value functions in terms of the maturity structure of bonds. Hence, value function is trivial in case of deciding the maturity structure of bond issuance.

Figure 9 shows the time series simulation of output, holdings of short-term and long-term bonds by the sovereign, and their associated prices. When the output is high and the economy is in boom, the sovereign faces higher borrowing capacity and therefore,



(a) with respect to short-term bond

(b) with respect to long-term bond

Figure 8: Value Functions

holds higher short-term and long-term bonds. The prices are also high because of lower default risk during a boom. During the periods of lower output and economic recessions, the sovereign cannot borrow due to lower borrowing capacity. However, in the periods when output decreases gradually, the sovereign participates in the credit market with opposite position on short-term and long-term bonds, i.e., the sovereign buys short-term bonds while issues long-term bond and vice versa during the period of declining output.

IV The Models without Dilution

This section compares different methods of eliminating the dilution problem in sovereign debt. The dilution problem is only involved with the long-term debt. However, holders of previously issued long-term debt get worse off regardless the sovereign issues new debt of any maturity structure. To compare among the methods, in this section, we assume that the sovereign issues new debt on either maturity structure, i.e., short- or long-term debt. This will also allow us to focus on the differences of social welfare among the methods.

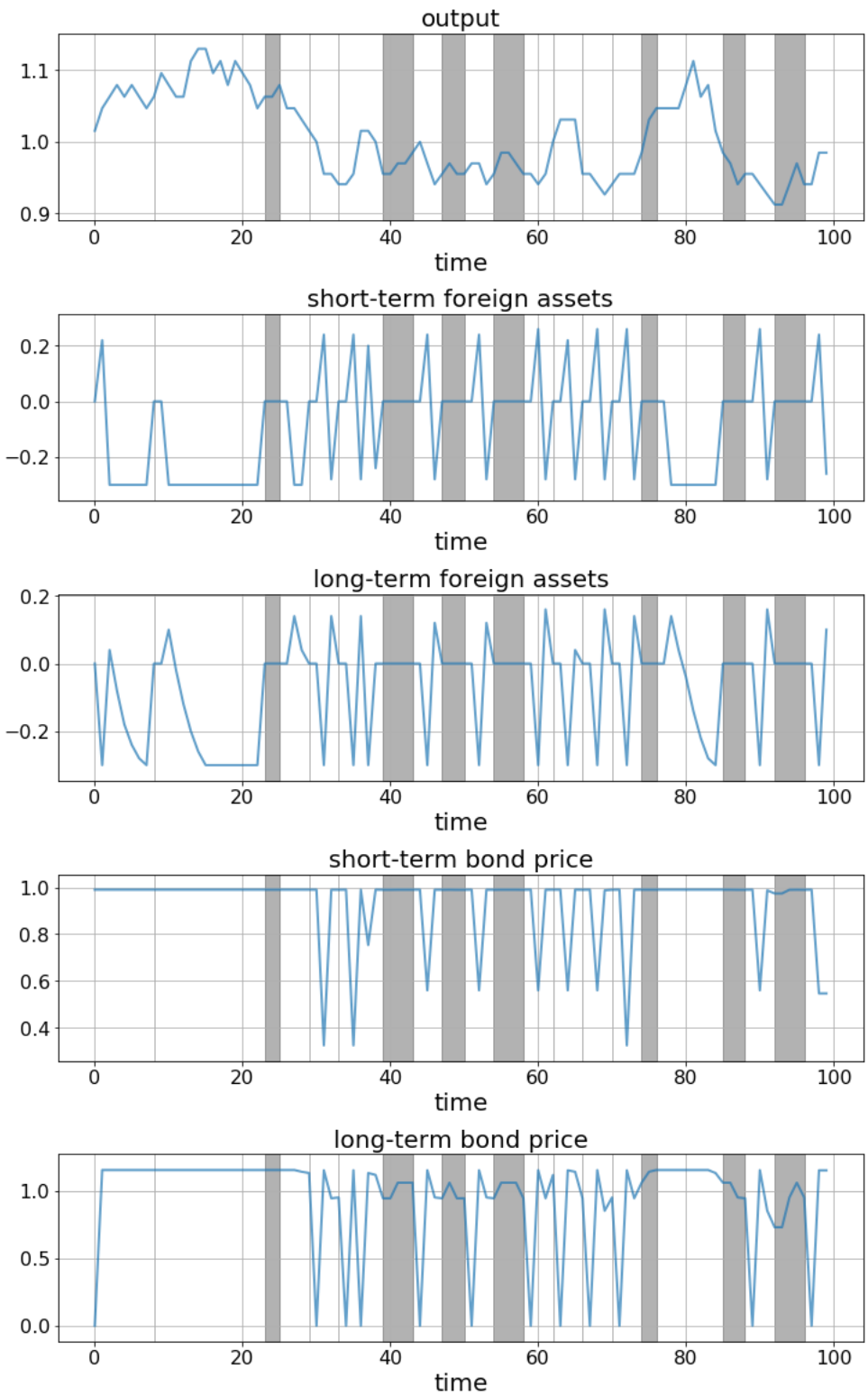


Figure 9: Simulation Results of Output, Foreign Assets, and Bond Prices

1 Compensation Covenant with Long-term Bonds

As in Hatchondo et al. (2016), this section proposes a compensation covenant which specifies that if the sovereign would like borrow more, it has to pay the difference between the observed bond price, $q(y, b')$ and the counterfactual bond price, $q(y, (1-\lambda)b)$ that would have been observed without any new borrowing by the sovereign in the current period to each holder of previously issued long-term bonds. This covenant makes the long-term bonds value independent of any borrowing in the future by the sovereign and thereby reduces the debt dilution problem.

