MEASUREMENT OF ULTRASONIC WAVESPEEDS IN OFF-AXIS DIRECTIONS OF COMPOSITE MATERIALS

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INTRODUCTION

The relationship of NDE with structural analysis is to provide quantitative information about material mechanical properties. For a composite structure (such as a rocket motor case) which is designed to handle in-plane loading, NDE should, ideally, provide information about the in-plane stiffness and strength properties of the structural material [1]. Because acoustic wave propagation depends on material elastic properties as well as being sensitive to material inhomogeneities, ultrasonic NDE has been nominated as a viable means of satisfying the needs of structural analytical modeling [2]. To address the need to detect in-plane properties, leaky Lamb wave [3,4] and non-normal incidence transmission [1] methods are being developed, for example. Development of composite ultrasonic NDE techniques, which are sensitive to material mechanical properties in the plane of a structure, required an understanding of acoustic wave propagation in anisotropic media. If a wave is introduced into the structure wall with an oblique angle of incidence less than critical angle, the refracted wave will travel in a non-principal or off-axis direction of the composite material. As a result, the wave energy will not generally travel in a direction normal to its phase fronts as it would in an isotropic medium. The acoustic wave energy or wave group propagates at a deviation angle, ψ, with respect to the phase front normal [5,6] as shown in Fig. 1. The deviation angle should be considered when measuring acoustic phase velocities from which the stiffnesses are calculated. The following sections discuss the effect of the group velocity propagation direction upon phase velocity measurements of quasi-longitudinal and quasi-shear waves propagating in non-principal directions in principal planes of orthotropic composite materials. Experimental results are shown for unidirectional graphite composite material samples.

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PHASE AND GROUP VELOCITY

Shear and longitudinal phase velocities can be expressed mathematically as functions of the material elastic stiffnesses, Q_{ij} [7]. For a composite material of orthotropic symmetry, there are nine independent stiffnesses. The phase velocity of a wave propagating in an arbitrary direction in the material will, in general, be dependent upon all nine stiffnesses. With anisotropic material, however, the wave will not propagate in a direction normal to its wave fronts except in special directions such as the three principal directions (1, 2 and 3) of the material. (The direction normal to the phase fronts is the direction that the wavevector, k, points and will be referred to as the k-direction.) This fact can lead to erroneous phase velocity measurements if care is not taken in the measurement or reduction of experimental data. For certain experimental conditions the equations used to calculate phase velocity from measured data are of the same form as they would be if the acoustic energy actually travels in the k-direction, i.e., $\psi = 0$. The effect on experimental measurements of the ultrasonic phase velocity due to the ultrasonic group propagating in a non-k-direction will be discussed below. To obtain an expression for the phase velocity in terms of the elastic stiffnesses, it is necessary to solve the Cristoffel equation for the wave mode (quasi-shear or quasi-longitudinal) of interest. The Cristoffel equation is given by [7]:

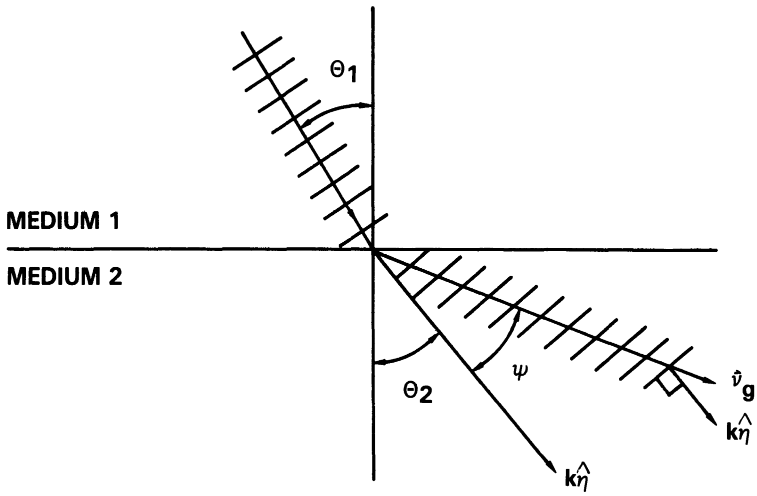

Figure 1. Refracted wave group.

