Supplementary Information to "Farm Management Optimization Under Uncertainty with Impacts on Water Quality and Economic Risk"

Görkem Emirhüseyinoğlu and Sarah M. Ryan
Department of Industrial and Manufacturing Systems Engineering
Iowa State University, Ames, IA, 50011, USA

January, 2022

S1 Available Information and Assumptions

Detailed assumptions considered while building the optimization model are provided in the supplementary Sections S1.1-S1.3.

S1.1 The Impact of N Application Rate on Yield

The online Corn Nitrogen Rate Calculator tool provides reliable information showing the impact of N rate on yield based on research trials (Sawyer et al., 2020). The tool generates data points indicating the percent of maximum yield given different N application rates for six midwestern states (Illinois, Iowa, Michigan, Minnesota, Ohio, and Wisconsin). Generated data points for Iowa, with integer-valued N application rates between 0 and 240 lbs/acre, are illustrated in Figure 3. Note that the N rate achieving 100% of the maximum yield is not necessarily the best selection for a farmer because maximum return to N (MRTN), the N rate where the economic net return to N application is maximized, can be different when fertilizer prices are taken into account (Sawyer et al., 2006). The tool currently does not elaborate on how precipitation affects the relationship between N rate and yield. Previous experiments demonstrate the need for N rate at higher than MRTN, yet current research is not reliable enough to indicate how much additional N would be needed (Sawyer, 2019). To preserve the linearity of the optimization model, we generate piecewise linear functions to approximate the data points displayed in Figure 3.

S1.2 The Impact of Planting and N Application Time Decisions on Yield

Optimal planting windows differ based on geographical region. Previous research includes elaborate experimental tests investigating how different time windows affect yields (Abendroth et al., 2017; Kucharik, 2008). Depending on the region, one can categorize planting windows based on their yield outcomes. Similarly, various studies examine the impact of N application timing on yield (Sawyer et al., 2016; Randall et al., 2008; Randall and Mulla, 2001). Using the information available in the literature, we define $\beta_{ij}(\omega, \gamma)$, as a fraction of maximum yield, to indicate the combined impact of the decisions, where i represents one of the N application timings considered in this study and j denotes the planting window. Harvested crop yield depends not only on those decisions but also on uncertain weather conditions. The random variables, ω and γ, symbolize the observed average growing season precipitation and temperature, respectively. We assume that fall fertilizer application is the default selection, and $\beta_{1,1}(\omega, \gamma) = 1$ under ideal weather conditions.

The cost of specialized equipment needed for sidedressing application is not considered in this study.

S1.3 The Impact of Precipitation and Temperature on Yield

Weather conditions influence both yield and hydrological processes, including N loss, by surface runoff and leaching. The weather effect on yield and N loss can be investigated under two time phases. The first phase goes from fall harvesting time until spring and the second spans spring until the next harvest.

In the literature, the fall fertilizer application is generally expected to result in lower yield and higher N loss compared to other applications. That is because additional N added to the soil during fall increases the chance of leaching, as no plant N uptake occurs until springtime. Experimental results support such claims and, as discussed in Section S1.2, we already take into account this particular yield impact through the parameter $\beta_{ij}(\omega, \gamma)$. This leaching rate, however, depends on fall precipitation. In reality, if the fall precipitation is significantly low in a given year, similar yield and N rates are expected from both fall and spring applications (and vice versa, high fall precipitation or mild winter can spike the N loss significantly during fall). Unfortunately, the experimental tests collected from the literature to calculate $\beta_{ij}(\omega, \gamma)$ do not include this inherent uncertainty. As a result, we lack enough information to calculate the impact of fall precipitation on N leaching and yield, and the fall precipitation uncertainty is not considered in this study.

Growing season weather uncertainty, on the other hand, is considered. In the literature, various studies examine the effect of precipitation and temperature during the growing season on yield (Li et al., 2019; Yamoah et al., 2000; Xu et al., 2016), and we account for the impact of those uncertainties multiplicatively, as they are independent of the investigated decisions.