The payment specified in the covenant is given by

$$\mathcal{C}(y, b, b') = \max\{q(y, (1-\lambda)b) - q(y, b'), 0\} \tag{22}$$

The government's budget constraint reads as

$$c \leq \begin{cases} y + q(y, b')[(1-\lambda)b - b'] + [\lambda + (1-\lambda)z]b + (1-\lambda)b\,\mathcal{C}(y, b, b') & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{23}$$

Without the dilution problem of sovereign debt through a compensation covenant for the existing bondholders before the sovereign can issue new bonds, there is a new term in the sovereign's budget constraint- $(1-\lambda)b\,\mathcal{C}(y, b, b')$ which implies that the sovereign must compensate the existing bondholders $(1-\lambda)$ portion of bonds with $\mathcal{C}(y, b, b')$. So, the total compensation becomes $(1-\lambda)b\,\mathcal{C}(y, b, b')$.

The price of a bond is given by

$$q(y, b') = \frac{1}{1+r^*} E_{\{y'|y\}}\Big[[1 - d(y', b')][\lambda + (1-\lambda)\{z + q(y', \tilde{b'}) + \mathcal{C}(y', b', \tilde{b'}\}]\Big]$$

Price of long-term bonds also adjusts the expected compensation $\mathcal{C}(y', b', \tilde{b'}$ in case of new borrowings by the sovereign in future.

The lifetime utility of a sovereign in a state of default is:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0)] \tag{24}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y, b)$, is given by

$$V^c(y, b) = \max_{\{b', c\}}[u(c) + \beta E_{\{y'|y\}} V^o(y', b')] \tag{25}$$

Finally,

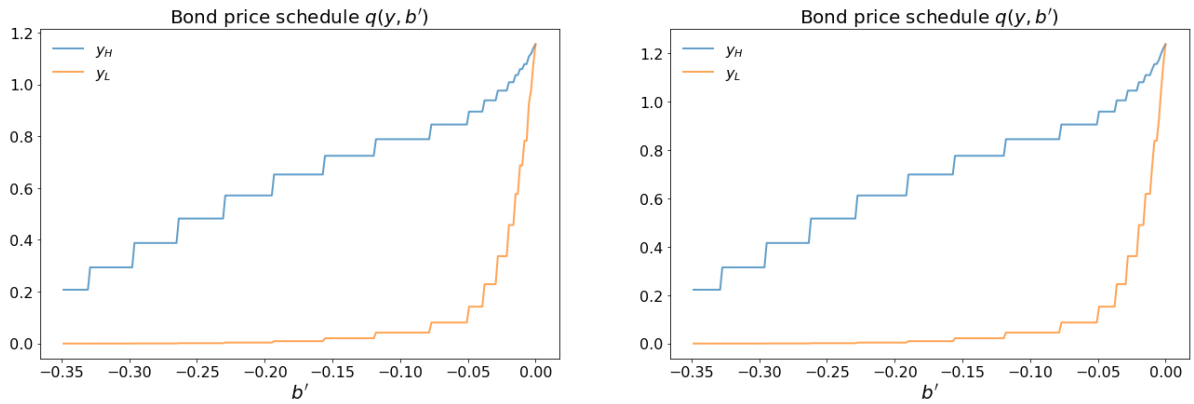
$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \tag{26}$$

1.1 Results

This section presents results in terms of bond prices, value functions, default probabilities, and simulation of output and foreign assets with the prices when the sovereign issues only long-term bonds with a compensation covenant as a solution to the debt dilution problem. This section also compares the results with the results when the debt dilution problem exists, i.e., without a compensation covenant for the existing bondholders.

Figure 10 shows the bond price schedule with dilution and without dilution with respect to the choice of new debt level under two output levels- high and low, when the sovereign issues only long-term bonds. The price schedule is pretty similar in both the cases. During low output period the bond prices go down sharply for a small amount of increase in borrowing and soon go close to zero. During economic boom the prices go down moderately and the difference is insignificant to observe visually. We will see the difference later when we compare the average prices with and without the dilution problem.

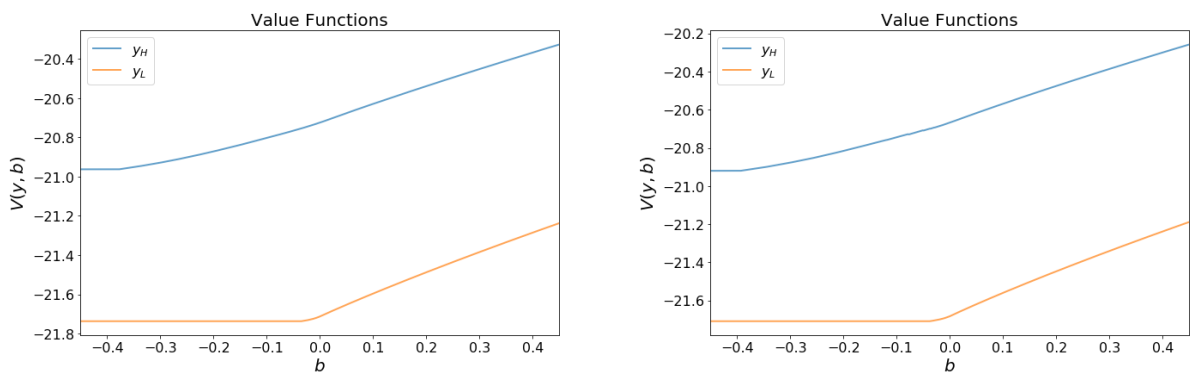
Figure 11 shows the value functions with dilution and without dilution with respect to the debt level under two output levels- high and low, when the sovereign issues only long-term bonds. The value functions are pretty similar in both the cases. During low



