Effects of dispersion on the inference of metal texture from $S_0$ plate mode measurements. Part I. Evaluation of dispersion correction methods

Yan Li and R. Bruce Thompson

Ames Laboratory, Iowa State University, Ames, Iowa 50011

(Received 3 June 1990; revised 17 May 1991; accepted 1 November 1991)

Ultrasonic $S_0$ waves (fundamental symmetric Lamb modes) are being considered in several laboratories for the nondestructive characterization of the texture (preferred grain orientation) and formability of metal sheets and plates. In a typical experimental setup, the velocities of the $S_0$ waves are measured as a function of wave propagation angle with respect to the rolling direction of the plate. However, the $S_0$ waves are known to be dispersive, and that dispersion must be considered in order to isolate the small, texture-induced shifts in the $S_0$ wave velocity. Currently, there are two approximate dispersion correction methods, one proposed by Thompson et al. [Met. Trans. A 20, 2431 (1989)] and the other introduced by Hirao and Fukuoka [J. Acoust. Soc. Am. 85, 2311 (1989)]. In this paper, these two methods will be evaluated using an exact theory for wave propagation in orthotropic plates. Through the evaluation, the limits of the current texture measurement techniques are established. It is found that when plate thickness to wavelength ratio is less than 0.15, both Thompson’s and Hirao’s methods work satisfactorily. When the thickness to wavelength ratio exceeds 0.3, neither Thompson’s nor Hirao’s dispersion correction method provides adequate corrections for the current texture measurement techniques. Within the range of 0.15–0.3, Thompson’s method is recommended for weakly anisotropic sheets and plates and Hirao’s method may be more appropriate for some strongly anisotropic cases.

PACS numbers: 43.20.Ks, 43.35.Cg, 43.40.Le, 43.20.Bi

INTRODUCTION

Texture is the nonrandom orientation of crystallites in a polycrystalline aggregate, often induced by manufacturing processes such as rolling and drawing. The texture is characterized by a set of dimensionless parameters $W_{lmn}$, called orientation distribution coefficients (ODCs). These coefficients, whose detailed definitions can be found in Refs. 1 and 2, specify the relative weights of generalized spherical harmonics in a series representation of the orientation distribution function (the probability that a grain has a particular orientation). Knowledge of texture information is particularly important in making formability predictions, i.e., predictions of the ability of a sheet to be converted to a complex shape by a deep drawing operation. For materials made of cubic crystallites, such as Al (fcc), Cu (fcc), and Fe (bcc), the most important ODCs in formability analysis are $W_{400}$, $W_{420}$, and $W_{440}$. Physically, $W_{420}$ and $W_{440}$ relate to the tendency of a metal sheet to form two and four ears, respectively, upon deep drawing. Here $W_{400}$ relates to overall capacity to withstand deep drawing. Typical values of these ODCs are on the order of $10^{-3}$.

Traditionally, these ODCs are obtained through x-ray or neutron diffraction techniques. In recent years, however, it has been shown that ultrasonic waves can be utilized to provide a nondestructive estimation of the texture of cubic polycrystals. The texture generally induces a weak elastic anisotropy that can be sensed by measurements of the velocity of waves propagating in different directions. When the sample is in the form of a sheet or plate, the observations can be easily interpreted in terms of the theory of guided modes. In contrast to the neutron or x-ray diffraction techniques, the ultrasonic techniques cannot obtain sufficient information to fully characterize the texture. However, they do provide information about $W_{400}$, $W_{420}$, and $W_{440}$, the ODCs most important in formability analysis.

Because of the richness of information that they can obtain, the x-ray and neutron diffraction techniques are generally preferred for laboratory studies of such phenomena as the evolution of texture during deformation of a material. However, there is also a need for simple nondestructive measurement techniques for field applications. The functions needed include both obtaining feedback information for process control by the primary manufacturers of the sheet and inspecting incoming material for quality control by the component fabricator. Here the neutron techniques are impractical because of the absence of portable sources of sufficient intensity. The x-ray techniques are being employed. However, they are rather expensive and introduce safety questions associated with the presence of radiation. Hence ultrasonic techniques have received considerable attention as an alternative technology for field monitoring of the texture in metal sheet in the last five years.

The ultrasonic techniques have utilized electromagnetic-acoustic transducers (EMATs) to excite Lamb waves propagating in the plane of the sheet with no need for a couplant. Since only three constants ($W_{400}$, $W_{420}$, and $W_{440}$) define the elastic anisotropy in the most commonly used approximations, only three velocities need to be measured,
typically those of $S_0$ modes propagating at 0°, 45°, and 90° with respect to the metal sheet. To minimize both the effects of dispersion on the velocity and the sensitivity of the measurement to the distance between the EMATs and the plate, the wavelength is generally chosen to be somewhat greater than the plate thickness. Nevertheless, because the texture-induced velocity shifts are small, the experimental data must be corrected for the dispersion if accurate predictions of the ODCs are to be made. The theory for wave propagation in anisotropic media is well known. However, since the anisotropy is unknown at the time of measurement, use of the full theory in interpreting data is cumbersome. Because of this fact, and the smallness of the dispersion at long wavelengths, the correction is generally made on the basis of an isotropic theory. The purpose of this paper is to examine the adequacy of the currently used approximate correction techniques with the exact theory being utilized to generate simulated data.

There are two aspects of the influence of dispersion of the $S_0$ waves on the velocity measurements. First, both the phase velocity and the group velocity are frequency dependent. Two dispersion correction methods have been proposed by Thompson et al. and Hirao and Fukuoka to remove the frequency dependence. In this paper, these two dispersion correction methods will be evaluated assuming perfect measurements of phase velocity. Another aspect of dispersion is the pulse distortion phenomenon; i.e., a pulse of a dispersive wave changes its shape and spreads out as it propagates. In Part II of this paper we discuss the influence of the pulse distortion on the accuracy of phase velocity measurements.