S2 Explanation of Constraints (6b)-(6n)

We introduce the yield protection plan indemnity, σ_1, and revenue protection plan indemnity, σ_2, in section 3.4.1 as:

$$\sigma_1 = \max\Big(\mu f_v r_0 - r_0 A, \quad 0\Big) \tag{S1}$$

$$\sigma_2 = \max\Big(\mu f_v r_0 - rA, \quad \mu f_v r - rA, \quad 0\Big) \tag{S2}$$

With respect to Equation (S1), the role of the disjunctive variable, q_1^s, and Constraints (6b)-(6d) can be explained as follows:

1. If $q_1^s = 0$, the yield protection plan is purchased for some v $\Big(\sum_v y_{v1} = 1\Big)$ and the first term of (S1) is greater than zero $\Big($i.e., $\sum_v \mu f_v r_0 y_{v1} - r_0 A^s > 0$ for some $s\Big)$. That means the farmer will receive some indemnity payment. Note that when $\sum_v \mu f_v r_0 y_{v1} - r_0 A^s > 0$ for some s, q_1^s cannot be equal to 1, because Constraints (6b) and (6d) will conflict. Constraint (6c) ensures that the insurance model is not unbounded by ensuring that the indemnity payment equals $\sum_v \mu f_v r_0 y_{v1} - r_0 A^s$.
2. If $q_1^s = 1$, this could indicate that either
    (a) The farmer did not purchase the yield protection plan, or

(b) The farmer purchased yield protection insurance for some v $\left(\sum_v y_{v1} = 1\right)$; however, the first term in Equation (S1) is less than zero $\left(\text{i.e., } \sum_v \mu f_v r_0 y_{v1} - r_0 A^s < 0 \text{ for some } s\right)$.

In either case, $\sum_v \mu f_v r_0 y_{v1} - r_0 A^s < 0$. Therefore, q_1^s cannot equal 0, because Constraint (6c) could not be satisfied.

3. Note that if $\sum_v \mu f_v r_0 y_{v1} - r_0 A^s = 0$ for some s $\left(\text{which can only happen if } \sum_v y_{v1} = 1\right)$, q_1^s could take either value of 0 or 1 without any impact on the solution.

Similarly, binary disjunctive variables q_2^s, q_3^s, q_4^s and Constraints (6e)-(6j) are introduced to calculate revenue protection plan indemnity, σ_2, with respect to Equation (S2). The logic of those variables and constraints are summarized as follows:

1. If $q_2^s = 0, q_3^s = 1,$ and $q_4^s = 1$, the revenue protection plan is purchased for some v, and the first term in Equation (S2) is the largest $\left(\text{i.e., } \sum_v \mu f_v r_0 y_{v2} - r^s A^s \text{ is larger than the other two terms for this } s\right)$. Note that when $\sum_v \mu f_v r_0 y_{v2} - r^s A^s$ is the largest term, q_3^s must be 1 to satisfy Constraint (6h), and q_4^s must be 1 to satisfy Constraint (6i). Since Constraint (6j) enforces the model to allow only one of q_2^s, q_3^s, q_4^s to be 0 for each s, q_2^s must equal 0 to so that Constraint (6f) prevents the model from being unbounded.

2. If $q_3^s = 0, q_2^s = 1,$ and $q_4^s = 1$, the revenue protection plan is purchased for some v, and the second term in Equation (S2) is largest $\left(\text{i.e, } \sum_v \mu f_v r^s y_{v2} - r^s A^s \text{ is the maximum for this } s\right)$. This logic is similar to that described in Item 1.

3. If $q_4^s = 0, q_2^s = 1,$ and $q_3^s = 1$, then either

   (a) The revenue protection plan is purchased for some v, and third term in Equation (S2) is the largest (i.e., the other two terms are negative; the logic is similar to that described in Item 1), or

   (b) The revenue protection plan is not purchased. In that case, both the first and second terms of Equation (S2) are negative. To satisfy Constraints (6f) and (6h), both q_2^s and q_3^s must equal 1. Constraint (6j) then forces q_4^s to equal 0.