(a) Price with Dilution

(b) Price without Dilution

Figure 10: Bond Price Schedule



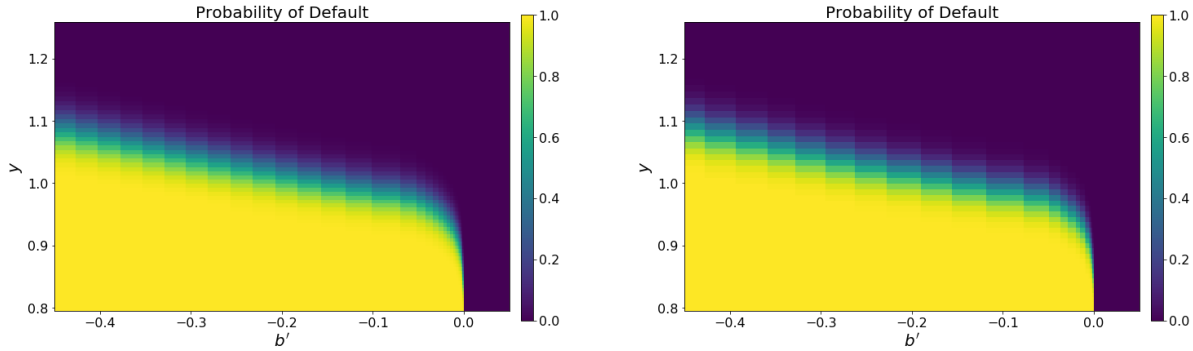
(a) Value Functions with Dilution

(b) Value Functions without Dilution

Figure 11: Value Functions

output period the value functions go down sharply for a small amount of increase in borrowing and soon become flat due to default decision by the sovereign. During economic boom the value functions go down moderately and the sovereign can borrow more before going for a default decision. The difference is insignificant to observe visually though.

Figure 12 shows the default probabilities with dilution and without dilution with respect to the choice of new debt levels and the levels of output, when the sovereign issues only long-term bonds. The default probabilities are pretty similar visually in both the



(a) Default Probability with Dilution

(b) Default Probability without Dilution

Figure 12: Default Probabilities

cases. We will see the difference later when we compare the average default probabilities with and without the dilution problem.

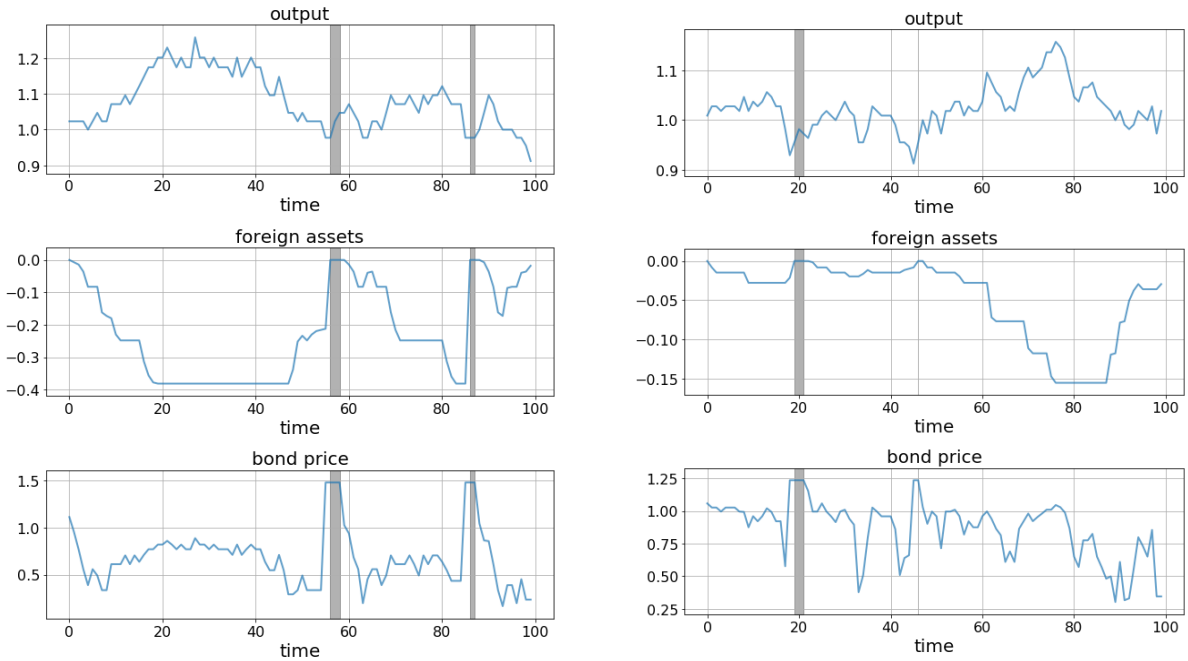
Figure 13 shows the simulation results of output, foreign assets, and bond prices with and without dilution, when the sovereign issues only long-term bonds. The results are pretty similar in both the cases. During low output period the bond prices go down and due to lower borrowing capacity borrowing also goes down. During economic boom the prices go up along with the borrowing due to higher borrowing capacity.

2 Compensation Covenant with Multiple Maturity Structure

In this section we assume that the sovereign can issue both short- and long-term bonds with a compensation covenant in the contract of long-term bonds. The compensation now depends on current period starting and next period levels of short- and long-term debts besides the stochastic income y. Others are as before.

