ANALYTICAL MODELING SOIL REACTION FORCES ON ROTARY TILLER

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ABSTRACT. Rotary tillers with flat tines are widely used in agriculture. In this study, the power requirement of an active tillage tool was investigated using a theoretical procedure. The goal of this work was to determine and analyze the soil resistance forces on the tine of rotary tiller. The laws and rules of theoretical mechanics were applied in the research. Theoretical studies obtained analytical relationships for determining the soil resistance forces acting on the chamfers and side surfaces of the tines of the rotary tiller, as well as the torques associated with these forces. The analysis of the obtained analytical expressions showed that the forces of the tiller tine is mainly influenced by the mode of operation of the tiller, the design parameters of the tine and the physical and mechanical properties of the soil. With an increase in the index of the kinematic mode of operation, $\lambda$, which is the ratio of the peripheral speed of the tine to the forward speed of the tiller, the resistance force of the flat tine decreases, and then takes a negative value. At $\lambda > 2$, the resistance force of the tine is directed towards direction of movement. In this case, a driving force is formed, the maximum value of which is reached at $\lambda = 2.2$

Keywords. modeling, reaction force, rotary tiller, tine, soil properties

INTRODUCTION

Rotary tillers are widely used in agriculture. Rotary tilling has a positive effect on the physical properties of the soil, water and nutrient regimes of plants [1]. In addition, active rotary tillers are used as an additional working body on plows [2-4],
deep-diggers and combined tillage machines [5-22]. Active working bodies such as rotary tillers of tillage machines, along with technological functions, will also perform the functions of movers and thus allows unloading a narrow link in the chain of energy transmission through the tractor's drive, and will also contribute to increasing the productivity and efficiency of the machine unit [2-6].

For loosening of compacted top soil layers and grinding plant residues, active rotary tillers with flat tines in combination with passive working bodies have been widely used [2-3]. Flat tines require low tillage energy and create crumbling of soil, because soil after tillage with rotary tiller is exposed to the passive working bodies of tillage machines. The flat tines of active rotary tiller can be installed radially, with the tine inclined from the radius to direction of rotation or backward [1]. When installing tines with a slope from the radius in the course of rotation, the pinching of the cut plant residues, so that does not consider the interaction of tines of this type of rotary tiller.

Researchers have modeled the soil-tool interaction of active tillage systems including both rotary harrows and and tillers. Most of the work has been associated with rotary tillers which generally use L-shaped blades rotating about a horizontal axis. Hendrick and Gill’s seminal work [25-29] described the effect of rotary tiller design parameters on tillage operation. Thakur and Godwin [30] summarized the models of soil reaction forces on the tines of rotary tillers. More recent work has investigated effects of kinematic and kinetic models on specific work [23] and the size of soil slices of rotary tillers [24].

The purpose of study is to predict analytically the draft forces of an active rotary tiller with straight tines as the tools interact with soil.

**METHODS AND MATERIALS**

Mathematical modeling based on theoretical agricultural soil mechanics were applied to predict draft forces from rotary tiller with straight tines. In the process of operation, the active rotary tine overcomes the reactive soil resistance forces and soil frictional forces that have arisen at the soil-blade/chamfers systems from the soil to soil and soil to tine surfaces (Fig. 1). By decomposing the soil resistance forces to cutting and frictional into horizontal and vertical components from the schematics in figure-1, we obtain:

\[
R_x = R_{hx} - F_{hx} + N_{fx} + T_x + F_x
\]

\[
P_y = P_{by} - P_{by} + N_{fy} + T_y + F_y
\]

where:

- \(R_{hx}, R_{hy}, F_{hx}, F_{hy}\) = horizontal and vertical components of the soil resistance and friction forces acting on the tine/knife blade;
- \(N_{fx}, N_{fy}, T_x, T_y\) = horizontal and vertical components of the soil resistance and friction forces acting on the knife chamfers;
- \(F_x, F_y\) = horizontal and vertical components of the soil resistance and friction forces that occur on the side surfaces of the knife/tine.

E.g.