Constraint (6k) ensures that the insurance indemnities are nonnegative, while Constraint (6l) ensures that only one insurance plan is selected. Finally, Constraints (6m) and (6n) are binary restrictions.

S3 Details about Computational Study

The study is designed to represent a typical corn farm in Iowa, where typically corn is grown in rotation with soybeans but sometimes is repeated year after year.

The cost of N fertilizer depends on the source, which can be urea, anhydrous ammonia, or urea ammonium nitrate (UAN; Sawyer et al. (2016)). In 2019, the cheapest anhydrous ammonia price was approximately \$0.30 - \$0.35 per lb N, while the most expensive UAN prices varied in the range of \$0.45 - \$0.50 per lb N. (In 2020, the pandemic caused anhydrous prices to fall as low as \$0.26 per lb N, and UAN dropped to \$0.40 per lb N.) To reflect typical conditions, we assume fertilizer cost to be \$0.40 per lb N.

Crop insurance premiums are calculated based on several factors, including the insured land area (acres); the projected price at harvest, as determined by the US Department of Agriculture Risk Management Agency and known to farmers when choosing a policy; the historical crop yield of the farm and trend (up to 10 years); and the county average yield. In Table S1, we present the key parameters used to formulate the model and generate crop insurance premiums for our baseline case. Because insurance premiums can be higher if the farm has had an increasing yield trend or yield expectation is significantly higher than the county average, we also generate alternative corn premiums where trend-adjusted crop yield for the next year is 10% higher. Although a considerable number of assumptions were necessary to generate insurance premiums, their baseline values are at the low end and alternative values are at the high end of the likely ranges.

In this case study, we assume that, under fall N application, both historical yield average (μ) and maximum achievable yield (H) equal 180 bu/acre. That is, we assume the farmer already utilizes their farm to its full potential and the investigated farm has a flat yield trend. We use the most recent projected corn price ($r_0 = \$3.88$) announced to farmers by the Risk Management Agency for 2020. Based on this information, the estimated premiums are obtained using a crop insurance decision support tool (Schnitkey, 2019).

Table S1: Crop insurance premiums per acre, approximated using Enterprise units, for Story County, Iowa. In the baseline case the yield trend is flat while in the alternative case the annual yield increase is 10%.

	Baseline Case		Alternative Case	
$\boldsymbol{f_v}$	c_{v1}	c_{v2}	c_{v1}	c_{v2}
50%	\$0.21	\$0.22	\$0.27	\$0.30
55%	\$0.27	\$0.30	\$0.39	\$0.48
60%	\$0.37	\$0.45	\$0.53	\$0.71
65%	\$0.5	\$0.66	\$0.76	\$1.10
70%	\$0.68	\$0.93	\$1.02	\$1.76
75%	\$1.04	\$1.68	\$1.66	\$3.38
80%	\$1.94	\$3.64	\$3.00	\$6.82
85%	\$3.83	\$8.10	\$5.66	\$14.08

Spring and sidedress applications are expected to result in higher yields due to their lower potential for N loss. We approximate the yield impact of N application timing decisions using field test results of Iowa State University et al. (2017); Randall et al. (2008). Accordingly, spring, split (40% preplant + 60% sidedress) and full summer sidedress applications are assumed to add +6%, +10% and +13%, respectively, to the yield relative to fall application.

To generate scenarios, r^s, for harvest corn prices, we use the past five years' official harvest prices (determined based on average futures price of Chicago Board of Trade in October for December) to calculate insurance indemnities. Because corn prices in the early 2010s were significantly higher than in the late 2010s, we limit the number of recent years to reflect the current corn market conditions. Since harvest prices for 2016 and 2017 were the same, the data-driven price scenarios are \$3.83, \$3.49, \$3.68 and \$3.90, with respective probabilities of 0.2, 0.4, 0.2 and 0.2.