The payment specified in the covenant is given by

$$\mathcal{C}(y, b_S, b_L, b'_S, b'_L) = \max\{q(y, b_S, (1-\lambda)b_L) - q(y, b'_S, b'_L), 0\} \tag{27}$$



(a) Simulation Results with Dilution

(b) Simulation Results without Dilution

Figure 13: Simulation Results of Output, Foreign Assets, and Bond Price

Government resource constraint is as follows:

$$c \leq \begin{cases} y - q_S(\cdot)b'_S + q_L(\cdot)[(1-\lambda)b_L - b'_L] + b_S + [\lambda + (1-\lambda)z]b_L + (1-\lambda)b_L\mathcal{C}(\cdot) & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{28}$$

where,

$$q_S(\cdot) = q_S(y, b'_S, b'_L)$$
$$q_L(\cdot) = q_L(y, b'_S, b'_L), \quad and$$
$$\mathcal{C}(\cdot) = \mathcal{C}(y, b_S, b_L, b'_S, b'_L)$$

The lifetime sovereign utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0, 0)] \tag{29}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y, b_S, b_L)$, is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'_S, b'_L)] \tag{30}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{31}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b_S, b_L)$ be the set of income levels for which the optimal decision is repayment when short- and long-term debt are b_S and b_L, such that

$$R(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \geq v^d(y)\}$$

and let $D(b_S, b_L) = \tilde{R}(b_S, b_L)$ be the set of income levels for which the optimal decision is default for the debts level of b_S, and b_L:

$$D(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q_S(y, b'_S, b'_L) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} f(y', y) dy', \tag{32}$$

$$q_L(y, b'_S, b'_L) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} [\lambda + (1 - \lambda)\{z + q_L(y', \tilde{b}'_S, \tilde{b}'_L) + \mathcal{C}(y', b'_S, b'_L, \tilde{b}'_S, \tilde{b}'_L)\}] f(y', y) dy' \tag{33}$$

where,

$$b'_S = \tilde{b}_S(y, b_S, b_L)$$
$$b'_L = \tilde{b}_L(y, b_S, b_L)$$

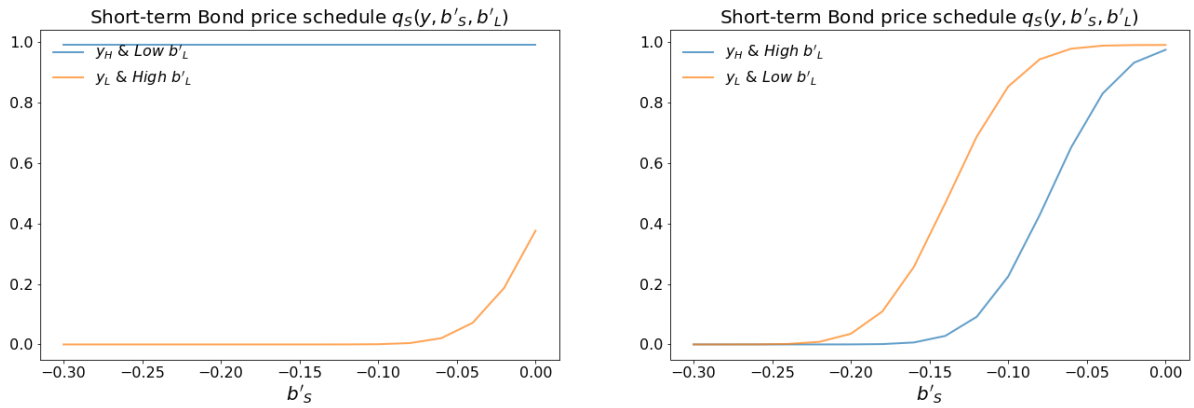
2.1 Results

This section presents results in terms of bond prices, value functions, default probabilities, and simulation of output and foreign assets with the prices when the sovereign issues

only both short- and long-term bonds with a compensation covenant as a remedy to the debt dilution problem.

Figure 14 shows the short-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term with a compensation covenant as a remedy to the debt dilution problem. The price schedule is shown as a function of the level of choice of short-term bonds, b'_S by the sovereign for the next period, given levels of output, y and the choice of long-term bonds, b'_L. The left panel shows the price schedule when output is high but the long-term bond level is low and when output is low but the level of long-term bond is high. During a recession the sovereign chooses to default after the price goes to zero after a small amount of bonds issuance. This is also because the sovereign has high level of long-term bonds already and further borrowing capacity is low. During economic boom, the price is close to one. The right panel shows the price schedule when output and the level of long-term bonds are either high or low together. This is interesting because the price is higher during a recession than during an economic boom due to a lower level of long-term bonds which frees up borrowing capacity with short-term bonds. Even in economic boom the sovereign goes to default with more than 15 percent short-term debt-to-output ratio since the borrowing capacity is low due to higher level of long-term debts.