$$\Omega(\mathbf{k},\omega) = \begin{vmatrix} (Y_{11} - \omega^2\varrho) & Y_{12} & Y_{13} \\ Y_{12} & (Y_{22} - \omega^2\varrho) & Y_{23} \\ Y_{13} & Y_{23} & (Y_{33} - \omega^2\varrho) \end{vmatrix} = 0 \qquad (1)$$

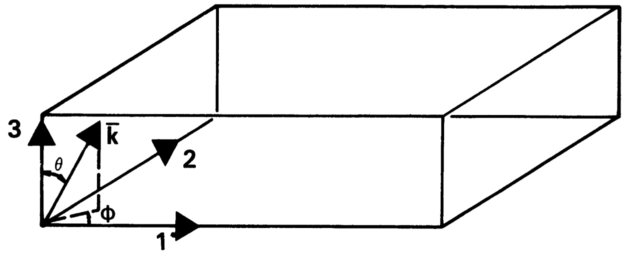

Figure 2. Laminate Coordinate System.

where

$$k_i = (\omega/c_p)n_i \quad i = 1,2,3 \tag{2}$$

and

$$n_1 = \sin(\theta)\cos(\phi)$$

$$n_2 = \sin(\theta)\sin(\phi)$$

$$n_3 = \cos(\theta) \tag{3}$$

n_i are the direction cosines for the wave vector k (see Fig. 2), c_p is the phase velocity, and ω is the angular frequency. Y_{ij} are the Cristoffel stiffnesses and, for an orthotropic material, are given by:

$$Y_{11} = k_1^2Q_{11} + k_2^2Q_{66} + k_3^2Q_{55}$$

$$Y_{12} = k_1k_2(Q_{12}+Q_{66})$$

$$Y_{13} = k_1k_3(Q_{13}+Q_{55})$$

(4)

$$Y_{22} = k_1^2Q_{66} + k_2^2Q_{22} + k_3^2Q_{44}$$

$$Y_{23} = k_2k_3(Q_{23}+Q_{44})$$

$$Y_{33} = k_1^2Q_{55} + k_2^2Q_{44} + k_3^2Q_{33}$$

Solutions to equation (1) for many special cases are found in reference 7 or 8, and will not all be enumerated here. The cases of interest in this paper are those for propagation in the 1-2 and 1-3 planes with particle motion in the respective planes. The solution to the Cristoffel equation for the 1-2 plane case is

$$\omega^2\rho - 1/2[k_1^2Q_{11} + k_2^2Q_{22} + Q_{66}] \pm 1/2\{[k_1^2Q_{11} + k_2^2Q_{22} + Q_{66}]^2$$
$$-4[Q_{66}(k_1^4Q_{11} + k_2^4Q_{22}) + k_1^2k_2^2(Q_{11}Q_{22} - 2Q_{12}Q_{66} - Q^2{}_{12})]\}^{1/2} = 0 \tag{5}$$

where (+) is for the quasi-longitudinal waves and (-) for the quasi-shear. The solution for the 1-3 plane is obtained from equation (5) by making the following substitutions:

$$Q_{11} \rightarrow Q_{11}$$
$$Q_{12} \rightarrow Q_{13}$$
$$Q_{22} \rightarrow Q_{33} \tag{6}$$
$$Q_{66} \rightarrow Q_{55}$$

The solutions to the Cristoffel equation give the acoustic phase velocity by making use of equation (2). The most general expression for the Cristoffel equation is in the form of a cubic equation in c_p^2 and has three solutions. These solutions correspond to two quasi-shear wave modes and one quasi-longitudinal mode. If the incident medium is a liquid and propagation is constrained to a principal plane such as the 1-3 plane of the second medium, two modes will be generated - one quasi-shear and one quasi-longitudinal.

The group velocity vector points in the direction that the ultrasonic wave energy actually propagates and is related to the Cristoffel equation by the following expression [6]

$$c_g = \frac{-\Delta_k \Omega_r}{2\Omega_r/2w} \tag{7}$$

where Ω_r is the root of the Cristoffel equation (equation (1)) corresponding to the wave mode of interest and is expressed in the form of equation (5), i.e.,

$$\Omega_r(k_i,\omega) = 0 \tag{8}$$

In general, the group velocity vector, c_g, does not point in the same direction as the phase velocity vector, c_p. (Note that c_p is parallel to k.) The group velocity deviation angle, ψ, is measured with respect to the k-direction and is given by [6]:

$$\mathbf{k}/k \cdot \mathbf{c}_g = c_p \tag{9}$$

or

$$c_p = c_g \cos \tag{10}$$

The phase and group velocities are only parallel for directions of high symmetry such as the principal directions of an orthotropic material. By combining equations (2), (3), (7), and (9), ψ can be obtained.