Several years ago, Thompson et al. and Lee et al. developed a theory that relates the aforementioned ODCs of textured sheets to the $S_0$ wave speeds in three different directions (0°, 45°, and 90°) with respect to the rolling direction. This theory assumes the ratio of sheet thickness to wavelength to be small. In this limit, the velocity of the $S_0$ wave asymptotically approaches a constant value, $V_{\text{lim}}$, and dispersion can be ignored. Upon comparison of experimental predictions of the ODCs based on this theory with the results from independent x-ray or neutron diffraction techniques, satisfactory agreement was found for $W_{420}$ and $W_{440}$, but not for $W_{400}$. In those measurements, the ratio of thickness to wavelength was about 0.1. Recently, Thompson et al. modified the theory and included dispersion correction in the calculation of the ODCs. In the modified theory, the dispersion correction is made on the basis of dispersion curves of the $S_0$ waves in the corresponding isotropic materials. It is assumed that the ratio $V_p/V_{\text{lim}}$ is not altered significantly by the presence of texture (weak anisotropy), where $V_p$ is the ultrasonic phase velocity measured at a specific frequency and $V_{\text{lim}}$ is the long wavelength limit of the $S_0$ wave phase velocity. Hirao and Fukuoka have proposed another method, which takes dispersion effects into account by making a Taylor series expansion of the dispersion curves of the $S_0$ mode propagating in weakly orthotropic media. In the expansion, only the first-order dispersion effects are retained.

The basis of our evaluation of the approximate dispersion correction methods is the exact theoretical equations recently developed for wave propagation in plates with orthotropic symmetry. The equations that express the dependence of wave propagation velocity upon texture parameters are very complex and a simple technique has not been developed to compute the ODCs from given wave velocities on the basis of these exact equations. These equations, however, can serve as an exact reference solution. In this paper, we will use the exact solution for the $S_0$ wave propagation in orthotropic plates to simulate experimental data and then use those data to test the accuracy of the two approximate dispersion correction methods.

One semantic difficulty should be resolved before proceeding. In the metallurgical community, the terms "sheet" and "plate" have connotations of different thicknesses, with the former appropriate to thicknesses on the order of a millimeter to which the ultrasonic approach has been most extensively applied. In the ultrasonic literature, the term "plate" is generally used to describe a solid medium having parallel surfaces, independent of thickness. In the remainder of this paper, the latter convention will be employed. However, in considering metallurgical applications, the "sheets" should be understood to be included.

I. GENERAL THEORY FOR WAVE PROPAGATION IN ORTHOTROPIC FREE PLATES

The authors of the present paper and Nayfeh and Chimenti have recently developed and reported the analytical dispersion equations for wave propagation in orthotropic or monoclinic free plates. For detailed derivation and discussions, the readers are encouraged to refer to these references. Here only the equations necessary to make this paper self-contained will be presented.

Consider the coordinate system shown in Fig. 1. Let us first make the following dimensionless definitions:

$$ K_1 = K \cos^2 \alpha, \quad K_2 = K \sin^2 \alpha, \quad K = \left[ \left( \frac{b}{r} \right)^2 \right]^\frac{1}{2}, $$

$$ W = \frac{\rho \alpha^2}{C_{66}} \left( \frac{b}{r} \right)^2, \quad R_i = \left( \frac{b}{r} r_i \right)^2 (i = 1, 2, 3), $$

where $b$ is the plate thickness, $\rho$ is the density, $C_q$ are the elastic constants of plate material, $\alpha$ is the wave propagation angle with respect to the rolling direction, $k$ is the wave vector in the propagation direction, $\omega$ is the angular frequency.

![FIG. 1. Definition of coordinates.](image-url)
of the propagating wave, and \( r_i \) are the eigenvalues of the associated Christoffel equations for plane wave propagation.

For a general elastic, homogeneous orthotropic free plate whose rolling, transverse, and normal directions coincide with the \( X_1 \), \( X_2 \), and \( X_3 \) axes, as shown in Fig. 1, the dispersion equation for symmetric modes propagating at 0° is

\[
Q_1 \{ \tan \left( \frac{\pi}{2} \sqrt{R_2} \right) \}^{-1} - Q_2 \{ \tan \left( \frac{\pi}{2} \sqrt{R_3} \right) \}^{-1} = 0, \tag{1}
\]

where \( Q_1 = Q(R_2,R_1) \), \( Q_2 = Q(R_3,R_2) \); and

\[
Q(X,Y) = \sqrt{Y} \left( C_{11}X - C_{11}K + C_{66}W \right) \left( C_{33}C_{55}Y + (C_{11}C_{33} - C_{13}C_{55} - C_{13})K - C_{33}C_{66}W \right),
\]

and \( R_2, R_3 \) are roots of the following equation for \( K_3 \):

\[
(C_{11}K + C_{55}K_3 - C_{66}W)(C_{55}K + C_{33}K_3 - C_{66}W) - (C_{13} + C_{55})^2K_3K = 0.
\]

The corresponding dispersion equation for symmetric modes propagating at 45° is (generalized Rayleigh–Lamb wave equation in orthotropic media)

\[
P_1 \left[ \tan \left( \frac{\pi}{2} \sqrt{R_1} \right) \right]^{-1} + P_2 \times \left[ \tan \left( \frac{\pi}{2} \sqrt{R_2} \right) \right]^{-1} + P_3 \left[ \tan \left( \frac{\pi}{2} \sqrt{R_3} \right) \right]^{-1} = 0, \tag{2}
\]

where \( P_1 = P(R_1,R_2,R_3) \), \( P_2 = P(R_2,R_3,R_1) \), \( P_3 = P(R_3,R_1,R_2) \).