Figure 15 shows the long-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long-term with a compensation covenant. The price schedule is shown as a function of the level of choice of long-term bonds, b'_L by the sovereign for the next period, given levels of output, y and the choice of short-term bonds, b'_S. The left panel shows the price schedule when output is high or low but the level of short-term bond is always low. The price goes down gradually after a certain level of long-term bonds during a recession which is expected because of lower borrowing capacity and more default risk. The prices are very close to one for 20 percent long-term debt-to-output ratio and go down afterwards. The right panel shows the price schedule



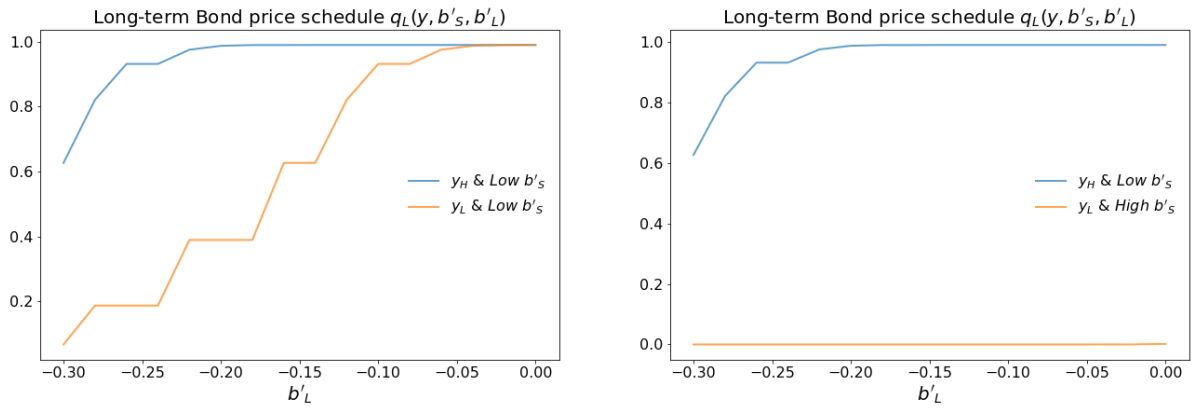
(a) (b)

Figure 14: Short-term Bond Price Schedule

when output is high but the level of short-term debt is low and when output is low but the level of short-term debt is high. During economic recessions when output is low, the sovereign does not have any borrowing capacity due to high short-term borrowing. During economic boom when output is high, the sovereign can issue significant amount of long-term bonds without a major price discount if the level of issuance of short-term bonds is low.

Figure 16 shows the value functions with respect to the debt level under two output levels- high and low, when the sovereign issues both short- and long-term bonds with a compensation covenant to mitigate the debt dilution problem. The left panel shows the value functions with respect to short-term bonds and the right panel shows the value functions with respect to long-term bonds. The value functions are pretty similar in both the cases. During low output period the value functions go down sharply for a small amount of increase in borrowing and soon become flat due to default decision by the sovereign. During economic boom the value functions go down moderately and the sovereign can borrow more. The difference is insignificant to observe visually though.

Figure 17 shows the time series simulation of output, holdings of short-term and long-term bonds by the sovereign, and their associated prices when bond issuance comes



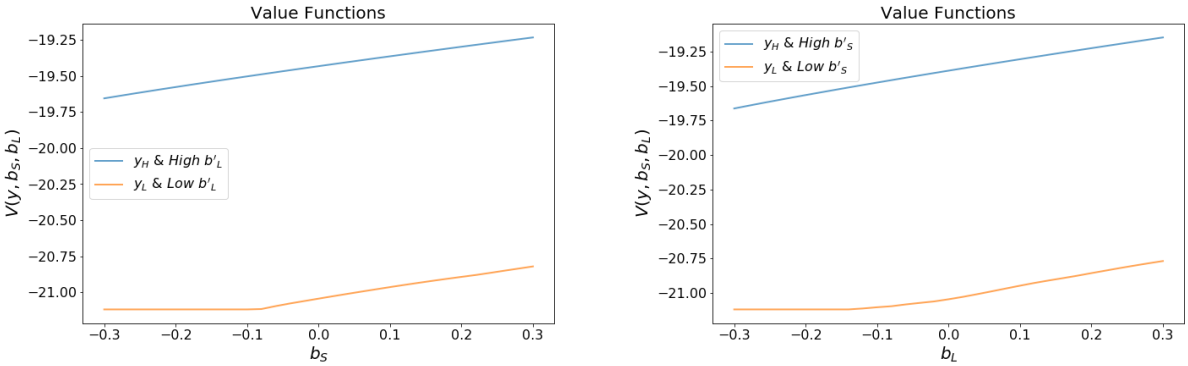
(a) (b)

Figure 15: Long-term Bond Price Schedule

with a compensation covenant. When the output is high and the economy is in boom, the sovereign faces higher borrowing capacity and therefore, holds higher short-term and long-term bonds. The prices are also high because of lower default risk during a boom. During the periods of lower output and economic recessions, the sovereign cannot borrow due to lower borrowing capacity. However, in the periods when output decreases gradually, the sovereign participates in the credit market with opposite position on short-term and long-term bonds, i.e., the sovereign buys short-term bonds while issues long-term bond and vice versa during the period of declining output.

V Comparative Analysis

Table 2 compares the models discussed in terms of the averages of default probability, short-term price, and long-term price when the value of λ is equal to 0.10. From table 2 it is evident that average default probability is the highest and therefore, average bond price is the lowest with only short-term bonds. Default probabilities go down when the sovereign has a probabilistic maturity structure for the bonds. In this type of bonds, average prices go up if the sovereign decides to include a compensation covenant in the bond contract that requires sovereign to compensate the existing bondholders before the gov-



(a) With respect to short-term bond

(b) with respect to long-term bond

Figure 16: Value Functions without Dilution

Table 2: Comparative Results ($\lambda = 0.10$)

Model	Avg. d	Avg. q_S	Avg. q_L
Only Short-term Bond	0.299	0.694	-
Only Long-term Bond	0.273	-	0.753
Only Long-term Bond & Compensation	0.266	-	0.815
Bonds with Multiple Maturity	0.193	0.799	0.832
Bonds with Multiple Maturity & Compensation	0.206	0.786	0.910

ernment can issue new bonds. The default probability further goes down if the sovereign issues both short- and long-term bonds. There are also significant increases in the bond prices as well. If the sovereign has a compensation covenant while issuing bonds with both the maturity structures then price for long-term bonds goes up while, interestingly enough, short-term bonds experience a decline in the price. This is largely because short-term bondholders do not get any compensation if the government decides to issue new bonds. Short-term bondholders either get the face value of the bond back in case of repayment or get nothing in case of default by the sovereign at the maturity of the bond. Moreover, default probability goes up with the inclusion of the compensation covenant in the bond contract and short-term bond price is largely related to the default probability of the sovereign.