The effect of growing season (May - October) mean temperature on yield is based on crop simulation model predictions (Xu et al., 2016). A normal distribution provides the best continuous fit to historical temperature means from 1894 to 2019 (Figure S1). Discrete scenarios consisting of z values equal to -1.029, 0 and 1.029, with respective probabilities of 0.3035, 0.3930 and 0.3035, are proved to be an optimal three-point approximation to a standard normal distribution (Pflug, 2001). However, approximately 5% of the temperature data corresponds to abnormally high growing season averages above 70°F. Therefore, we also include a worst-case temperature alternative with probability 0.05. We normalize the probabilities provided by Pflug (2001) to sum to 0.95 and, thus, generate four probabilistic outcomes for mean growing-season temperature, as shown in Table S2. Note that only higher-than-average temperatures diminish the yield.

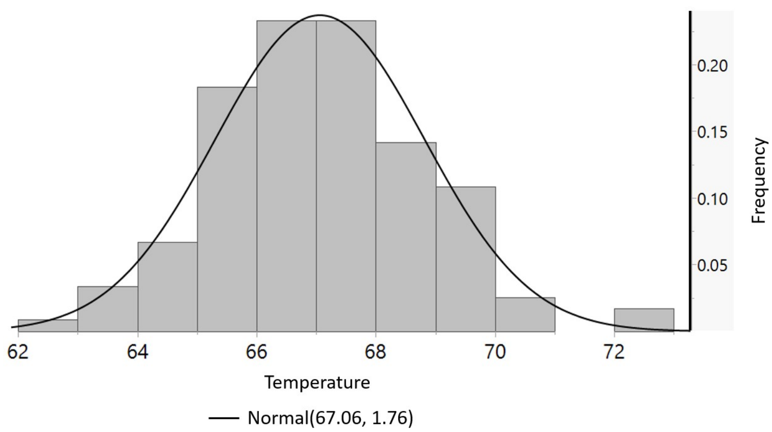

Figure S1: Frequency of temperature averages (°F) for Iowa from 1894 to 2019 between May and October

Table S2: Growing season mean temperature outcomes (γ)

	Low	Medium	High	Worst
Value	65.26°F	67.06°F	68.87°F	72.02°F
Probability	0.29	0.37	0.29	0.05
Yield Impact	-	-	-2.62%	-8.00%

Li et al. (2019) demonstrates that, in the Midwest, prediction model estimates given growing season precipitation from May to August are significantly different from actual yield observations. The study generates 14 bins of standardized precipitation intensity with width 0.5σ and tails defined as $< -2.5\sigma$ and $> +3.5\sigma$, and summarize the observed yields at each bin in the Midwest. Considering the similarities in yield outcomes and low probability of occurrence in certain categories, we aggregated the potential growing season precipitation outcomes to four as shown in Table S3. The probability of occurrence for each discrete scenario directly reflects historical growing season precipitation in Iowa between May and August from 1980 to 2019. The yield impact of each discrete outcome is simply the weighted average of the selected precipitation intensity range calculated according to the yield impact information of bin provided by Li et al. (2019). Finally, the probability of occurrence for each bin is generated based on Iowa precipitation data.

Table S3: Growing season precipitation outcomes (ω)

	Very Dry	Dry	Regular	Wet
Standardized value range	$(-\infty, -2\sigma]$	$(-2\sigma, \sigma]$	$(\sigma, 2\sigma)$	$[2\sigma, \infty)$
Probability	0.025	0.100	0.825	0.050
Yield impact, Iowa	-25.18%	-7.87%	-	-33.05%