- \(R_{hx}\) = horizontal soil resistance;
- \(N_{fx}, N_{fy}, T_x, T_y\) = horizontal and vertical components of the soil resistance and friction forces acting on the tine/knife chamfers;
- \(F_x, F_y\) = horizontal and vertical components of the soil resistance and friction forces that occur on the side surfaces of the knife/tine.
Figure 1. Scheme for determining the soil resistance forces acting on the chamfers and side surfaces of the tine/knife

When cutting soil and plant residues with a knife/tine, a force $N_b$, occurs on its blade, directed along the normal $N - N$ (Fig. 1 and 2).

$$N_b = \sigma_b \delta l_b,$$

where:
- $\sigma_b$ = breaking contact tension;
- $\delta$ = blade thickness;
- $l_b$ = blade working length

Figure 2. Scheme for determining the soil resistance forces acting on the knife/tine blade

From Fig. 1 we have

$$l_b = KL = l_n - \frac{2(R - h_l \sin \varphi)}{2 \sin(\varphi - \theta)},$$

where
- $R$ = rotary tiller radius;
- $h$ = depth of soil loosening;
- $l_n$ = tine/knife length
Substituting of $l_b$ value to expression (4) in (3) and decomposing the force $N_b$ into horizontal and vertical components, we have

$$R_{bx} = \sigma_l \delta \left[ l_n - \frac{2(R - h - r_1 \sin \varphi_i) + b \cos(\varphi_1 - \theta)}{2\sin(\varphi_1 - \theta)} \right] \cos(\varphi - \theta),$$

$$R_{by} = \sigma_l \delta \left[ l_n - \frac{2(R - h - r_1 \sin \varphi_i) + b \cos(\varphi_1 - \theta)}{2\sin(\varphi_1 - \theta)} \right] \sin(\varphi_1 - \theta).$$

On the blade of the knife, there are frictional forces, the value of which depends on the value of the angle $\alpha$ between the normal $N-N$ and the absolute velocity $\upsilon_a$. When $\alpha \geq \varphi$, $F_b = fN_b = N_\alpha \tan \varphi$, and when $\alpha < \varphi$, $F_b = N_\alpha \tan \alpha$. At the same time, for the corresponding position of the knife at the specified $\lambda$, $R$, $h$, $\theta$, and $r_1$, it is necessary to determine the length of the blade $l_\alpha$, on which $\alpha \geq \varphi$ and the length of the blade $l_\varphi$, on which $\alpha < \varphi$. After that, possible define the components $F_x$ and $F_y$ of total friction force $F^\wedge$.

For the part of the blade on which $\alpha \geq \varphi$:

$$F_{bx} = \sigma_\alpha \delta \alpha \tan \varphi \sin(\varphi_1 - \theta),$$

$$F_{by} = \sigma_\alpha \delta \alpha \tan \varphi \cos(\varphi_1 - \theta).$$

For the part of the blade on which $\alpha < \varphi$:

$$F_{bx} = \sigma_\alpha \delta \alpha \tan \alpha \sin(\varphi_1 - \theta),$$

$$F_{by} = \sigma_\alpha \delta \alpha \tan \alpha \cos(\varphi_1 - \theta).$$

Resultant of the normal soil pressures $N_c$ acting on the tine/knife chamfers

$$N_c = 2N_{c1} \sin \beta = \rho a l_b,$$

where $a = \text{knife/tine thickness}$

By decomposing $N_c$ into horizontal and vertical components we get:

$$N_{cx} = \rho a \left[ l_n - \frac{2(R - h - r_1 \sin \varphi_i) + b \cos(\varphi_1 - \theta)}{2\sin(\varphi_1 - \theta)} \right] \cos(\varphi_1 - \theta),$$

$$N_{cy} = \rho a \left[ l_n - \frac{2(R - h - r_1 \sin \varphi_i) + b \cos(\varphi_1 - \theta)}{2\sin(\varphi_1 - \theta)} \right] \sin(\varphi_1 - \theta).$$

From the normal force $N_c$ on the chamfers of the tine/knife, friction forces $T_c$ arise. The elementary force $dT_c$ is directed in the direction opposite to the direction of the absolute velocity of the center point of the elementary chamfer site.