For an orthotropic material, there are, in general, nine independent elastic constants \( C_{ij} \). When the plate is made of cubic crystallites, these elastic constants are not all independent. They are related to the elastic constants of single crystallites and texture parameters. The relations, published by Hirao et al., are as follows:

\[
\begin{align*}
C_{11} &= \lambda + 2\mu + \left( \frac{12\nu^2\pi^2}{35} \right) \\
C_{22} &= \lambda + 2\mu + \left( \frac{12\nu^2\pi^2}{35} \right) \\
C_{33} &= \lambda + 2\mu + \left( \frac{32\nu^2\pi^2/35}{W_{400}} \right) \\
C_{44} &= \mu - \left( \frac{16\nu^2\pi^2/35}{W_{400} + \sqrt{5/2}W_{420}} \right) \\
C_{55} &= \mu - \left( \frac{16\nu^2\pi^2/35}{W_{400} - \sqrt{5/2}W_{420}} \right) \\
C_{66} &= \mu + \left( \frac{4\nu^2\pi^2/35}{W_{400} - \sqrt{5/2}W_{440}} \right) \\
C_{23} &= \lambda - \left( \frac{16\nu^2\pi^2/35}{W_{400} + \sqrt{5/2}W_{420}} \right) \\
C_{12} &= \lambda - \left( \frac{16\nu^2\pi^2/35}{W_{400} - \sqrt{5/2}W_{420}} \right) \\
\end{align*}
\]

where \( \lambda \) and \( \mu \) are Lamé constants for the corresponding isotropic (texture-free) material and \( \nu \) is a measure of the elastic anisotropy of the individual crystallites. The isotropic Lamé constants and the anisotropy constant can be obtained from single crystal elastic constants \( c_0 \) via different averaging methods. Voigt, Hill, and Reuss averaging methods are commonly used in texture study, owning to their simplicity. The Lamé constants for these averaging methods are given in Ref. 8 as

\[
(\lambda + 2\mu)_V = c_{11} - 2c_{13}/5, \quad (\lambda + 2\mu)_H = [ (\lambda + 2\mu)_V + (\lambda + 2\mu)_R ]/2, \tag{4}
\]

and \( s_{ij} \) are elastic compliances for single crystallites.

From Eqs. (1)–(4), with a given set of ODCs, solutions for frequency \( \omega \) can be computed for each wave number \( k \). Figure 2 shows an isotropic dispersion curve for the \( S_0 \) wave in an aluminum plate. References 23 and 24 include some examples of similar dispersion curve plots with wider ranges of \( k \) and \( \omega \) for different orthotropic materials. The phase velocities, defined as \( \omega/k \), can then be calculated for different frequencies easily.
waves ($SV$) only; they are decoupled from the horizontally polarized shear partial waves ($SH$). On the other hand, at $45^\circ$ angle, all $L$, $SV$, and $SH$ partial waves are coupled together to form the $S_0$ waves. Because of this, the dispersion equation for the $S_0$ wave propagating at the $45^\circ$ angle is significantly more complicated, as can be seen in Eq. (2).

Although the dispersion relations given by Eqs. (1) and (2) can be used to compute ultrasonic wave speeds from given texture parameters, the inverse problem cannot be solved analytically. Therefore, the community has developed approximate procedures to obtain texture parameters from experimental data. In the following section, we will use the exact dispersion relations to evaluate the accuracy of the two dispersion correction methods currently known to the authors.

II. APPROXIMATE THEORIES FOR DISPERSION CORRECTIONS

Thompson et al. and Lee et al. applied the theory of wave propagation to texture characterization of cubic polycrystalline aggregates in plates with infinitesimal thickness. Under that assumption, the propagating wave is not dispersive and the relation between its speed and the elastic constants of the plate is (after dropping higher-order texture and the stress related terms):

$$\rho V^2_{S_0}(\alpha) = \tilde{C}_L + \frac{2}{3} \alpha \tilde{C}_L \cos 2\alpha - \frac{2}{3} \tilde{C}_T (1 - \cos 4\alpha),$$  

where

$$\tilde{C}_L = \frac{C_{11} + C_{22}}{2} - \frac{C_{12} + C_{21}}{2C_{33}}, \quad \tilde{C}_T = C_{66},$$

$$\tilde{\alpha} = \frac{\{(C_{11} - C_{22}) - [(C_{13} - C_{23})/C_{33}]\}}{\tilde{C}_L},$$

$$\tilde{\beta} = \frac{\{(C_{11} + C_{22})/2 - C_{12} - (C_{13} - C_{23})^2/2C_{33}\}/2 - C_{66}}{\tilde{C}_T}.$$

To first order in anisotropy, the velocity is then given by

$$V_{S_0}(\alpha) = \left(\frac{\tilde{C}_L}{\rho}\right)^{1/2} \times \left[1 + \frac{\tilde{\alpha}}{4} \cos 2\alpha - \left(\frac{\tilde{\beta} \tilde{C}_T}{4 \tilde{C}_L}\right) (1 - \cos 4\alpha)\right].$$  

(6)

To express the velocity in terms of ODCs [using Eqs. (3) and (4)], after certain approximations involving moving the $W_{400}$ terms in $C_{33}$ to numerators by means of first-order Taylor approximation, Eq. (5) can be reduced to

$$\rho V^2(\alpha) = \left(1 - \frac{P^2}{L^2}\right) L + \frac{4\pi^2 c}{35} \times \left[\sqrt{2} \left(3 + 8 \frac{P}{L} + 8 \frac{P^2}{L^2}\right) W_{400}\right.\]

$$- 4\sqrt{5} \left(1 + 2 \frac{P}{L}\right) W_{420} \cos 2\alpha\]

$$+ 2\sqrt{35} W_{440} \cos 4\alpha\right],$$

(7)

where $L = \lambda + 2\mu$, $P = \lambda$, and $c$ is an anisotropy constant also defined in Eqs. (4). Similar approximation to Eq. (6) leads to

$$V(\alpha) = \sqrt{\left(1 - \frac{P^2}{L^2}\right) \frac{L}{\rho}} + \frac{2\pi^2 c}{35} \sqrt{\frac{\rho L}{(L^2 - P^2)}} \left[\sqrt{2} \left(3 + 8 \frac{P}{L} + 8 \frac{P^2}{L^2}\right) W_{400}\right.\]

$$\times W_{420} \cos 2\alpha + 2\sqrt{35} W_{440} \cos 4\alpha\right].$$

(8)

From Eq. (7) or (8), linear combinations of velocities measured for $\alpha = 0^\circ, 45^\circ$, and $90^\circ$ can then be taken to obtain the values of $W_{400}$, $W_{420}$, and $W_{440}$.