EXPERIMENTAL APPLICATIONS

To experimentally apply the above theory to a composite sample, the sample geometry and ultrasonic transducer arrangement must be taken into consideration. Two applications are considered in the following subsections - quasi-longitudinal wave propagation in the 1-2 plane and quasi-shear wave propagation in the 1-3 plane.

Propagation in the 1-2 Plane

Several samples of unidirectional composite material were cut out of a panel in the manner illustrated in Fig. 3a. The quasi-longitudinal wave speed was obtained by measuring the time-of-flight of a pulse propagating through the width of each sample. For the samples with off-axis fiber direction, the acoustic pulse propagates through the width of the sample with deviation angle, ψ, as shown in Fig. 3b. The phase velocity for the off-axis samples is calculated in the following manner.

The group velocity is given by the distance traveled divided by the transit time of the pulse, or

$$c_g = h/t\cos\psi \tag{11}$$

Using equation (10), the phase velocity is simply given by

$$c_p = h/t \tag{12}$$

Thus, it is seen that c_p is calculated from the same relation that it would be if $\psi = 0$. This implies that measuring the group velocity at point A in Fig. 3b is equivalent to measuring the phase velocity at point B in Fig. 3b if the wave were to propagate directly across the sample with no deviation angle. However, the transducer must be placed at point A in order to correctly measure the transit time, t. The results for the phase velocity calculation are shown in Fig. 4a. The solid curve in Fig. 4a was produced by performing a least-squares fit of the correctly measured phase velocity data to equation (5). The fitting parameters are the stiffnesses, Q_{11}, Q_{12}, Q_{22}, and Q_{66} which were obtained and found in good agreement with values calculated from mixture rules. A least squares fit to the incorrectly measured phase velocities produced unphysical values for some of the stiffnesses. The measured and calculated deviation angles are presented in Fig. 4b for each of the respective samples for which the phase velocities were measured. The Q_{ij}'s obtained from the least-squares fit were used to produce the calculated deviation angle curve.

Propagation in the 1-3 Plane

A more practical problem lies with non-normal incidence of a longitudinal wave coupled into a planar structure such as a plate by a water coupling medium. If access is available to both sides of the plate wall, a wave may be transmitted through it and received on the opposite side. For an obliquely incident longitudinal wave, mode conversion will occur upon refraction into the second medium. Two quasi-shear modes and one quasi-longitudinal mode will be produced, in general. If the incident medium is liquid and if propagation is constrained to a principal plane of the material such as the 1-3 plane, assuming the symmetry is orthotropic or higher, only two modes will be generated - one quasi-shear and one quasi-longitudinal. Both modes will have particle motion in the plane of propagation. If the plate is rotated in the transducer beam, it is

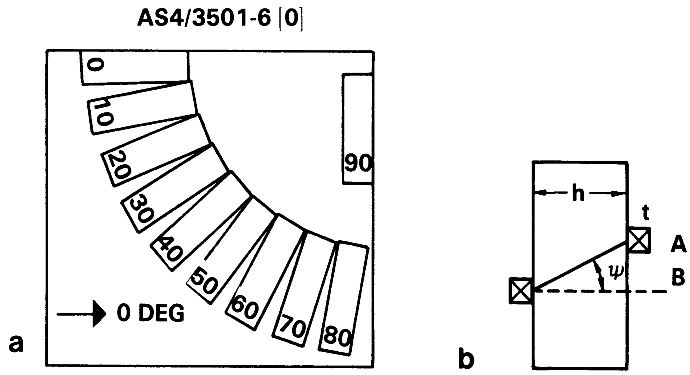

Figure 3. (a) Samples cut from AS4/3501-6 panel.
(b) Group velocity vector deviation in off-axis samples.