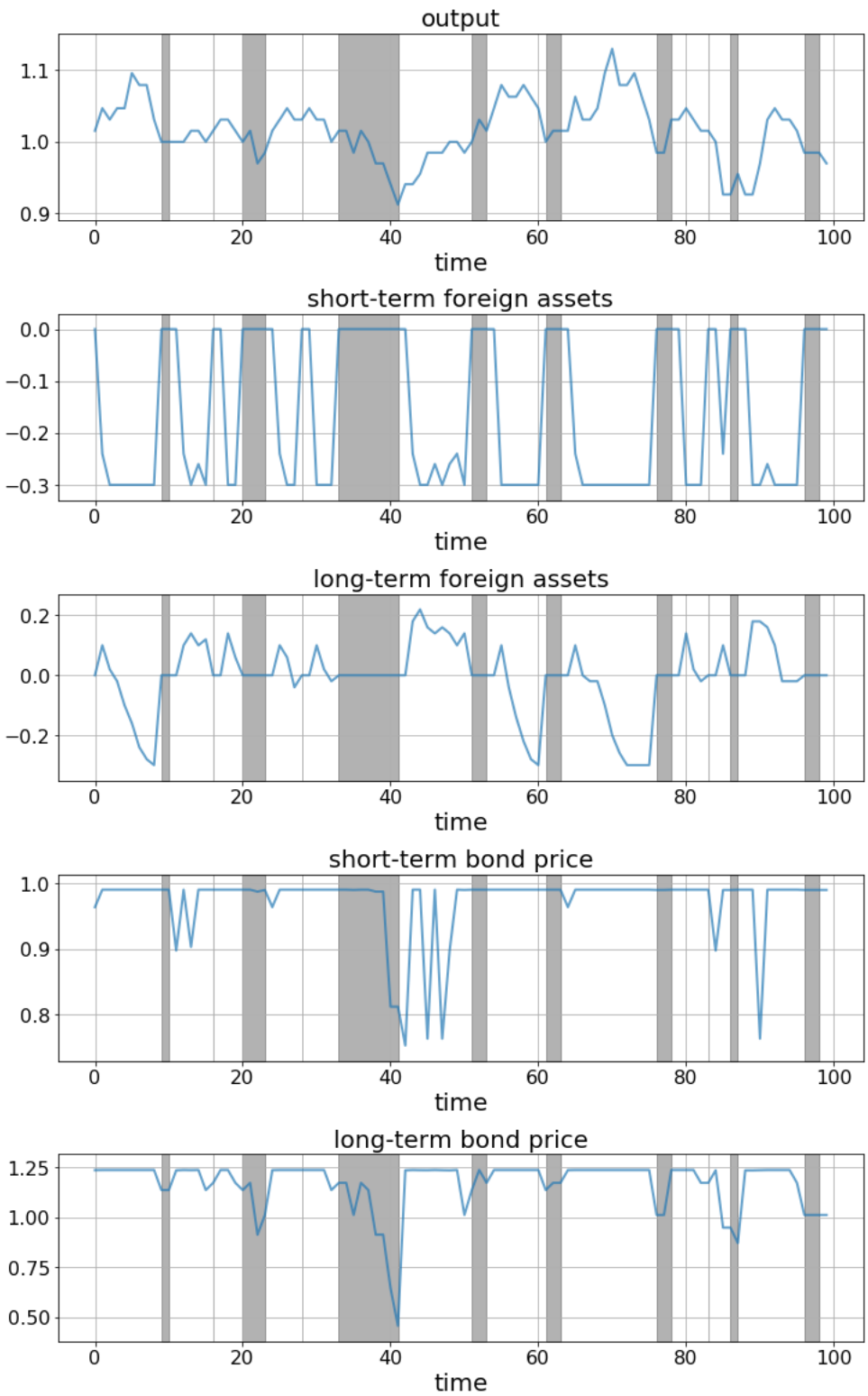


Figure 17: Simulation Results of Output, Foreign Assets, and Bond Prices

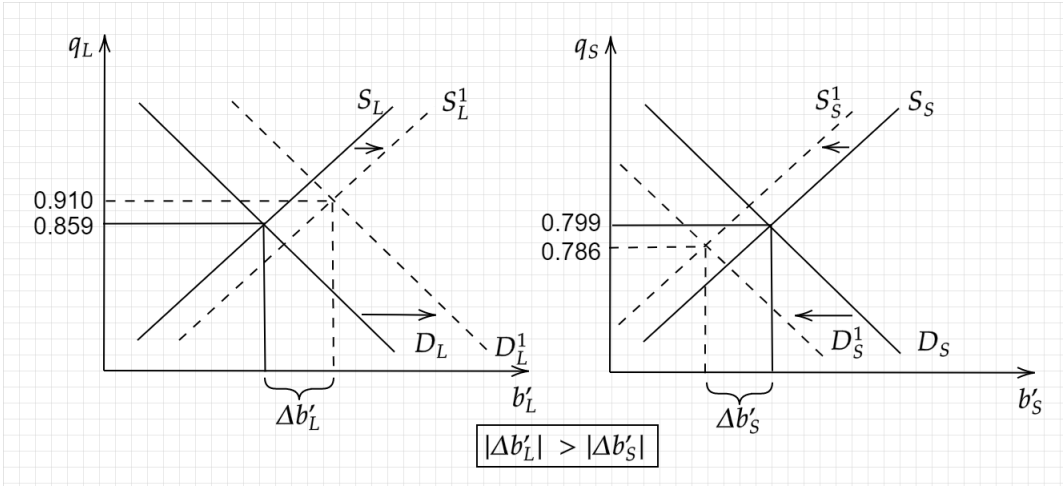


Figure 18: Effects of Compensation Covenant on Prices

Figure 18 shows the effect of compensation covenant on prices of long-term bonds on the left panel and short-term bonds on the right panel. Due to the compensation covenant the lenders would like to increase the supply of long-term bonds while decrease the supply of short-term bonds. From the result of long-term bond price with compensation covenant we can infer that the demand for long-term bonds also goes up. This makes the overall price increase for long-term bonds. The demand for short-term bonds goes down resulting in the lower price of short-term bonds. Finally, since the default probability depends on the levels of bonds, the increase in long-term bonds is higher than the decrease in short-term bonds which is consistent to the higher default probability with compensation covenant when $\lambda = 0.10$.

According to OECD Sovereign Borrowing Outlook 2019, the average sovereign debt maturity is 8 years. Therefore, we need to increase the maturity of long-term bonds in out model to compare the results with those of lower maturity debts $(\lambda = 0.10)$. Table 3 compares the models discussed in terms of the averages of default probability, short-term price, and long-term price when the value of λ is equal to 0.03. From table 3 it is evident that default probabilities go down when the sovereign has a probabilistic maturity struc-

Table 3: Comparative Results ($\lambda = 0.03$)

Model	Avg. d	Avg. q_S	Avg. q_L
Only Short-term Bond	0.299	0.694	-
Only Long-term Bond	0.243	-	0.919
Only Long-term Bond & Compensation	0.245	-	1.067
Bonds with Multiple Maturity	0.210	0.782	0.982
Bonds with Multiple Maturity & Compensation	0.173	0.819	1.233

ture for the bonds. Comparing with table 2, it is also evident that the default probability goes down to 24.3 percent from 26.8 percent with an increase in the average maturity of long-term bonds from 10 periods to 33 periods. Long-term prices also improve in case of both with and without compensation covenant with an increase in the average maturity of bonds. In fact, when the sovereign issues both the short- and long-term bonds, the average price of long-term bonds goes over one, allowing the sovereign to issue premium bonds. Two other interesting findings are the improvement of both the average default probability and the average short-term bond price after the inclusion of the compensation covenant when the average maturity of long-term bonds is high ($\lambda = 0.03$). When the average maturity of long-term bonds is low ($\lambda = 0.10$), the average default probability goes up from 19.3 percent to 20.6 percent after inclusion of the compensation