Optimal crop planting dates depend on weather and soil conditions. Previous studies show that optimal planting dates vary across Iowa, ranging from mid-April until the second week of May, depending on the location (Elmore, 2012; Abendroth et al., 2017). However, except for the southern parts of the state, any planting after May 1 commonly results in lower yields. Therefore, in the case study, we assume that the farmer strives to complete any springtime farming operations before May 1, and failure to do so results in a 5% yield reduction. Because spring farming operations include not only fertilizer application but also other activities such as planting, we assume any fertilizer application should be completed within the first three weeks of April to avoid a planting delay. Weather and soil conditions are again the main factors determining whether a day is suitable for fieldwork depending on the emerging soil moisture at a given date. For a suitable fieldwork day, the soil must be not wet but also not too dry. The number of days, D, needed to apply fertilizer depends on several factors and is approximated by Hanna (2016) using equation (S3).

$$D = \frac{\text{field size(acre)}}{\text{daily working hours} \times \text{field capacity(acre/hrs)}} \tag{S3}$$

Field size represents the total area which needs to be covered during the fertilizer application, and the formula for estimating the field capacity is:

$$\text{field capacity(acre/hrs)} = \frac{\text{width(ft)} \times \text{speed(mph)} \times \text{field efficiency(\%)}}{\frac{43{,}560\text{(sq ft/acre)}}{5280\text{(ft/mile)}}} \tag{S4}$$

Here, "width" refers to actual implement width, "speed" represents how fast the machinery can travel while performing the operation, and finally "field efficiency" represents the percent of effective working time by taking into account the time lost while turning around, slowing down, etc. Assuming 10 working hours per day, a 1000-acre farm will need approximately 10 working days to apply the fertilizer if the width of the implement, speed and field efficiency are 20 ft, 5 mph and 0.8 respectively. In the case study, we assume $D = 10$. However, we recognize this number may vary greatly depending on the unique conditions of the investigated farm.

Hanna (2014) summarizes the probabilities of a day to be suitable for fieldwork in Iowa, by week, from April until October. According to the study, the probabilities that a given day in the first, second, and third week of April is suitable for field work are 0.33, 0.43, and 0.45, respectively. We average the weekly probabilities over this three-week window and approximate the number of days suitable for fieldwork as binomial:

$$\Pr(\tau_1 \in B) \equiv \Pr(\tau_1 \geq D) = \sum_{d=D}^{21} \binom{21}{d} 0.4^d 0.6^{(21-d)} \tag{S5}$$

In the case study, we assume that the number of days needed to apply the fertilizer, D, is equal to 10 (see the supplement for more information), and $\Pr(\tau_1 \geq 10) = 0.32$. Hence, we generate two discrete outcomes for τ_1 where, if the farmer selects spring or split application, a planting delay will occur with probability 0.68, or will not occur otherwise.

On the other hand, recall that if some portion of the fertilizer is planned to be applied during summer and there are not enough suitable workdays during this summer feed, there will be no choice but to apply less fertilizer to the soil than the preselected value of t. We assume that this summer N application will occur before the start of the V8 stage. The corn growth stage calendar depends on the planting date and weather conditions observed in a given year. The Corn Split N decision support tool (Gramig et al., 2017) estimates the V8 stage date for May 1 planting as approximately June 14. By using this approximation, we assume that the fertilizer application should be completed approximately two weeks before this date. By following the same logic used for spring application, we use the average probability (approximately 0.65) for a day to be suitable for fieldwork during the first two weeks of July from Hanna (2014), and calculate the potential outcomes for τ_2 as presented in Table S4. Note that we neglect potential outcomes with probability very close to 0.

Table S4: τ_2 outcomes where $D = 10$

	Value of τ_2	Probability	k_3^s	k_4^s
Outcome 1	5	$\Pr(\tau_2 = 5) = 0.017$	94%	50%
Outcome 2	6	$\Pr(\tau_2 = 6) = 0.048$	100%	60%
Outcome 3	7	$\Pr(\tau_2 = 7) = 0.103$	100%	70%
Outcome 4	8	$\Pr(\tau_2 = 8) = 0.172$	100%	80%
Outcome 5	9	$\Pr(\tau_2 = 9) = 0.217$	100%	90%
Outcome 6	$\geq$10	$\Pr(\tau_2 \geq 10) = 0.438$	100%	100%

References

Abendroth, L. J., K. P. Woli, A. J. Myers, and R. W. Elmore (2017). Yield-based corn planting date recommendation windows for Iowa. *Crop, Forage & Turfgrass Management 3*(1), 1–7.