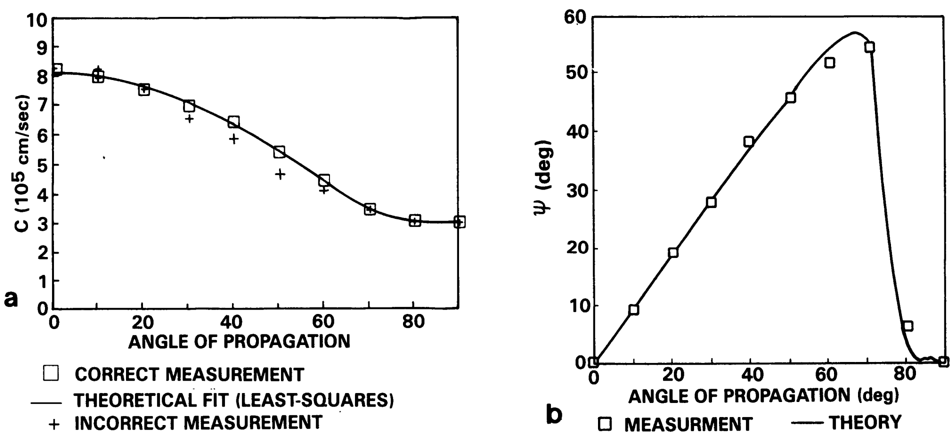

Figure 4. (a) Quasi-longitudinal phase velocity in 1-2 plane of uni-directional AS4/3501-6 prepreg composite. (b) Group velocity deviation angle for same material.

possible to measure the phase velocity as a function of angle of propagation in the 1-3 plane in a localized region of the plate. Such an experimental set up is shown in Fig. 5 with all of the relevant geometric parameters. The following equations can be obtained by referring to Fig. 5:

$$c_g = \left[\frac{\cos\beta}{c_w} - \frac{\Delta t}{y} \sin\beta \right]^{-1} \qquad (13)$$

and

$$\beta = \theta_2 - \theta_1 + \psi \qquad (14)$$

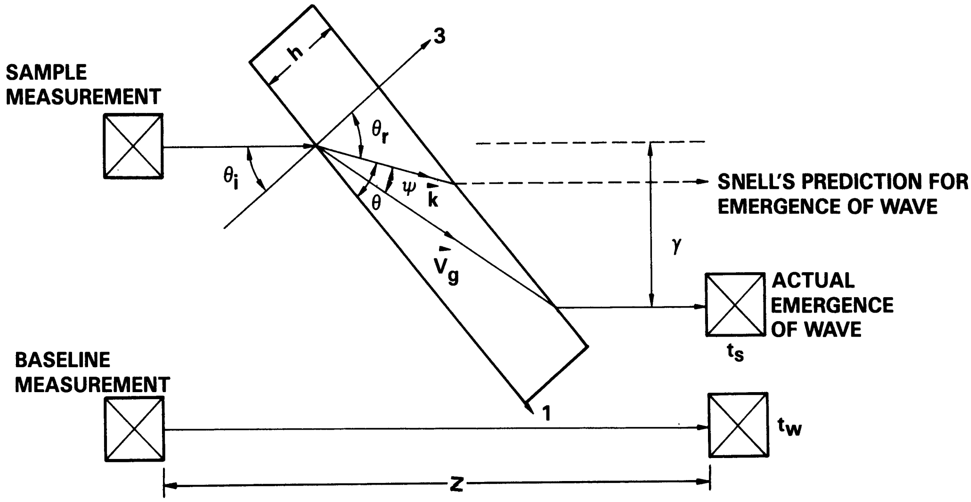

Figure 5. Setup for sample rotation experiment.

where

$$\Delta t = t_w - t_s \tag{15}$$

By combining equations (10), (13), and (14), the following can be obtained.

$$c_p = \frac{\cos(\beta+\theta_1)\cos\theta_2 + \sin(\beta+\theta_1)\sin\theta_2}{\dfrac{\cos\beta}{c_w} - \dfrac{\Delta t}{y}\sin\beta} \tag{16}$$

This equation cannot be used to calculate c_p because θ_2, the refracted angle, depends on c_p through Snell's law. (Note that the phase velocity vector obeys Snell's law, but the group velocity vector does not in an anisotropic medium [9].) Thus,

$$\frac{\sin\theta_2}{c_p} = \frac{\sin\theta_1}{c_w} \tag{17}$$

Making use of the relation (refer to Fig. 5)

$$\tan\beta = \frac{y\cos\theta_1}{h + y\sin\theta_1} \tag{18}$$

and Snell's law (equation 17), it is possible to obtain the following relation for c_p:

$$c_p = \left[\frac{1}{c_w^2} - \frac{2\Delta t\cos\theta_1}{hc_w} + \frac{\Delta t^2}{h^2}\right]^{-1/2} \tag{19}$$

It is interesting to note that this resultant expression for c_p is independent of both y and ψ, and in fact is the same result that would be obtained if $\psi = 0$ were assumed on the outset of the derivation [10]. This explains why some literature measurements of the phase velocity are correct for off-axis directions of composites [10,11].