Since no plate is infinitesimally thick, wave propagation in a plate is always dispersive. Although this effect is small for thin plates, so is the effect of texture. Thus the experimental data must be corrected for dispersion if quantitative values of the texture parameters are to be obtained.

The $S_0$ waves used in texture studies are generally weakly dispersive, with a typical measurement frequency of 500 KHz and plate thickness of a few millimeters. In order to reduce the error introduced by the dispersion, Thompson et
al. suggested a simple dispersion correction approach. Starting from the measured phase velocity \( V_p \), the data were corrected to estimate the long wavelength limit of that velocity, \( V_{\text{lim}} \), by assuming the ratio \( V_p / V_{\text{lim}} \) to be the same in the weakly anisotropic plate as it would be in an isotropic plate of the same thickness. The corrected velocities (long wavelength limits) were then used in Eq. (7) or (8). In our experimental work, \(^9\) the dispersion correction normally amounts to less than 10% of the measured velocities, and the dispersion correction method described above was intuitively believed to be reasonable. This correction improved the accuracy of estimates of the ODCs, particularly on \( W_{400} \), as expected. However, no rigorous evaluation of the range of accuracy of this approach was made.

Hirao and Fukuoka \(^{20}\) have proposed another dispersion correction method. They have developed a dispersion equation for wave propagation in orthotropic plates under a perturbation frame that neglects the involvement of \( SH \) partial waves for wave propagation in nonsymmetry directions. The dispersion equation has a form that resembles Eq. (1) and reduces to it when the wave propagation direction is in a symmetry direction. To develop an explicit relation between ODCs and wave speeds in different propagation directions, they then made a Taylor series expansion at zero frequency and included one higher-order term to approximate dispersion effects at low frequencies. After dropping higher-order terms in \( W_{lmn} \), the equation for the square of the velocity is

\[
V_s^2(\alpha) = V_o^2[(1 - \Delta) + (2c/\rho)V_o] [s_0 + d_0 \Delta] W_{400} + (s_2 + d_2 \Delta) W_{420} \cos(2\alpha)
\]

(9)

(This equation was not published in Ref. 20, but is an intermediate step.) After a further approximation, the final equation is

\[
V_s(\alpha) = V_o \sqrt{(1 - \Delta) + (c/\rho)V_o} [(s_0 + d_0 \Delta) W_{400} + (s_2 + d_2 \Delta) W_{420} \cos(2\alpha)] + s_4 W_{440} \cos(4\alpha)
\]

(10)

where

\[
s_0 = (2\nu^22\pi/35) [3 + 16\lambda(\lambda + \mu)/(\lambda + 2\mu)^2],
\]

\[
s_2 = -(8\nu^25\pi^2/35) (3\lambda + 2\mu)/\lambda + 2\mu),
\]

\[
s_4 = 4\nu^2/35,
\]

\[
d_0 = (16\nu^22\pi^2V_o^2) (3\lambda + 2\mu)/[35\lambda(\lambda + 2\mu)],
\]

\[
d_2 = -16 \nu^2 2\pi^2 V_o^2 (3\lambda + 2\mu)/[35\lambda(\lambda + 2\mu)],
\]

with

\[
V_o = \sqrt{4\mu(\lambda + \mu)/\rho(\lambda + 2\mu)}
\]

being the isotropic velocity at \( kb = 0 \) and \( \Delta = \lambda^2/\lambda + 2\mu \)^2/(kb/2)^2/3 describing the dispersion. For either of these expressions, solution for the \( W_{lmn} \) in terms of the velocities at 0°, 45°, and 90° is straightforward.

Since Eq. (9) was derived via a Taylor expansion in wave vector, it is expected to be valid (or provide good approximation) for small \( kb \). This is confirmed by comparing Eqs. (7) and (9), which reveals that they are identical for plates of zero thickness. The approximation made in the derivation of Eq. (10) involved a second Taylor series in the small variables \( W_{lmn} \), to eliminate the squares in velocities, an approximation similar to that made in going from Eq. (5) to Eq. (6) or from Eq. (7) to Eq. (8). Thus, Eq. (10) reduces to Eq. (8), but not Eq. (9), for zero thickness. However, Eqs. (7)–(10) are identical in the absence of anisotropy (texture free) in the long wavelength limit. The effects of this further approximation will be discussed in the next section. For the convenience of discussion later on, we shall call the Thompson’s dispersion correction method applied to Eqs. (7) and (8) and the Hirao’s dispersion correction method using Eqs. (9) and (10) as Thompson’s-A, Thompson’s-B, Hirao’s-A, and Hirao’s-B schemes, respectively.

In summary, Thompson’s schemes neglect the small deviation of the \( S_o \) dispersion curves of textured plates from that of the isotropic ones and Hirao’s schemes use a parabolic approximation to the anisotropic dispersion curves to replace the exact ones that are not suitable for the estimation of texture parameters.