Elmore, R. W. (2012). Progress of corn planting and corn emergence. *Integrated Crop Management News 187*. https://dr.lib.iastate.edu/handle/20.500.12876/18160.

Gramig, B. M., R. Massey, and S. Do Yun (2017). Nitrogen application decision-making under climate risk in the US Corn Belt. *Climate Risk Management 15*, 82–89.

Hanna, M. (2014). Fieldwork days in Iowa. *Ag Decision Maker Information Files 7*. https://lib.dr.iastate.edu/pubs_agdm/7.

Hanna, M. (2016). Estimating the field capacity of farm machines. *Ag Decision Maker Information Files 8*. https://lib.dr.iastate.edu/pubs_agdm/8.

Iowa State University, Iowa Department of Agriculture and Land Stewardship, and Iowa Department of Natural Resources (2017). Iowa nutrient reduction strategy: A science and technology-based framework to assess and reduce nutrients to Iowa waters and the Gulf of Mexico. http://www.nutrientstrategy.iastate.edu/sites/default/files/documents/2017%20INRS%20Complete_Revised%202017_12_11.pdf.

Kucharik, C. J. (2008). Contribution of planting date trends to increased maize yields in the Central United States. *Agronomy Journal 100*(2), 328–336.

Li, Y., K. Guan, G. D. Schnitkey, E. DeLucia, and B. Peng (2019). Excessive rainfall leads to maize yield loss of a comparable magnitude to extreme drought in the United States. *Global Change Biology 25*(7), 2325–2337.

Pflug, G. C. (2001). Scenario tree generation for multiperiod financial optimization by optimal discretization. *Mathematical Programming 89*(2), 251–271.

Randall, G. W. et al. (2008). Nitrogen application timing, forms, and additives. In *Final Report: Gulf Hypoxia and Local Water Quality Concerns Workshop*, St. Joseph, MI, pp. 73–85. American Society of Agricultural and Biological Engineers.

Randall, G. W. and D. J. Mulla (2001). Nitrate nitrogen in surface waters as influenced by climatic conditions and agricultural practices. *Journal of Environmental Quality 30*(2), 337–344.

Sawyer, J., E. Nafziger, J. Camberato, K. Steinke, D. Kaiser, S. Culman, and C. Laboski (2020). Corn nitrogen rate calculator. `http://cnrc.agron.iastate.edu/`.

Sawyer, J., E. Nafziger, G. Randall, L. Bundy, G. Rehm, B. Joern, et al. (2006). *Concepts and Rationale for Regional Nitrogen Rate Guidelines for Corn.* Ames, IA: Iowa State University Extension.

Sawyer, J. E. (2019). Springtime precipitation – a tool for estimating nitrogen application need for corn. *Integrated Crop Management News*, 2563.

Sawyer, J. E., D. W. Barker, and J. P. Lundvall (2016). Impact of nitrogen application timing on corn production. *Agronomy Conference Proceedings and Presentations 61.*

Schnitkey, G. (2019). 2019 crop insurance premiums and the crop insurance decision tool. *Farmdoc Daily 9*(12).

Xu, H., T. E. Twine, and E. Girvetz (2016). Climate change and maize yield in Iowa. *PLOS One 11*(5), e0156083.

Yamoah, C., D. Walters, C. Shapiro, C. Francis, and M. Hayes (2000). Standardized precipitation index and nitrogen rate effects on crop yields and risk distribution in maize. *Agriculture, Ecosystems & Environment 80*(1-2), 113–120.