The projection of the friction forces $dT_c$ on the plane of the knife is equal to

$$dT = 2dT_c \cos \beta_\alpha = \rho a \frac{\cos \beta_\alpha}{\sin \beta_\alpha} \cos \theta_1 d\rho.$$

Then

$$dT_x = \rho a \cos \theta_1 \cos \beta_\alpha d\rho,$$

$$dT_y = \rho a \sin \theta_1 \sin \beta_\alpha d\rho,$$

where $\beta_\alpha = \text{the angle of the knife sharpening in the plane deviated from the normal plane to the cutting edge of the knife by the angle } \alpha$. To simplify the calculations, we accept $\beta_\alpha = \beta$.

From the triangle $ABC$ (Fig. 1) we have
\[
\cos \theta_1 = \frac{BC}{AC}, \quad \sin \theta_1 = \frac{AB}{AC},
\]
\[
BC = \rho \sin(\varphi_1 - \theta) + r_i \sin \varphi_1 - r, \quad (17)
\]
\[
AB = \rho \cos(\varphi_1 - \theta) + r_i \cos \varphi_1 - r, \quad (18)
\]
\[
AC = \sqrt{\rho^2 + 2\rho \left[ r_i \cos(\varphi_1 - \theta) \cos \varphi + \sin(\varphi_1 - \theta)(r_i - \sin \varphi_1 - r) \right] + r_i^2 - 2rr_i \sin \varphi_1 + r^2}, \quad (19)
\]

where 
\(r = \) the distance from the axis of rotation of the knife to the instantaneous center of rotation, that is, to the point \(C\).

Denoting
\[
m = r_i \cos(\varphi_1 - \theta) \cos \varphi_1 + (r_i \sin \varphi_1 - r) \sin(\varphi_1 - \theta), \quad (20)
\]
\[
n = r_i^2 - 2rr_i \sin \varphi_1 + r^2 \quad (21)
\]
we have
\[
AC = \sqrt{\rho^2 + 2\rho m + n}. \quad (22)
\]

Then
\[
T_x = f P \tan \beta \int_{\frac{l_1}{L_1}} \frac{\rho \sin(\varphi_1 - \theta) + r_i \sin \varphi_1 - r}{\rho^2 + 2\rho m + n} d\rho, \quad (23)
\]
\[
T_y = f P \tan \beta \int_{\frac{l_1}{L_1}} \frac{\rho \cos(\varphi_1 - \theta) + r_i \cos \varphi_1 - r}{\rho^2 + 2\rho m + n} d\rho \quad (24)
\]

Calculating the integrals obtained, we get
\[
T_x = f P \tan \beta \left\{ \sin(\varphi_1 - \theta)(A_2 - A_1) + \left[ r_i \sin \varphi_1 - r - m \sin(\varphi_1 - \theta) \right] \ln \left( \frac{\frac{L_2 + m + A_2}{l_1 + m + A_1}} \right) \right\}, \quad (13, 25)
\]
\[
T_y = f P \tan \beta \left\{ \cos(\varphi_1 - \theta)(A_2 - A_1) + \left[ r_i \cos \varphi_1 - r - m \cos(\varphi_1 - \theta) \right] \ln \left( \frac{\frac{L_2 + m + A_2}{l_1 + m + A_1}} \right) \right\}, \quad (14, 26)
\]

where
\[
A_1 = \sqrt{\rho^2 + 2m \rho + n}, \quad A_2 = \sqrt{\rho^2 + 2m \rho + n}. \quad (27)
\]

By selecting the elementary site \(dS\) on the surface of the knife we determine the components of the elementary reactions of the soil from the forces of friction and adhesion
\[
dF = 2fPdS = 2fPb_0 d\rho, \quad (28)
\]
\[
dF_x = 2fPb_0 \cos \theta_1 d\rho, \quad (15, 28)
\]
\[
dF_y = 2fPb_0 \sin \theta_1 d\rho, \quad (16, 29)
\]

where 
\(b_0 = \) knife base width

From fig. 1 we have
\[
b_0 = \frac{2\tan \beta - a}{2\tan \beta}. \quad (30)
\]

By inserting the values \(\cos \theta_1\) and \(\sin \theta_1\) to (28) and (29) we get
\[ F_x = 2 f p b_0 \int_{l_2} l_2 \rho \sin(\varphi - \theta) + r_i \sin \varphi_1 - r_i d\rho, \quad (31) \]

\[ F_y = 2 f p b_0 \int_{l_2} l_2 \rho \cos(\varphi - \theta) + r_i \cos \varphi_1 - r_i d\rho, \quad (32) \]

\[ l_2 = l_n - \frac{R - h - r \sin \varphi_1}{\sin(\varphi_1 - \theta)}. \quad (33) \]