### III. EVALUATION OF DISPERSION CORRECTION SCHEMES

To evaluate the performance of the dispersion correction schemes mentioned above, we calculated the \( S_o \) wave speeds for four selected groups of ODCs as a function of plate thickness to wavelength ratio using the exact dispersion relations presented in Sec. I. These speeds were then used as input to the dispersion correction schemes to obtain estimates of the ODCs. The first step can be considered as a forward problem while the second step is an inverse problem. The initial values for the ODCs are listed in Table I. The values of the ODCs chosen here for the simulations are realistic representations of values encountered in textured plates. Groups I and II correspond to relatively strong textures and groups III and IV correspond to relatively weak textures. Simulations have been run for the three commonly used cubic materials: Al, Cu, and Fe. The densities and the single crystal elastic constants for the three materials are given in Table II. For all the simulation runs, the Hill averaging method was employed because it is known to be more accurate than either the Voigt or Reuss averaging method, which, respectively, provide upper or lower bounds to the isotropic moduli. The isotropic and anisotropic elastic constants and Poisson ratios for the polycrystalline materials are listed in Table III for the Hill averaging method. For the purpose of this paper, we neglect any errors in the Hill ap-

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**Table I. Initial ODCs for computer simulations.**

<table>
<thead>
<tr>
<th>Group</th>
<th>( W_{400} )</th>
<th>( W_{420} )</th>
<th>( W_{440} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.01</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>II</td>
<td>0.005</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>III</td>
<td>0.0075</td>
<td>0.0075</td>
<td>0.004</td>
</tr>
<tr>
<td>IV</td>
<td>0.005</td>
<td>0.005</td>
<td>0.003</td>
</tr>
</tbody>
</table>

---

**Table II. Densities and elastic constants of materials for single crystals.**

<table>
<thead>
<tr>
<th>Material</th>
<th>( C_{11} ) (GPa)</th>
<th>( C_{12} ) (GPa)</th>
<th>( C_{44} ) (GPa)</th>
<th>( \rho ) (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>108.0</td>
<td>62.0</td>
<td>28.3</td>
<td>2.71</td>
</tr>
<tr>
<td>Cu</td>
<td>169.0</td>
<td>122.0</td>
<td>75.3</td>
<td>8.9</td>
</tr>
<tr>
<td>Fe</td>
<td>229.0</td>
<td>134.0</td>
<td>144.0</td>
<td>7.8</td>
</tr>
</tbody>
</table>
Table III. Isotropic and anisotropic elastic constants and Poisson ratios of polycrystalline materials using the Hill averaging method.

<table>
<thead>
<tr>
<th>Material</th>
<th>$L = \lambda + 2\mu$ (GPa)</th>
<th>$P = \lambda$ (GPa)</th>
<th>$T = \mu$ (GPa)</th>
<th>$c$ (GPa)</th>
<th>$c/\mu$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>112.06</td>
<td>59.97</td>
<td>26.05</td>
<td>-10.77</td>
<td>-0.41</td>
<td>0.3486</td>
</tr>
<tr>
<td>Cu</td>
<td>200.73</td>
<td>106.13</td>
<td>47.30</td>
<td>-9.76</td>
<td>-2.07</td>
<td>0.3459</td>
</tr>
<tr>
<td>Fe</td>
<td>272.65</td>
<td>112.17</td>
<td>80.24</td>
<td>-132.08</td>
<td>-1.65</td>
<td>0.2915</td>
</tr>
</tbody>
</table>

Approximation. Note that the anisotropy constant to isotropic shear modulus ratio $c/\mu$ in Table III for Cu or Fe is about four to five times larger than that for Al. Since the anisotropy of a polycrystal aggregate arises from the anisotropy within the single crystals and $W_{\text{inm}}$ are the only orientation description parameters of the aggregate, the same set of $W_{\text{inm}}$ represents different degrees of anisotropy for different materials. For the four groups of ODCs we used in our study, groups I and II for Cu and Fe exhibit the strongest anisotropy. All the rest are more weakly anisotropic, even though groups I and II for Al have the same texture values as for the strongly anisotropic Cu and Fe cases. A feeling of the strength of the anisotropy for the four sets of ODCs in the simulation can be obtained from the elastic constants given in Table IV.

Figures 3-5 show the results from the inversion step. Here the values of the ODCs that would be predicted on the basis of different dispersion correction schemes are plotted as functions of plate thickness to wavelength ratio $b/D$, where $D$ denotes the wavelength. Please note the scales for the ordinates are different for the predictions of each of the three ODCs. For the current texture measurement configuration, the wavelength of the $S_0$ waves is about 10 mm; the range 0-0.5 for $b/D$ represents a plate thickness of 0-5 mm.

In addition to the four curves representing the predictions from Thompson's and Hiraö's schemes described in the previous section, there are three horizontal straight lines in each figure, identifying the value of the ODCs assumed in the forward calculations and the target error bounds (to be discussed shortly). To see how dispersion correction schemes influence the prediction of ODCs, the results calculated directly from Eq. (7) without any dispersion corrections are also included in Figs. 3-5. These results are represented by an extra dash-dotted curve in the $W_{440}$ and $W_{420}$ figures of Figs. 3-5. In the $W_{440}$ figures of Figs. 3-5, this extra curve is not plotted; it would fall on top of the curve representing the response from the Hiraö's-A scheme, which uses Eq. (9). A comparison between Eqs. (7) and (9) shows that in Hiraö's-A scheme, dispersion correction plays no role in the prediction of $W_{440}$; therefore, the responses for $W_{440}$ using Eqs. (7) and (9) are identical.

Consider first the performance of the predictions of $W_{440}$ [parts (c), (f), (i), and (l) of Figs. 3-5]. For the weakly anisotropic cases [parts (c), (f), (i), and (l) of Fig. 3, and figures (i) and (l) of Figs. 4 and 5], the performances of Thompson's schemes are practically equivalent, and Hiraö's-B scheme is found to be better than Hiraö's-A scheme, providing a wider range of reliable predictions. When the anisotropy becomes strong [parts (c) and (f) of Figs. 4 and 5], both Thompson's-B and Hiraö's-B schemes may produce predictions of $W_{440}$ with relatively large errors for small thickness to wavelength ratio. This is especially true for Thompson's-B scheme, although it may sometimes give good predictions at some large thickness to wavelength ratio. This error is the consequence of the approximations made in going from Eq. (5) to Eq. (8) and from Eq. (9) to Eq. (10), which suggests that Eqs. (8) and (10) are not favorable for such cases. Over the range of thickness to wavelength ratio plotted, Thompson's-A scheme has a longer flat region and is generally better than Hiraö's-A scheme for

Table IV. Elastic constants of the textured plates for the $W_{\text{inm}}$ given in Table I (in GPa).