covenant. However, with a sufficiently large maturity ($\lambda = 0.03$), the default probability goes down to 17.3 percent from 21 percent after including the compensation covenant when the sovereign issues both short- and long-term bonds. With the increase of the maturity of long-term bonds, the average price of short-term bonds goes down to 0.782 from 0.799 without the compensation covenant in the model of bonds with both maturity structure. However, with inclusion of the compensation covenant, the average short-term price, in fact, goes up from 0.786 to 0.819. This confirms that the compensation covenant solves the debt dilution problem in a larger scale if the maturity of long-term bonds is sufficiently high.

VI Conclusion

Maturity structure plays an important role in sovereign debt. Moreover, a sovereign's lack of commitment to the action of future governments regarding future borrowing and default give birth to the debt-dilution problem and eventually long-term bonds become costly. In the calibrated model we show that short-term bonds do not face the debt dilution problem since the sovereign issues such bonds only once in a period. However, rollover crisis is higher in case of short-term bonds. We also show that default frequency and debt-dilution problem reduce significantly when the sovereign issues both short- and long-term bonds.

Debt dilution problem has several remedies, such as, compensation package for the existing bondholders (Hatchondo et al., 2016) or a seniority arrangement for the bondholders (Chatterjee and Eyigungor, 2015). However, none of these solutions are comprehensive to protect the bondholders completely from higher default probability of the sovereign due to raising the debt level. With the compensation covenant in multiple maturity structure in debt issuance, the existing creditors will be compensated before any new issuance of bonds by the sovereign. This mechanism has twofold benefits. First, it provides the existing creditors a compensation during further borrowing by the sovereign. Second, it reduces the default risk when the maturity of long-term bonds is sufficiently high.

Future direction of the research could be to improve the debt covenants by specifying the exact compensation packages and bargaining protocols for the bondholders. Endogenizing the maturity of long-term bonds would be another possible interesting work. Moreover, it would be worthwhile to see these cases under both Eaton and Gersovitz (1981) baseline framework and Cole and Kehoe (2000) self-fulfilling rollover crises. Since the financial sector is also involved with and impacted by sovereign debt defualts (Bolton, 2016; Gennaioli et al., 2014), it would also be interesting to revisit the linkage with and without the existence of the compensation covenant for the bondholders.