After calculating the obtained integral expressions, we will have

\[ F_x = 2 f p b_0 \left\{ \sin(\varphi_1 - \theta)(A_2 - A_1) + \left[r_i \sin \varphi_1 - r - m \sin(\varphi_1 - \theta)\right]\ln\left(\frac{l_n + m + A_2}{l_2 + m + A_1}\right) \right\}, \quad (17, 34) \]

\[ F_y = 2 f p b_0 \left\{ \cos(\varphi_1 - \theta)(A_2 - A_1) + \left[r_i \cos \varphi_1 - r - m \cos(\varphi_1 - \theta)\right]\ln\left(\frac{l_n + m + A_2}{l_2 + m + A_1}\right) \right\}. \quad (18, 35) \]

The obtained analytical expressions allow us to determine the components of the soil resistance forces acting on the rotary tiller tine/knife under different operating modes, depending on the design parameters of the tine/knife and the physical and mechanical properties of the soil.

The torque \( M_c \) supplied to the rotary tiller from the tractor power take-off shaft is spent on overcoming the moments from the reactive resistance forces and soil friction forces acting on the blade, chamfers and side surfaces of the tine/knife, that is

\[ M_c = M_R + M_{fb} + M_n + M_T + M_F, \quad (36) \]

where

\( M_R \) and \( M_{fb} = \) moments from the soil’s resistance and friction forces acting on the tine/knife blade;

\( M_n \) and \( M_T = \) moments from the resistance and friction forces acting on the chamfers of the tine/knife;

\( M_F = \) moments from the friction forces acting on the side surfaces

From fig. 3 we have

\[ dM_R = dR_b h_b = \sigma_b \delta h_b d\rho, \quad (37) \]

\[ dM_N = dN_{eb} h_b = p a h_b d\rho, \quad (38) \]

where

\[ h_b = r_i \cos \theta + \rho. \quad (39) \]
Figure 3. Scheme for determining the moments from the soil resistance forces acting on the cutter blade

Then

\[ M_R = \int_{l_i}^{l_f} \sigma_b \delta(r_i \cos \theta + \rho)d\rho, \quad (40) \]

\[ M_N = \int_{l_i}^{l_f} pb (r_i \cos \theta + \rho)d\rho. \quad (41) \]

After calculating the integral expressions, we get

\[ M_R = \sigma_b \delta_1 \left[ \cos \theta (l_n - l_i) + (l_n - l_i)^2 \right] \quad (19, 42) \]

\[ M_N = pbr_1 \left[ \cos \theta (l_n - l_i) + (l_n - l_i)^2 \right] \quad (20, 43) \]

The moment \( M_{Fb} \) of the soil friction force acting on the blade is determined by the following expression

\[ M_{Fb} = F_b h_{Fb} F_b (r_1 \sin \theta + \frac{b}{2}). \quad (21, 44) \]

The elementary friction forces \( dt \) and \( dF \) acting on the chamfers and the side surfaces of the tine/knife create elementary moments of resistance equal to

\[ dM_T = dTh_T = fpactg\beta_{cp} h_1 d\rho, \quad (22, 45) \]

\[ dM_T = dTh_T = fpactg\beta_{cp} h_1 d\rho. \quad (23, 46) \]

From the fig. 3 we have

\[ h_T = AC + r \cos \theta \quad (47) \]

After substituting the values of \( AC \) and \( \cos \theta \) to dependence (45) and (46), we will have:

\[ M_T = fpactg\beta_{cp} \left[ \int_{l_i}^{l_f} \frac{\rho \sin(\varphi_T - \theta) + r_1 \sin \varphi_T - r}{\sqrt{\rho^2 + 2m\rho + n}} d\rho \right] \quad (24, 48) \]

\[ M_F = 2fpb_0 \left[ \int_{l_i}^{l_f} \frac{\rho \sin(\varphi_T - \theta) + r_1 \sin \varphi_T - r}{\sqrt{\rho^2 + 2m\rho + n}} d\rho \right] \quad (49) \]