<table>
<thead>
<tr>
<th>Material</th>
<th>Group</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{13}$</th>
<th>$C_{23}$</th>
<th>$C_{33}$</th>
<th>$C_{12}$</th>
<th>$C_{13}$</th>
<th>$C_{23}$</th>
<th>$C_{24}$</th>
<th>$C_{55}$</th>
<th>$C_{66}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>I</td>
<td>110.96</td>
<td>112.04</td>
<td>113.44</td>
<td>58.74</td>
<td>59.83</td>
<td>61.22</td>
<td>24.82</td>
<td>25.90</td>
<td>27.30</td>
<td>27.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>109.93</td>
<td>111.01</td>
<td>110.69</td>
<td>60.11</td>
<td>61.20</td>
<td>60.88</td>
<td>26.19</td>
<td>27.28</td>
<td>26.95</td>
<td>26.95</td>
<td></td>
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<tr>
<td></td>
<td>III</td>
<td>113.22</td>
<td>112.57</td>
<td>112.75</td>
<td>59.95</td>
<td>59.30</td>
<td>59.48</td>
<td>26.03</td>
<td>25.38</td>
<td>25.56</td>
<td>25.56</td>
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</tr>
<tr>
<td></td>
<td>IV</td>
<td>112.70</td>
<td>112.05</td>
<td>111.37</td>
<td>60.64</td>
<td>59.99</td>
<td>59.31</td>
<td>26.72</td>
<td>26.06</td>
<td>25.39</td>
<td>25.39</td>
<td></td>
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<tr>
<td>Cu</td>
<td>I</td>
<td>190.60</td>
<td>200.55</td>
<td>213.32</td>
<td>94.86</td>
<td>104.82</td>
<td>117.59</td>
<td>36.02</td>
<td>45.98</td>
<td>58.75</td>
<td>58.75</td>
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<tr>
<td></td>
<td>II</td>
<td>181.15</td>
<td>191.11</td>
<td>188.14</td>
<td>107.45</td>
<td>117.41</td>
<td>114.44</td>
<td>48.62</td>
<td>58.57</td>
<td>55.60</td>
<td>55.60</td>
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<tr>
<td></td>
<td>III</td>
<td>211.35</td>
<td>205.37</td>
<td>207.03</td>
<td>105.97</td>
<td>101.65</td>
<td>47.14</td>
<td>41.16</td>
<td>42.82</td>
<td>42.82</td>
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<tr>
<td></td>
<td>IV</td>
<td>206.62</td>
<td>200.65</td>
<td>194.44</td>
<td>112.27</td>
<td>106.30</td>
<td>53.43</td>
<td>47.46</td>
<td>41.24</td>
<td>41.24</td>
<td>41.24</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>I</td>
<td>259.09</td>
<td>272.41</td>
<td>289.51</td>
<td>97.08</td>
<td>110.41</td>
<td>127.50</td>
<td>65.15</td>
<td>78.47</td>
<td>95.57</td>
<td>95.57</td>
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<tr>
<td></td>
<td>II</td>
<td>246.45</td>
<td>259.77</td>
<td>255.80</td>
<td>113.94</td>
<td>127.26</td>
<td>123.29</td>
<td>82.00</td>
<td>95.33</td>
<td>91.35</td>
<td>91.35</td>
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<tr>
<td></td>
<td>III</td>
<td>286.86</td>
<td>278.87</td>
<td>281.08</td>
<td>111.96</td>
<td>103.96</td>
<td>106.18</td>
<td>80.02</td>
<td>72.03</td>
<td>74.24</td>
<td>74.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>280.54</td>
<td>272.54</td>
<td>264.22</td>
<td>120.39</td>
<td>112.39</td>
<td>104.07</td>
<td>88.45</td>
<td>80.45</td>
<td>72.13</td>
<td>72.13</td>
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</tbody>
</table>
FIG. 3. Comparison of dispersion correction schemes in predicting ODCs for Al. (a)-(c): group I; (d)-(f): group II; (g)-(i): group III; (j)-(l): group IV. --, Thompson's-A scheme; ---, Thompson's-B scheme; ----, Hirao's-A scheme; -----, Hirao's-B scheme; ——, no dispersion correction; ———, exact value; —-——, target error bounds. The target errors are 0.001, 0.0005, and 0.0005 for $W_{400}$, $W_{420}$, and $W_{440}$. 
FIG. 4. Comparison of dispersion correction schemes in predicting ODCs for Cu. (a)–(c): group I; (d)–(f): group II; (g)–(i): group III; (j)–(l): group IV. – Thompson's-A scheme; --- Thompson's-B scheme; -- Hirao's-A scheme; --- Hirao's-B scheme; -- no dispersion correction; --- exact value; .... target error bounds. The target errors are 0.001, 0.0005, and 0.0005 for $W_{400}$, $W_{420}$, and $W_{440}$. 

FIG. 5. Comparison of dispersion correction schemes in predicting ODCs for Fe. (a)–(c): group I; (d)–(f): group II; (g)–(i): group III; (j)–(l): group IV. ---, Thompson's-A scheme; ---, Thompson's-B scheme; ---, Hirao's-A scheme; ---, Hirao's-B scheme; ----, no dispersion correction; ----, exact value; --- ---, target error bounds. The target errors are 0.001, 0.0005, and 0.0005 for $W_{400}$, $W_{420}$, and $W_{440}$.
cases studied. In fact, in the region $b/D = 0.0-0.25$, the errors associated with the prediction by Thompson's-A scheme are very small. Since in Hirao's schemes $W_{440}$ is not corrected for dispersion, when compared to Thompson's schemes, one finds that improvement can be made with the inclusion of the dispersion effect, although this effect is not as strong as that for $W_{400}$ (see the discussions on $W_{400}$).