Calculations of the quasi-shear phase velocity from experimental data using equation (19) are shown in Fig. 6a for the 1-3 plane of a unidirectional sample of AS4/3501-6 [0] material. The expected (theoretical)

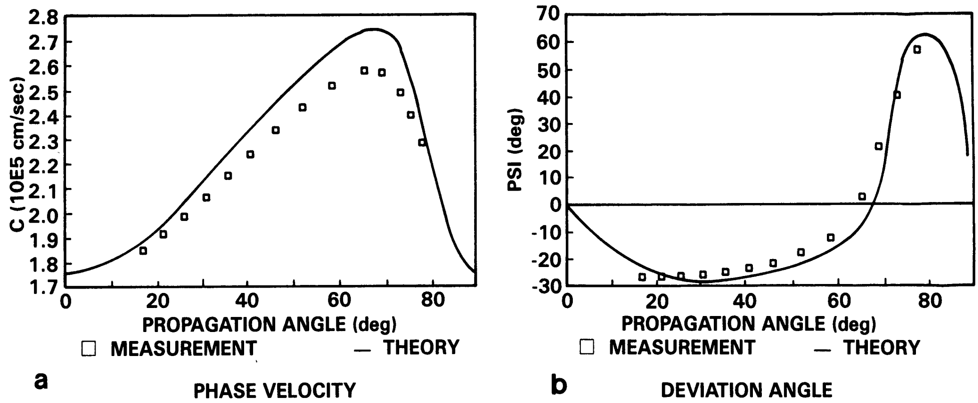

Figure 6. (a) Quasi-shear phase velocity for 1-3 plane of unidirectional AS4/3501-6 material. (b) Group velocity deviation angle for 1-3 plane of same material.

curve was calculated using the Q_{ij}'s calculated from the least-squares fit discussed above. This is justified by the assumption that the unidirectior laminate is transverse isotropic in symmetry and hence, $Q_{13} = Q_{12}$, $Q_{55} = Q_{66}$, and $Q_{33} = Q_{22}$. There are at least two reasons for the disagreement between the measured and theoretical results shown in Fig. 6a; 1) the unidirectional sample probably varies somewhat from transverse isotropy and so the Q_{ij}'s used are not quite correct, and 2) the transit time, t_s, must be measured at the y position where the energy actually emerges. This is difficult, particularly at wide angles of incidence, for a bounded beam because the anisotropy (in phase velocity and attenuation coefficient) of the sample distorts the refracted beam. This causes the further complicati(of phase cancellation [12] in the receiving transducer. These conditions add up to making it difficult to find the y position corresponding to the position of maximum energy immergence.

Figure 6b shows the measured and expected group velocity deviation angles for the quasi-shear wave corresponding to the phase velocity measurements in Fig. 6a. Note that at wide angles of propagation the deviation angle is negative in sign. This has the effect of allowing the wave to pass through the sample with little or no apparent refraction. Data were not obtained near normal or near critical angle incidence because the quasi-shear wave transmission coefficient is zero or almost zero at these extremes.

CONCLUSIONS

For both experimental applications described in this paper, it was found that the theory used to calculate c_p from experimental data was the same that would be obtained if the deviation angle for the group velocity vector were set to zero and only the phase velocity vector were to be considered in the derivation. It was also pointed out that in order to make an accurate transit-time measurement it is important to place the receiving ultrasonic transducer in the position where the energy emerges from the sample, not in an anticipated position based on Snell's law. This is particularly true for shear waves because, for certain angles of propagation, the group velocity deviation angle is negative in sign with respect to the Snell's law angle of refraction. Further improvements on the method described in the section on propagation in the 1-3 plane will require the development of a bounded beam pulse transmission model to explain the effects of wave distortion and phase cancellation on the measured y position of the immerging energy.

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