After calculating the obtained integral expressions, we have

\[ M_T = fpactg\beta_{cp} \left\{ \frac{l_n + m}{2} A_2 - \frac{l_1 + m}{2} A_1 + \frac{n - m_2}{l_1 + m + A_i} \ln \left( \frac{l_n + m + A_i}{l_1 + m + A_i} \right) r \cos(\varphi_1 - \theta)(A_2 - A_1) + \right. \]

\[ + r [r \cos \varphi_1 - m \cos(\varphi_1 - \theta)] \ln \left( \frac{l_n + m + A_i}{l_1 + m + A_i} \right) \}, \quad (25, 50) \]

\[ M_F = 2fpb_0 \left\{ \frac{l_n + m}{2} A_2 - \frac{l_1 + m}{2} A_1 + \frac{n - m_2}{l_1 + m + A_i} \ln \left( \frac{l_n + m + A_i}{l_1 + m + A_i} \right) r \cos(\varphi_1 - \theta)(A_2 - A_1) + \right. \]

\[ + r [r \cos \varphi_1 - m \cos(\varphi_1 - \theta)] \ln \left( \frac{l_n + m + A_i}{l_1 + m + A_i} \right) \}, \quad (51) \]

The algebraic sum of expressions (42-44, 48, 50) will represent the resistance moment \( M_s \) of the tiller when tine/knife interacts with the soil. The obtained dependences allow us to analyze the nature of change in \( M_s \) depending on position of the tine/knife in the soil (\( \varphi_1 \)), design parameters and operating modes of the tiller, as well as physical and mechanical
properties of the soil.

The results of determining the physical, mechanical and technological properties of soils are presented. It is established that before plowing the fields have a pronounced uneven relief, characterized by the presence of ridges and irrigation furrows. The average height of the ridges in fields with row spacing of 90 and 60 cm is 17.1 and 12.8 cm, respectively. The physical, mechanical and technological properties of the soil of the arable and sub-arable layer of the cotton field along the wheel track differ significantly from the soil between the rows without the wheel track and the ridge: the density of the soil along the wheel track of the soil reaches 1.65 g/cm³, which is correspondingly greater than the density of the ridge soil by 0.24; the density, hardness and resistance of the soil to various deformations has a maximum value in the layers of 15-25 cm along the track of the tractor wheels.

RESULTS AND DISCUSSION

In fig 4 and 5 are graphs of horizontal soil reaction dependence of the soil interacting with the tiller blade $R_x$ on angle of rotation of tiller $\varphi_1$ and the kinematic parameter $\lambda$ of the tiller at $\alpha=10$ mm, $b=65$ mm, $\beta=200$, $R=300$ mm, $r=100$ mm, $r=150$ mm, $l_o=210$ mm, $f=0.5$, $p=6010^4$ Pa, $\sigma_b=35010^4$ Pa, $\delta=100$ microns, $h=150$ mm.

![Figure 4. Change in the total soil horizontal reaction acting on the rotary tiller tine from the angle of rotation](image)

From fig. 4, it can be seen that each rotary tiller tine at $\lambda=2$ creates a pushing force $R_x=0.58$ kN. The maximum pushing force is created at $\lambda=2.2-2.4$ (fig 5). A further increase in $\lambda$ slightly increases the value of $R_x$. This reduces the total pushing force due to the movement of the adjacent tine/knife on the loosened soil. Thus, at $\lambda=2.2-2.4$, the maximum pushing force is created, soil spraying is excluded, and the required energy for the rotary tiller drive is reduced.

![Figure 5. Determination total horizontal reaction of soil acting on rotary tiller tine from the kinematic mode of operation](image)

CONCLUSIONS

From this research, the following conclusions can be drawn:

1. Analytical dependences for determining soil resistance forces acting on the blade, chamfers and side surfaces of knife/tine of active rotary tiller, as well as moments from these forces, were obtained. The analysis of obtained analytical expressions showed that main influence on traction resistance of tiler tine/knife is mode of operation of tiller, the design parameters of tine/knife and physical and mechanical properties of the soil.

2. It is established that the maximum pushing force of the rotary tiller tine/knife is created when kinematic mode of operation equal to $\lambda=2.2-2.4$.

3. The impact of the force from the rotary cultivator in the soil bin and field to verify the mathematical prediction
will be made in future researches

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