Now consider the performance of the predictions of $W_{420}$. As can be seen in parts (b), (e), (h), and (k) of Figs. 3-5, Thompson's and Hirao's schemes influence the prediction of $W_{420}$ in opposite directions in all the cases studied, although the amount of influence is about the same in the range of $b/D = 0.0-0.25$. Except for the strongly anisotropic cases, Hirao's-B scheme generally gives better predictions than Thompson's schemes and Hirao's-A scheme. Similar to the predictions of $W_{440}$, Thompson's-B and Hirao's-B schemes may give unacceptable errors to the prediction of $W_{420}$ when the anisotropy of the plate gets strong. However, when compared to the predictions without dispersion correction, one finds that neither Thompson's nor Hirao's schemes are as good as the uncorrected predictions for all the cases studied. This clearly indicates that dispersion correction is really not necessary for $W_{420}$. As a matter of fact, the curves representing the predictions of $W_{420}$ without the dispersion correction in general have a very flat region for $b/D = 0.0-0.25$. These results are not fully understood. It can be argued that, since $W_{420}$ is known to be a measure of in-plane anisotropy, having little to do with the plate thickness that strongly influences the dispersion characteristics, no correction is needed. However, this same argument would apply to predictions of $W_{440}$. Since Hirao's-A scheme, corresponding to no correction, gives the poorest results for $W_{440}$, further factors must be involved.

The situation is somewhat different for the prediction of $W_{400}$, which is rather sensitive to the way in which a correction is made for dispersion. A glance of parts (a), (d), (g), and (j) of Figs. 3-5 reveals that both Hirao's and Thompson's schemes improve the estimation of $W_{400}$ significantly. Noting the compressed scale of these plots we see that the errors are considerably greater than in the predictions of $W_{420}$ and $W_{440}$. Hirao's-A and Hirao's-B schemes generally exhibit similar performance, with the former being some-

what more accurate. Thompson's schemes also exhibit similar performance, particularly when the anisotropy is not strong. Depending on the sign of $W_{400}$, Thompson's and Hirao's schemes may affect the prediction of $W_{400}$ in either the same or the opposite direction. For weakly anisotropic textured plates, Thompson's schemes generally predict $W_{400}$ with smaller errors. When the anisotropy becomes stronger, Hirao's schemes can be superior than Thompson's schemes. This fact is due to the nature of the approximations made in the Thompson's and Hirao's schemes. For Hirao's schemes, the accuracy of the prediction is closely related to the value of $b/D$; it is relatively insensitive to the degree of anisotropy. The performance of Thompson's schemes, on the other hand, depend on the smallness of the difference between the isotropic and anisotropic dispersion curves. For most of rolling and annealing textures, particularly on Al plates, where the anisotropy is not very strong, this difference is indeed small. In this case, Thompson's schemes may be more appropriate for $b/D > 0.15$. The greater sensitivity of the predictions of $W_{400}$ to the details of the dispersion correction occurs because $W_{400}$ depends on the absolute, rather than relative values of measured velocities.\(^9\)

To see how Thompson's and Hirao's schemes correct for the dispersion quantitatively, we set up the following target error bounds for each group: $|\delta W_{400}| < 0.001$, $|\delta W_{420}|$, and $|\delta W_{440}| < 0.0005$. These error bounds are chosen from a practical point of view, as they represent the experimentally observed differences between ultrasonic and diffraction (x-ray or neutron) predictions of the ODCs.\(^9\) Table V shows the acceptable limits of thickness to wavelength ratio for Thompson's-A and Hirao's-A schemes as well as from the uncorrected equation, if one wishes to stay within these bounds for the cases studied. It gives a guideline for the validity range of the current experimental configuration and dispersion correction schemes. For most metal sheets of interest in texture and formability prediction, the plate thickness is less than 2.5 mm. This thickness is about the limit of the present techniques if the wavelength is around the typical 10 mm value. From Table V, it is readily seen that, for the prediction of $W_{400}$, Hirao's-A scheme is not favorable for Al when the plate thickness to wavelength ratio is larger than 0.17. With the exception of group II in Cu and Fe, Thomp-

| TABLE V. Acceptable limits of thickness to wavelength ratio for the error bounds: $|\delta W_{400}| < 0.001$, $|\delta W_{420}|$, and $|\delta W_{440}| < 0.0005$ (N, no dispersion correction; T, Thompson's-A scheme; H, Hirao's-A scheme). |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\delta W_{400}$ | $\delta W_{420}$ | $\delta W_{440}$ |
| Group I | Group II | Group III | Group IV |
| $N$ | $T$ | $H$ | $N$ | $T$ | $H$ | $N$ | $T$ | $H$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Al $W_{400}$ | 0.041 | 0.212 | 0.154 | 0.038 | 0.194 | 0.173 | 0.043 | 0.281 | 0.161 | 0.042 | 0.276 | 0.171 |
| $W_{420}$ | 0.394 | 0.307 | 0.278 | 0.395 | 0.281 | 0.293 | 0.438 | 0.500 | 0.344 | 0.438 | 0.500 | 0.347 |
| $W_{440}$ | 0.237 | 0.361 | 0.237 | 0.230 | 0.346 | 0.230 | 0.289 | 0.406 | 0.289 | 0.284 | 0.400 | 0.284 |
| Cu $W_{400}$ | 0.075 | 0.244 | 0.180 | 0.063 | 0.141 | 0.271 | 0.101 | 0.289 | 0.231 | 0.094 | 0.266 | 0.270 |
| $W_{420}$ | 0.393 | 0.348 | 0.220 | 0.400 | 0.193 | 0.311 | 0.441 | 0.500 | 0.344 | 0.437 | 0.500 | 0.354 |
| $W_{440}$ | 0.268 | 0.406 | 0.268 | 0.236 | 0.334 | 0.236 | 0.290 | 0.407 | 0.290 | 0.265 | 0.366 | 0.265 |
| Fe $W_{400}$ | 0.085 | 0.242 | 0.192 | 0.065 | 0.137 | 0.218 | 0.108 | 0.281 | 0.231 | 0.099 | 0.254 | 0.243 |
| $W_{420}$ | 0.472 | 0.330 | 0.286 | 0.464 | 0.170 | 0.365 | 0.494 | 0.490 | 0.403 | 0.489 | 0.471 | 0.406 |
| $W_{440}$ | 0.305 | 0.402 | 0.305 | 0.264 | 0.333 | 0.264 | 0.325 | 0.407 | 0.325 | 0.304 | 0.372 | 0.304 |
TABLE VI. Errors for zero thickness plates ($\times 0.01$) (For Thompson’s-A and Hirao’s-A schemes).