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APPENDIX

A The Model with Debt Threshold

A debt threshold covenant requires the sovereign to compensate the holders of each previously issued long-term bonds the difference between the new debt level and a debt threshold before issuing any new debt. This covenant penalizes the sovereign for new borrowing and thus induces a lower debt level. This covenant eliminates the debt dilution problem because the creditors are assured at the beginning of the contract that before the sovereign can borrow from the international credit market, they will get their compensation equal to the new debt level after new borrowings and a certain debt threshold, $\bar{b}$. Thus, the the new compensation covenant is given by

$$\mathcal{C}_b(b') = \psi \max\{\bar{b} - b', 0\} \tag{34}$$

Except this debt threshold compensation, everything is the same as in the section where there is a discussion of the model without debt dilution.

B The Model with Price Threshold

In price threshold covenant the creditors do not need to depend on the counterfactual price of bonds that would be observed without any new debt issuance in a period. The covenant, rather, specifies a compensation for the creditors for a decline in the price of bonds from a certain threshold. This price threshold requires the sovereign to pay the holders of previously issued long-term bonds the difference between a constant threshold price, $\bar{q}$ and the market price of long-term bonds after new issuance, $q(y, b')$. Thus, the promised compensation in the price threshold covenants of long-term bonds is

$$\mathcal{C}_q(y, b, b') = \begin{cases} \max\{\bar{q} - q(y, b'), 0\} & \text{if } b' < (1-\lambda)b \\ 0 & \text{Otherwise} \end{cases} \tag{35}$$

C The Model with Income Contingent Debt

The government's budget constraint reads as

$$c \leq \begin{cases} y + [\lambda + (1-\lambda)z]b + E_{\{y'|y\}}\{q(y, b'(y'))[(1-\lambda)b - b'(y')]\} & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{36}$$

where,

$$b'(y') \leq \bar{b}'(y') = \sup\{\tilde{b}' : V^c(y', \tilde{b}') \geq V^d(y') \ \ \forall y\}$$

The lifetime sovereign utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0)] \tag{37}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y, b)$, is given by

$$V^c(y, b) = \max_{\{b'(y'), c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'(y'))] \tag{38}$$

Finally,

$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \tag{39}$$

D The Sovereign Debt with Settlement

Government resource constraint will still hold.

$$c \leq \begin{cases} y - q_S(y, b'_S, b'_L)b'_S + q_L(y, b'_S, b'_L)[(1-\lambda)b_L - b'_L] + b_S + [\lambda + (1-\lambda)z]b_L & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{40}$$

The sovereign's lifetime utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', G_S(y'), G_L(y')] \tag{41}$$

The payoff of the sovereign from repaying the debt, $V^c(y, b_S, b_L)$, and finally, $V^o(y, b_S, b_L)$ will still be the same. The payoff of the sovereign from repaying the debt, denoted $V^c(y, b_S, b_L)$, is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'_S, b'_L)] \tag{42}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{43}$$

With some abuse of notation, the default decision rule of the sovereign, $d(y, b_S, b_L)$ is implicitly determined by this equation, where $d = 1$ means default is the optimal decision and 0 otherwise.

Under competition, the pricing equations of a unit bond are as follows:

$$q_S(y, b'_S, b'_L) = \frac{1}{1+r^*} E_{\{y'|y\}} \left[[1 - d(y', b'_S, b'_L)] + d(y', b'_S, b'_L) \frac{P_S(y', G_S(y'), G_L(y'))}{-b'_S} \right] \tag{44}$$

$$q_L(y, b'_S, b'_L) = \frac{1}{1+r^*} E_{\{y'|y\}} \Bigg[[1 - d(y', b'_S, b'_L)][\lambda + (1-\lambda)\{z + q_L(y', \tilde{b'}_S, \tilde{b'}_L)\}] \tag{45}$$
$$+ d(y', b'_S, b'_L) \frac{P_L(y', G_S(y'), G_L(y'))}{-b'_L} \Bigg]$$

where, as in previous section,

$$b'_S = \tilde{b}_S(y, b_S, b_L)$$
$$b'_L = \tilde{b}_L(y, b_S, b_L)$$

and, $P_S(y, G_S(y), G_L(y))$ and $P_L(y, G_S(y), G_L(y))$ are the aggregate values of expected repayments on the defaulted short- and long-term debts conditional on output being y and, settlement values are $G_S(y)$ and $G_L(y)$, respectively for short- and long-term debts. Since the aggregate values are equally distributed across all bonds in each maturity class, in expectation, each short-term bond will receive $E_{\{y'|y\}} P_S(y', G_S(y'), G_L(y'))/b'_S$ and each long-term bond will receive $E_{\{y'|y\}} P_L(y', G_S(y'), G_L(y'))/b'_L$.

$P_S(y, G_S(y), G_L(y))$ and $P_L(y, G_S(y), G_L(y))$ are given by:

$$P_S(y, G_S(y), G_L(y)) = \frac{1}{1+r^*} E_{\{y'|y\}} \Big[(1-\xi) P_S(y', G_S(y'), G_L(y')) + \xi q_S(y', G_S(y'), G_L(y'))(-G_S(y')) \Big] \tag{46}$$

$$P_L(y, G_S(y), G_L(y)) = \frac{1}{1+r^*} E_{\{y'|y\}} \Big[(1-\xi) P_L(y', G_S(y'), G_L(y')) + \xi q_L(y', G_S(y'), G_L(y'))(-G_L(y')) \Big] \tag{47}$$

In case, settlement is reached between the sovereign and the creditors next period, then the creditors as groups receive settlement with aggregate values of $q_S(y', G_S(y'), G_L(y'))(-G_S(y'))$ for short-term bonds and $q_L(y', G_S(y'), G_L(y'))(-G_L(y'))$ for long-term bonds.