<table>
<thead>
<tr>
<th></th>
<th>$\delta W_{400}$</th>
<th></th>
<th>$\delta W_{420}$</th>
<th></th>
<th>$\delta W_{440}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Al</td>
<td>Cu</td>
<td>Fe</td>
<td>Al</td>
<td>Cu</td>
</tr>
<tr>
<td>Group I</td>
<td>0.009</td>
<td>0.050</td>
<td>0.050</td>
<td>0.006</td>
<td>0.029</td>
</tr>
<tr>
<td>Group II</td>
<td>0.019</td>
<td>0.054</td>
<td>0.054</td>
<td>0.007</td>
<td>0.033</td>
</tr>
<tr>
<td>Group III</td>
<td>0.002</td>
<td>0.012</td>
<td>0.012</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td>Group IV</td>
<td>0.001</td>
<td>0.011</td>
<td>0.012</td>
<td>0.002</td>
<td>0.010</td>
</tr>
</tbody>
</table>

TABLE VII. Errors for zero thickness plates ($\times 0.01$) (For Thompson’s-B and Hirao’s-B schemes).

<table>
<thead>
<tr>
<th></th>
<th>$\delta W_{400}$</th>
<th></th>
<th>$\delta W_{420}$</th>
<th></th>
<th>$\delta W_{440}$</th>
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<tbody>
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<td>Al</td>
<td>Cu</td>
<td>Fe</td>
<td>Al</td>
<td>Cu</td>
</tr>
<tr>
<td>Group I</td>
<td>0.014</td>
<td>0.087</td>
<td>0.081</td>
<td>0.007</td>
<td>0.025</td>
</tr>
<tr>
<td>Group II</td>
<td>0.015</td>
<td>0.108</td>
<td>0.095</td>
<td>0.016</td>
<td>0.089</td>
</tr>
<tr>
<td>Group III</td>
<td>0.001</td>
<td>0.022</td>
<td>0.020</td>
<td>0.005</td>
<td>0.021</td>
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<tr>
<td>Group IV</td>
<td>0.001</td>
<td>0.024</td>
<td>0.021</td>
<td>0.002</td>
<td>0.012</td>
</tr>
</tbody>
</table>

son’s scheme provides a wider range of valid dispersion corrections for $W_{400}$. Both Thompson’s-A and Hirao’s-A scheme, however, significantly improve the prediction of $W_{400}$. On the other hand, the valid ranges for both Thompson’s-A and Hirao’s-A schemes for the prediction of $W_{420}$ are narrower than that from the equation without dispersion correction. It also can be seen from Table V that the valid range of $b/D$ for Thompson’s-A scheme for the prediction of $W_{440}$ is from 0 up to about 0.35–0.4. The corresponding range for Hirao’s-A scheme (equivalent to no correction) is 0.23–0.3.

From Figs. 3–5, one cannot fail to see that even when the plate thickness approaches zero, where the dispersion corrections are zero for all schemes, the results from the inverse process do not give the right answers. This is not surprising. The errors for Thompson’s-A and Hirao’s-A schemes are due to the approximations made when developing Eqs. (5), (7), and (9). These errors, however, are, in general, tolerable, as they are well within the target error bounds. These errors are given in Table VI for all groups. For Thompson’s-B and Hirao’s-B schemes, the errors in predictions of $W_{420}$ and $W_{440}$ at zero thickness for Cu and Fe can be large, exceeding the target error bounds. Table VII lists the errors from these two schemes. A comparison of the values in Table VI to those in Table VII clearly indicates that, with few exceptions, Thompson’s-A and Hirao’s-A schemes are better than Thompson’s-B and Hirao’s-B schemes for thin plates.

IV. CONCLUSIONS

We have evaluated the two available dispersion correction methods using numerical simulations to estimate the range of validity of each correction scheme. In general, both Thompson’s-A and Hirao’s-A schemes work well for plates with thickness to wavelength ratio less than 0.15. Thompson’s-B and Hirao’s-B schemes also lead to satisfactory results in this region, except for problems at small thickness to wavelength ratios of highly textured plates of Fe and Cu. Depending on the details of texture, preference may be given to a particular scheme. For thickness to wavelength ratios larger than 0.15, each technique begins to breakdown. Thompson’s schemes usually have a greater range of validity for $W_{400}$ for weakly anisotropic materials while Hirao’s schemes may be superior in the prediction when the materials anisotropy is strong. None of these schemes, however, provides adequate corrections for $W_{400}$ when the ratio exceeds 0.3. Therefore, one should be very cautious when applying the current experiment configuration to plates that give a thickness to wavelength ratio larger than 0.3. For the prediction of $W_{420}$, the use of neither Thompson’s schemes nor Hirao’s schemes is encouraged, as they all reduce the valid range for the prediction. For $W_{440}$, Thompson’s-A and Hirao’s-A schemes are practically equivalent for plates with a thickness to wavelength ratio less than 0.2. When this ratio exceeds 0.2, Thompson’s-A scheme is recommended. In either case, the dispersion correction effects are not as dominant as for $W_{400}$. Finally, Thompson’s-B and Hirao’s-B schemes should be avoided when the plate anisotropy is very strong, due to the relatively large errors at the small thickness to wavelength ratio.

ACKNOWLEDGMENTS

Ames Laboratory is operated for the U. S. Department of Energy by the Iowa State University under Contract No. W-7405-Eng-82. This work was supported by the Director for Energy Research, Office of Basic Energy Sciences.

5C. M. Sayers, Ultrasonics 24, 289 (1986).